Mathematical and Biological Representations in Derivative Analysis

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Abstract: This paper aims at presenting the analytical model for derivative analyses (MDA) as it now stands, with the improvements that have recently been implemented. MDA essential purpose is to systematically analyze music which has been organically composed, according to the Schoenberg's principle of *Grundgestalt* and developing variation. This systematization is mainly accomplished through associations between musical events and mathematical formalization, geometrical representations and concepts derived from the fields of Evolutionary Biology and Genetics. In the final section of this paper, a practical application on Brahms's *Violin Sonata* Op.78 is presented.

Keywords: Derivative Analysis. Mathematical Formalization. Organic Construction. Musical and Biological Variation.

Representações Matemáticas e Biológicas em Análise Derivativa

Resumo: Este artigo tem como objetivo apresentar o modelo analítico para análise derivativa (MDA) em sua versão atual com os aperfeiçoamentos que foram recentemente implementados. O propósito essencial do MDA é analisar sistematicamente música organicamente composta, de acordo com os princípios schoenberguianos da *Grudgestalt* e variação progressiva. O processo de sistematização foi concretizado à partir de associações entre eventos musicais e formalização matemática, representações geométricas e conceitos derivados dos campos da Evolução Biológica e da Genética. Na parte final do artigo, uma aplicação prática na Sonata para Violino Op.78 de Brahms é apresentada.

Palavras-chave: Análise Derivativa. Formalização Matemática. Construção Orgânica. Variação Musical e Biológica.

1. Background

The Model of Derivative Analysis (MDA) is part of a research project initiated in 2011 and intended to produce systematic studies on musical variation under analytical, as well as compositional perspectives. Since then, several analyzes have been made, encompassing a varied group of organically-constructed works,[1] which resulted in considerable improvements to the research. This paper aims at presenting the MDA's current version, which has been consolidated with new basic assumptions, derived theoretical formulations, and a conceptual corpus constructed from these studies. Presently, MDA has achieved a high level of systematization (the very objective of the research), which became possible through a new, and consistent biological bias (more specifically, considering the areas of Genetics and Evolution Theory) and mathematical formalization of procedures and elements. This led to the

[1] ALMADA (2011a; 2011b; 2013a; 2013b; 2016), MAYR (2015) and MAYR & ALMADA (2016; 2017b).

creation of new terminology, symbology, and graphic analytical resources, whose main characteristics will be summarily exposed.

2. Basic overview

The research is theoretically grounded on the principles of developing variation and *Grundgestalt*, elaborated by Arnold Schoenberg. In short, a *Grundgestalt* can be viewed as a primordial idea from which a whole composition can be obtained, through progressive transformation of its elements. These derivative processes are associated with the concept of developing variation, which differs from "ordinary", non-consequential variation, by the fact that it gradually promotes divergent transformation from the basic material, producing new, though related ideas.[2]

Both principles arouse in Schoenberg's mind as a consequence of an organic conception of the artistic creation, normally known as Organicism, a philosophical-scientific-artistic trend that influenced part of 19th century German composers, especially Beethoven, Brahms, as well as the young Schoenberg (MEYER, 1989; FREITAS, 2012; GRIMES, 2012). Music built according to organic terms can be basically defined as presenting a balance of forces associated with the apparently contradictory attributes of economy (or coherence) and variety and unfolds like the growth of a living being.[3]

MDA was conceived aiming at explaining the constructive processes that are employed for such development in pieces that present organic architecture. Its main focus is in the thematic structures that result from the application of developing variation techniques to the *Grundgestalt*'s basic components. One of the main drawbacks that an analysis of this sort faces is the difficulty of mapping accurately all the variations applied in a given segment of the piece, since normally this process is made with idiosyncratic derivative techniques, and not rarely involving combined transformations of musical parameters in different degrees. Even in the cases where the analyst is able to identify the adopted compositional strategy, a precise, objective, and concise explanation becomes normally a hard (if not impossible) task. This problem motivated the elaboration of MDA's basic premise: that the derivative process can be decomposed into two distinct streams of musical information: pitch and

[2] For more about these principles, see, among others, DUDEQUE (2005, p. 135-172).
[3] This subject is more deeply examined in MAYR & ALMADA (2017a).

rhythm.[4] They are classified as *domains* in MDA, integrating an abstract plan of analysis, where processes of *developing variation of first order* (DV1) are performed.[5] This article is specifically intended to expose the processes and material considered for the abstract analysis.

3. Levels and Definitions

An MDA analysis of a given organically-constructed piece must be preceded by a very important phase, namely the localization and delimitation of the piece's *Grundgestalt* (in short, G). Already in this earlier stage, the analyst must be profoundly familiarized with the work in question, through many hearings of its recordings and a careful examination of the score. More precisely, one must pay special attention to the transformation of the musical materials along the piece and, gradually, be able to trace back their correlations to an eventual original source (which is, in most cases, located in the initial bars of the piece). Just after having done this, and, evidently, if one is successful in the task, the analyst is capable of properly identifying the referential unity (G) from which presumedly most of the piece's derivative relationships emanate, and further analytical interpretations will be produced.

A thematic derivative analysis of a multi-movement musical work in MDA is performed considering four levels:

I. Intra-genetic – corresponds to: (a) identification and labeling of the potential abstract referential forms for further derivative processes, with corresponding attribution of permanent and/or temporary variables; (b) detection of eventual derivative relations between components (and subcomponents). This examination is exclusively employed in the (main) *Grundgestalt*;

II. Inter-thematic – corresponds to: (a) explanation of the thematic construction based on derivative relations from the first-level referential forms; (b) determination of eventual correlations between the themes (and/or thematic fragments). In this approach, involving the content of isolated movements, the themes' formal functions and tonal contexts play an important role;

III. Inter-movement (or inter-genetic) – corresponds to analysis on the formation of the second-order *Grungestalten*, considering their possible derivation from the

[4] Other parameters can be considered as well, like tonal relations and metrics (see below).
[5] Conversely, there is also a developing variation of second order (DV2), associated to the concrete plan of transformations. For a limitation of focus and space, this plan will not be examined in this study.

elements present in the (main) *Grundgestalt* and eventual influences from themes of precedent movements. This approach is only applied to non-first movements;

IV. Intra-work – corresponds to highest-level abstract analysis, considering two different investigations: (1) about tonal relations that may emanate from the *Grundgestalt* (from the seminal study by Carpenter, 1983); (2) about the existence of structural metric dissonance and, consequently, a metric *Grungestalt* (based on theoretical proposal by Ng, 2012).

The present article focuses on the first level and the procedures associated with it. Some preliminary definitions are necessary:

1 **Domain** – Is the musical attribute subject to be abstracted for variation purposes. This study considers two domains: pitch (p) and rhythm (r).[6]

2 **Sub-domain** – Are categories contained in a domain. There are three pitch sub-domains (ambit, intervallic sequence and melodic contour) and two rhythmic sub-domains (rhythmic sequence and metric contour). The sub-domains are formally described as algebraic vectors. Considering a given melodic fragment *F* (Example 1), its sub-domains can be presented as follows:

2.1 Ambit (*/F): is the intervallic distance between the first and last pitches. It is measured in semitones. Ex: <-5> (descending perfect fourth);

2.2 Interval sequence (i/F): displays the ordered directions and intervallic contents that form the fragment. Ex: <-8+1+4-2>.

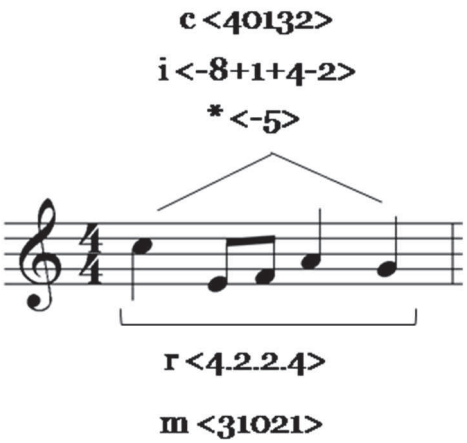
2.3 Melodic contour (c/F): presents an abstracted version of the intervallic sequence, indicating only the directions and relative positions of the notes (the lower point is always labeled as "0" and the higher point is numbered as "n-1", being n the number of onsets present in the fragment).[7] Ex: <2130>;

2.4 Rhythmic sequence (r/F): expresses not the sequence of durations of the fragment, but its *inter-onset intervals*, or IOIs (TEMPERLEY, 2001), i.e., the distances between each note attack. In this way, the duration of the last note is not considered, just its point of attack (moreover, this fits the intuition that rests and ties can be considered relatively irrelevant for the

[6] Evidently, other domains could be considered, as harmony, timbre, texture, etc.
[7] For more detailed information about Contour Theory, see MORRIS (1987).

determination of rhythmic similarity). The IOIs are measured in sixteenth notes and separated by dots. Ex: r <4.2.2.4>;

2.5 Metric contour (m/F): like the melodic contour, indicates the relative metric strength of each note of the fragment, according to the metric context.[8]



Example 1: Sub-domain labeling.

3 **Components** – are delimited segments considered for an analysis. In the case of the *Grundgestalt*, the components are labeled according to the formula: G.A, G.B, G.C, ... and may eventually be segmented into subcomponents (labeled with subscripts integers);

4 **Abstraction** – is a formal process employed for isolating a musical characteristic of a given melodic fragment, according to some domain/sub-domain.

5 **Second-order abstraction** – is the application of an abstraction-process to an already abstracted structure. The application of this process is indicated by a "2" exponent on the corresponding sub-domain to be recurrently abstracted. Ex: i<+2+1+1+2> → i^2<(ascending scale)>;

6 **Operation** – is an algorithm applied to a given referential form in order to transform it onto a variant;

7 **Divergence** – is a special case of developing variation in which a derived form becomes an autonomous, divergent unity, initiating a new ramification of an existent lineage. Divergence can be analogously compared to the process by which a new species arouses from a referential life form through cumulative variations;

[8] This concept is explained in MAYR & ALMADA (2017a).

8 **Variables** – are used to describe forms that are derivatively relevant aiming to avoid long labels. There are two types of variables:

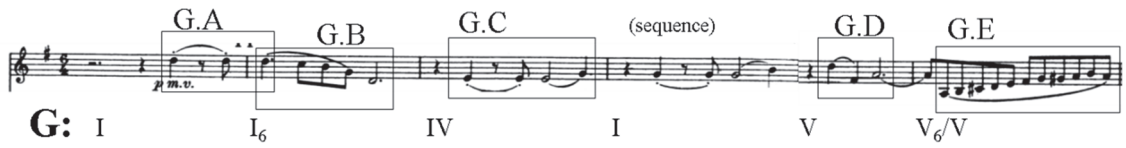
3.1 Permanent – attributed to those variants that influence derivative processes beyond the boundary of the movement where they are created. They are labeled with the final letters of the alphabet, in reverse order of occurrence, in upper case (Z, Y, X, ...);

3.2 Temporary – attributed to those variants that influence derivative processes only inside the movement where they are created. They are labeled with final letters of the alphabet, in reverse order of occurrence, italicized, and in lower case (*z*, *y*, *x*, ...).

4. A Practical Application

In this section, the procedures that characterize the first-level MDA are presented as an example of application, considering a real case, the analysis of some *Grundgestalt*'s elements from the first movement of Brahms's *Violin Sonata* Op.78.[9]

The *Grundgestalt* (G) is presented by the violin in the first six bars of the piece, and is formed by five distinct components (G.A-G.E), as shown in Example 2.

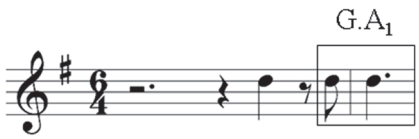


Example 2: Brahms – *Violin Sonata* Op.78 (mm. 1-6).

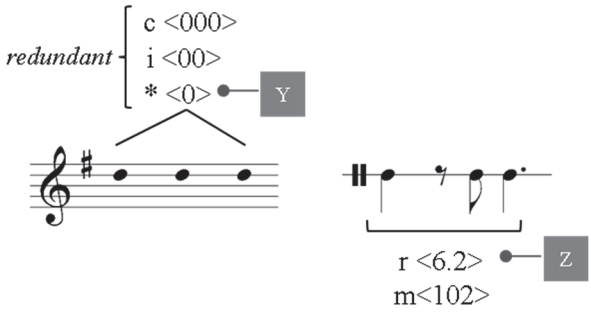
Each component represents a special, peculiar motivic characteristic with potential germinative power (albeit distinct disseminative capacities). In the next step of the analysis, the seven components are separately examined (some of them present subcomponents), in order to evidence their specific abstract properties. Considering the available limits of space in this paper, we will select just two of the five components (in this case, G.A and G.B) for demonstrating the application of the analytical method. Example 3 presents the segmentation of G.A.

The analysis of the components (and subcomponents, as well) take into account the respective five sub-domains abstractions (Example 4).

[9] This piece and Miguéz's Violin Sonata Op.14 form the object of study of the research which is associated this paper.



Example 3: *Grundgestalt* / component G.A.



Example 4: Abstractions of component G.A.

The first permanent variable (Z) is attributed to the sub-domain "r", due to the almost omnipresence of this motivic unity along the work (considering the three movements). As it can also be observed in Example 4, the three pitch sub-domains (c, *, i) present mutual redundancy. The ambit was selected as the domain's representative, and is also labeled with a permanent variable (Y), considering its pervasiveness in the piece.

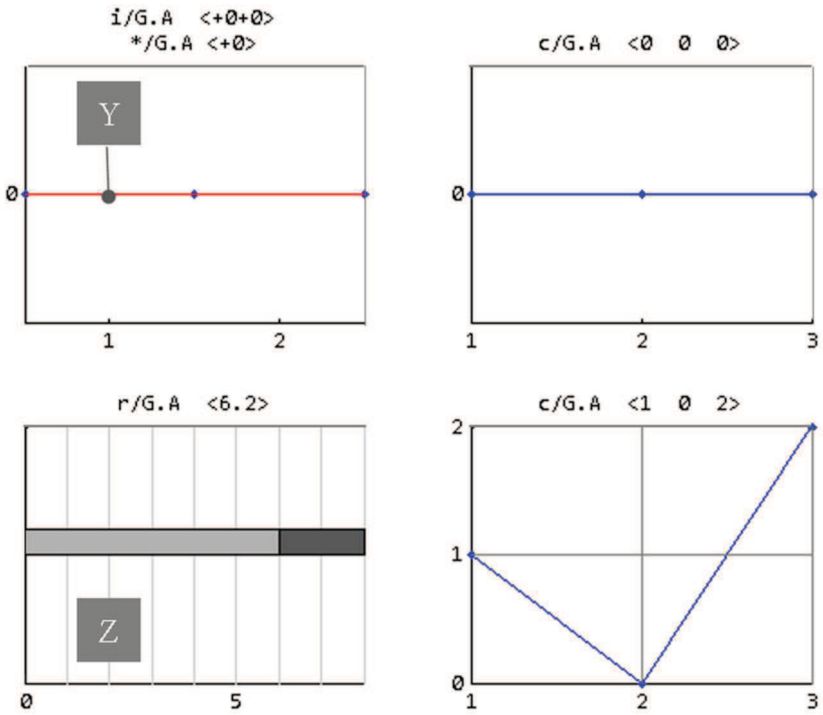
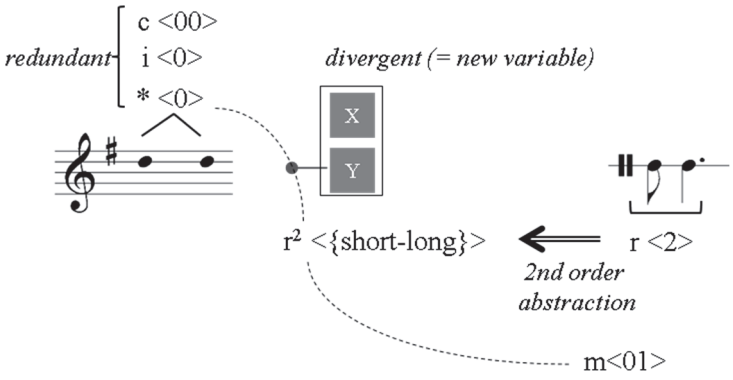


Figure 1: Graphic representation of abstracted characteristics of component G.A. Top left: intervallic sequence (blue line) / ambit (red line); top right: melodic contour; bottom left: IOIs; bottom right: metric contour. Variables associated to subdomains are properly identified.

An alternative view of the abstract characteristics of a given component is provided by a graphic representation. The main advantage of this method in comparison to the conventional musical notation is its neutral, systematic character. Figure 1 presents the group of G.A's five abstractions, grouped in four plots.[10]

The two top graphs allow us to confirm the redundant character of the pitch material in G.A. The IOI graph (bottom left) represents, in horizontal bars, the distance between the onsets, considering sixteenth notes as unity of measurement. The last graph depicts the pattern of hierarchical metric fluctuations in the motive, as a contour (c.f. MAYR & ALMADA, 2017a).

Example 5 presents the abstract analysis of subcomponent G.A_1.



Example 5: Abstractions of subcomponent G.A_1

A conjunction of three abstracted characteristics – pitch repetition (* <0>) + metric profile "weak-strong" (m <01>) + second-order, abstracted rhythmic configuration (r^2 <{short-long}>) – can be viewed as a subset of Y (or, better, a special case of this). In the graphical representation of G.A_1 (Figure 2), we can observe from another perspective the triple origin of variable X. The second-order abstraction of the IOIs sequence is added below as a stereotypical graphic scheme representing the pattern "short-long".

The segmentation and the analysis of the abstractions of component G.B are shown in Example 6. We observe in Example 6b the first case of abstract transformation: the application of an intervallic *complementation* (operation **com**)[11] to the unison <0> (Y) results in a descending octave <-12> (Y_1). The graphic representation of G.B is shown in Figure 3

[10] Graphs plotted by the software MDA, implemented in Matlab.
[11] In the operation **com,** a given intervallic content is substituted by its complement.

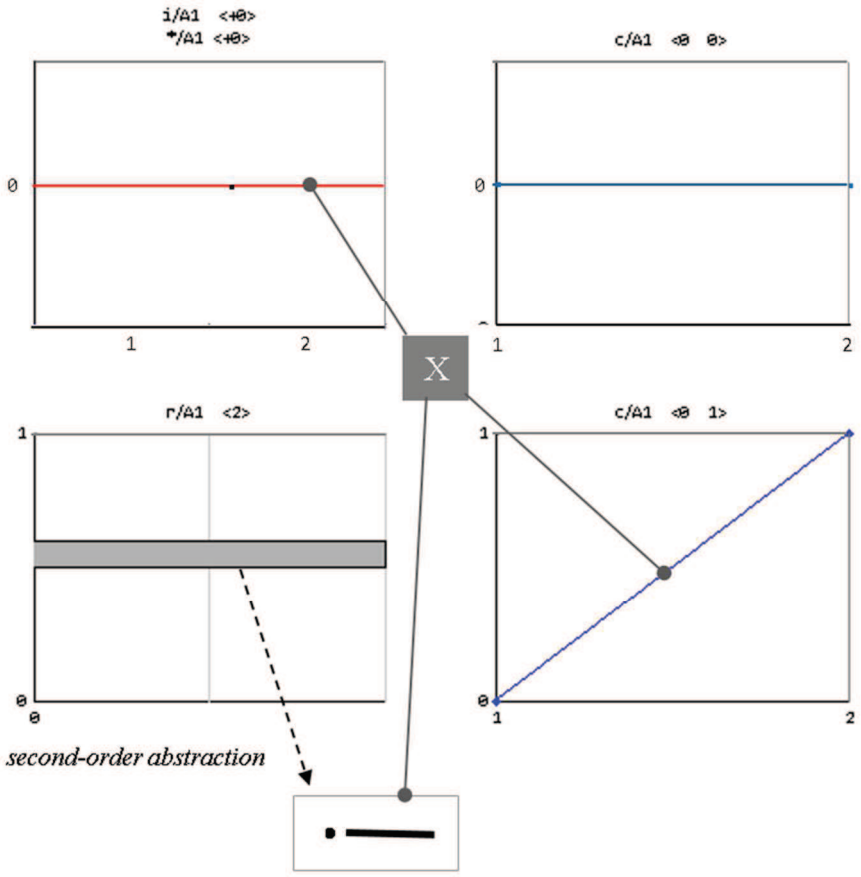
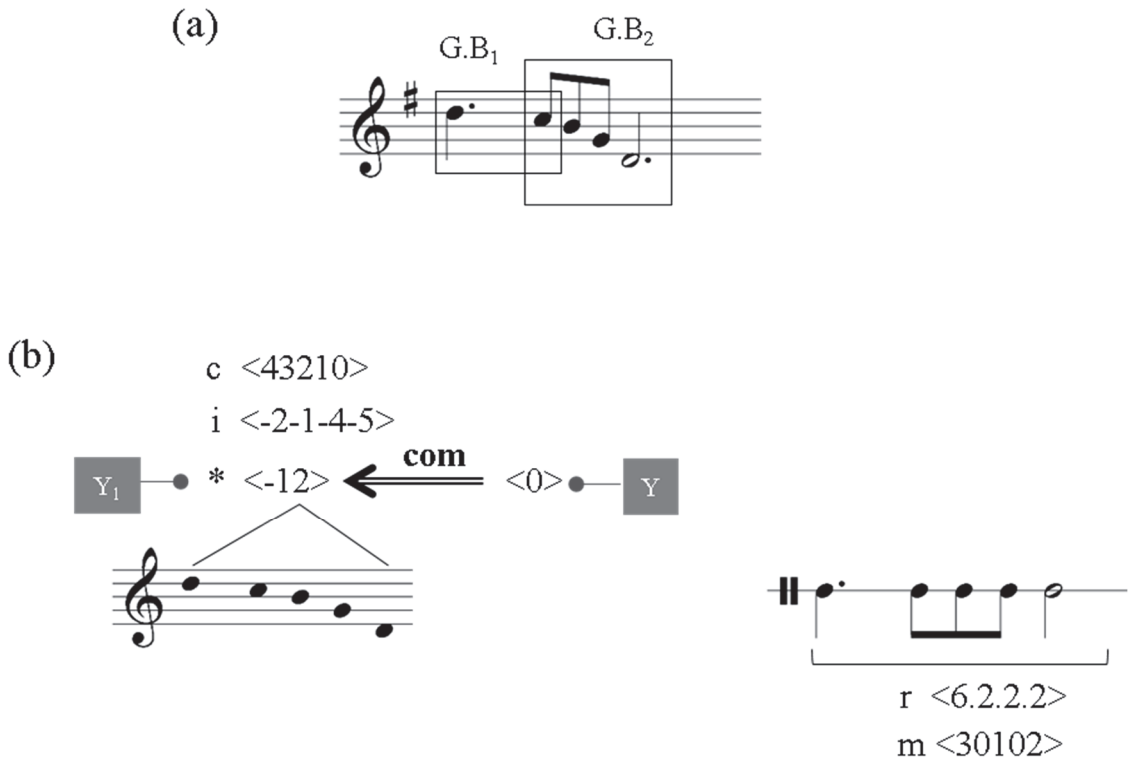


Figure 2: Graphic representation of abstracted characteristics of component G.A$_1$.



Example 6: Segmentation of component G.B (a); G.B's abstractions (b).

Like the previous cases, the same information can also be graphically represented (Figure 3).

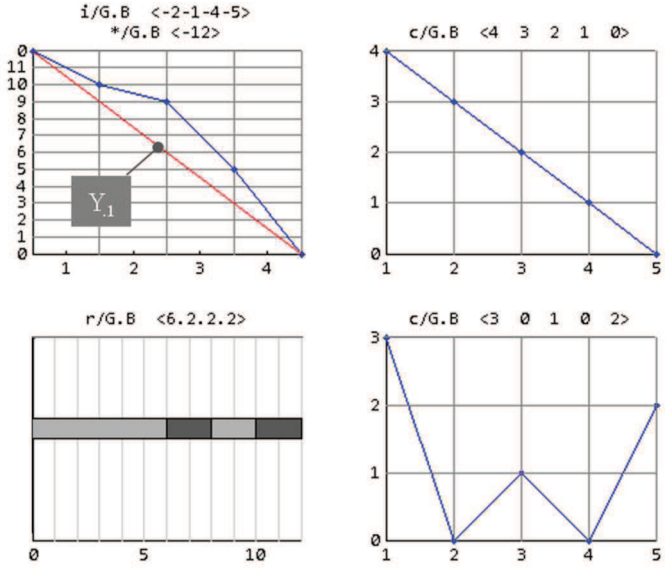
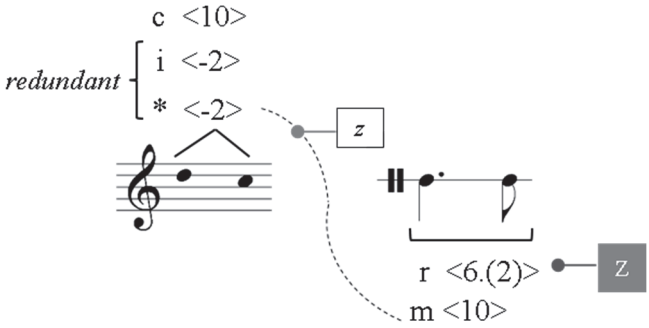
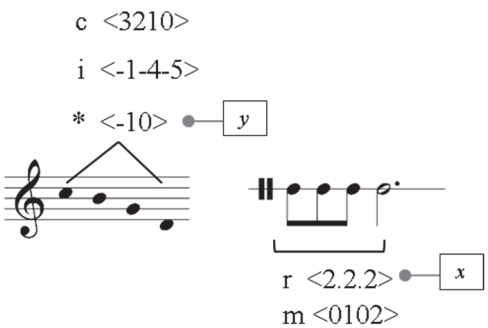


Figure 3: Graphic representation of abstracted characteristics of component G.B.

Examples 7 and 8 present the analysis of the abstractions that form subcomponents G.B$_1$ and G.B$_2$, whose graphic representations are shown Figures 4 and 5.



Example 7: Abstractions of subcomponent G.B$_1$



Example 8: Abstractions of subcomponent G.B$_2$

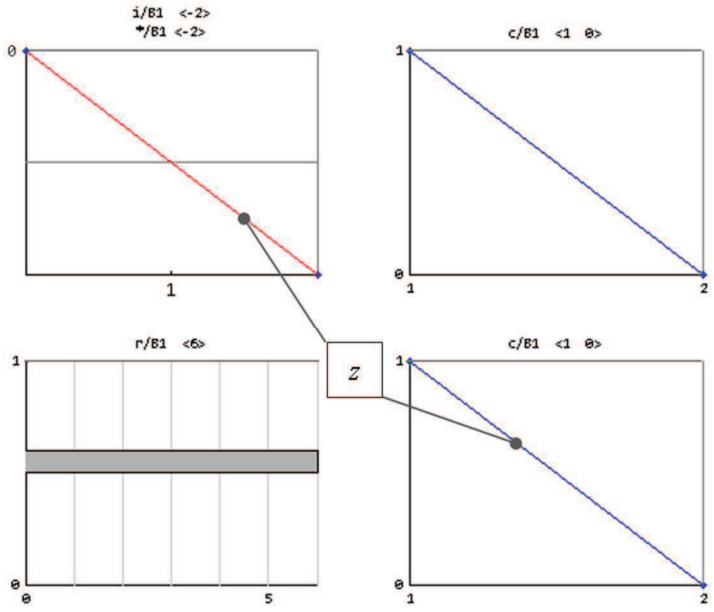


Figure 4: Graphic representation of abstracted characteristics of component G.B$_1$.

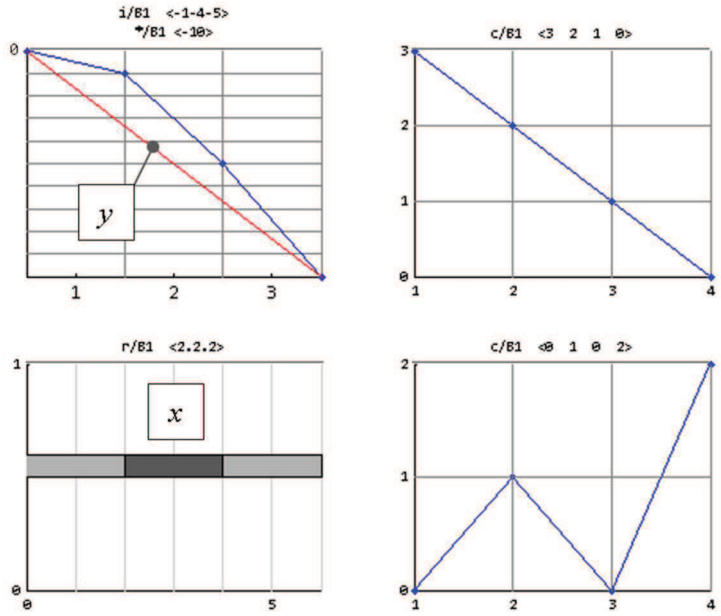


Figure 5: Graphic representation of abstracted characteristics of component G.B$_2$.

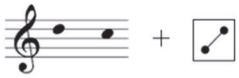
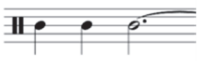
Tables 1 and 2 summarize the variables present in Op.78's *Grundgestalt* (including that referent to components G.C, G.D, and G.E, whose analyzes were omitted in this paper). The variables ultimately represent the referential abstract forms for MDA's second level, namely the inter-thematic analysis. The tables show three types of representation for the variables: formal (associated with their "genetic" origins), graphic (combining both musical and graphic descriptions of

components/subcomponents, resulting in a kind of hybrid, ideographic notation), and algebraic, as vectors.

Table 1: Permanent variables.

Variable	Formal representation	Graphic representation	Vector representation
Z	r/G.A		<6.2>
Y	*/G.A		<0>
X	{* m r^2}/G.A_1		<0> <01> <{short-long}>
W	*/G.C_2		<+3>
V	*/G.E_1		<2.2.2.2>

Table 2: Temporary variables.

Variable	Formal representation	Graphic representation	Vector representation
z	{* m}/G.B_1		<-2> <01>
y	*/G.B_2		<-10>
x	r/G.B_2		<2.2.2>
w	r/G.C_2		<8.(4)>
v	c/G.D		<201>
u	*/G.D		<-5>
t	r/G.D		<4.4>
s	c^2/G.E		<{asc.scale}>

With the definition of the variables as referential forms for the thematic construction, it is also possible to propose *genomic* representations for the unities of analysis, preparing the next analytical stage. The term "genomic" is employed in this context (reinforcing the metaphorical relations between music and biology of this approach) as a graphic representation in which musical notation is substituted by strings of variables (that act analogously to *genes*) organized according to the considered domains, pitch, and rhythm (our *chromosomes*). The genomic representation of the *Grundgestalt* (that can also be considered as a *unity of analysis* #0) is shown in Fig.6. As it can be observed, the graph depicts a high concentration of "genetic" information in a relatively short span of time: all of these "microscopic" unities, in greater or lesser extent, will play some role in the further organic process for producing the subsequent themes of the movement. Maintaining our "biomusic" terminological correspondence, it is also possible to consider the concrete, musical structures under analysis as *phenotypic-like* representations, since a phenotype can be shortly defined as the real expression of some group of genetic instructions.[12]

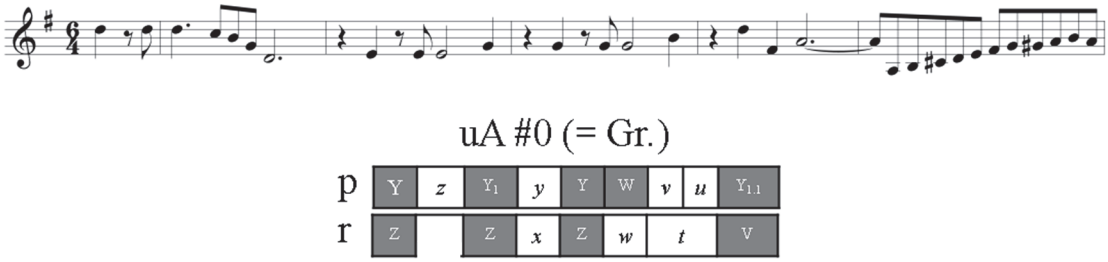


Figure 6: Phenotypic and genomic representations of Gr (unity of analysis #0).

5. Concluding Remarks

This study presented a number of innovations introduced in the current version of MDA, which can be subdivided into the following categories: (a) expansion of the original terminology and conceptual corpus; (b) mathematical formalization and vector descriptions of abstract elements; (c) use of concepts associated with biological/genetic phenomena; (d) graphic representations (of several types) as complementary information. As it was demonstrated, these innovations decisively contributed for the systematization of the analytical process (the primordial objective of the research), making it more precise and accurate.

[12] For more about this subject, see DAWKINS (2006).

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