Game of Tones: A Simple Schoenbergian Serial Music-Making Model

José Fornari
Universidade Estadual de Campinas (UNICAMP)
fornari@unicamp.br

Abstract: Automatic music making models are a unique type of game-like aesthetic algorithms that have been explored for a long time and used to create music without a musician, where the actual composer is the model's maker. Chaotic behavioral models offer an enticing chance for game-like music making as they allow the creation of similar yet novel music to be listened to or even to be later played by humans. Here is presented an undecidable model based on Game of Life and the Twelve-Tone technique to make non-supervised autonomous melodies as a musical Turing machine.
Keywords: Game of life. Schenberg. Twelve-tone technique. Undecidability. Automatic Composition.

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Resumo: Modelos automáticos de criação musical são algoritmos estéticos semelhantes de jogos usados para compor música onde o compositor real é o criador do modelo. Modelos caóticos oferecem uma oportunidade interessante deste tipo de criação, pois permitem a criação automática de música semelhante mas original, que pode ser ouvida ou mesmo tocada posteriormente por humanos. Aqui é apresentado um destes modelos baseado no Jogo da Vida e na Técnica dos Doze Tons que cria melodias de modo autônomo e não supervisionado, como uma máquina de Turing musical.
Palavras-chave: Jogo da Vida. Técnica dos Doze Tons. Indecibilidade. Composição Automática.

1. Introduction

The current coronavirus pandemic brought many losses to society, also impacting science and the academic community, such as with the passing away of John Horton Conway, in 2020[1]. Conway was a notorious mathematician, active in many fields of expertise, including theory of finite groups, number theory and combinatorial game theory. This is the case of his most popular achievement, in the branch of recreational mathematics, more specifically his contribution in the 1970s with the invention and development of a simple yet wondrous cellular automation known as the "Game of Life" [2].

Cellular automation is a class of computational models initially conceived in the 1940s, by Stanislaw Ulam and John von Neumann, belonging to the field of Automation Theory, the study of abstract self-operating machines. Cellular automations normally consist of a regular grid of cells that can assume a finite number of states. In the case of Game of Life, the states of each cell are binary, meaning that they can assume only two states: either "alive" (on) or "dead" (off) which is decided by

[1] Roberts, Siobhan. "John Horton Conway, a 'Magical Genius' in Math, Dies at 82". New York Times. April, 15th 2020.
[2] https://youtu.be/R9Plq-D1gEk

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simple deterministic rules (if too many or too few neighbors are alive, the cell dies; in a proper amount of neighbors, the cell thrives or is born). This simple automata is also known to be computationally universal, or Turing complete, meaning that it can simulate a Turing machine, which endows full equivalence with any other Turing complete system, such as virtually all modern programming languages (e.g. C, C++, C#, Java, Javascript, Lua, Python and Pure Data), some well known commercial softwares, such as MS PowerPoint, Excel and even a few video games, such as Minecraft (ZWINKAU, 2020).

Although Game of Life is called a "game" the only parameter a player can control (if any) is the initial binary exclusive states (alive or dead) of each cell, which are represented by squares in a two-dimensional grid, also called population. After a grid's initial state is settled, a Turing complete automatic simulation based on simple deterministic rules carried over the entire population in regular periods (also known as a generations) where the new states of all cells in the entire population are updated together, thus showing at each generation which cells are born, which ones are still alive and which ones have died. Despite its strictly deterministic principle of operation and rules, counterintuitively Game of Life is also known to be mathematically Undecidable. This means that it is impossible to have a formal proof or to create an algorithm capable of calculating, from the grid's initial state, the outcome of its runs, so to accurately determine if, for a given grid's initial arrangement, if the model will halt, extinguish (meaning that all cells will eventually die), loop (periodically repeating the same sequence of patterns) or just go on forever, always producing new and unexpected patterns. This is equivalent to say that some runs of Game of Life will eventually present chaotic behavior, which can be artistically used in computer-assisted music making models, such as the simple algorithm previously presented by (FORNARI, 2020). The approach currently presented here goes one step further from the previous one that used the chaotic behavior of Logistic maps to compose simple melodies and timbres. The current model takes advantage of the undecidable behavior of Game of Life, which also implies the partaking of the famous Turing's halting problem (DEKKER, 1959). This means that there is no formal way of predicting if the music making model presented here, giving a random generated initial melodic sequence, will run until its completion (thus halting) or simply keep on making melodies forever.

For me, this seems to be particularly enticing for the aesthetic exploration of new forms of algorithmic composition, which also follows the tradition of game-like musical composition models, such as the *Musikalisches Würfelspiel* (German for "Musical dice game"). These are a set of music compositional algorithms implemented and used much before our digital computer era. They are also based on simple rules and random sequences (initially provided by tossing dices, thus the term "dice game"). Probably the most famous one in this category is the "Mozart dice game", an interesting algorithmic music making model debatably whose musical excerpts were supposedly created by the famous 18th century composer. This model is nowadays widespread and even freely available online[3] (HEDGES, 1978, p. 180). This is a musical game consisting of 272 precomposed quaternary measures arranged in two plan grids (6 by 16 length for *minuets* and 11 by 16 length for *trios*). By tossing two dices, the next measure of each grid is chosen, resulting in 16 excerpts of *minuets* and *trios*. Although the musical result of this algorithm will always be somehow auditorily similar, with this approach it is possible to generate a dazzling amount of 6^{16} different *minuets* and 11^{16} different *trios*.

2. Twelve-tone games

The music games previously described are for me in the aesthetic vicinity of a well-known compositional method created by Arnold Schoenberg, in 1923, namely the Twelve-Tone technique. Schoenberg himself defined this technique as a "method of composing with twelve tones which are related only with one another" (SPINNER, 1983, p. 2). The idea behind this technique seems to be based on the pursuit of tonalism's transcendence, bypassing its psychoacoustic's hierarchical structure given by the acoustic phenomenon known as harmonic series (formally defined by a divergent infinite series of the overtones naturally generated by many tonal musical instruments). The Twelve-Tone technique thus ignores this psychophysical principle by defining a musical series of twelve tones where all of them are equal, ought to be played no more no less than once for each series (also called row), share the same importance, have the same musical rights and are ultimately ruled only by the common relationship among themselves for each row, which is decided by a game-like set of rules that defines the bases of serial music composition, such as the use of a row in its original (prime) form, or after simple transformations such as its intervalar inversion and retrograde movement.

For me these algorithmic and interactive compositional approaches brought music making, which was traditionally more like a non semantic type of "storytelling", closer to the distributed interactivity of game playing. (STENROS, 2016) presents a review of more than 60 definitions of characteristics presented by games, underlining ten points of interest by which games are usually defined. These features are not exclusive or sufficient to define any game and different games may convey different amounts of each feature. They are: 1) Rules, 2) Purpose, 3) Activity, 4) Simulation, 5) Role, 6) Leisure (the opposite of working or being somehow directly productive), 7) Conflict (competition), 8) Goal (halting), 9) Construction of category (theme), 10) Coherence (fair play). As I see it, both the Game of Life and the game-like features of the Twelve-Tone technique, present at least: 1) Rules, 2) Purpose, 3) Activity and 4) Coherence.

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[3] http://www.playonlinedicegames.com/mozart

This paper presents a simple game-like melody making model inspired by the previously mentioned Conway's Game of Life and the famous Schoenberg's Twelve-Tone technique for serial music composition. Here called "Game of tones" (for the sake of a pun) its epic goal is to pursue an undecidable thus mysterious melodic path paved on simple formal rules applied on an intriguing initial randomly generated schenbergian-extended series of tones. The prototype of this model has been able to generate modal melodies through the strict non-supervised application of coherent deterministic rules that create unpredictable yet enticing humanistic-like melodic outcomes of seeming idiomatic improvisation.

3. An undecidable problem

The rules used here in the prototypical model of Game of Tones are the same ones designed by Conway and colleagues for Game of Life. They are stated as follows: Given a theoretical infinite two-dimensional plan (grid) made of squares (cells) that can assume only two mutually exclusive states, 1 (alive) or 0 (dead). Each cell has 8 possible neighbours (which are the other cells in its immediate surrounding). The cells' states are updated altogether in a regular period of time (generation). For each generation, four deterministic rules are applied for all cells in the grid: 1) If the cell has 4 or more neighbours alive, in the next generation this cell will be dead. 2) If it has 3 neighbours alive, this cell will survive (if already alive) or be born (if dead). 3) If it has 2 neighbours alive, this cell will keep its current state. 4) If it has 1 or 0 neighbours alive, this cell will die.

When implemented as a computer simulation, such as many ones freely available online[4], these simple deterministic rules create an undecidable problem which defies the limits of formal logic and mathematics[5]. This paradox-like situation, where the result of the recurrent application of formal rules can not be formally calculated, extends at least since the end of the 19th century, with the investigations of Georg Cantor for a strictly formal definition of infinity and its demonstration that there are real numbers that can not be counted (proved by the famous Cantor's diagonal argument[6]). This kindled a great debate in the field of mathematics that, at the beginning of the 20th century, divided mathematicians in two groups: the formalists that praised Cantor's contribution (led by Hilbert) and the intuitionists (led by Poincaré) that tried to disregard it as nonsensical. However, contradicting Hilbert's expectations that, despite Cantor's findings, mathematics would still hold to be complete, consistent and decidable, Kurt Gödel in the 1930s proved through his famous "incompleteness theorems" that mathematics can neither be complete nor consistent.

[4] https://playgameoflife.com/

[5] There is an interesting BBC documentary about this subject, called "Dangerous Knowledge", from (2007).

[6] https://mathworld.wolfram.com/CantorDiagonalMethod.html

With the previously mentioned Turing halting problem, some deterministic methods, and thus some mathematical formal models, were also proved to be undecidable. In philosophical terms, this relates to the Paradox of Self-reference[7], which is based on a statement that refers to itself or to its statements, which necessarily leads to undecidability. To give an example, let's consider these three hypothetical statements: 1) "X is my enemy", 2) "X is also its own enemy", 3) "All enemies of my enemies are my friends". This leads to a situation of undecidability, as stating that X is its own enemy (for its erratic self-destructing behaviour) creates a self-referentialism in which it is impossible to decide whether X is my enemy (for hypothesis 1) or not (for 2 and 3). Game of Life is so special and interesting because, through its self-referential formal rules, it turns into a very simple, elegant and therefore effective way to observe undecidability in action.

4. From intuitive complete tonalism to a formally incomplete atonalism

By the same time that mathematics was dealing with such turmoil and profound epistemological changes, a similar revolution was also happening in formal music. By the end of the 19th century, the bases of tonalism, inherited by the perceptual proportions of the harmonic series, were challenged. Many composers such as Franz Liszt were already flirting with atonal possibilities, such as in his composition *Bagatelle sans tonalité*[8] and Richard Wagner's opera *Tristan und Isolde* with a dubious cadence in its overture, known today as "Tristan Chord", that for some musicologists, represents the end of strict tonalism (ROSS, 2007). The modernist movement in the beginning of the 20th century reshaped the foundations of philosophy and arts, steering from form to concept, thus allowing the surfacing of more abstract types of artistic expressions. In music, this is well represented in the already mentioned formal work of Arnold Schoenberg's Twelve-Tone Technique that culminated into a 20th century revolution in formal composition. In a moment of such engagement, movements and jargons, this came to be known as serialism[9]. Schoenberg's serialist attempt to bring formalism to atonality distanced formal music composition from the intuitive atavistic auditory tone understanding given by the linear frequency proportions of harmonic series and equipped it with a formal compositional interactive strategy that helped composers to transcend, in an interactive game-like manner, the previous tonal mindset and even to take advantage of its undecidability, more as an inspirational compass than an unbroken rule, as it is known that many serial composers, including Schoenberg himself, along their carriers have broken serial rules in their own compositions.

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[7] https://plato.stanford.edu/entries/self-reference/

[8] https://youtu.be/Zx_Wolki0dc

[9] https://www.musictheoryacademy.com/understanding-music/serialism/

5. A zero performer melodic game

Game of Life is often referred to as a "zero-player game" which means that there is no interaction of human players during the development of each game's run. In this sense, the prototype here presented as Game of Tones is also a zero-player game, or better yet, a zero human composer (or even a zero human performer) music game. Each run automatically composes a melody whose length depends on its undecidable halting. Meanwhile the model will keep populating an extended series of tones according to simple deterministic rules inspired by the ones from Game of Life, as explained in the next section. A tone is here understood as one element belonging to the twelve-tone equal temperament system, which divides the octave (the frequency range between a fundamental f and its double 2f) in 12 equally proportioned parts on a logarithmic scale. This is equivalent to the 12 steps of a geometric progression with a ratio equal to the 12th root of 2 ($2^{\frac{1}{12}} \approx 1.05946...$). So, each step of frequency represents a variation of about 5,95% of the previous one, increasing as the note gets higher (goes up on the pentagram or to the right side on the piano's keyboard) or decreasing otherwise. As octaves are perceptually similar, their chromatic distinction is ignored (meaning that $0.5f = f = 2f$). In this implementation there is actually an extra step included in the 12 original ones. They represent the 12 notes within 1 octave of the chromatic scale and one extra silent step, representing a musical rest. As actual musical melodies are made of more than just an ordered set of pitches (as represented in the original Twelve Tone technique), here, besides the 12 pitch steps (plus 1 resting step), another two attributes were also included to each tone: duration and intensity. For the sake of simplicity and proximity with traditional music notation, the duration of the notes has only 5 levels. They are arranged as powers of 2 (4, 2, 1, 0.5 and 0.25 respectively representing the musical note durations: whole, half, quarter, eight and sixteenth). The degree of intensity of the notes has 3 levels, representing: soft, medium and loud; or in musical terms: *p*, m*p or mf*, *f*. Each tone is therefore defined by a triple made of fundamental frequency (12 notes' pitches and 1 rest), duration (5 steps) and intensity (3 steps). Each individual tone t is thus defined as $t(f,d,i)$ where: $f \in [1,2,3,4,5,6,7,8,9,10,11,12,13]$, $d \in [1,2,3,4,5]$ and $i \in [1,2,3]$.

For me, the motivation behind the Twelve-Tone technique seems to be drawn from an idealist premise of equality, unanimously empowering all tones with the same right of being expressed (played no more no less than once). On the other hand, the rules of Game of Life seem to lean towards a more realistic, empirical premise of natural selection, which is based on the fact that individuals seek and compete for resources. They are born, strive and die, both by excess or lack of others around. In the original twelve-tone technique, all 12 tones in a series have the same duration and all are played once. It disregards tonal hierarchy, given by their order (based on which tone was played first), inheritance (tonal relation in regard to the harmonic series of the previous tone) and opportunity (the many possibilities of intervalar resolutions).

On the other hand, in the Game of Life, opportunity plays a major role in the thriving of each individual, despite the fact that the same rules are applied to all. In other words, in the twelve-tone technique, individuals (tones) have the same rights. In the Game of Life, individuals (cells) have the same opportunity. For me, the philosophical standpoints expressed by the twelve-tone technique and the Game of Life seem to allow an interesting aesthetic merging; the egalitarian, cooperative approach of Schoenberg's music making technique with the darwinian, competitive approach of Conway's Game of Life.

6. The Game of Tones

As said, in the Game of Tones, an individual is an extended form of the Schoenbergian tone, instead of representing only pitch (fundamental frequency); here a tone has frequency, duration and intensity; represented by t(f,d,i). However, their neighbours are actually very limited. There are, at most, only two neighbours in the vicinity of each middle tone, the one immediately preceding it and the one immediately succeeding it. However, as each tone is given by a triple of discrete dimensions (f, d and i), there are many possible neighborhoods, even with only 2 neighbors. A neighborhood is here determined by the occurrence between consecutive tones where each one of their coincident dimensions are equal or at most one degree apart. Thus, there are 7 possible neighbourhoods of t(f,d,i). They can be: 1) For one dimension: t(f±1,d,i), t(f,d±1,i), t(f,d,i±1). 2) For two dimensions: t(f±1,d±1,i), t(f±1,d,i±1), t(f,d±1,i±1); 3) For three dimensions: t(f±1,d±1,i±1)

Which brings to the following set of possible neighbourhoods: 1) For one dimension, there are 6 possible neighbours: t(f+1,d,i), t(f-1,d,i), t(f,d+1,i), t(f,d-1,i), t(f,d,i+1), t(f,d,i-1). 2) For two dimensions, there are 12 possible neighbourhoods: t(f+1,d+1,i), t(f+1,d-1,i), t(f-1,d+1,i), t(f-1,d-1,i), t(f+1,d,i+1), t(f+1,d,i-1), t(f-1,d,i+1), t(f-1,d,i-1), t(f,d+1,i+1), t(f,d+1,i-1), t(f,d-1,i+1), t(f,d-1,i-1), 3) For three dimensions, there are 8 possible neighbourhoods: t(f+1,d+1,i+1), t(f+1,d+1,i-1), t(f+1,d-1,i+1), t(f+1,d-1,i-1), t(f-1,d+1,i+1), t(f-1,d+1,i-1), t(f-1,d-1,i+1), t(f-1,d-1,i-1)

In this sense, there can be up to 26 possible neighborhoods for each middle tone. Considering neighborhoods instead of neighbours the relationship of survival, birth and death of each individual (tone) can follow the same rules of Game of Life. Therefore, for each tone: If there are 0 or 1 neighborhoods, the tone dies (for lack of company). If there are 2 neighborhoods, the tone survives to the next generation. If there are 3 neighborhoods, a new tone is born to the next generation. If there are 4 or more neighborhoods, the tone dies (for lack of resources).

In this current implementation the initial twelve-tone series is automatically calculated randomly. This is made of 13 sequentially ordered non repeating extended tones triples (t1, t2, t3, t4, t5, t6, t7, t8, t9, t10, t12, t13) where only one of them is a resting, meaning that its frequency f is zero, or t(0,d,i). In the initial series, this is the case for the middle tones (t2, t3, t4, t5, t6, t7, t8, t9, t10, t11, t12) which have two neighbour tones (t-1 and t+1). Its extreme tones (t1 and t13) have only one neighbour. As said, tones can have neighborhoods in all their 3 dimensions (f, d and i). For instance, given t5(f,d,i), the preceding tone t4 can have with t5 up to 3 neighborhoods, such as in: t4(f±1,d±1,i±1), or even coincidental values of duration and intensity, as in t4(f±1,d,i) once that dimensions d and i are not mutually exclusive, as it is f for the initial series (that, by Schoenberg's definition, must have different values of f for all twelve tones in the prime series). So, from the previous example, other possible neighborhoods with t5(f,d,i) would be t4(f±1,d,i). For the middle tones with 2 neighbours, there are up to 6 possible neighborhoods (3 for each neighbour) which allows the application of Game of Life rules as previously described, in terms of neighborhoods instead of neighbors.

A prototypical model of Game of Tones was programmed using Pure Data (Pd) visual programming language[10]. Pd is a free software framework for the creation of real time audio control, processing and synthesis. This model is made of 2 parts, each one implemented as an independent Pd subpatch. They are: 1) a first subpatch that creates a random series of extended tones t(f,d,i) where f is a random non repeating value from 0 to 12 (f=0 representing a musical rest, where the value of i is disregarded); 2) a second subpatch that applies the deterministic rules from Game of Life according to the neighborhoods of the tones within the extended series, that for each generation removes, maintains and creates new tones. The result of one run of the Game of Tones is presented here as a musical example of the melodic possibilities of this simple model. The following link has an MP3 file of the entire non-supervised melody composition described in the next section, together with the Pd model that generated it and a PDF file of its music notation[11].

As expected, the model is undecidable, meaning that it is impossible to predict or calculate whether and whenever it will halt, repeat or keep generating an infinite melody. In the same way as the Game of Life, oftentimes the runs of Game of Tones present repeating melodic patterns which reinforce listener's cognitive understanding of a seeming tonality, thus emerging a perception of musical idiomatism which was unexpected. This behaviour is automatically generated by this undecidable model that seems to self-organise a tonal referentialism emerging from the atonal origin of the primordial randomly generated schoenbegian series of tones. The inclusion of durations and intensities seems to have made the melodic outcome more palatable and musically appealing. As it can be heard in the provided example, I chose a timbre of oboe for this run as it seemed to me as a nice fit for the type of melody automatically generated by the model at that specific run. This melody is presented as generated,

[10] www.puredata.info

[11] https://drive.google.com/drive/folders/10EfS7AubtOy8AIoCnY9Om7_SzORpnVAA?usp=sharing

without editions and in its entirety, exactly as it was created by the model, from the input of the original series until its unexpected halt.

7. Undecidable coda with unexpected tonality

To exemplify the type of melodies that can be generated by this simple model, the music notation of the run here presented is depicted below. As said, the original series is initially randomly generated, following Schoenberg's rules (of having all twelve non-repeating tones in it plus one rest) and an extended representation of tones with durations and intensities forming a tridimensional series of thirteen tones. For this specific run, the notation of the original series, as generated by the first subpatch is:

Example 1: Random generated extended Schoenbergian twelve-tone series (barlines included automatically by the music notation software)

Following that, the second subpatch applied the rules from Game of Life in the extended tones' neighborhoods, as described in section 6. Each run presents a different and unexpected melodic result that, at each generation, creates, maintains or removes tones from their sequence, as in a Turing machine, moving along a virtual infinite sequential row, writing, reading, keeping or removing symbols until an undecidable halt. As in the Turing machine, it is also undecidable if and when Game of Tones loops, runs forever or ends in a musical coda. In this specific case, the model halted after about 50 seconds of melody (the MIDI file of the original series of extended tones generated by the first subpatch has a melodic length of about 5 seconds). It is interesting to realize that, differently from the schoenbergian original series, the one generated by Game of Tones in this run (and many others generated by this model) presented many repeating patterns, is often enriched with spontaneous musical ornaments and motifs automatically generated by the model itself that, although based on a primordial atonal series, populated it with non-supervised but wellcome (at least for me) human-like musical improvisational stylizations. The "humanized" (quantized) musical notation of the first 9 measures (or 2 phrases) of this specific run is shown below:

Example 2: First two melodic phrases automatically generated from the series depicted in Example 1.

A MIDI file was generated in real time by the Pd model (more specifically by its second subpatch) and recorded with Aria Maestosa[12], a simple yet powerful free software for MIDI files edition. After some simple data scrubbing (replacing overlapping notes and snapping the whole sequence to the begin of an initial measure) the resulting MIDI files were imported to MuseScore[13], a cross-platform free software for score writing. This step also required extra data scrubbing specially in terms of voice reduction, and ornament replacements, from complex notes and rhythmic structures to the well-known symbols of ornaments and grace notes. This made the score (as presented above) much easier to be read by a human musician, while preserving the automatically generated articulations. This data scrubbing seems to me to remain as an unavoidable step of this otherwise automatic non-supervised process, necessarily requiring the participation of a human musician to reshape the automatic generated music notation, considering that this one is intended to be further performed by a human musician. Here, the original (non-humanized) generated score was read as a MIDI file by MuseScore and exported directly from MIDI to audio generated by its default software synthesizer, and finally converted as digital audio file in MP3 format.

Further research on this subject can also explore different priorities of neighborhoods for the Game of Tones' rules, in terms of pitch, duration and intensity. Now, they are as egalitarian as Schoenberg envisioned for pitch, in his Twelve-Tone technique, but perceptually these dimensions may endow a cognitive priority in its auditory processing by the human mind. The prototype model presented here also limited the tone range generation within only one octave. Further developments may explore new rules for the generated melody to be extended within more than one single chromatic scale, for instance, making it closer to the average range of human voice. Finally, further developments of this serial music making model can be expanded to explore the generation of voice's counterpoints, harmonic cadences or pitch clusters, which might transcend the current aesthetic expressivity of Game of Tones as presented here, eventually allowing it to cross a musical frontier from its current individualistic monotonic perspective to a more socialist polyphonic kin of automatic generated non-supervised computer music models.

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[12] https://ariamaestosa.github.io/ariamaestosa/docs/index.html

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