A Chord's Third Nature, and How it Models and Generalizes Omnibus Progressions

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Abstract: *Richard Cohn and Dmitri Tymoczko have explored how certain chords, already endowed with a "first nature" characterized by an acoustic consonance relatively high for the chord's size, also possess a "second nature" that permits them to voice lead relatively smoothly to their pitch-class transpositions using contrary motion. I propose a "third nature" enjoyed by some chords that also enables smooth contrary motion to their pitch-class transpositions, but adds step inertia, harmonic variety, and the virtue of "going somewhere" to the list of compositional achievements. This perspective on a chord's "third nature" facilitates both a codification and generalization of the long version of the familiar omnibus progression, as demonstrated through analyses of music by Nikolai Medtner, Ferdinand Ries, Giuseppe Verdi, and especially Dmitri Klebanov, and the composition of a passage in the style of an etude by György Ligeti.*
Keywords*: harmonic consonance, voice leading smoothness, chordal evenness, omnibus progression, Klebanov.*
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1. Introduction

Richard Cohn [3] and Dmitri Tymoczko [14] have explored how certain chords, already endowed with a "first nature" characterized by an acoustic consonance relatively high for the chord's size, also possess a "second nature." Figure 1 generalizes and schematizes this second nature, visualizing a progression of two chords each of size n within a chromatic universe of size c, where $\frac{c}{n} \in \mathbb{Z}^+$, $c > 2$, and $n > 1$. A double-lined box encloses each chord, with the one on the left progressing as usual to the one on the right, although this diagram should also be understood to also account for a reversal of the two chords in time. Each circle represents a pitch class within each chord. To extend pitch-class equivalence throughout Figure 1, its reader should consider the top and bottom parts of the image as glued together, each arrow's transposition (**T**) as a pitch-class transposition, and negative subscripts as calculated modulo c. The dashed portions of the boxes

*I thank both Hayden Pyle, whose research on the omnibus progression inspired the current paper, and to Katrina L. Mitchell, who brought Mr. Pyle's research to my attention. I dedicate this paper to the memory of Robert Gauldin (1931–2025).

and arrows visually accommodate different sizes of chords, although, for any duplicate pitch locations within a chord that result from small-n realizations of Figure 1 (e.g. "Note 2" and "Note $n-1$" are the same when $n = 3$), only the pitch location at the top of the chord's box (i.e. with the variable) should be used.

Music theorists call each of the two chords of Figure 1 "nearly even" (NE), because one of its pitches, highlighted by the circle filled with gray, has been shifted via the dotted arrow up one chromatic step in pitch-class terms (that is, $\mathbf{T_1}$) from its position in a "perfectly even" (PE) chord, which is a chord completely generated by $\mathbf{T}_{\frac{c}{n}}$. For Figure 1 to account for a shift *down* a chromatic step, which equally produces a NE chord, all of the transpositional subscripts should be inverted, that is, multiplied by –1. I will henceforth assume that Figure 1 generalizes to both scenarios although it only displays one of them; an asterisk (*) signifies the relevance of the inverted Figure 1 when the need arises in later text. This shift of one pitch class by $\mathbf{T_1}$ changes exactly two of the chord-generating $\mathbf{T}_{\frac{c}{n}}$ transpositions of the PE chord to $\mathbf{T}_{\frac{c}{n}+1}$ and $\mathbf{T}_{\frac{c}{n}-1}$ in the NE chord.

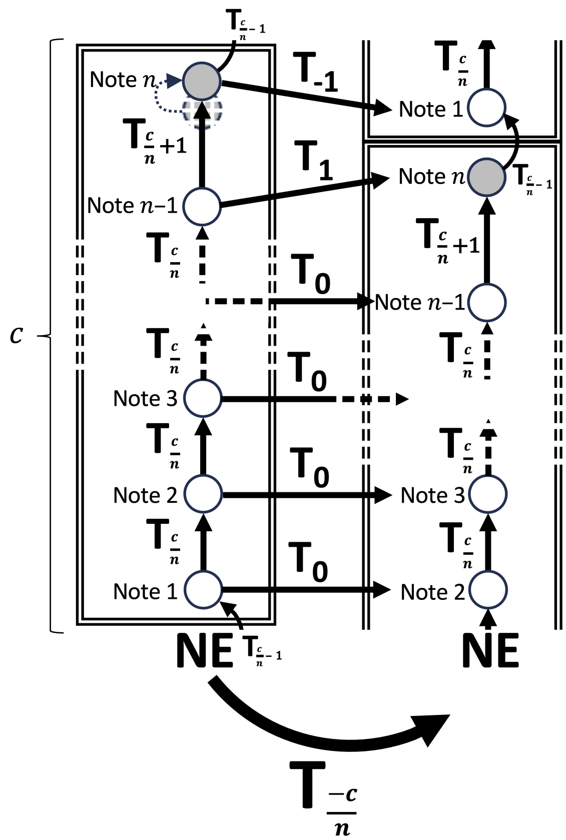

Figure 1: *Graph generalizing a progression demonstrating a nearly even (NE) chord's second nature.*

As shown with the large arrow at the bottom, the second NE chord is a $\mathbf{T}_{\frac{-c}{n}}$ transposition of the first NE chord. This has the net effect of reducing every chord-generating $\mathbf{T}_{\frac{c}{n}}$ transposition to $\mathbf{T_0}$, and the $\mathbf{T}_{\frac{c}{n}+1}$ and $\mathbf{T}_{\frac{c}{n}-1}$ transpositions to $\mathbf{T_1}$ and $\mathbf{T_{-1}}$, respectively. Therefore, such a $\mathbf{T}_{\frac{-c}{n}}$ or $\mathbf{T}_{\frac{c}{n}}$* transposition between two nearly-even chords of size n is thus capable of achieving what I will call for this generalized scenario "highly smooth voice leading," whereby no voice-leading motion exceeds a single chromatic step. However, this achievement must meet two conditions in the realization of this transposition. First, n voices must bijectively map the n pitches of the first chord to the pitches of the second chord with a cyclic permutation that undoes the cyclic permutation of the first chord's transpositional generation. Figure 1 renders this permutational undoing by shifting the second chord-box "down one note," in the opposite direction of the upward pointing arrows that generate each chord. The horizontal voice-leading arrows display this bijective voice leading. Second, reminded that each of Figure 1's transpositions only represent pitch-class transformations, each of these bijective pitch-class transpositions of $\mathbf{T_0}$, $\mathbf{T_1}$, and $\mathbf{T_{-1}}$ must be realized in register as a common tone (i.e. unison), one chromatic step up, and one chromatic step down, respectively, in order to realize the highly smooth voice-leading potential inherent in nearly-even chords. In this realization, the two moving voices also achieve contrary motion.

Setting c to 12, the size of the usual chromatic scale, Figures 2 and 3 offer pitch-class realizations of Figure 1 with $n = 3$ and $n = 4$, respectively, producing minor triads and half-diminished seventh chords as the nearly-even chord types. Figures 4 and 5 offer pitch-class realizations of the inverted* form of Figure 1 with $n = 3$ and $n = 4$, respectively, producing major triads and dominant seventh chords as the nearly-even chord types. Figures 6 and 7 then each begins with a highly smooth realization of the corresponding pitch-class progression of Figures 2 and 5, and continues by dovetailing the progression with another similar progression, iterating the same procedure several times. Cohn and others find these realizations suggestive, for they show how certain chords that already possess at least a fairly high degree of acoustical harmonic consonance—their "first nature"—can also express two particular part-writing desiderata—highly smooth voice leading and contrary motion—that are coveted by some common-practice styles, thus making these chords "overdetermined."

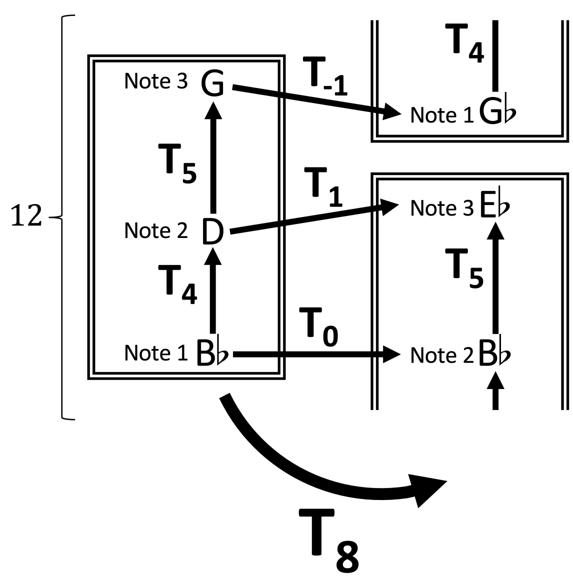

Figure 2: *One graphic, pitch-class realization of Figure 1 with $c = 12$ and $n = 3$.*

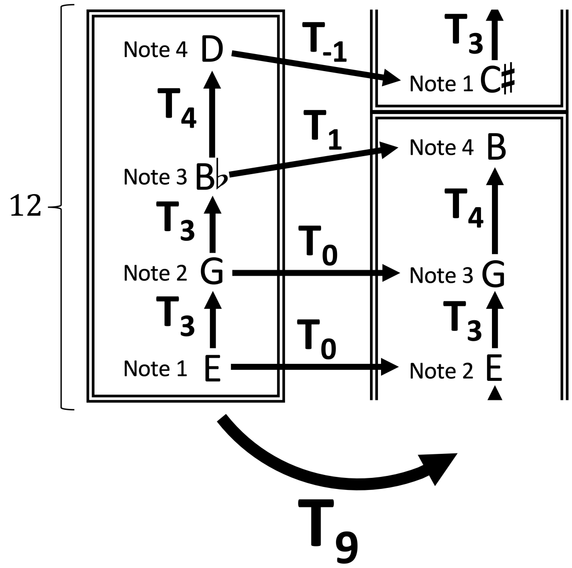

Figure 3: *One graphic, pitch-class realization of Figure 1 with* $c = 12$ *and* $n = 4$.

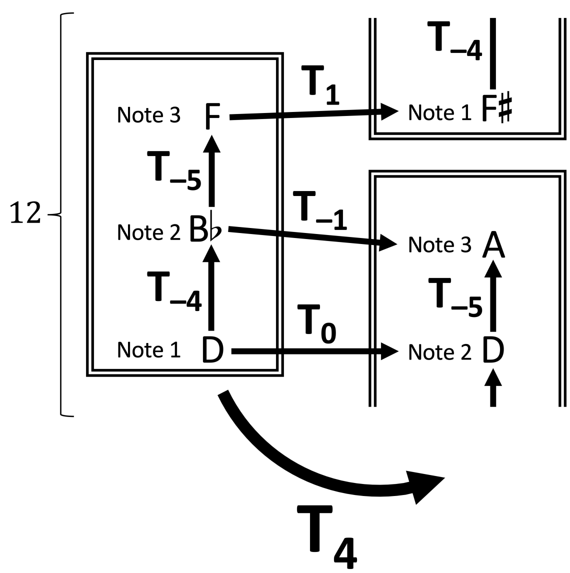

Figure 4: *One graphic, pitch-class realization of an inversion* of Figure 1 with* $c = 12$ *and* $n = 3$.

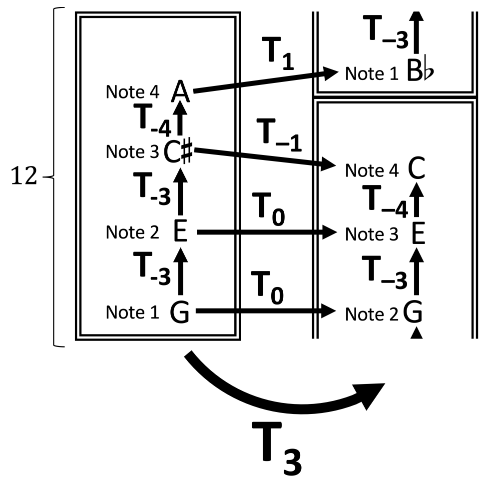

Figure 5: *One graphic, pitch-class realization of an inversion* of Figure 1 with* $c = 12$ *and* $n = 4$.

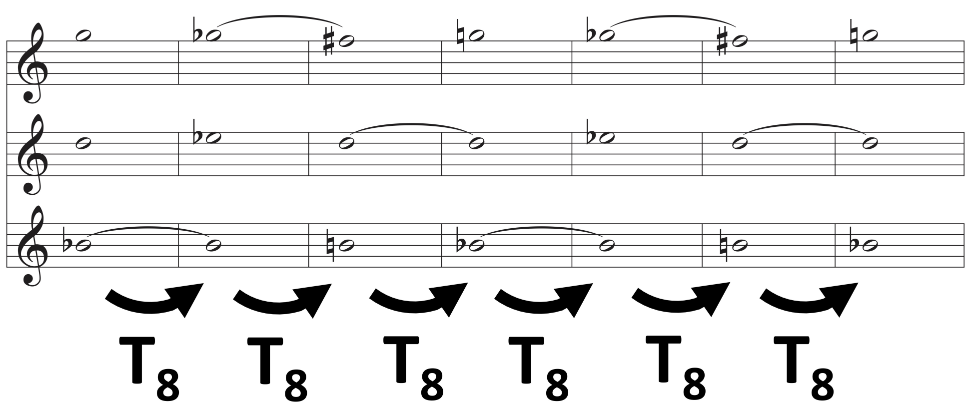

Figure 6: *One notated, registral realization of Figure 2, followed by five dovetailed iterations.*

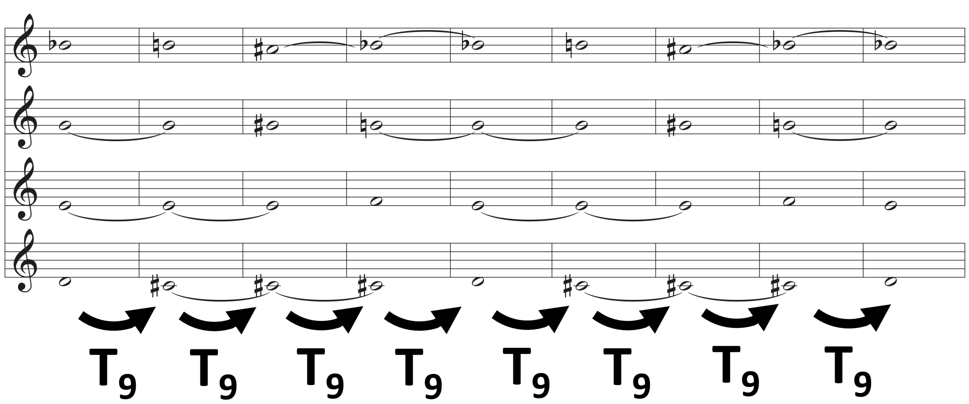

Figure 7: *One notated, registral realization of Figure 3, followed by seven dovetailed iterations*.*

While Cohn's discovery of a NE chord's second nature is significant, it would be premature to consider it as musical overdetermination's last word, for two reasons. First, common-practice desiderata are many [13], and the triad's second nature cannot, of course, satisfy them all. For one example, each of the individual lines in Figures 6 and 7 oscillate between two different pitches. This "slow trill" structure contravenes common-practice linear idioms, codified by both the prevalence of foreground linear progressions in Schenkerian analytical graphs and empirical music theory's step inertia [16] [17]: both structures model the common linear design where a step is followed by a step in the same direction. The oscillatory nature of the lines of Figures 4 and 5 further means that, despite multiple iterations, the music "goes nowhere": chords instead come back in their same registral voicings, and a broad sweep through pitch space is not possible without violating the smoothness that these chords' second natures afford. Furthermore, a realization of Figure 1 requires the immediate adjacency of chords of the same transpositional type. However, if a chord is defined simply as the persistence of a set of pitches over some span of time, and not instead as an abstracted set of pitches that emerges after some form of reduction, such as using only downbeat pitches or removing pitches like passing tones, at least some common-practice musical styles tend to use only infrequently an immediate adjacency of chords of the same transpositional type [5]. This puts a lot of music outside of the second nature's purview.

A second reason to continue research into musical overdetermination beyond a chord's second nature is that there exist multiple ways for a PE chord generated by $\mathbf{T}_{\frac{c}{n}}$ to undergo minimal alterations that translate into favored voice leading from one such altered chord to its $\mathbf{T}_{\frac{c}{n}}$* or $\mathbf{T}_{\frac{-c}{n}}$ transposition. This essay proposes one idea that merges the second reason with the aforementioned details regarding the first reason: the realization of a tweak of a PE chord, but different than that of Figure 1, that retains the same contrary motion and high level of smoothness displayed in Figures 6 and 7 but adds step inertia, harmonic variety, and the virtue of "going somewhere." Moreover, composers have employed one of these realizations since the eighteenth century and scholars have known the category of music to which this realization belongs well enough to give it a name: omnibus.

2. Near Cyclicity, a Chord's Third Nature, and the Omnibus

To produce a NE chord from a PE chord, a single pitch is shifted up or down* by a single chromatic step. But what if this pitch is not shifted but removed? The result is what I prefer to call a "nearly cyclic" (NC) chord.[1] More formally, a NC pitch-class set of size n is a subset of PE set of size $n + 1$. There is a one-to-one mapping from a NC chord to the PE chord that includes it. And what if a pitch in a PE chord was shifted up or down* not by a single chromatic step but by any number of chromatic steps? The result is what I prefer to call a "incremented nearly cyclic" (INC) chord. More formally, an INC pitch-class set of size n is a superset (including a multiset) of a NC set of size n–1. For now, I call the pitch class that is in an INC set but not in its NC subset "untethered," although I will switch to a different name after discussing Figure 8's relevance to the omnibus. This untethering permits a one-to-many mapping from a NC chord to the multiple INC chords that include it. INC chords can also be PE, NE, NC with one pitch class doubled, or none of these.

Figure 8 displays a progression of two INC chords, with each NC subset in a double box. The symbols of Figure 8's schematic have the same meaning and level of generalization as those in Figure 1 described in the first paragraph of this essay, with one exception: courtesy of inversional symmetry of the NC subset and the variability of the untethered pitch class, one does not need to invert transpositional subscripts in order to account for inverted INC chords in Figure 8 as one

[1]"Nearly cyclic" is not equivalent to Dmitri Tymoczko's "near interval cycle." [15]

must* in Figure 1. The NC subset's telltale sign is the transposition $\mathbf{T}_{\frac{2c}{n}}$, which spans the gap left by the untethering of a pitch class in a PE chord generated by $\mathbf{T}_{\frac{c}{n}}$. This untethered pitch, shaded in gray, is relocated by $\mathbf{T}_{\partial}$ to make the INC chord. The single deletion to make a NC subset is much like the single chromatic-step shift to make a NE chord, in that both alterations impose a minimal tweak of a PE chord. However, whereas the NE chord's tweak manifests in the PE chord's otherwise uniform transpositional network as a perturbation by a single chromatic step ($\mathbf{T}_{\frac{c}{n}} \to \mathbf{T}_{\frac{c}{n}+1}$ and $\mathbf{T}_{\frac{c}{n}-1}$), the NC chord's tweak manifests as a perturbation by a single step in the PE chord-as-scale ($\mathbf{T}_{\frac{c}{n}} \to \mathbf{T}_{\frac{2c}{n}}$).

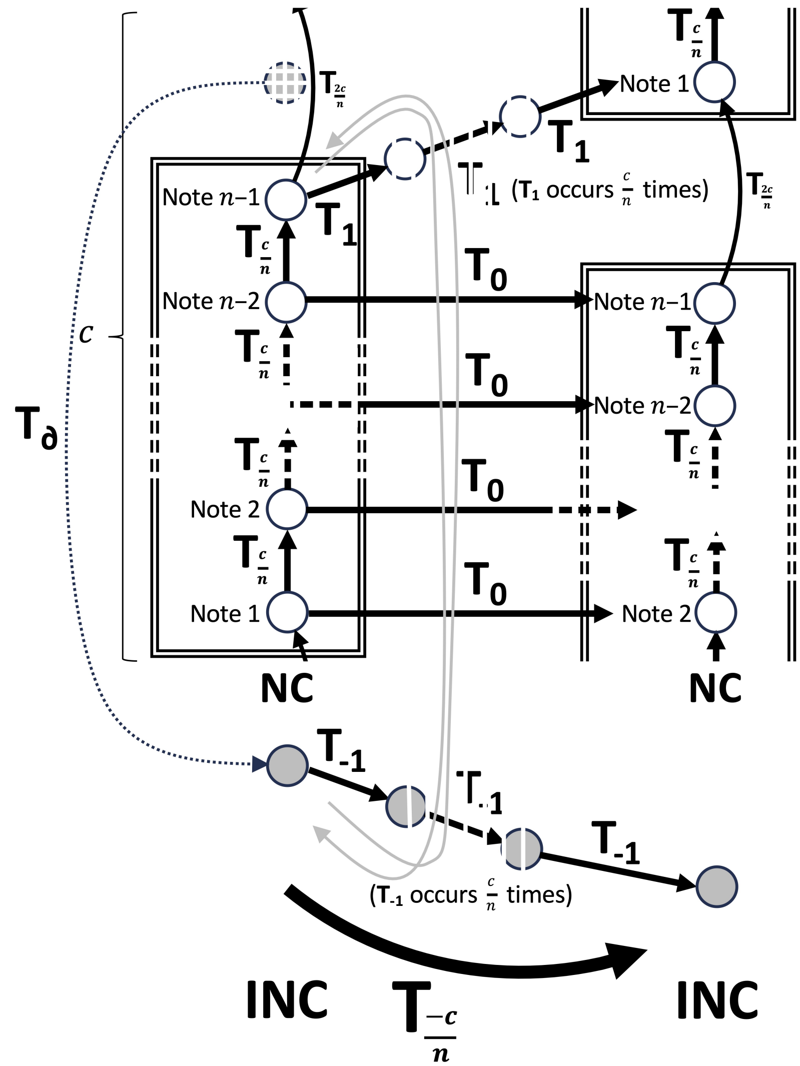

Figure 8: *Graph generalizing a progression demonstrating an incremented nearly cyclic (INC) chord's third nature.*

When the NC subset undergoes a $\mathbf{T}_{\frac{-c}{n}}$ transposition, its transpositions $\mathbf{T}_{\frac{c}{n}}$ reduce to $\mathbf{T_0}$ as before, but its single $\mathbf{T}_{\frac{2c}{n}}$ reduces to $\mathbf{T}_{\frac{c}{n}}$. Assuming that $\mathbf{T_1}$ does not generate the NC chord (i.e. assuming that $c \neq n$), this single non-common-tone voice-leading interval of $\mathbf{T}_{\frac{c}{n}}$ is not a single chromatic step and thus lacks the smoothness of the voice-leading intervals enabled by second-nature progressions. But $\mathbf{T}_{\frac{c}{n}}$ can be smoothed out, embellished as $\mathbf{T_1}$ executed $\frac{c}{n}$ times. The result of this embellishment is a line that not only moves maximally smoothly just as in the lines of all second-nature progressions, but also exhibits step inertia and produces harmonic variety as the moving voice passes over the other pitches of the NC subset held as common tones (again, assuming $c \neq n$). But what of the second nature's potential for contrary motion? This potential is built directly into an INC chord's structure: as shown near the bottom of Figure 8, one simply assigns the untethered pitch class to a voice that moves in lock step with the embellishing voice, but in the opposite direction: $\mathbf{T_{-1}}$ executed $\frac{c}{n}$ times. This line's overall transposition is $\mathbf{T}_{\frac{-c}{n}}$, which is exactly the transposition of the NC subset. This means that, as already allowed, this added voice may start anywhere: $0 \leq \partial \leq c-1$. In fact, other voices may join the added voice in parallel motion while still supporting a transposition of the initial chord.

Figure 8 does not show this last option in order to make more clear the correspondence between Cohn's observation and mine, a correspondence that is one of this essay's primary discoveries: in a chromatic universe of size c, both a NE chord of size n and a INC chord of size n can progress to its $\mathbf{T}^*_{\frac{c}{n}}$ or $\mathbf{T}_{-\frac{c}{n}}$ by moving two voices in contrary, chromatic-stepwise motion and keeping all of the other pitches fixed as common tones. Assuming $\frac{c}{n} > 1$, the only difference in these two scenarios is that the two moving voices execute a chromatic step multiple times in succession when connecting INC chords, but only once when connecting NE chords. Since some INC chords are also NE chords, which possess a "second nature," and some NE chords also enjoy the "first nature" of acoustical consonance, it seems proper to designate the potential demonstrated in Figure 8 as revealing a INC chord's "third nature."

Figure 9 offers a pitch-class realization of Figure 8 with $c = 12$ and $n = 4$, which makes the diminished triad as the type of NC subset, here placed with close spacing in the treble clef. With $\partial = -1$ (equivalently, $\partial = 11$), the INC chord type is the dominant-seventh chord. Figure 9 begins with a highly smooth realization of the correlating pitch-class progression of Figure 8, making the line of untethered pitch classes the bass line, and continues by dovetailing the progression with another like progression, iterating the same procedure several times. This music matches what Robert Wason calls an "extended omnibus" [16] and what Paula Telesco calls a "omnibus cycle" [13]. These scholars append these qualifiers because, as coined by Victor Fell Yellin [17], the default term "omnibus" signifies some transposition of the shorter five-chord progression offered in Figure 10. Figure 9's longer version comes about through what Yellin calls "mutations" that overlap the first four of the five chords with itself at $\mathbf{T_9}$. This essay adopts a position that—at least temporarily—flips this default, postulating Figure 8 as the omnibus's structural source, and suggesting that music like Figure 10 comes about by proceeding halfway through Figure 8 but then backtracking through the same pitch classes with a swapping of the two moving voices' lines, shown with two gray arrows in Figure 8. Therefore, this essay, given its focus on a chord's third nature, concerns itself with what I will neutrally call the "long" omnibus (thus designating Figure 10 as an example of the "short" omnibus) and the transpositional design and iterability of the long omnibus's embellished two-chord progression as its backbone. Furthermore, as the highly generalized Figure 8 contains several adjustable parts that can move while maintaining the integrity of this transpositional backbone, this focus also facilitates relating omnibus progressions like those of Figures 9 and 10—which, when $c = 12$, $n = 4$, and $\partial = -1$, I will call "standard"—to other similar but non-standard structures found in both musical theory and practice, as the following section demonstrates.

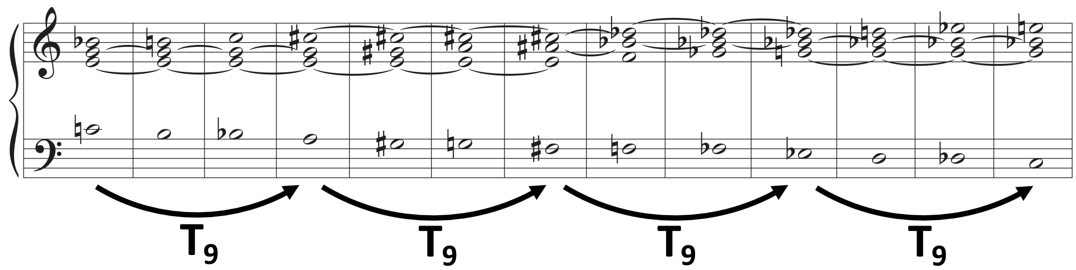

Figure 9: *One notated, registral realization of Figure 8 ($c = 12$, $n = 4$, $\partial = -1$), followed by three dovetailed iterations, creating the standard "long" omnibus progression.*

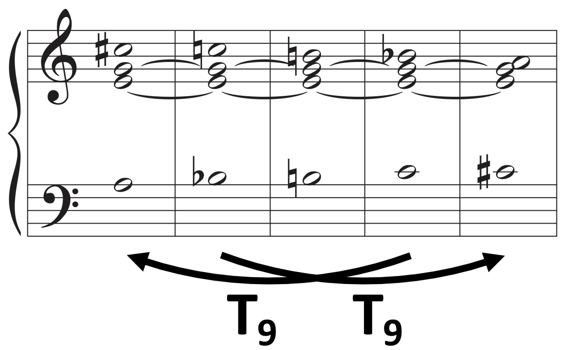

Figure 10: *A "short" omnibus realization of Figure 8.*

One detail from the previous paragraph warrants a quick sidebar before concluding this section. The line of untethered pitch classes certainly does not have to be realized as the bass line, but in most realizations of Figure 8 in the literature—in the standard omnibus progression, in all of the examples in this essay, and beyond—it is. Such an assignment of the untethered line to the bass line comports with certain common-practice conventions, especially as Figure 8 iterates. In this extended scenario, the untethered line continuously moves in a beeline, covering the most registral ground of all of the voices, a degree of latitude that the bass line typically enjoys more than upper voices, especially inner voices. Furthermore, the definitions of many harmonically-based tonal idioms—cadences, non-cadences, embellishing chords, augmented sixths, Neapolitan sixths, and so forth—tolerate voice permutations that do not involve the bass line much more than permutations that do. Assigning the untethered line to the bass line means that what does and does not return with each iteration of Figure 8 corresponds with this level of tolerance. Therefore, henceforth I use the word "bass" as a stand-in for the term "untethered" when referring to the gray pitches in Figure 8, as already reflected with their position at the bottom of the image.

3. Generalizing the Omnibus

Figure 8 furnishes four independent levels of generalization: three of them are represented by the variables ∂, n, and c. Each of the first, third and fourth sections below focuses on one of these levels of generalization, respectively, that are already pre-installed into Figure 8; the second section focuses on a level of generalization not explicitly diagrammed in Figure 8.

3.1 Changing the Bass Note

One level of generalization concerns ∂, the variable that defines the relationship between the INC chord and its NC subset. ∂ in Figure 8 can range from 0 to c–1, including any ∂ that results in a doubling up of a pitch class. Figure 11 offers a glimpse into this variability, using the same three upper voices from Figure 9 keeping $c = 12$ and $n = 4$, but proposing twelve different bass lines, each labeled by ∂, each as the line with the pitch classes outside of the NC chord's generated subset. (This array of options also accounts for the retrograde and any transposition of any of these twelve options.) The standard long omnibus, with its dominant-seventh chords and fifth-doubled minor triads, uses a ∂ of –1. A ∂ of 1 yields the inverted form of the omnibus, what Joti Rockwell calls the "Ziehn Inverted Omnibus," with its half-diminished-seventh chords and root-doubled major triads [11] [18].

There exist ten other solutions. The solution with ∂ of 0 creates fully-diminished-seventh chords and major seventh chords. It also provides a backdrop for re-examining a transitional passage from the second piece of *Forgotten Melodies* op. 39 (1919–20) by the Russian composer and pianist Nikolai Medtner analyzed by Hon Ki Cheung [3]. Figure 12 reproduces this passage, which links the end of the opening theme in F minor to the beginning of a second theme also in F minor. Preceding this music, mm. 21–28 prolong a V_7 chord in F minor with tortuous chromatic embellishments but clearly expressive of the notated 3/4 meter. The pickup to m. 29 begins a series of ten six-triplet-eighth broken-chord figures that cut against the triple meter with a half-note pulse. The first of these wave-like arpeggiations installs an A♭ at its crest, placing this note on a downbeat immediately followed by G a step below. The preceding prolongation of the dominant of F minor plus non-chord conventions recommends this Ab as a non-chord tone metrically displacing the first arpeggiation's fully-diminished-seventh chord of $E^{\circ 7}$. These crested appoggiaturas continue in parallel fashion through m. 28, reducing to the ten chords as beamed together in Figure 12, and homophonically presented and enumerated in Figure 13. Cheung's reduction breaks from this parallelism at the final chord, which she interprets as an undisplaced D half-diminished-seventh chord.

As reinforced with brackets below Figure 13, the ten chords of this passage gather into three successive groups of three, three, and four chords each, not only because of the upward leaps in the top part but also because of the harmonic content: the first and last groups each uses only fully-diminished seventh and major seventh chords, whereas the second group includes neither of these chord types, opting instead for a half-diminished seventh chord, augmented triad with doubled pitch, and a minor seventh chord. The third and final group realizes a retrograde of Figure 8 with c as 12, n as 4, ∂ as 0, and the two NC subsets as F-A♭-B and D-F-A♭, and thus expresses the third nature of the INC of $B^{\circ 7}$ with a $\mathbf{T_3}$ transposition from Chord 7 to Chord 10 (a retrograde of the $\mathbf{T_9}$ transposition of Figure 8's configuration, as the bass is in ascent instead of descent). This realization can be construed as the payment of a promissory note issued at the beginning of the passage. With only three chords, the first group falls one chord short of realizing the third nature of its initial INC of $E^{\circ 7}$; Figure 14 shows this hypothetical continuation.

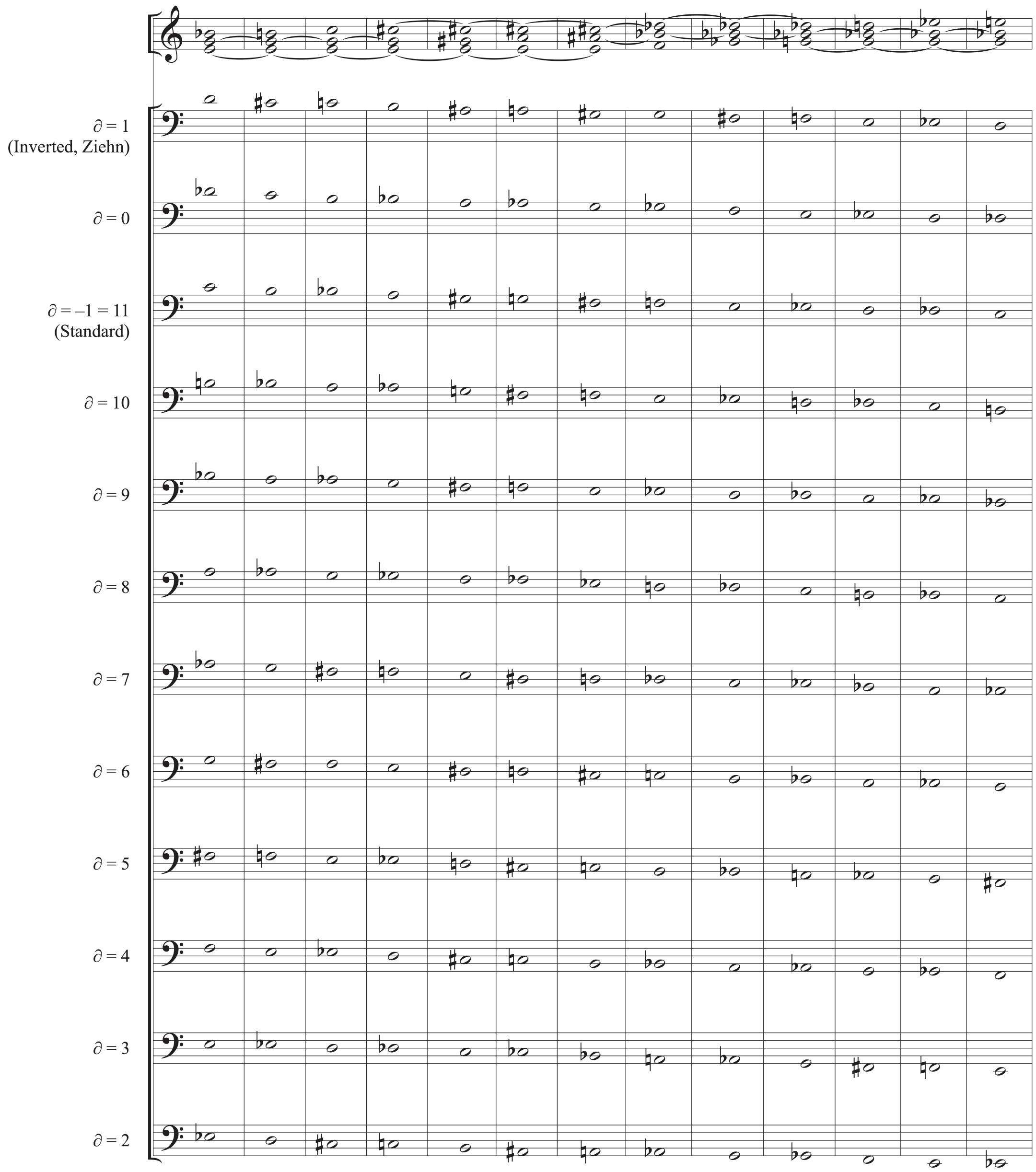

Figure 11: *The progression of NC triads from Figure 9 above twelve possible bass lines* ($0 \leq \partial \leq 11$).

Figure 12: *Nikolai Medtner,* Forgotten Melodies *op. 39 no. 2, mm. 25–41, with annotations.*

Figure 13: *Harmonic reduction of Medtner*, Forgotten Melodies *op. 39 no. 2, mm. 28–35, and relationship of chords to the relevant portion of Figure 11.*

Figure 14: *Reduction of hypothetical continuation of Medtner*, Forgotten Melodies *op. 39 no. 2, mm. 28–31.*

3.2 Permuting the Voice Leading

The visual layout of Figure 8's music emphasizes its first and last pitch events as primarily belonging to chords and its pitch events in between as primarily belong to voices; music, either notated or performed, in which the common tones (usually in inner voices) lack fresh onsets can support a similar impression. However, the realizations of Figure 8 in Figures 9 and 10 demonstrates that one can conceive of and manifest the medial events as belonging to chords just as much as the first and last events, resulting in a total of $\frac{c}{n} + 1$ chords that can be numbered Chord 0, Chord 1, and so forth to Chord $\frac{c}{n}$. Of these chords, only the initial Chord 0 and final Chord $\frac{c}{n}$ are necessarily INC chords. However, the medial chords still possess a third nature, but one that may require a realization that transcends the design of Figure 8. When the last chord of a realization of Figure 8 becomes the first chord of a later realization of Figure 8—what I have been calling a dovetailed iteration—it is not only that Chord 0 of the second realization is a $\mathbf{T}_{\frac{-c}{n}}$ transposition of Chord 0 of the first realization, and that Chord $\frac{c}{n}$ of the second realization is a $\mathbf{T}_{\frac{-c}{n}}$ transposition of Chord $\frac{c}{n}$ of the first realization, but also that every Chord m ($m \in \mathbb{Z}^+, 0 \leq m \leq c/n$) of the second realization is a $\mathbf{T}_{\frac{-c}{n}}$ transposition of Chord m of the first realization. Put another way, the $\mathbf{T_9}$ transformations in Figure 9 apply from *any* chord to the chord three measures later, not just to the chords in the first, fourth, seventh, and tenth measures as shown. This means that any chord—initial, medial, final—that ensues from a realization of Figure 8 has a third nature, but, during this chord's journey to its $\mathbf{T}_{\frac{c}{n}}$ transposition, *more than two voices may change pitch*, although no more than two voices will each move by a chromatic step at any one time, and always in contrary motion.

For example, in the standard long omnibus progression of Figure 9, the four-chord progression from C7 in m. 1 to A7 in m. 4 is a realization of Figure 8, such that $c = 12$, $n = 4$, $\partial = -1$, and the first bass pitch class is C. This four-chord progression involves the soprano voice ascending through three chromatic steps and the bass voice simultaneously descending through three chromatic steps, while the two inner voices do not change pitch. The extraction from Figure 9 of these four chords as a formal unit recommends the fifth-doubled second-inversion Em in m. 2 as a passing chord that embellishes and smooths out the $\mathbf{T_9}$ transposition from C7 to A7; a similar recommendation would apply to the similarly configured C♯m in m. 5 as a passing chord from A7 to F♯7 in m. 7 of Figure 9. However, what if the four-chord progression from Em in m. 2 to C♯m in m. 5 were extracted as a formal unit instead, reversing figure and ground? Every aspect about Figure 8 would still pertain—exactly two voices move at a time with contrary and chromatic-stepwise motion—with two exceptions. First, the bookending chords are no longer INC. Second, *three* different voices move *at some point* during the transit from m. 2 to m. 5: only the tenor stays put (on the pitch E4 in Figure 9's realization) during this time.

One could express the ordering of voice motions in the upper voices (that is, the voices other than the untethered bass voice) of mm. 1–4 as S (B♭→B), S (B→C), S (C→C♯) or S-S-S for short, and the same of mm. 2–5 as S (B→C), S (C→C♯), A (G→G♯) or S-S-A for short, with S for soprano and A for alto. The ordering S-S-A has something S-S-S (or A-A-A, and so forth) does not: the potential to be permuted into a different sequence—S-A-S or A-S-S—that preserves the omnibus's core voice-leading features while modifying some of its harmonic content.[2] Figure 15 shows a realization of each of these two permuted upper-voice progressions, below the omnibus's original, which has been transposed and spelled to correspond with an upcoming analytical subject. The annotations showing each upper-voice chord's harmonic type indicate that the NC diminished-triad subset, the non-untethered portion of the omnibus's dominant-seventh INC, has been replaced with a PE augmented-triad subset; Cohn [7] would say that these two subsets are in the same sum class. In fact, the S-A-S and A-S-S progressions map one-to-one to the two paths, up to transposition, from one augmented triad to an adjacent augmented triad through the Cube Dance graph that Cohn features in his research. Of course, to complete the omnibus-like music, a fourth line placed in one of the twelve possible positions moves continuously in the opposite direction of the upper voices' trajectory.

This voice-leading variation on Figure 8's omnibus model can be shown to undergird a transitional passage that first appears early in the first movement of the String Quartet No. 4 (1946) by the Ukrainian composer and violinist Dmitri Klebanov (1907–1987).[3] An examination of the music surrounding this passage supplies an important backstory for its analysis. Klebanov dedicated his fourth string quartet to the memory of Mykola Leontovych (1877–1921), a Ukrainian

[2]This permutation bears a resemblance to a permutation Tymoczko uses in [15].

[3]Klebanov reuses this passage later in the movement in mm. 64–71, transposed down a whole step.

composer best known for his arrangement of the traditional Ukrainian folk song "Shchedryk," which is known to English speakers as the Christmas-affiliated "Carol of the Bells." Lasting 36 measures of 3/4, the quartet's *Allegro moderato* introduction delivers a straightforward instrumental rendition of one verse of Leontovych's arrangement in D minor. Figure 16 provides the first twelve measures of the quartet, which begins with a F-E-F-D motive (labeled x) and its repetition as an ostinato. A recurring idea in Leontovych's arrangement, common when projecting material around sustained pedals or motivic ostinati, is voice exchange. The first of many instances of such is shown with diagonal lines in Figure 16: compared to the second violin's D-C-B♭-A entry in m. 5, the viola's four-note entry in m. 9 moves the B♭-A to the beginning and the D to the end. Figure 17 spotlights three more voice exchanges in the 36-measure introduction; each assigns the Shchedryk ostinato to a different instrument.

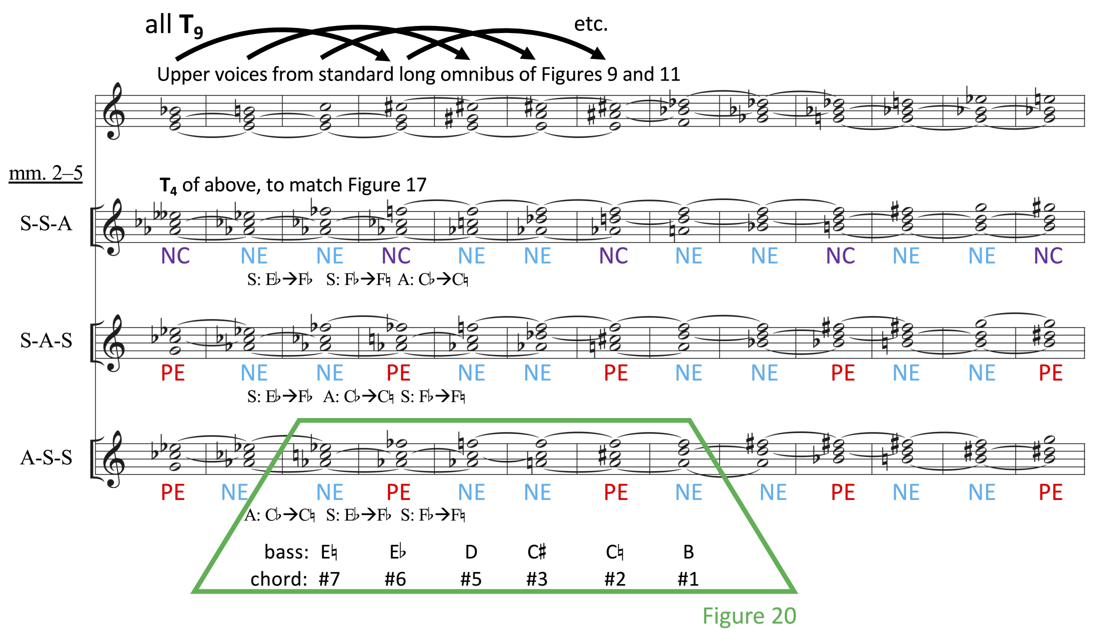

Figure 15: *Two voice-leading permutation variations on the standard long omnibus, with harmonic categories in colored text, and relationship of one part of one of the variations to the music of Figure 20.*

A new D-minor theme transcribed in Figure 18 follows this introduction, whose opening motive varies some of the intervallic, accentual, and durational content of the Shchedryk ostinato's motive while preserving its down-a-second, up-a second, down-a-third diatonic shape; Figure 18 labels this varied motive as x'.[4] The *Allegro* 12/8 tempo and meter translates the Shchedryk's four-measure hypermeter and one-measure harmonic rhythm into a four-beat meter and one-beat harmonic rhythm. Six measures into this theme, a highly smooth G7→B♭7 fragment of the standard omnibus progression in viola and cello concludes a two-measure idea, provided in Figure 19, in which the two violins share in the manner of a hocket a line that passes back and forth between D and F, then swap places in the next measure through an exchange more substantial than the preceding voice exchanges. This immediately precedes the four-measure transitional passage under scrutiny, which begins the excerpt in Figure 20. Immediately afterwards, and also

[4]The $\hat{5}$-♯$\hat{4}$-$\hat{5}$-♭$\hat{3}$ variant here, although short and not uncommon especially in Russian music, also matches the scale degrees that open Leontovych's final composition "Smert′" (Death).

shown in Figure 20, the music with its stepwise descending bass line passes through nine of the twelve chords of the standard long omnibus beginning with a root-position G7 and ending with a root-position B♭7 resolving to a second-inversion Dm triad that elides with the return of the D-minor theme.

The four-measure transitional passage (Figure 20, mm. 43–46) comprises a measure and its transposition up an octave, followed by these two measures transposed up a minor third. Downbeat position recommends reducing these four measures to two chords: $\mathrm{B}^{\varnothing 7}$ for mm. 43–44 and $\mathrm{D}^{\varnothing 7}$ for mm. 45–46, as symbolized with the slurs below the chord symbols in Figure 20. Transformation annotations added to Figures 19 and 20 indicate that progression of these two half-diminished chords relates to the G7→B♭7 progression that immediately precedes this passage on the foreground and immediately follows this passage in the middleground: $\mathrm{B}^{\varnothing 7} \rightarrow \mathrm{D}^{\varnothing 7}$ is the retrograde-inversion of G7→B♭7 around D and F, the two pitch classes emphasized in the Shchedryk ostinato. Put another way, these four seventh chords are the only members of set-class [0258] (Cohn's "Tristan genus" [7]) that contain D and F. This reduction to two four-note INC chords separated by a minor third ($\mathbf{T}_{\frac{12 \text{ notes in the chromatic scale}}{4 \text{ notes in the INC chord}}}$) also resonates with all of the omnibus activity before, after, and even during this passage. In each of the transitional passage's four measures, the top voice continues the hocket-like distribution of notes between the two violins begun two measures earlier, but now descending chromatically at a steady quarter-note rate, embellished by the Shchedryk ostinato flattened into a down-up-down contour by semitones, labeled as *x*″ in Figure 20. The cello rises at the same quarter-note rate and the same chromatic pace, all while the viola sustains two pitch classes.

Figure 16: *Dmitri Klebanov, String Quartet No. 4, I, mm. 1–12, with motivic and voice-exchange annotations.*

But as much as the voice-leading structure of mm. 43–46 resembles the omnibus, and although

$B^{\varnothing 7} \rightarrow D^{\varnothing 7}$ could be easily realized with a part of the inverted (Ziehn) omnibus, mm. 43–46 cannot be generated by Figure 8. Nonetheless, one could relate the generalized omnibus idea of Figure 8 to Klebanov's music in a couple of ways. First, Klebanov's Chords #1–4 in mm. 43–44 match Medtner Chords #7–10 analyzed in the previous section (Figures 12 and 13), down to the same pitch classes and voice assignments, with one exception: Medtner sustains an A flat instead of an A natural. This type of substitution is common among analyses of occurrence of the standard omnibus in the repertoire when a fully-diminished seventh stands in for a dominant-seventh chord.

Second, one may interpret most of Klebanov's passage as a A-S-S permutation of the voice-leading motions connecting the NC subsets in an inverted-omnibus progression ($c = 12$, $n = 4$, $\partial = 1$), already adumbrated on the bottom of Figure 15 and fleshed out in Figure 21. Of the three Ziehn-inverted omnibus progressions, this is the only one produced by inverting the standard long omnibus that immediately follows this passage around D and F, the Shchedryk ostinato's prolonged pitch classes. Assuming Klebanov's choice of $B^{\varnothing 7} \rightarrow D^{\varnothing 7}$ for the foundation of these four measures as fixed, there are multiple ways in which this permutational variation connects to Leontovych's music. One way is more subtle and theoretical: this particular cyclic permutation, in which an ordered voice-leading pair <D→C♯, C→♯C> at the end is shifted to the beginning, displacing the beginning A→A♭ to the end, is the exact same permutation in mm. 9–12 that relates Leontovych's first linear harmonization of the Shchedryk ostinato to the second, in which the concluding B♭ and A is moved to the beginning and the initiating D is moved to the end, shown in green in Figure 21. Even the three pitch classes or voice leadings involved in both permutations form the same (015) set type: {A, B♭, D} for mm. 9–12 and {A→A♭, C♯→C, D→C♯} for mm. 43 and 44.

Figure 17: *Examples of voice exchanges in Dmitri Klebanov, String Quartet No. 4, I (a) mm. 13–16 (b) mm. 21–24 (c) mm. 29–32 (part with ostinato in boldface).*

Figure 18: *Dmitri Klebanov, String Quartet No. 4, I, mm. 37–8, with motivic annotations.*

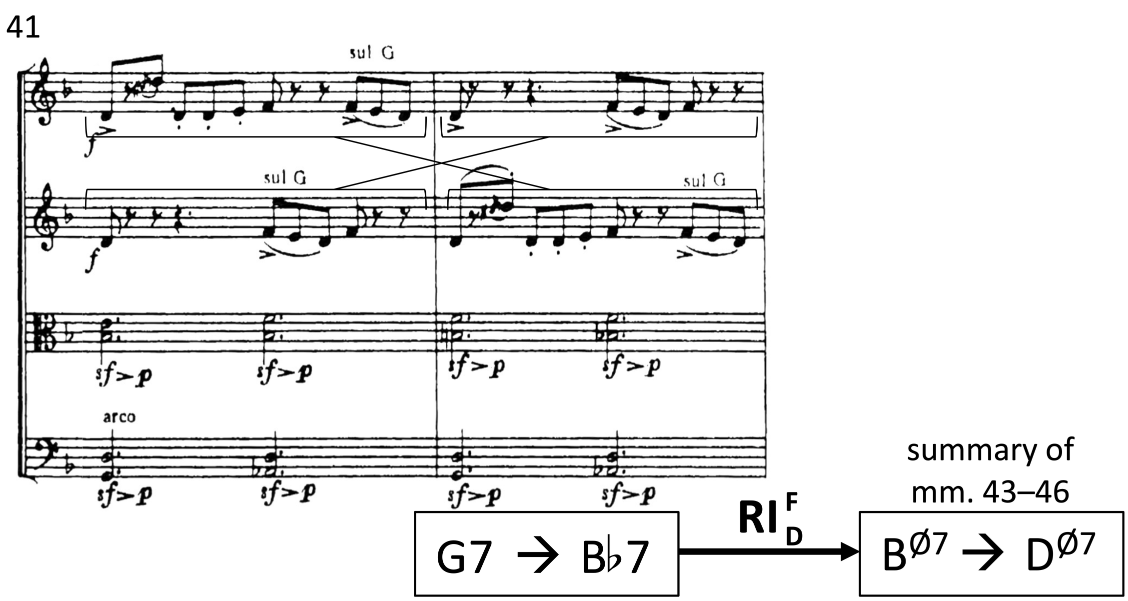

Figure 19: *Dmitri Klebanov, String Quartet No. 4, I, mm. 41–2, with motivic exchange and transformational annotations.*

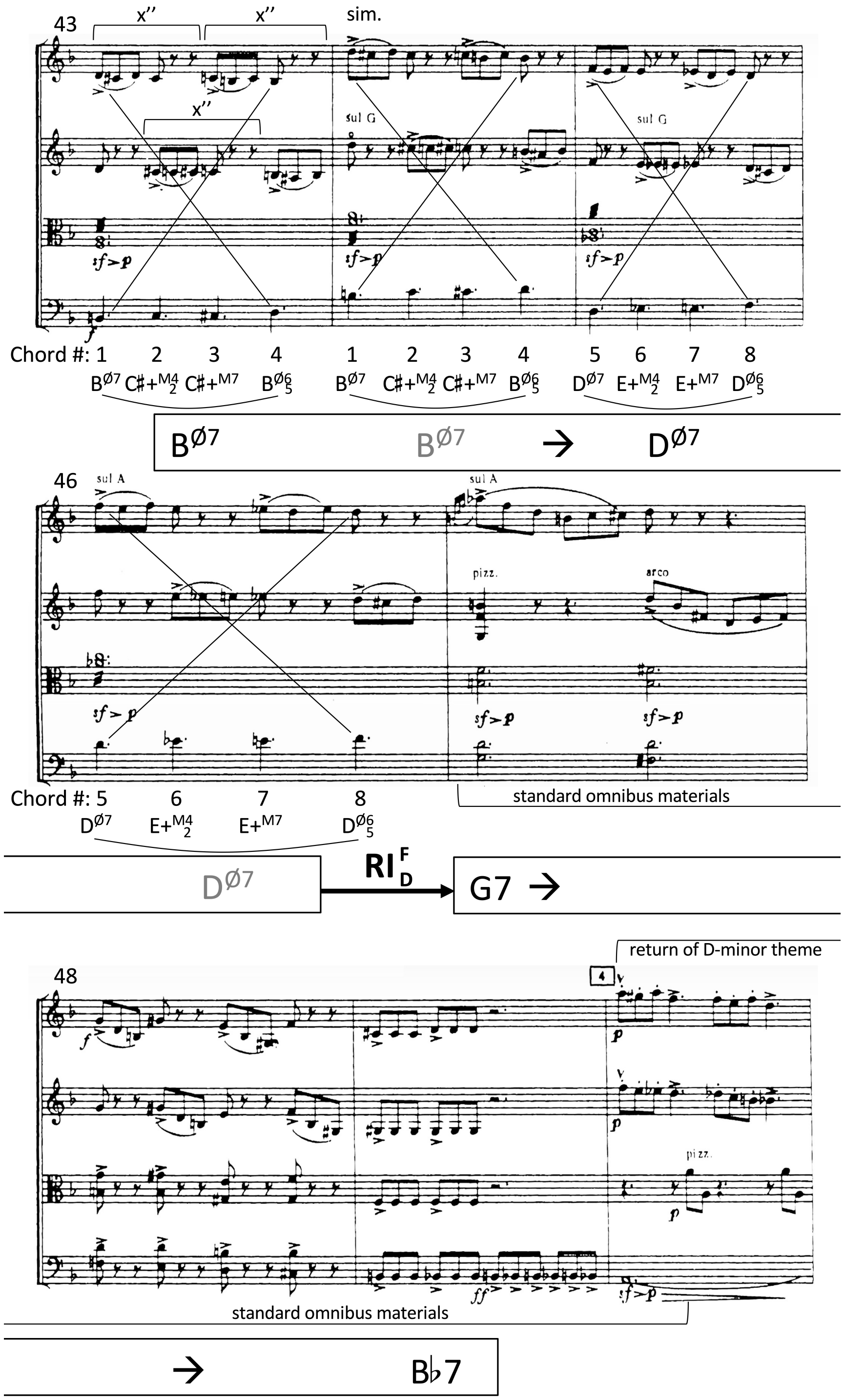

Figure 20: *Dmitri Klebanov, String Quartet No. 4, I, mm. 43–50, with chord enumeration (used in Figure 15) and other annotations.*

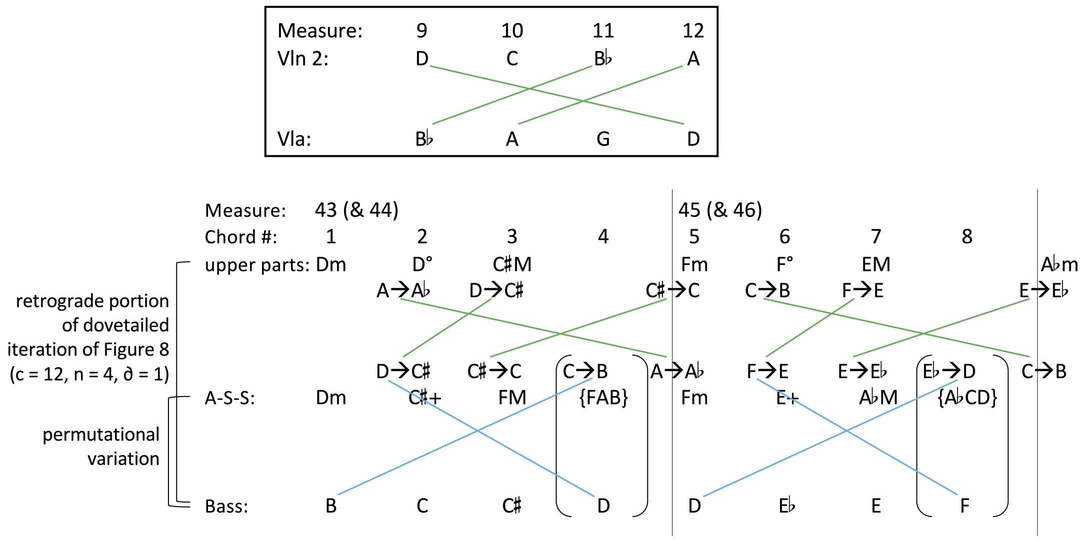

Figure 21: *Comparison between a permutation variation of a realization of Figure 8 and Dmitri Klebanov, String Quartet No. 4, I, mm. 43–46, and a comparison between the permutation in mm. 43–46 and a permutation earlier in the quartet.*

Another way is more conspicuous and practical. Unlike Figure 1, Figure 8 is latently metric. Placing the initial and medial chords on strong (hyper)beats, and isochronously distributing the medial chords in between, realizes this metric latency. When $c = 12$ and $n = 4$, the latent meter is triple, regardless of ∂ or any voice-leading permutation. The passage by Medtner analyzed in the previous section effortlessly accommodates this triple meter, although it unfolds on a hemolic layer twice as slow as the notated 3/4. The 4/4 meter of Klebanov's *Allegro* requires some adjustment of its four-voice omnibus material. Measures 47–49 accommodate the long omnibus's inherent triple-ness by either deleting every third chord—the 6/4 minor triad in particular—and maintaining the quarter-note harmonic rhythm (mm. 48-49), or deviating from this harmonic rhythm but keeping the progression unabridged (m. 47).

The four-measure transitional passage beginning Figure 17 finds yet another solution. Chords #4 and #8, each on a fourth beat, lie outside of a permutational variation of an inverted long omnibus; brackets in Figure 21 indicate this isolation. Rather, each chord is a return to the same pitch classes on the preceding first beat, but in chordal inversion: the two violins' hocket line on top and the cello's bass line on bottom execute a voice exchange, brought to the fore with diagonal lines in Figure 20 and blue lines in Figure 21. This return not only perpetuates the introduction's pervasive voice-exchange trope, but also bolsters the previously mentioned interpretation that each measure harmonically reduces to its downbeat chord. The chord reused in the voice exchange flanks the measure on its first and last beats, adumbrating what Cohn generally calls a "switchback" metric-hierarchical structure (ABA | ABA or ABA | BAB), in contrast to the "parallel" structure of Figure 8's latent metrical realization (AB | AB) [14]. This switchback design also underpins the short omnibus in Figure 10. As aforementioned, Figure 8 generates some transposition of Figure 10 by setting n to 12, c to 4, ∂ to –1, and going through three of the five chords of Figure 8 but then, to reuse Cohn's language, performing both a switchback in time—the fourth chord (Chord 3, using the zero-based numbering chosen earlier) is the second chord (Chord 1), and the fifth chord (Chord 4) is the first chord (Chord 1)—and a swap between the two moving voices. While this harmonic switchback scheme fits in nicely with a duple or quadruple metric parallel scheme (ABCB | ABCB | A...), it only squares with a quintuple metric switchback scheme (ABCBA | ABCBA | A...). For some version of Figure 8 with $n = 4$ to deliver switchbacks of both harmony and quadruple meter, creating the four-chord version of the five-chord Figure 10 (ABBA | ABBA), either the composer must use $\partial = 0$ or must permute the voice leading. The former works because the INC chord is a fully-diminished seventh chord, whose pitch classes all return under T9, as in Medtner's Chords #7–10 of Figure 13, although Medtner metrically realizes these four chords as metrically parallel, in that the two $B^{\circ 7}$ chords fall in the same metric position: beat 3 in the triple meter. But this option works poorly with the minor-third sequence of Klebanov's music: the same four pitch classes would have crossed the barline into m. 45, missing the opportunity to align new chord tones with a downbeat. Klebanov's use of a permutation of omnibus voice leading—the latter option—manages the 4/4 meter well, in that just two voices—one upper voice, and the bass voice—are needed for the small-omnibus switchback routine, saving the third voice's replacement of A with A♭ for the long-omnibus-powered transposition and the downbeat of m. 45.

Nonetheless, the challenges of reconciling the standard omnibus progression's latent triple periodicity with its isochronous realization in a (qua)d(r)uple meter are completely reconstituted when the size of the chord changes but the fundamental mechanism of Figure 8 does not, as the next section lays out.

3.3 Changing the Size of the Chord

A constraint on both Figures 1 and 8 is that n, the size of the chord, is an integer factor of c, the size of the chromatic universe. Cohn's "generalized Weitzmann regions" theoretically allow for different values of both c and n for Figure 1 ([7][p. 166]), although his book concentrates on nearly-even chords of size 3 (major and minor triads) and 4 (dominant sevenths and half-diminished sevenths; the "Tristan genus"), with a sidebar entertaining applications to chords of size 6 (set class 6–34), all within the usual 12-note chromatic scale. Figure 8 may be similarly generalized, starting with chord sizes of 2, 3, 6, and 12, the factors of 12 greater than 1 that have not yet been explored in this essay, because the standard omnibus employs four-note chords. The PE chords of these sizes are the tritone, augmented triad, the whole-tone scale, and the chromatic scale, which allows the NC subsets of Figure 1's INC chords when $c = 12$ as the singleton [0] for $n = 2$, the major third [04] for $n = 3$, the dominant ninth ♯11 chord [02468] for $n = 6$, and the aggregate minus one member [0123456789t] for $n = 12$. Each of these chords can uniformly transpose by $\mathbf{T}_{\frac{-12}{n}}$ through the individual transposition of one of its pitch classes by $\mathbf{T}_{\frac{12}{n}}$, or, equivalently, $\mathbf{T_1}$ $\frac{12}{n}$ times, creating motion that is potentially smooth and, excepting $n = 12$, linearly inertial in register. Synchronizing this motion with a smooth line moving by $\mathbf{T_1}$ $\frac{12}{n}$ times in the opposite direction extends the familiar omnibus design to four other symmetrical divisions of the chromatic scale besides the minor third.

The top staves of Figures 22a, b, c, and d provide for each of these four options an analogue of the top staff of Figure 11 (or, equivalently, Figure 15), which exhibited the NC subsets of the standard ($n = 4$) long omnibus progression. (For ease of notation, the inverted parts of the image in Figure 22d represents the complement operation; for example, the first measure of Figure 22d contains all pitches from F4 to F5 minus the F5.) Each progression of NC chords can then combine with one of 12 chromatically descending bass lines ($0 \leq \partial \leq 11$, none shown) to make a unique omnibus-like progression up to transposition and rotation, with the exception of $n = 2$, for which only two unique bass lines exist. The bottom two staves of each of Figures 22b and c provide an analogue of the two bottom staves of Figure 15, indicating the two possible voice-leading variations that maintain the major-third and major-second transpositional structure but permute the upper voices' steps in time. (The $n = 2$ and $n = 12$ versions allow for no permutational variations as such.) Therefore, allowing ∂ and n to vary, and the voice leading to permute, there are 121 possible generalized omnibus progressions within the 12-note chromatic universe ($c = 12$), of which 85 are in addition to the 36 $n = 4$ solutions introduced in the previous section.

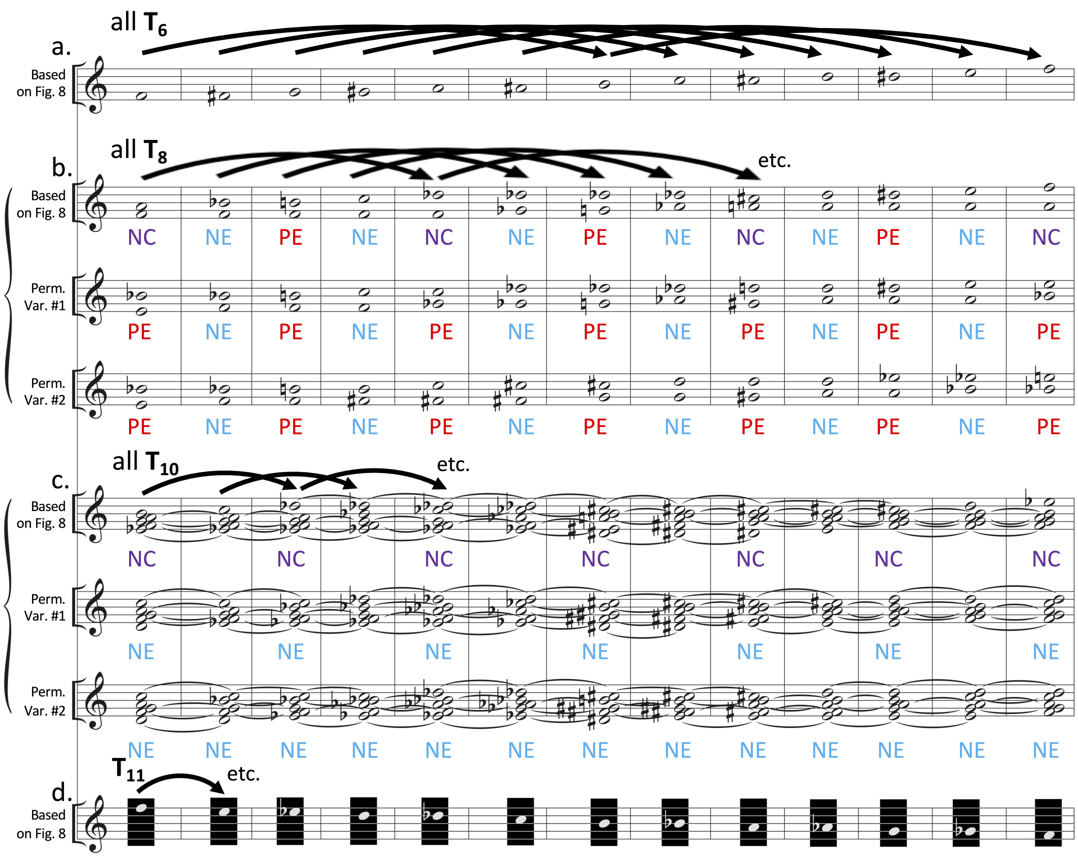

Figure 22: *One notated, registral realization of the NC subsets of 8 ($c = 12$) with (a) $n = 2$, (b) $n = 3$, (c) $n = 6$, and (d) $n = 12$, followed by $n - 1$ dovetailed iterations, and the two voice-leading permutation variations each for $n = 3$ and $n = 6$, with harmonic categories in colored text.*

One of these 85 solutions closely pertains to three theoretical studies as well as musical literature. Figure 23a shows a realization of Figure 8, plus its dovetailed iterations that complete a transpositional cycle, where $c = 12$, $n = 3$, $\partial = 2$, and the first bass note is A flat (thus matching the top staff of Figure 22b); the voice-leading motions are not permuted. To detail the realization: the opening E♭-F-A chord derives first from the PE augmented triad of D♭-F-A, which is generated by $\mathbf{T_4}$ ($\mathbf{T}_{\frac{12}{3}\,\frac{(c)}{(n)}}$). This PE chord is intermittently made NC by omitting the D♭, then ultimately made INC by reintroducing the D♭, but up two semitones ($\partial = 2$) to E♭ and in the bass. The voices with E♭ and A then wedge outward by contrary semitonal motion four ($\frac{12}{3}$) times, with the F as a held pitch, arriving at a $\mathbf{T_4}$ transposition that is then recycled as the first chord of Figure 8 to repeat the pattern. For each dovetailed iteration of Figure 8, this particular bass line for $n = 3$ outlines in three voices not only a $\mathrm{V}^7 \rightarrow \mathrm{I}$ progression but also the three-chord voice-exchange portion of the standard omnibus, as annotated.

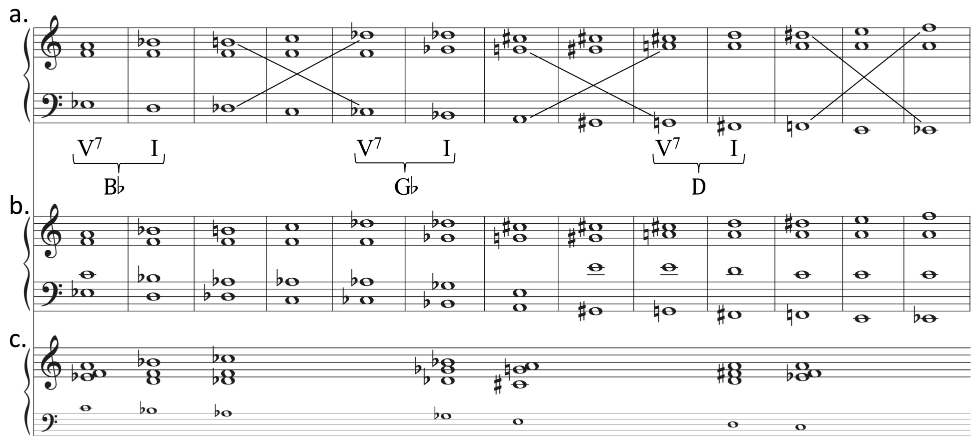

Figure 23: *(a) Major-third omnibus progression combining top staff of Figure 22b with bass line, compared to (b) progression from Gauldin ([5], Example 6a, $\mathbf{T_6}$), and (c) progression from Santa ([12], Figure 3, $\mathbf{T_{10}}$).*

This music closely resembles a progression explored by Matthew Santa [12], which weaves together $\mathbf{T_4}$ cycles of **T**-type [026] and [047] chords such that the smoothest voice leading is contrary semitonal motion in two voices throughout. This "nonatonic" progression—so-called because its six-chord cycle uses all and only all of the members of a nonatonic collection (9–12)—is provided in one transpositional and registral realization on the bottom grand staff of Figure 23 (Figure 23c) for easy comparison to the first grand staff. The $n = 3$ omnibus progression composes-out each of the nonatonic [026] trichords with a voice exchange, or, equivalently, the nonatonic progression lacks the aforementioned voice-exchange portion of the $n = 3$ omnibus progression, reducing the number of chromatic steps in each of its moving lines from four to two. Santa shows how the addition of a fourth whole-tone line, which Figure 23c puts in smaller font, completes each incomplete dominant seventh chord and doubles the root of each major triad. Although Santa analytically applies the nonatonic progression to passages from Liszt, Beethoven, and Schubert, this progression and Santa's variations on it furnish more of an unordered harmonic vocabulary and less of an ordered syntax: the linear stepwise unidirectionality available within the progression rarely manifests in Santa's examples.

A source that more clearly spotlights this linear stepwise unidirectionality and the inertial advantages it brings is Robert Gauldin's study of chromatic wedge progressions [10]. Like Figure 8, Gauldin moves beyond the standard omnibus in accounting for a variety of transpositions that frame two voices moving in contrary motion by step through a chromatic scale. He goes even further, consider transpositional intervals that are not factors of 12 (e.g. $\mathbf{T_5}$), because, unlike Figure 8, he does not yoke together chord size and transposition; rather, he defaults to four-voice harmonizations of these wedges. The second grand staff of Figure 23 (Figure 23b) provides a transposition of one of Gauldin's $\mathbf{T_4}$ harmonizations (Example 6a) that hybridizes the other two grand staves: it includes the voice-exchange component that preserves the wedge design, but it also uses four voices like Santa, although it relocates Santa's added bass to an inner voice. Concentrating on the music of Wagner and Tchaikovsky, the example Gauldin provides that best matches the entirety of Figure 18c is some music from *Tristan und Isolde*'s Act II Love Duet (which is reused in the Act III Transfiguration). While the three-chord voice-exchange portion recurs at $\mathbf{T_8}$ once and partially twice, the $\mathrm{V}^7 \rightarrow \mathrm{I}$ portion neither occurs nor recurs—unsurprisingly, given the composer's restraint from tonality's most common routines—due to chordal insertions and deletions.

Figure 24 includes an example from earlier and more conservative music that complements Gauldin's example: it repeats the $V^7 \rightarrow I$ portion at $\mathbf{T_8}$ but does not repeat the three-chord voice exchange, and it moves strictly incrementally through a transposition of Figure 23a without insertions or deletions but only through one half of it. The German composer and pianist Ferdinand Ries (1784–1839) completed his second symphony in 1814, and dedicated it to Beethoven, of whom Ries served as copyist, secretary, friend, and promoter. Figure 24 provides an excerpt near the end of the finale of the symphony; beyond the excerpt's twenty-four measures, only fifteen more measures remain of the 309-measure movement. These twenty-four measures divide into a twelve-measure half and a varied repetition of this half, and each twelve-measure half further divides into three four-measure segments exhibiting presentation, continuation, and cadential functions as annotated. Both continuation segments pare down to three parts: a chromatically ascending treble line from D or E to A flat, a chromatically descending bass line from B or B flat to F, and a more static middle line that stays on C until a shift to D flat. While this number of voices counters tonality's four-voice standard, it also transparently realizes Figure 8 with $n = 3$, $c = 12$, $\partial = 2$, and the first bass note as B flat (a $\mathbf{T_7}$ transposition of the beginning of Figure 23a). This facilitates the INC fifthless C7 to proceed with stepwise contrary motion to its $\mathbf{T_8}$ transposition: the INC fifthless A♭7. Both secondary dominant sevenths resolve to their secondary tonics of IV and ♭II ($\mathbf{T_8}$ of IV) within the symphony's key of C minor; the Neapolitan-sixth chord then form-functionally pivots to the first chord of the ensuing cadential harmonic progression. Since $n = 3$, the meter is 2/4, and the chromatic motion uses an isochronous quarter-note pace, each continuation segment can place both of its fifthless dominant sevenths and their resolutions in parallel metrical positions within Ries's duple meter, sidestepping the mismatches underscored in the previous analysis of Klebanov's music. The first continuation segment puts each fifthless dominant seventh on a downbeat, but Ries breaks the isochrony by delaying the D-flat major triad (foregrounded with the tip-stretched arrow), ostensibly to save the resolution to the tonic of ♭II to the next downbeat. The second continuation segment appears to respond to this incongruity: Ries omits the upbeat G chord of m. 275 from the corresponding m. 287, relocating both dominant-to-tonic resolutions to downbeats without recourse to isochrony distention. Notice that the continuation segments' most active line, played by the violins and condensed into the top staff of mm. 275–78 and 287–90 comprises both treble and middle lines in the manner of a compound melody, connecting with a smooth semitonal stroke (E-F-F♯-G-A♭) the NC {C, E} to its $\mathbf{T_8}$, the NC {C, A flat}.

Figure 25a provides a small portion of the closing material for the "Triumphal March" from the second act of Verdi's *Aida*, one of the most well-known operatic numbers by the composer and one of the most well-known triumphal marches by any composer. The choral text at this moment is composed of repetitions of "Gloria!" and "Grazie rendete a gli Dei!" [Give thanks to the gods!], celebrating the Egyptian army's victory over the Ethiopians. Both outer voices sweep incrementally and isochronously outward, each through an B♭-to-B♭ octave in eleven instead of twelve steps, disallowing each voice from moving strictly chromatically. The two voices make up for this single-semitonal difference from a complete chromatic scale in different ways: the bass part simply injects a single whole step near the end ($1+1+1+1+1+1+1+2+1+1+1 = 12$), whereas the treble voice injects two whole steps at the beginning and a unison near the end ($2+2+1+1+1+1+0+1+1+1+1 = 12$); the temporal placements of these injections situate members of the E-flat tonic triads (E flat, G flat, G, B flat) in the outer voices on downbeats. These injections divide this four-measure portion into three segments: a beginning and ending segment each whose outer voices employ what Gauldin calls a hybrid wedge, blending diatonic and chromatic steps, and a middle segment whose outer voices are completely semitonal in their motion.

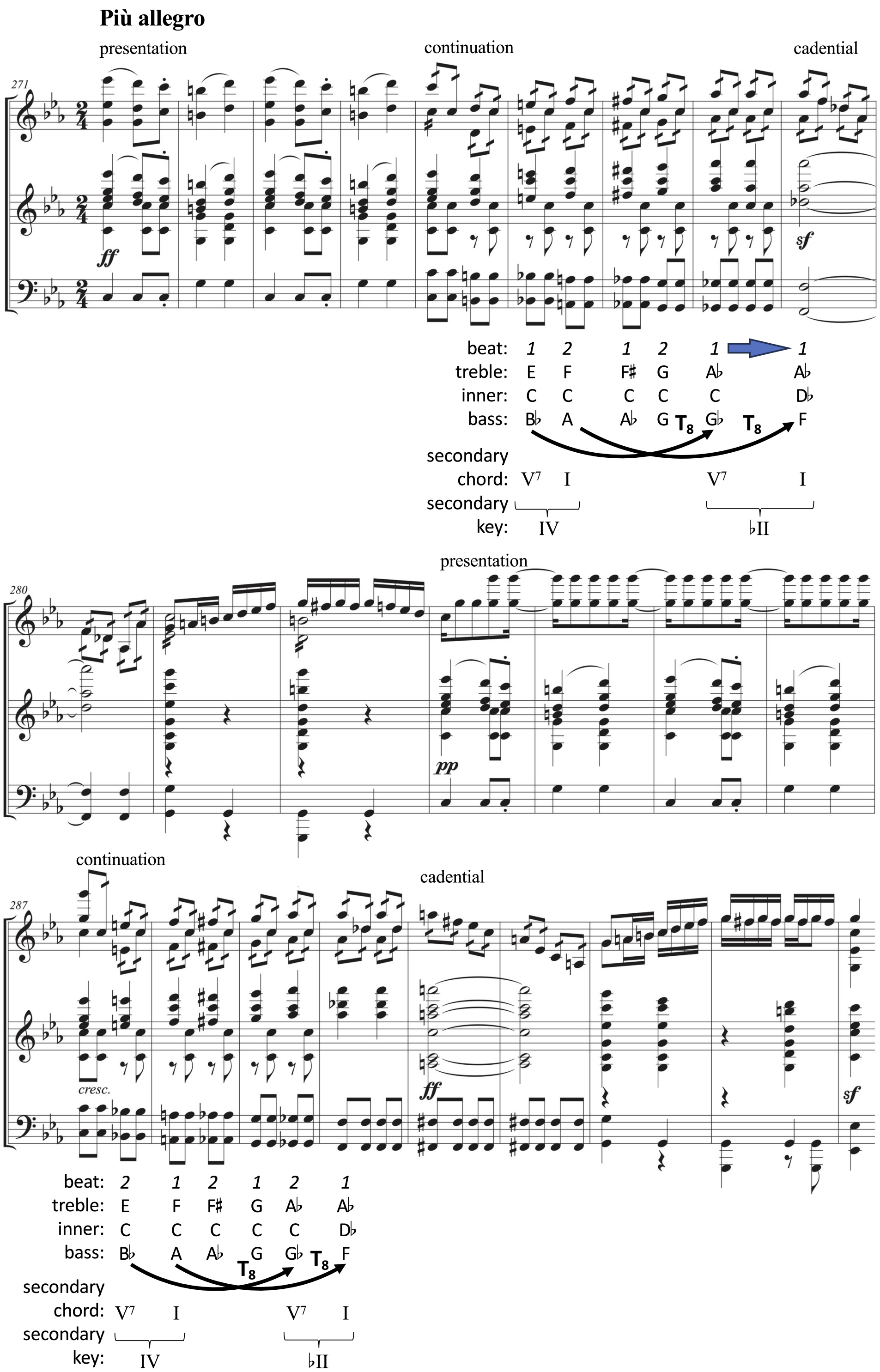

Figure 24: *(a) Ferdinand Ries, Symphony No. 2, IV, mm. 271–95, with annotations.*

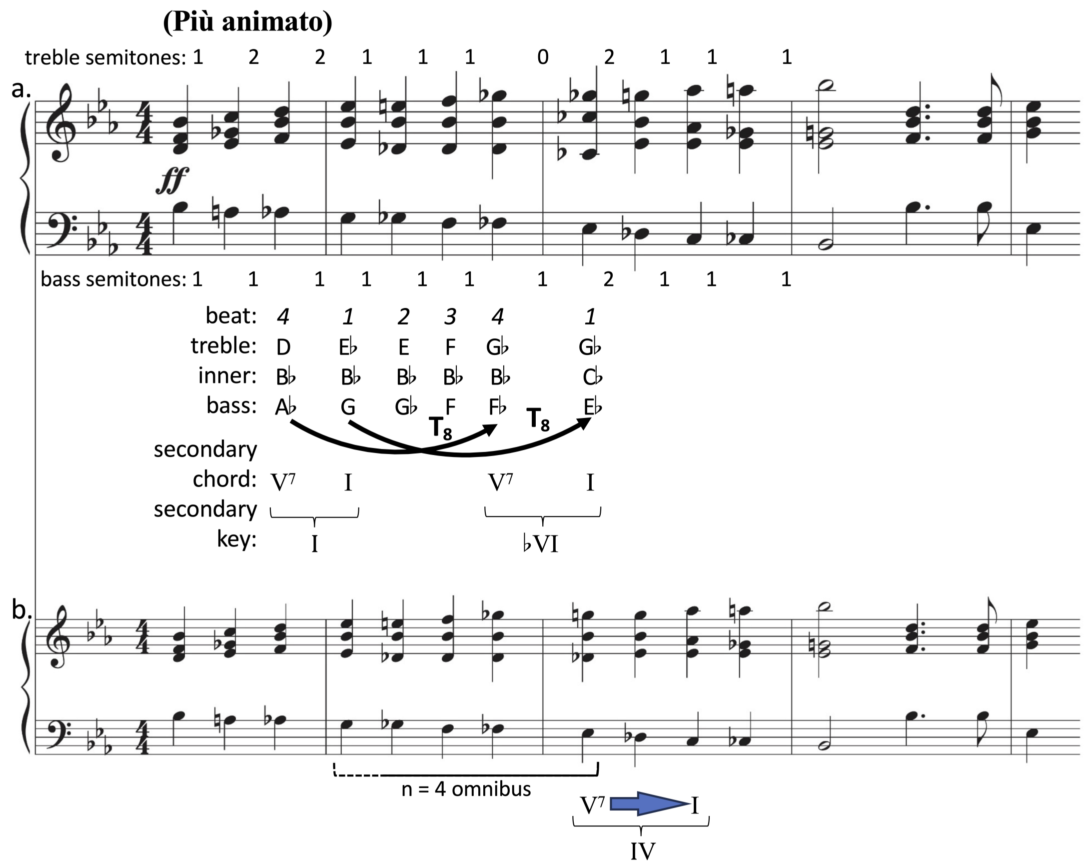

Figure 25: *Giuseppe Verdi,* Aida*, Act II, Gran Finale, rehearsal L+10, with annotations.*

This chromatic middle segment of Figure 25a matches the continuation segment of the music in Figure 19 in multiple ways: both realize Figure 8 with $n = 3$, $c = 12$, $\partial = 2$ and a isochronous harmonic rhythm, both occur during an thrilling quickened coda of an already exciting, fast-paced movement, and both are in (qua)d(r)uple meter, setting up a parallel relationship between the inherent four-ness of a $n = 3$ realization of Figure 8 and the 4/4 time signature. However, they differ in other ways: besides a first bass note on B flat, the realization in Figure 25a fills out the texture to (at least) four voices, and the $\mathbf{T_8}$ transposition occurs between the secondary keys of I and ♭VI instead of Ries's IV and ♭II, although notice how both composers place both resolutions to tonic on downbeats. In his monographic study of the augmented-sixth chord, Mark Ellis cites the music in Figure 20 as an exemplar of a "triumphant" progression, which features "outward-moving scalic lines (often chromatic), a crescendo or series of 'terraced' dynamic increases, last inversion dominant seventh chords (often irregularly resolved) and augmented sixth chords. . . The overall effect is often of a series of rapid modulations" [4, p. 185]. Omnibus generalization helps to connect this example from Verdi with other examples of the triumphal progression that Ellis cites by Beethoven (Symphonies Nos. 1 and 2), Mendelssohn (Overture to *Die Hochzeit des Camacho*), and Tchaikovsky (Violin Concerto), because excerpts from these pieces realize distinctive parts of Figure 8 but with an n of 4. Omnibus generalization also speculates a glimpse inside of Verdi's craftsmanship: the repeat of the G flat in the top part across the second barline not only follows naturally from two $n = 3$ realizations of Figure 8 dovetailed together, but also provides the second stage in the solution to the aforementioned octave-in-eleven-steps discrepancy, compensating for the first stage's overshooting two whole steps within the top part's opening 5-6-7-8. If the middle segment stemmed from the $n = 4$ omnibus instead, as shown with the smaller staff in Figure 25b, this would have delivered a generative mechanism for all four voices of Verdi's chosen texture, but it would also have required a shift in the discrepancy solution that would both briefly stall the regular quarter-note harmonic rhythm—with two E-flat-rooted chords back-to-back, as foregrounded with the tip-stretched arrow—and reduce the number of modulations whose multiplicity Ellis recognizes as distinctive of these progressions.

This article offers no examples from the literature that realizes Figure 8 with $c = 12$ and $n = 3$ but a different ∂, or with $c = 12$ but $n = 6$. I suspect that examples of the latter lurk in the same *fin-de-siècle* musical repertoire in which realizations of Figure 1 with $c = 12$ and $n = 6$ can be found; Cohn spends a little time with a passage by Scriabin and the second nature of the mystic chord [013579] in his book. However, among all of the possible different $c = 12$ realizations of Figure 8 up to transposition, voice assignment, register, and so forth—which total 121 as enumerated earlier—the standard omnibus progression along with its inverse ($n = 4$, $\partial = 1$ or –1) is perhaps the most frequent because it is the only one that actualizes both the third nature of a NC three-note chord and the second nature of a NE four-note chord.

4. Changing the Size of the Universe

Exempting limiting cases in which $n = 1$ or $n = c$, c cannot be prime for Figure 8 to be realized, but this still leaves many musically actionable choices for c beyond 12. Given the prevalence of the twelve-note chromatic as western music's most common equal division of the octave, the majority of practical $c \neq 12$ applications of both Figures 1 and 8 reside in the domain of rhythm rather than pitch, whereby c is the length of a recurrent, evenly divided time span—such as a measure—and beat classes replace pitch classes.[5] However, this essay has already presented one $c \neq 12$ realization in the pitch domain, hiding in plain sight. A fifthless dominant seventh (a T-type [026]) is not only INC in a $c = 12$ universe, with a NC of [04], but, in a $c = 6$ universe where [026] becomes the T-type of [013], this trichord is also both NE, as it is a single-step perturbation of [024], and INC, as it is the union of a NC [02] and a singleton. Thus, in the $c = 6$ universe, it possess both second and third natures. Its second nature materializes when extracting the fifthless dominant sevenths that appear as every other chord in Santa's nonatonic progression, provided earlier as the top staff of Figure 23c. Within a parent whole-tone scale, one gets from one mod-6 [013] to another by moving by a single scale step two of its three pitches that do not span a mod-6 [02] in opposite directions. One may understand Santa's full nonatonic progression as this realization transported into a $c = 12$ universe, in which the two single whole-tone steps are each subdivided into two semitonal steps, although this perspective curiously treats the resulting consonant triads as harmonic byproducts fleshing out a backbone of [026]s.

The musical passage in Figure 26, which I composed in the style of an etude by György Ligeti, demonstrates the third nature of the INC mod-6 [013] in both pitch and rhythm. The first measure clearly establishes a metric modulus of six units, notated as eighth notes, and emphasizes the downbeat—beat class 0—with both an accent mark and a dominant-tonic F-to-B-flat resolution courtesy of the chromatic motion: 6 time points yields 5 intervals, which, if the intervals are all descending semitones, yields the wraparound 5-semitone interval of an ascending perfect fourth.

[5] In "Cohn Functions" [12], David Lewin uses a rhythmic example to realize (and extend) Cohn's maximally smooth property to a $c = 9$ universe. This property is not equivalent, but closely related, to the property demonstrated in Figure 1. An exercise for the reader is to fashion a graph comparable to Figure 1 that sufficiently displays this maximally smooth property; it should include both transposition and inversion operations.

The second and third measures introduce two more parts imitating the first in close stretto at a time offset of one and three time units, producing an three-onset rhythm of [013] mod 6, and an analogous pitch offset of one and three steps in the whole-tone scale, producing a pitch-class set of [013] mod 6. The mod-6 numbers written above or below the accented notes may be read both as beat classes, with the downbeat as 0 in the sextuple meter, and as pitch classes, with B flat as 0 in the even-numbered whole-tone scale.

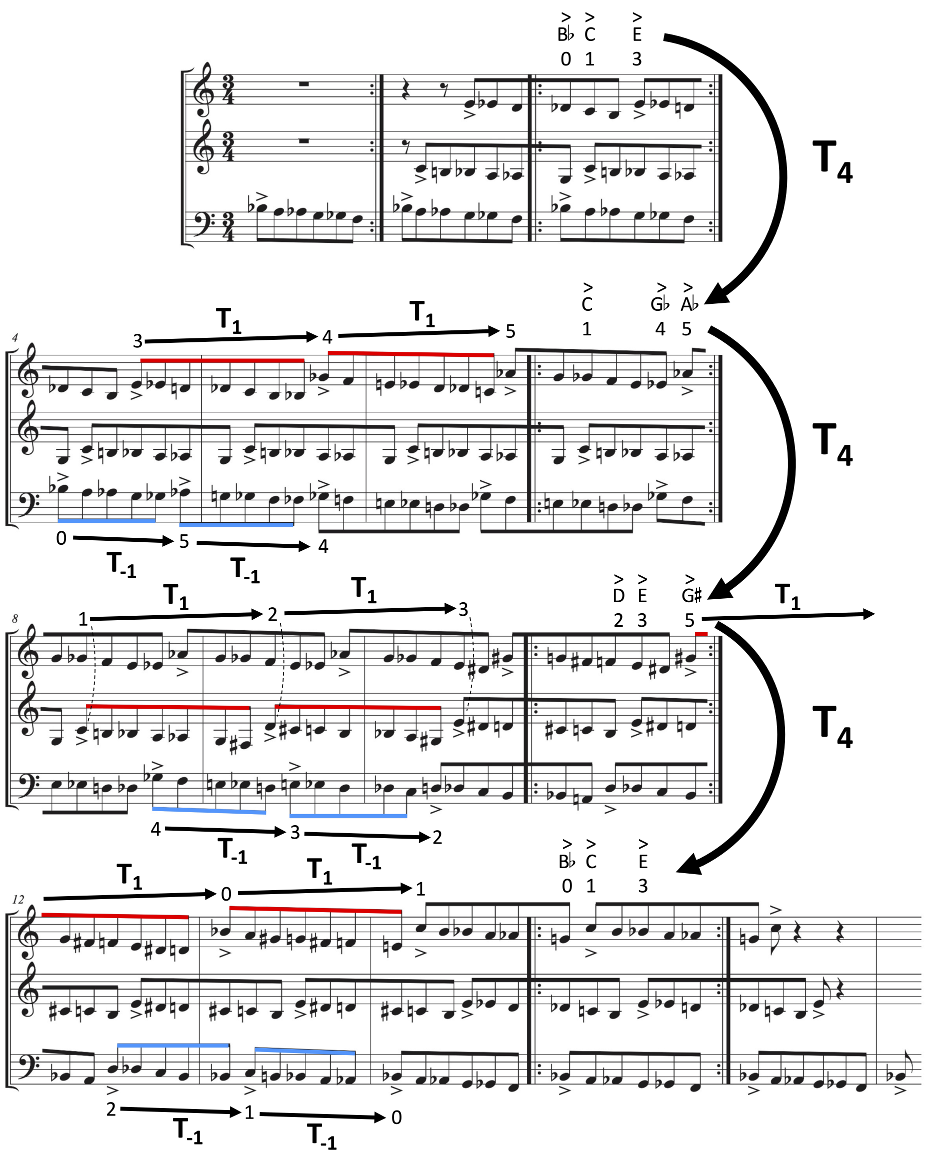

Figure 26: *Newly composed Ligeti-like music that demonstrates a full cycle of dovetailed realizations of Figure 8 with* $c = 6$, $n = 3$, *and* $\partial = 1$ *in both pitch and rhythm.*

Both the pitch content and the rhythmic content of this opening idea undergoes a full omnibus cycle in the $c = 6$ universe. The progression from the end of one system to the end of the next realizes Figure 8 with $n = 3$ and $\partial = 1$ and no permutational variation.[6] The bottom staff serves as both bass part and "bass part": not only do the accented pitches continuously descend through a whole-tone scale, but they also shift backwards relative to the measure each time a shorter five-note line, beamed in blue, replaces the usual six-note grouping, eventually completing the cycle in the manner of Reich's phase music. The top two lines take turns interpolating two successive seven-note groupings, beamed in red, amid the usual six-note groupings. For both upper and lowest voices, putting two $\mathbf{T_{\pm 1}}$s back-to-back conveys a rhythmic analog to the idiomatic "naturalness" of step inertia: the performer can lock into a quintuple or septuple periodicity briefly before returning to the notated meter or some displacement thereof. (Such a "lock" would be even more pronounced in a situation where $\frac{c}{n} > 2$.) The result of these concomitant and slight contractions and expansions of the sextuple figure is a wholesale $\mathbf{T_4}$ shift of the [013] mod 6 accent rhythm; that is, forward four eighth notes (equivalently, backward two eighth notes): this is possible because [013] mod 6 is INC.

5. Conclusion

Although I have been saying that a INC chord possesses a third nature, this cardinal numbering is arbitrary: it is merely another nature that some chords possess in the abstract, and may express in a musical composition. In addition, there may be more natures for a scholar interested in overdetermined phenomena to uncover: some musical element or process already marked as advantageous for some reason (practical, aesthetic, etc.) contains another property that is able fulfill some other musical preference. But I also recommend the exploration of overdetermination's inverse: "underdetermination." Although it may seem more sensible that a composer would rather embrace overdetermined materials and procedures that accommodate multiple predilections that they bring to their art, the comprises and trade-offs that underdetermined materials and procedures inherently require impart another important potential: variety, both between different styles, between different works in the same style, and between different moments in the same work.

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