Generalized Omnibus Progressions: A Mathematical Formalization of Harmonic Progressions Exhibiting Semitonal Contrary Motion and Tertian Sonorities

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Abstract: *The proposed mathematical framework offers a set-theoretic model of describing semitonal contrary motion found in the omnibus progression in terms of omnibus functions (and their mirrored versions). These functions reveal the novel observation that any generalized omnibus progression will have at least three, but at most six, unique sonorities before an extension occurs. This study examines all four-voice triads and seventh chords built on major/minor thirds in connection with generalized omnibus progressions. An excerpt from the fourth string quartet of Dmitri Klebanov (1907-1987) serves as the primary example, which includes the unusual tertian sonority of augmented-major seventh chords. Finally, this study identifies all the generalized omnibus progressions with tertian sonorities as graphical networks.*

Keywords: *Semitonal contrary motion. Tertian harmony. Omnibus progression. Dmitri Klebanov.*

I. Preliminaries

Angled brackets will denote *ordered pitch-class multisets* or *pc segments*; the generic form of pc segments is $\langle p_1, p_2, \ldots, p_{n-1}, p_n \rangle$, $n \in \mathbb{N}$ with pitch classes $p_1, p_2, \ldots, p_{n-1}, p_n \in \mathbb{Z}_{12}$. Only pc segments of size four are considered, such as $\langle w, x, y, z \rangle$, where w corresponds to the bass voice, x corresponds to the tenor voice, y corresponds to the alto voice, and z corresponds to the soprano voice. Using pc segments as opposed to unordered sets gives the versatility of representing unique voicings; for example, $\{0, 4, 7, 11\} = \{0, 7, 4, 11\}$, but $\langle 0, 4, 7, 11 \rangle \neq \langle 0, 7, 4, 11 \rangle$.

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The set of aurally distinct musical intervals will be denoted as $I = \{$minor second, major second, minor third, major third, perfect fourth, tritone, perfect fifth, minor sixth, major sixth, minor seventh, major seventh, perfect octave$\}$, and each of these will correspond to the number of half-steps needed to create the interval.

$$\begin{array}{lll} 1' = \text{minor second} & 2' = \text{major second} & 3' = \text{minor third} \\ \vdots \quad \vdots & \vdots \quad \vdots & \vdots \quad \vdots \\ 11' = \text{major seventh} & 12' = \text{perfect octave} \stackrel{\text{def}}{=} & 0' (= \text{perfect unison}). \end{array}$$

The $'$ symbol is added to the numbers to distinguish them as intervals rather than pitch classes. Thus, $I = \{0', 1', 2', 3', 4', 5', 6', 7', 8', 9', 10', 11'\}$. While a perfect unison and a perfect octave are not the same aural interval, using generic pitch classes makes octave designations irrelevant, so it is defined that any interval having a difference of $0'$ is at least one octave apart on a musical staff.

Definition 1. *For any $p_i, p_j \in \langle p_1, p_2, \ldots, p_{n-1}, p_n \rangle$ and $i, j \in \{1, 2, \ldots, n-1, n\}$, an **interval** is the modular difference $p_i - p_j$ for $i > j$. This interval will be denoted as $p_i - p_j = k$, where $k \in I$.*

Definition 2. *For any $p_i, p_{i-1} \in \langle p_1, p_2, \ldots, p_{n-1}, p_n \rangle$ and $i, i-1 \in \{1, 2, \ldots, n-1, n\}$, a **consecutive interval (c.i.)** is the modular difference $p_i - p_{i-1}$.*

Definition 3. *A **consecutive interval vector (CIV)** is defined as the representation $(k_1 - k_2 - \cdots - k_{n-1})$ for consecutive intervals $k_i \in I$ found in a pc segment with n pitches.*

Example 1. *Consider $\langle 0, 4, 7, 0 \rangle$ and $\langle 0, 0, 4, 7 \rangle$, both of which are a root position C major triad. The CIVs are $(4' - 3' - 5')$ and $(0' - 4' - 3')$, respectively. Thus, even though each CIV generically represents a root position major triad, the CIV $(4' - 3' - 5')$ describes the triad in stacked thirds in the lower three voices with the root doubled in the soprano voice, whereas the CIV $(0' - 4' - 3')$ describes the triad in stacked thirds in the upper three voices with the root doubled in the tenor voice (see Figure 1).*

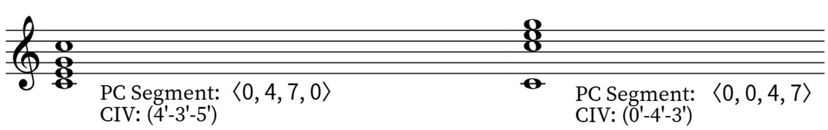

Figure 1: *Root position major triads with root of C, and their associated pc segments and each CIV.*

Eric Regener explores a very similar concept in [8], where his *interval notation* (IN) is a parenthetical representation of the consecutive intervals and the interval between the highest pitch and the lowest pitch. As a consequence, his IN representation includes n digits as opposed to $n - 1$ digits in a CIV for some given pc segment containing n pitches. Due to the requirement of using pitch-class sets, duplicated pitches in different octaves are ignored in Regener's mathematical framework (see Figure 2).

Definition 4. *Let $w, x, y, z \in \mathbb{Z}_{12}$. The pc segment $\langle w, x, y, z \rangle$ is a **four-voice triad** if the elements of $\langle w, x, y, z \rangle$ can be reordered such that exactly one consecutive interval in the reordered pc segment is $0'$, and the other two consecutive intervals are a) both $4'$, b) both $3'$, or c) one is $4'$ and one is $3'$.*

Definition 5. *Let $w, x, y, z \in \mathbb{Z}_{12}$ such that w, x, y, z are all distinct. The pc segment $\langle w, x, y, z \rangle$ is a **seventh chord** if the elements of $\langle w, x, y, z \rangle$ can be reordered such that each consecutive interval is either $3'$ or $4'$.*

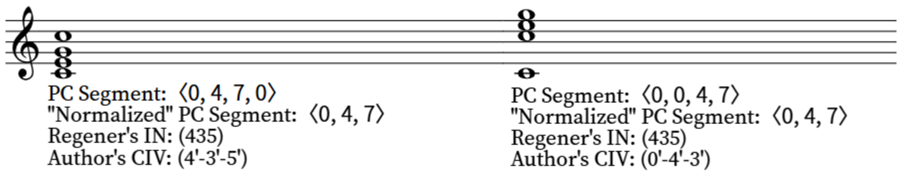

Figure 2: *Distinctions in the notations of Regener's IN and the CIV. Certain pc segments lend itself to an alignment between the two notations, whereas others do not.*

Example 2. *Consider $\langle 4,7,10,0\rangle$, $\langle 10,3,10,7\rangle$, and $\langle 0,2,7,10\rangle$. The first pc segment is a seventh chord because all of its elements are distinct and its elements can be reordered as pc segment $\langle 0,4,7,10\rangle$, a C dominant seventh chord, with a CIV of $(4'-3'-3')$. The second pc segment is a four-voice triad because its elements can be reordered as pc segment $\langle 3,7,10,10\rangle$, an E♭ major triad with B♭ doubled, with a CIV of $(4'-3'-0')$. Finally, the last pc segment is neither a seventh chord nor a four-voice triad since no reordering of its elements can satisfy either definition. In fact, ordering the elements in the most compact form reveals a G minor triad with an added C; in mathematical notation, $\langle 0,2,7,10\rangle \implies \langle 7,10,0,2\rangle$, with a CIV of $(3'-2'-2')$.*[1]

The possible sonorities of seventh chords are augmented-major seventh ($+^{M7}$), major-major seventh (^{M7}), major-minor seventh (dominant seventh; ^{7}), minor-major seventh (m^{M7}), minor-minor seventh (m^{7}), half-diminished seventh (^{ø7}), and fully diminished seventh (^{o7}). See Figure 3. Lead-sheet symbols and/or figured bass will be utilized in musical examples, while in-text discussion will state *root position*, *first inversion*, *second inversion*, and *third inversion*.

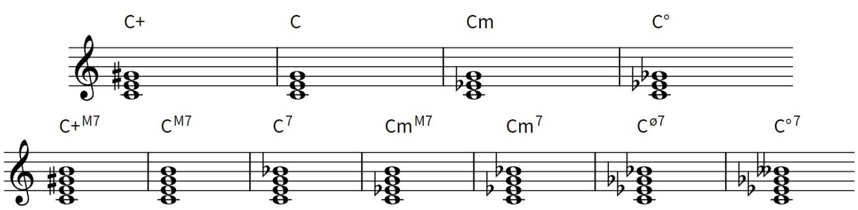

Figure 3: *The permissible chord types with a root of C, given Definition 4 and Definition 5.*

Remark 1. *Augmented triads present ambiguity in terms of the root of the chord (and even the spelling of the chord, given enharmonicism of generic pitch classes). To demonstrate, upon initial inspection, the following pc segment $\langle 0,4,8,0\rangle$ could be interpreted as C+ with C doubled, E+ with B♯ doubled, or A♭+ with C doubled. Fully diminished seventh chords have this root/spelling ambiguity, too. For clarity, the root of the chord in the pc segment will be underlined, a technique used in [2]. For triads where the root is doubled, only the lowest voice containing the root will be underlined.*

II. The Omnibus Progression

In [11], Victor Yellin defines the omnibus progression as a five-chord progression, beginning with a dominant seventh chord in first inversion (root in the soprano), moving to a root position dominant seventh chord, to a minor triad in second inversion, to the same dominant seventh

[1] The author has developed a Jupyter Notebook (available in a GitHub repository) containing algorithms where a user can input 4 elements of $\mathbb{Z}_{12}$ and determine if there is such an ordering where either Definition 4 or 5 is satisfied.

chord as the second chord but in third inversion, and finally to a root position dominant seventh chord with the same root as the first chord. The progression occurs when the two outer voices of each chord move in contrary motion in four semitonal steps, while the two inner voices form an interval of a minor third and serve as a double pedal tone (see Figure 4).

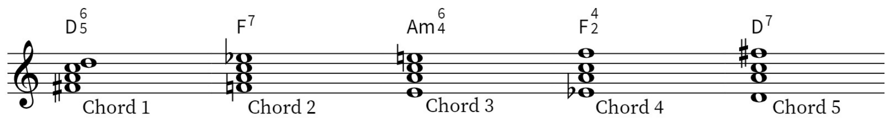

Figure 4: *Yellin's realization of the omnibus progression with lead-sheet symbols.*

This definition inspires a mathematical framework that captures the omnibus progression's mechanism of semitonal contrary motion limited to tertian sonorities. The possible harmonic progressions can be abstractly represented as a series of function compositions, which can also be visually represented through graphical networks. For some root $\underline{n} \in \mathbb{Z}_{12}$, the following generalized pc segments and CIVs describe the chords of the omnibus progression (for clarity, note that $\underline{n} = \underline{n+0}$). The example in Figure 4 is listed as pc segments after the CIV entries. The omnibus' contrapuntal idea is then mathematically defined.

1. $\langle n+4, n+7, n+10, \underline{n+0}\rangle$ CIV: $(3' - 3' - 2')$ Figure 4 chord 1: $\langle 6, 9, 0, \underline{2}\rangle$
2. $\langle \underline{n+3}, n+7, n+10, n+1\rangle$ CIV: $(4' - 3' - 3')$ Figure 4 chord 2: $\langle \underline{5}, 9, 0, 3\rangle$
3. $\langle n+2, \underline{n+7}, n+10, n+2\rangle$ CIV: $(5' - 3' - 4')$ Figure 4 chord 3: $\langle 4, \underline{9}, 0, 4\rangle$
4. $\langle n+1, n+7, n+10, \underline{n+3}\rangle$ CIV: $(6' - 3' - 5')$ Figure 4 chord 4: $\langle 3, 9, 0, \underline{5}\rangle$
5. $\langle \underline{n+0}, n+7, n+10, n+4\rangle$ CIV: $(7' - 3' - 6')$ Figure 4 chord 5: $\langle \underline{2}, 9, 0, 6\rangle$

Definition 6 (Omnibus Functions)**.** *Let pc segment $c = \langle w, x, y, z\rangle$ be a triad or a seventh chord as defined in Definitions 4 and 5. Let Q be the set of all possible triads and seventh chords. Let R be the set of all pc segments of size four. Let $V = \{S, A, T\}$, where S, A, T represent the soprano, alto, and tenor voices of pc segment c. Define the set of* ***omnibus functions****, where $f_V : Q \to R$ for each $v \in V$, as*

$$f_V(c) = \Big\{\{f_S(c)\}, \{f_A(c)\}, \{f_T(c)\}\Big\}, \text{ where}$$

$$f_S\Big(\langle w, x, y, z\rangle\Big) := \langle w-1,\ x,\ y,\ z+1\rangle$$

$$f_A\Big(\langle w, x, y, z\rangle\Big) := \langle w-1,\ x,\ y+1,\ z\rangle$$

$$f_T\Big(\langle w, x, y, z\rangle\Big) := \langle w-1,\ x+1,\ y,\ z\rangle.$$

Example 3. *Any chord of Yellin's definition can be represented as a series of compositions of f_S. The last chord of Figure 4 would be the following compositions of f_S:*

$$f_S f_S f_S f_S\Big(\langle 6, 9, 0, \underline{2}\rangle\Big) = f_S f_S f_S\Big(\langle \underline{5}, 9, 0, 3\rangle\Big) = f_S f_S\Big(\langle 4, \underline{9}, 0, 4\rangle\Big) = f_S\Big(\langle 3, 9, 0, \underline{5}\rangle\Big) = \langle \underline{2}, 9, 0, 6\rangle.$$

The function f_V is defined as a set of functions, so if Example 3 was examined in terms of f_V, the result would be a set of three pc segments (see Figure 5). Since the resulting pc segments from compositions of f_A and f_T do not yield triads or seventh chords per Definitions 4 and 5, not every omnibus function of f_V will yield one of the permissible chord types for some arbitrary initial chord. Additionally, the resulting pc segments for compositions of f_A and f_T do not mathematically capture crossed voices.

$$f_V f_V f_V f_V\Big(\langle 6,9,0,\underline{2}\rangle\Big) = \Big\{ f_S f_S f_S f_S\Big(\langle 6,9,0,\underline{2}\rangle\Big),\ f_A f_A f_A f_A\Big(\langle 6,9,0,\underline{2}\rangle\Big),\ f_T f_T f_T f_T\Big(\langle 6,9,0,\underline{2}\rangle\Big)\Big\}$$
$$= \Big\{\langle \underline{2},9,0,6\rangle, \langle 2,9,4,2\rangle, \langle 2,1,0,2\rangle\Big\}.$$

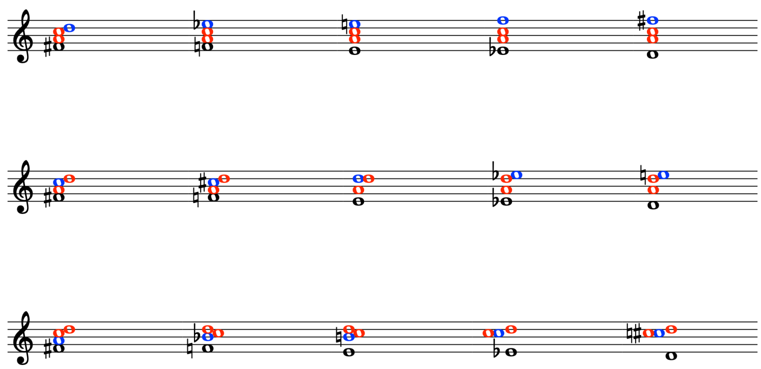

Figure 5: *The musical representation of each composition of f_S on the first system, f_A on the second, and f_T on the third. Note the blue pitch represents the ascending semitonal line and indicates the particular voice that corresponds to each respective omnibus function. The red pitches correspond to the double pedal tones, which are not exclusively minor thirds, as in the case of f_A and f_T.*

The function f_V is not surjective for any $v \in V$ since for $\langle 0,0,0,0\rangle \in R$, $\langle 0,0,0,0\rangle \neq f_v(c)$ for some pc segment $c \in Q$. Thus, "inverse effect" of the omnibus function has to be alternatively defined as *mirror omnibus functions*.

Definition 7 (Mirror Omnibus Functions)**.** *Let pc segment $c = \langle w,x,y,z\rangle$ and sets Q, R, and V be defined as in Omnibus Functions. Define the set of **mirror omnibus functions**, where $g_V : Q \to R$ for each $v \in V$, as*

$$g_V(c) = \Big\{\{g_S(c)\}, \{g_A(c)\}, \{g_T(c)\}\Big\},\ \text{where}$$
$$g_S\Big(\langle w,x,y,z\rangle\Big) := \langle w+1,\ x,\ y,\ z-1\rangle$$
$$g_A\Big(\langle w,x,y,z\rangle\Big) := \langle w+1,\ x,\ y-1,\ z\rangle$$
$$g_T\Big(\langle w,x,y,z\rangle\Big) := \langle w+1,\ x-1,\ y,\ z\rangle.$$

Proposition 1. *Consider sets Q, R, V, f_V, and g_V as in Definition 7. For some pc segments $c_1 = \langle w_1,x_1,y_1,z_1\rangle$ and $c_2 = \langle w_2,x_2,y_2,z_2\rangle$, f_V and g_V are injective for each $v \in V$.*

Proof. Let Q, R, V, f_V, and g_V be defined as in Definition 7. This proof demonstrates that $f_S \in f_V$ is injective. Let $c_1 = \langle w_1,x_1,y_1,z_1\rangle$ and $c_2 = \langle w_2,x_2,y_2,z_2\rangle$. Suppose

$$f_S(c_1) = f_S(c_2) \implies$$
$$f_S\Big(\langle w_1,x_1,y_1,z_1\rangle\Big) = f_S\Big(\langle w_2,x_2,y_2,z_2\rangle\Big) \implies$$
$$\langle w_1-1,x_1,y_1,z_1+1\rangle = \langle w_2-1,x_2,y_2,z_2+1\rangle.$$

Since these are equivalent pc segments, each voice of the first pc segment is equivalent to its corresponding voice in the second pc segment. Thus,

$$w_1 - 1 - (w_2 - 1) \equiv 0 \pmod{12}$$
$$w_1 - w_2 \equiv 0 \pmod{12} \Rightarrow w_1 \equiv w_2.$$

Similarly, $z_1 \equiv z_2$. It can be shown that $x_1 \equiv x_2$ and $y_1 \equiv y_2$, and since equivalence classes cannot contain elements congruent to a different equivalent class, f_S is therefore injective. It can be similarly shown that f_A, f_T, g_S, g_A, and g_T are injective functions. ■

III. Extensions of the Omnibus Progression

Yellin defines a *mutation* in [11] as a "continuation of the ascending or descending motion of the semitonal bass voice of the omnibus beyond the normal ambit of five half-steps, by means of the perception of a new double pedal of the minor third (or a major sixth) in a seventh chord sonority." A mutation of the omnibus progression may be thought of as an *extension* (this term is preferred to generalize the discussion of multiple mutations of the progression), where the fifth chord of the original omnibus sequence becomes the second chord in a new omnibus sequence.

Example 4. *Consider Figure 6. The fifth chord is $\langle \underline{4}, 11, 2, 8 \rangle$. This becomes the second chord of the omnibus progression that now has a major sixth as the double pedal tone, which starts a cycle of three distinct sonorities: a root position dominant seventh chord, a second inversion minor triad, and a third inversion dominant seventh chord with the same root as the root position seventh chord.*

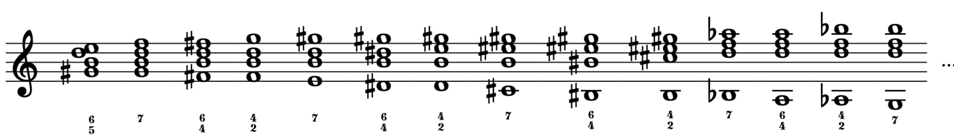

Figure 6: *Yellin's definition of an omnibus progression for the first five chords, then a mutation occurs at $\langle \underline{4}, 11, 2, 8 \rangle$, proceeding with a new yet repetitive sequence every three chords and may continue indefinitely.*

Example 5. *At rehearsal M in the first movement of his first symphony, Norwegian composer Johan Svendsen provides a musical passage that includes both Yellin's five-chord definition and four omnibus extensions, outlining the chromatic scale in the bass voice. Figure 7 displays a textural reduction of this passage, supplied by [7]. Harmonically, this can be analyzed as dominant prolongation to the original key, which is the standard harmonic function of omnibus progressions.*

In the omnibus progression (and the omnibus functions), the bass voice is never part of the double pedal tone because it continuously moves by semitone. The two upper voices that construct the initial double pedal tone in the second chord each ascend only by an interval of $3'$, and the other upper voice in the second chord ascends by an interval of $6'$ to get to a permutation of the original pc segment, despite the order of the upper three voices (see Figures 8 and 9).

Figure 7: *Johan Svendsen, Symphony No. 1 (1867), I. Molto allegro, Rehearsal M (textural reduction).*

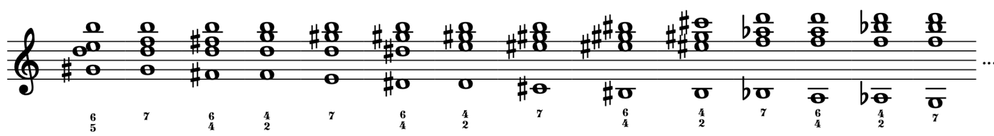

Figure 8: *The same omnibus progression as Figure 6 but with an alternate voicing of the upper three voices.*

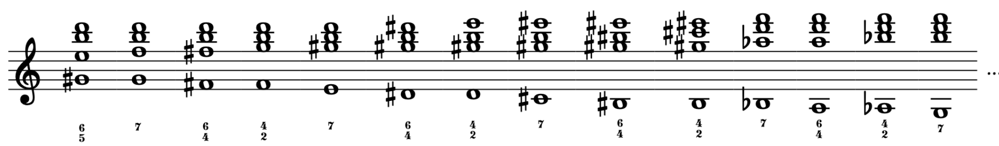

Figure 9: *The same omnibus progression as Figure 6 but with another alternate voicing of the upper three voices.*

In Figure 6, the second chord and the final chord have the same root and are both in root position. The upper three voices are different only in order between those two chords. Since there are three inversions of triads, all generalized omnibus progressions require $3 \cdot 12 = 36$ iterations to reach the original triadic voicing in the upper three voices. One can "see" this by isolating the upper three voices from the second chord onward and observing that the root position diminished triad formed by the tenor, alto, and soprano voice in the second chord is in first inversion twelve chords later. After twelve more chords, that diminished triad will be in second inversion, and consequently, twelve more yields the original root position, as seen in the last chord of Figure

9. The complete chromatic scale in the bass and the root relationships by minor third in the intermediate steps are the reasons for this pattern between the second/last chord in Figure 6.

Figures 8 and 9 contain extensions of Figure 6 if the first chord of each figure is disregarded. Figures 6, 8, and 9 outline an *exhaustive* omnibus progression where the second chord of Figure 6 and the final chord of Figure 9 have identical pc segments. Since there are $3! = 6$ ways to order the upper three voices in the first chord, the other three possible voicings for the first chord indicate that there is another exhaustive yet disjoint omnibus progression contained in those voicings. This paper now has the musical motivation and mathematical framework to consider generalized omnibus progressions.

IV. Generalized Omnibus Progressions

German-American music theorist Bernhard Ziehn (1845-1912) was the first to exploit an omnibus progression that did not contain dominant seventh chords and minor triads. In [12], he wrote two omnibus-inspired canons that consist of root position half-diminished seventh chords, root position major triads, and third inversion half-diminished seventh chords. In [9], Joti Rockwell calls this the *Ziehn inverted omnibus progression* since it can be derived by reflecting (inverting) the original chords across an imaginary axis on a circle of chromatic pitches (see Figure 10). However, Rockwell acknowledges in [9] that the Ziehn inverted omnibus progression is much more difficult to incorporate in a tonal context, and that even Wagner, a pioneer of unusual harmonic progressions involving half-diminished seventh chords, seemed to only give fleeting impressions of the progression that Ziehn fully developed in [12].

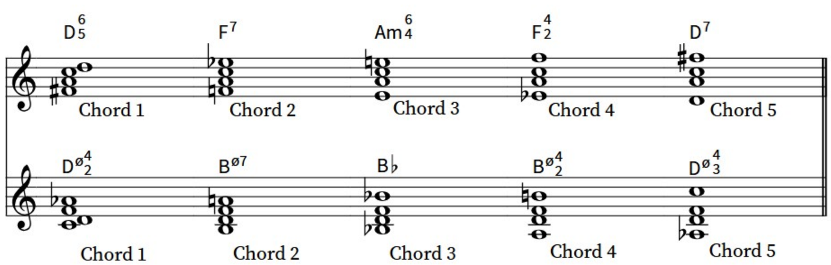

Figure 10: *A realization of Yellin's formal definition and an inversionally-related realization based off of Ziehn's second canon. The third inversion and second inversion D half-diminished seventh chords are not present in [12].*

For the sake of generalization, suppose a root position half-diminished seventh chord with root $\underline{n}$ has the following voicing: $\langle \underline{n}, n+3, n+6, n+10 \rangle$—like the root position B half-diminished seventh chord in Figure 10. The Ziehn inverted omnibus progression can be described with omnibus functions as shown in Figure 11 by the red path. Lead-sheet notation is utilized in the following networks, with the (M) indicating major triads in root position.

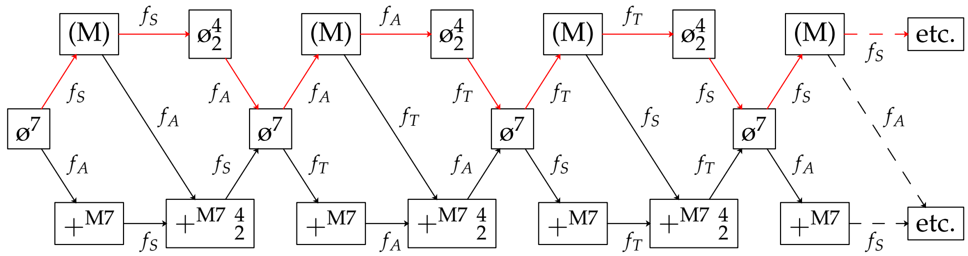

Figure 11: *Network of possible voice-leading paths with initial voicing* $\langle \underline{n}, n+3, n+6, n+10 \rangle$ *in terms of chord sonorities. The Ziehn inverted omnibus progression is outlined by the red path.*

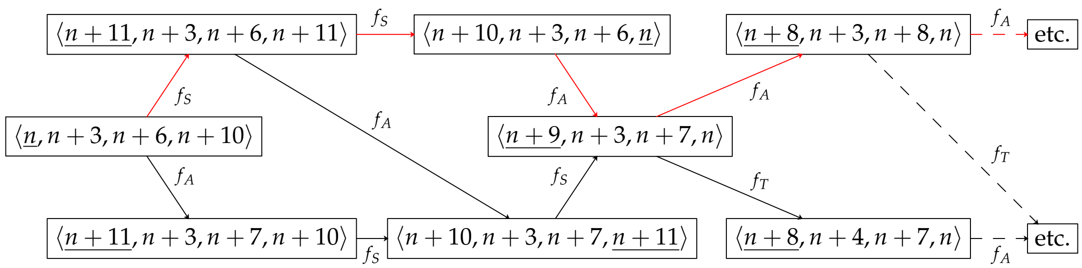

Figure 12: *Network of possible voice-leading paths with initial voicing* $\langle \underline{n}, n+3, n+6, n+10 \rangle$ *in terms of pc segments. The Ziehn inverted omnibus progression is outlined by the red path.*

Figure 11 outlines the possible voice-leading paths with initial pc segment $\langle \underline{n}, n+3, n+6, n+10 \rangle$ in terms of tertian sonorities. These two networks reveal that there are three distinct omnibus progressions that can be extended every three chords.[2] These are:

- the Ziehn inverted omnibus progression (red path),
- the progression including a root position half-diminished seventh chord, a root position augmented-major seventh chord, and a third inversion augmented-major seventh chord, and
- the progression including a root position half-diminished seventh chord, a root position major triad, and a third inversion augmented-major seventh chord.

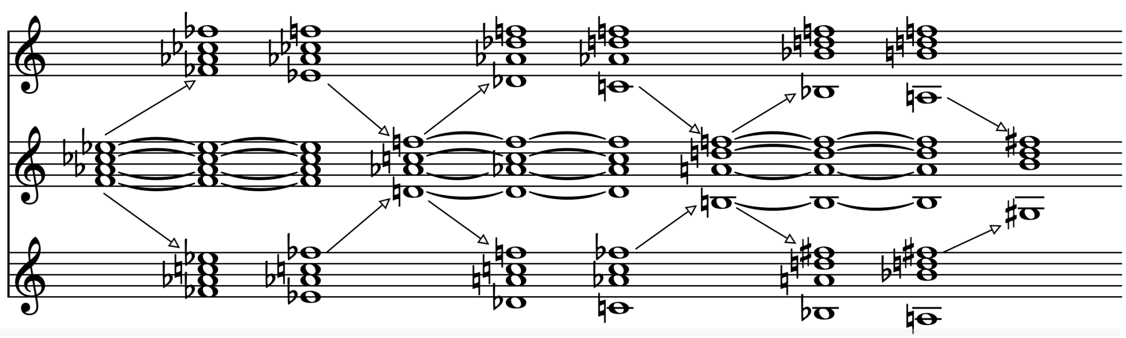

Figure 13: *Musical realization of Figure 11 with initial pc segment* $\langle 5, 8, 11, 3 \rangle$.

[2]The visual aid to where the extensions occur lie in the creation of a new hexagonal cell, starting and ending with the root position half-diminished seventh chord.

Because the possibility exists for a root position half-diminished seventh chord to approach two different chords, this generalized omnibus progression need not be limited to extend only every three chords. In fact, the number of chords before an extension occurs may be any multiple of three (see Figure 14). The ordering of the upper three voices is independent of this fact; the only thing that would change in the network of Figure 11 would be the f_v for some $v \in \{S, A, T\}$. The observation from Section III implies that the initial voicing of $\langle \underline{n}, n+3, n+6, n+10 \rangle$ contain the permutations $\langle \underline{n}, n+6, n+10, n+3 \rangle$ and $\langle \underline{n}, n+10, n+3, n+6 \rangle$ while the voicing $\langle \underline{n}, n+3, n+10, n+6 \rangle$ contain $\langle \underline{n}, n+6, n+3, n+10 \rangle$ and $\langle \underline{n}, n+10, n+6, n+3 \rangle$.

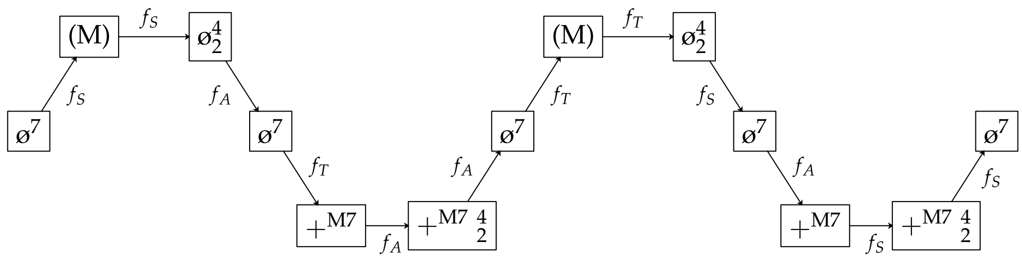

Figure 14: *Network of root position half-diminished seventh chords, root position major triads, third inversion half-diminished seventh chords, root position augmented-major seventh chords, and third inversion augmented-major seventh chords demonstrating an extension after six chords. This path can be found in Figure 11.*

Example 6. *A truncated example of the progression containing augmented-major seventh chords be found in the fourth string quartet of Dmitri Klebanov (see Figure 15). Beginning at Rehearsal 6 (m. 64), the sonorities present are a root position half-diminished chord on beat 1, a third inversion augmented-major seventh chord on beat 2, the root position of that same augmented-major seventh chord on beat 3, and finally the first inversion of the same half-diminished chord on beat 4, followed by an octave transposition upward in m. 65. Then, mm. 66-67 continues right where m. 64 ended with an upward minor third transposition. Because of the contracting nature of this generalized omnibus progression, this path travels from right-to-left in the graphical network Figure 11 (or, equivalently, uses the mirror omnibus functions). The pc segments for mm. 64 and 66, the mirror omnibus function compositions, and their associated CIVs are given.*

$$
\begin{array}{rll}
 & \text{m. 64, beat 1 } \langle \underline{9}, 3, 7, 0 \rangle & \text{CIV: (6'-4'-5')} \\
g_S(\langle \underline{9}, 3, 7, 0 \rangle) = & \text{m. 64, beat 2 } \langle 10, 3, 7, \underline{11} \rangle & \text{CIV: (5'-4'-4')} \\
g_S g_S(\langle \underline{9}, 3, 7, 0 \rangle) = & \text{m. 64, beat 3 } \langle \underline{11}, 3, 7, 10 \rangle & \text{CIV: (4'-4'-3')} \\
g_S g_S g_S(\langle \underline{9}, 3, 7, 0 \rangle) = & \text{m. 64, beat 4 } \langle 0, 3, 7, \underline{9} \rangle & \text{CIV: (3'-4'-2')} \\
T_3(\langle \underline{9}, 3, 7, 0 \rangle) = & \text{m. 66, beat 1 } \langle \underline{0}, 6, 10, 3 \rangle & \text{CIV: (6'-4'-5')} \\
g_S(\langle \underline{0}, 6, 10, 3 \rangle) = & \text{m. 66, beat 2 } \langle 1, 6, 10, \underline{2} \rangle & \text{CIV: (5'-4'-4')} \\
g_S g_S(\langle \underline{0}, 6, 10, 3 \rangle) = & \text{m. 66, beat 3 } \langle \underline{2}, 6, 10, 1 \rangle & \text{CIV: (4'-4'-3')} \\
g_S g_S g_S(\langle \underline{0}, 6, 10, 3 \rangle) = & \text{m. 66, beat 4 } \langle 3, 6, 10, \underline{0} \rangle & \text{CIV: (3'-4'-2')}
\end{array}
$$

This example does not capture the cyclical nature of generalized omnibus progressions, especially since the progression is 4 chords long, with the first and fourth chords having the same root. For the progression to follow the cyclical pattern, the last beat of m. 64 (and m. 65) and the first beat of m. 66 (and m. 67) would need to be a root position C half-diminished seventh chord rather than a first inversion A half-diminished seventh chord. In terms of omnibus functions, simply changing $g_S g_S g_S(\langle \underline{9}, 3, 7, 0\rangle) = \langle 0, 3, 7, \underline{9}\rangle$ to $g_A g_S g_S(\langle \underline{9}, 3, 7, 0\rangle) = \langle \underline{0}, 3, 6, 10\rangle$ would provide the desired result. Despite the break from the cyclical pattern, Klebanov made an unusual choice to use augmented-major seventh sonorities as the resulting harmony of semitonal contrary motion being applied in an omnibus-like fashion when he could have chosen Ziehn's inverted path. He made the same voice-leading choices earlier in mm. 44-47, with everything being transposed up by a major second.

Figure 15: *Dmitri Klebanov, String Quartet No. 4 (1946), I. Allegro moderato – Allegro – Poco sostenuto, mm. 64–67.*

There are other examples of progressions where there are multiple voice-leading possibilities for some chords, but there are other generalized omnibus progressions that exclusively can be extended every three chords due to their limited voice-leading options. For example, Rockwell finds in [9] a progression involving root position augmented triads, root position minor-minor seventh chords, and third inversion minor-minor seventh chords. He acknowledges that augmented triads have aural ambiguity with non-distinct transpositions, so he does not include the possibility of inversions for augmented triads. However, given the root distinction by underlining, this study can differentiate between various augmented triad inversions strictly from an notational (visual) stance, not a sonority (aural) stance. While including all of these progressions may seem contradictory to an earlier assumption of representing a distinct sonority in one way (i.e. A4 is spelled differently than d5, but they are identical sonorities considered as a tritone, or 6′), the analytical (or even compositional) implications are worth exploring by considering different types of root movement or transposition cycles for the augmented sonorities. This reveals the following notationally distinct generalized omnibus progressions:

- the progression including a *root position* augmented triad with the root doubled, a third inversion minor-minor seventh chord, and a root position minor-minor seventh chord,
- the progression including a *first inversion* augmented triad with the third doubled, a third inversion minor-minor seventh chord, and a root position minor-minor seventh chord, and
- the progression including a *second inversion* augmented triad with the fifth doubled, a third inversion minor-minor seventh chord, and a root position minor-minor seventh chord.

For the sake of generalization, suppose a generic augmented triad has the following voicing: $\langle n, n+4, n+8, n\rangle$. The above progressions form a one-dimensional network since there is only one possible omnibus function that successfully transforms each chord into a permissible result. However, different analytical possibilities are given for each augmented sonority, and the above progressions can still be extended every $3n$ chords, $n \in \mathbb{N}$. This would involve re-interpretation of the roots of the augmented triads every third chord (see Figure 16).

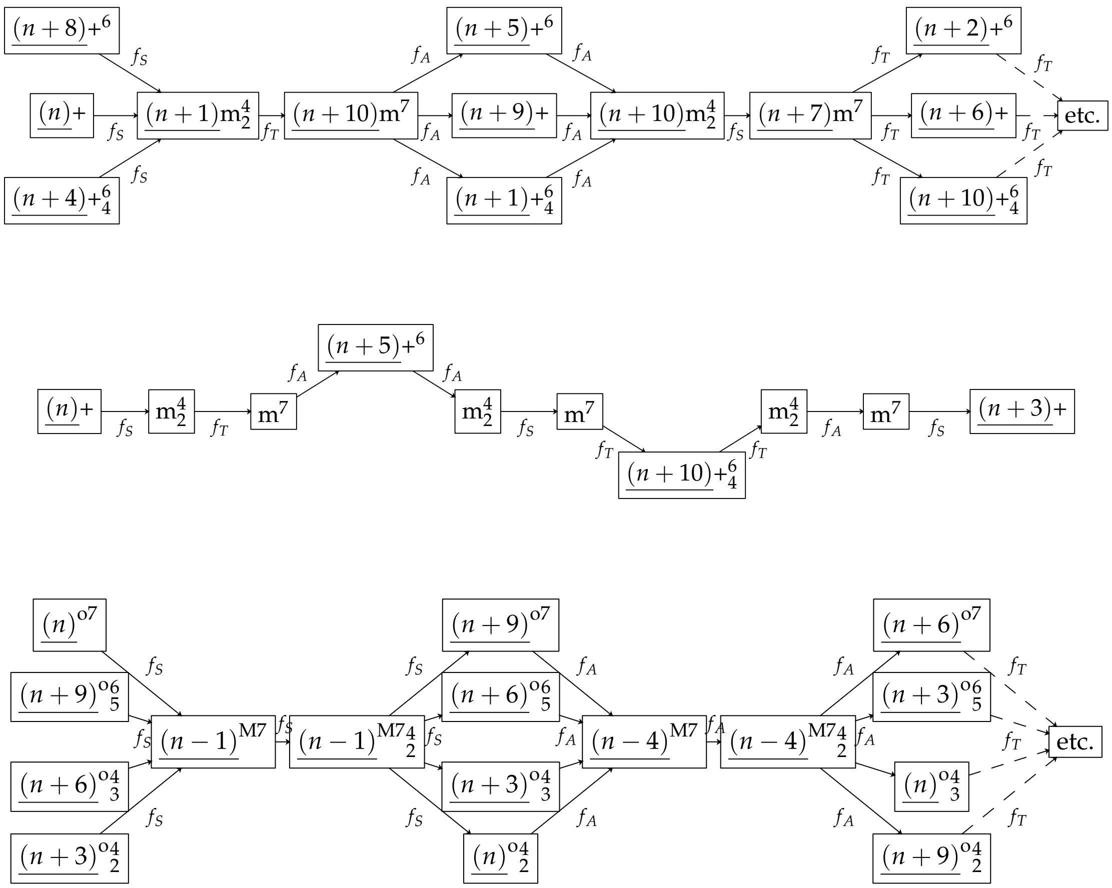

Figure 16: *From top to bottom: a network of augmented triads, root position minor-minor seventh chords, and third inversion minor-minor seventh chords demonstrating an extension after nine chords; a network of the same chord types as the previous one, demonstrating the enharmonic spellings of augmented triads; a network of fully diminished seventh chords, root position major-major seventh chords, and third inversion major-major seventh chords, demonstrating the enharmonic spellings of fully diminished seventh chords.*

Another possible generalized omnibus progression that can be extended every three chords involves fully diminished seventh chords and major-major seventh chords. Again, fully diminished seventh chords only have three aurally unique transpositions, but given the underlining technique, one can distinguish between all of the inversions of a fully diminished seventh chord, providing four possible generalized omnibus progressions. Similarly, those progressions could be arranged in such a way where the progression could be extended every $3n$ chords, $n \in \mathbb{N}$. This would involve re-interpretation of the roots of the fully diminished seventh chords every third chord. Suppose a generic fully diminished seventh chord has the following voicing: $\langle n, n+3, n+6, n+9 \rangle$ – the bottommost graphical network in Figure 16 shows the various enharmonic spellings every three chords..

Although these two progressions can be arranged in such a way that they mutate every $3n$ chords, the difference between these two sets of progressions and the set of progressions contained in Figure 11 is the fact that the progressions in Figure 11 have voice-leading *options* whereas these other two sets do not. Only one f_v for some $v \in \{S, A, T\}$ is possible for the progressions involving augmented triads and fully diminished seventh chords.

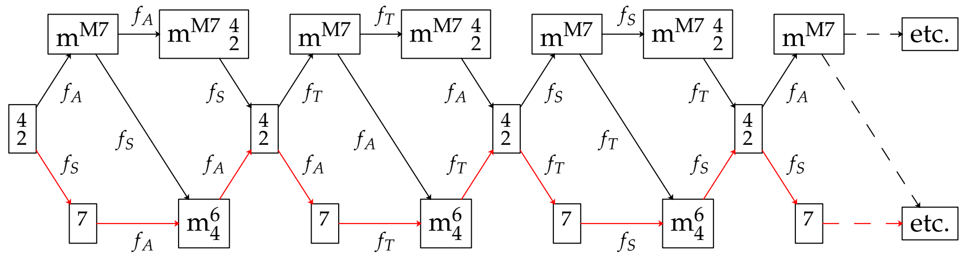

Figure 17: *Network of possible voice-leading paths with initial voicing* $\langle n+10, n+4, n+7, \underline{n} \rangle$*. Yellin's omnibus progression is outlined by the red path.*

The final generalized omnibus progression incorporates Yellin's omnibus progression and the unusual tertian sonority of minor-major seventh chords. The third inversion major-minor seventh chord has the same property as a root position half-diminished seventh chord in the sense that it can approach two different chords. A similar network to Figure 11 can be realized for initial pc segment $\langle n+10, n+4, n+7, \underline{n} \rangle$ in Figure 17. The three omnibus progressions that can be extended every three chords include:

- Yellin's omnibus progression,
- the progression including a third inversion major-minor seventh chord, a root position minor-major seventh chord, and a third inversion minor-major seventh chord, and
- the progression including a third inversion major-minor seventh chord, a root position minor-major seventh chord, and a second inversion minor triad.

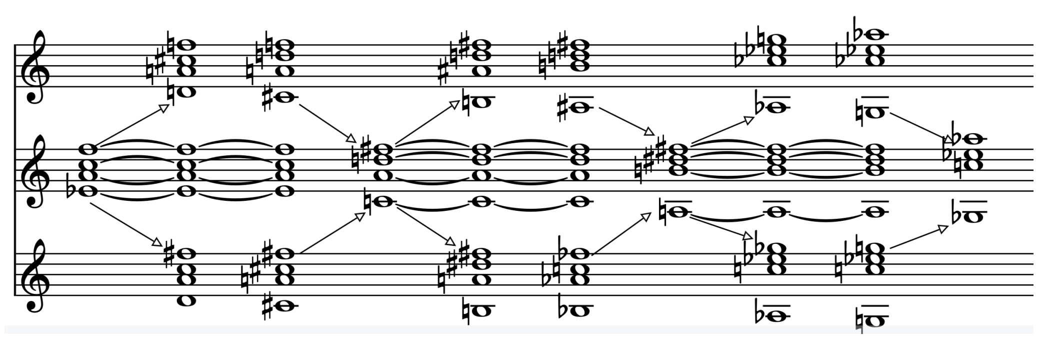

Figure 18: *Musical realization of Figure 17 with initial pc segment* $\langle 3, 9, 0, 5 \rangle$*.*

However, the third inversion minor-major seventh chord is another chord that has the property of being able to approach two different chords; it can approach a third inversion major-minor seventh chord or a second inversion major-minor seventh chord. Thus, the network in Figure 17 can be expanded to a tree diagram as seen in Figure 19 with a second inversion major-minor seventh chord beginning the tree, which has six unique sonorities.

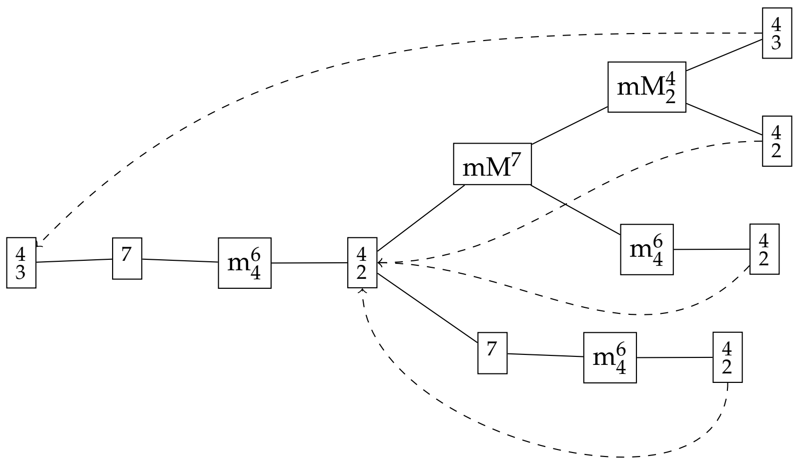

Figure 19: *Tree diagram where each dashed arrow indicates that the tree repeats itself at that node, and its descendants can be found by the dashed arrow's destination.*

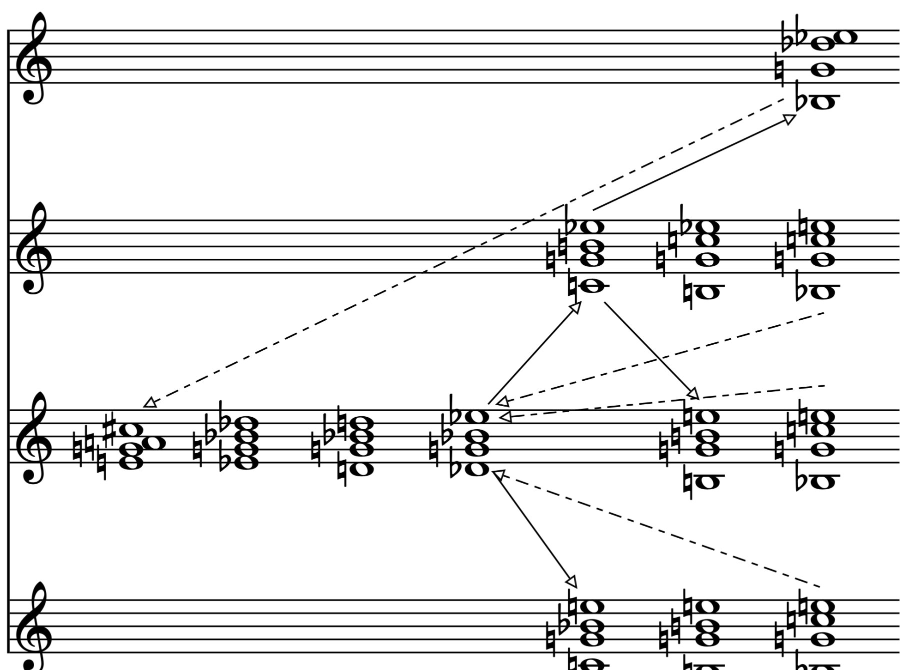

Figure 20: *Musical realization of Figure 19 with initial pc segment* $\langle 4, 7, 9, 1 \rangle$.

V. Current Problems and Future Research

Conjecture 1. *Given the definitions of triads/seventh chords and omnibus functions, a generalized omnibus progression must have at least three, but at most six, unique sonorities before an extension occurs.*

Conjecture 2. *For $n \in \mathbb{N}$, a generalized omnibus progression occurs precisely when there exists at least $3n$ compositions of f_v for some $v \in V$ such that each $f_v \in Q \cap \mathrm{Im}\, f_V$.*

The set R has $12^4 = 20,736$ distinct elements, and the set Q has $3,240$ distinct elements, but what is the size of Im f_V, especially given that for some $u, v \in V$, $f_u(\langle w_1, x_1, y_1, z_1 \rangle) = f_v(\langle w_2, x_2, y_2, z_2 \rangle)$? How many (and which) chords in Q are even approachable by an omnibus function? Are there connections to the realm of $\mathbb{Z}_7$, or, more generally, $\mathbb{Z}_n$? Because voice leading in contrary motion can occur in any scale system (diatonic, chromatic, microtonal, etc.), is there an analogous set of definitions and patterns to any given n? Finally, how does this proposed mathematical framework connect with other models of voice-leading, such as *OPTIC*-spaces, voice-leading spaces, contextual-inversion spaces, *Cube Dance* and *Power Towers*, and/or transformational music theory? (See [1], [6], [3], [5], and [4] for further exposition.)

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