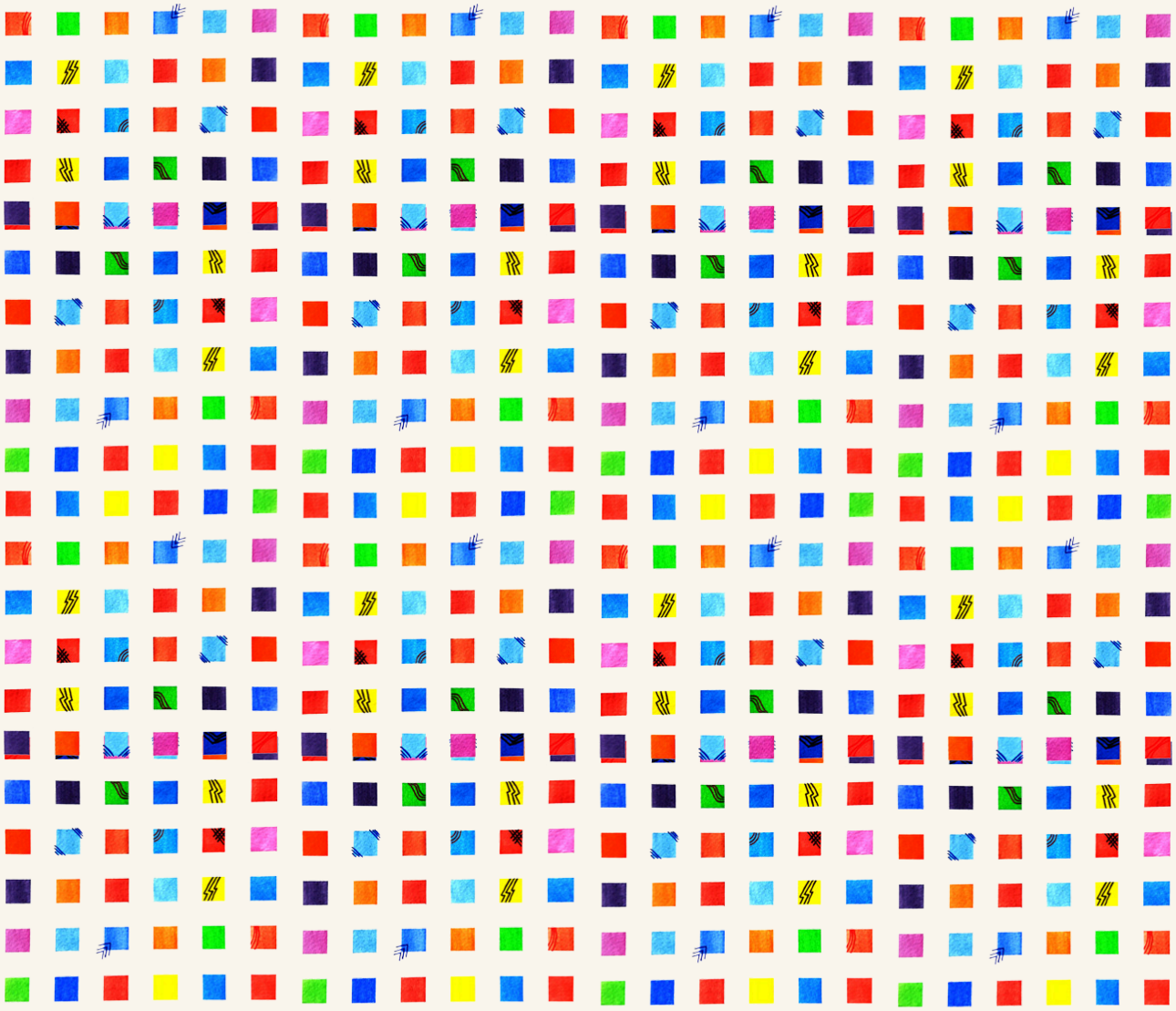
MUSMAT SERIES VOL. I

SCHOENBERG: MUSIC AND THEORY

Carlos Almada, Carlos Mathias, Cecília Saraiva,
Daniel Moreira, Hugo Carvalho, and Liduino Pitombeira
Editors



MusMat
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Federal University of Rio de Janeiro (UFRJ)
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Preface

In 2021 the world was at the height of the Covid crisis. Travel was discouraged at best, forbidden entirely at worst. Had Covid made its deadly debut early in the computer revolution, say, 1985, there was little that could have been done to keep the wheels of international academic research spinning. But Covid did not make its debut in 1985; it entered onto the world stage in 2020, where a bevy of video communication programs awaited, programs specifically designed to foster communication between people who could not meet in person. Thus, the 6th International Conference MusMat 2021, *Schoenberg: Music and Mathematics ("Hommage to Arnold Schoenberg")*, was presented as an online international conference with the participants at their home or office computers in Brazil, the USA, Europe, and the Middle East. The present book has 12 chapters, most of which were delivered as papers at the online conference.

It is fitting that a conference on music and mathematics focus on the music and ideas of Arnold Schoenberg. Not that Schoenberg himself explored any of the intersections between music and mathematics in his own theoretical writings. But Schoenberg's music, and in particular the twelve-tone music, prompted a torrent of theoretical and analytical writings that used numbers in heretofore unimagined ways. To be sure, numbers had already been used in many different ways in music theory – from Euclid and Boethius to Rameau and Messiaen. But there was little precedent for the type of writing that began appearing in (mostly) North American theory circles, with the writings of such important figures as Milton Babbitt, George Perle, and Allen Forte. But just as much as it became possible to use numerical or arithmetic operations to explain aspects of Schoenberg's music, it has not become an exclusive pathway. It is still possible to use analytical or historiographical tools that have no connection to mathematics at all – as is amply demonstrated by some of the essays in the present book.

In 2024 we mark Schoenberg's 150th birthday. One can expect that the birthday will stimulate renewed interest in Schoenberg's works and ideas. The present collection of essays has gotten the ball rolling. We look forward to seeing additional contributions in the near future.

Ethan Haimo
May 2024
Ra'anana, Israel

Foreword

In 2021, the MusMat Research group promoted the 6th International Conference on Music and Mathematics honoring Arnold Schoenberg (1874-1951). We had the presence of a significant number of world-renowned Brazilian and foreign researchers, invited to lecture on their most recent research associated with the theoretical and artistic work of this important composer. The conference was conducted entirely online, through video conferences, keynotes, virtual debates, and concerts.

This book is a result of this conference, and its release coincides with the 150th anniversary of Schoenberg's birth, joining a myriad of tribute initiatives honoring his legacy around the world. It presents a comprehensive prospection of Arnold Schoenberg's music, paintings, as well as his theoretical writings. Thus, each chapter provides a unique perspective on different aspects of his extraordinary multifaceted work presented by a team of esteemed scholars and experts who delve deep into the intricacies of his groundbreaking contributions (theoretical and artistic).

Robert Morris, in chapter 1, analyzes the third of Schoenberg's Opus 23 piano pieces, discussing its pivotal (or transitional) role in Schoenberg's music from free atonal to twelve-tone composition. In chapter 2, **Jack Boss** explores Schoenberg's concept of "musical idea" and its implications on the reorderings of, additions to, and omissions from the twelve-tone row in his Op. 33a. **Luigi Verdi**, in turn, discusses in chapter 3 the notion of "kaleidocycle" techniques in musical composition, exploring the transformation of chords through transposition cycles and their application in rhythmic-melodic canons. **Robert Peck**'s study in chapter 4 focuses on inversional combinatoriality (I-combinatoriality) in Schoenberg's serial music, producing a graph-theoretical model to explore the space of I-combinatorial hexachords. In chapter 5, **Marco Feitosa** delves into the application of group theory in twelve-tone music, focusing on Schoenberg's usage of interval structures and combinatorial operations within tone rows. **Carlos Almada**, in chapter 6, examines Schoenberg's principles of *Grundgestalt* and developing variation, exploring their implications in music theory and composition, as well as their possible correlations with biological-evolutionary processes. In the seventh chapter, **Guilherme Bueno** investigates Arnold Schoenberg's visual artistry and its connections to European *avant-garde* movements, highlighting the parallels between Schoenberg's music and painting in his artistic pursuits. **Ernesto**

Hartmann, in chapter 8, compares Arnold Schoenberg's ideas of musical education with traditional approaches, emphasizing similarities between Schoenberg's pedagogical project and the Perennialist perspective. **Severine Neff**, in turn, discusses in chapter 9 the symmetries of Schoenberg's Serenade, drawing parallels to Wilhelm Werker's discoveries in Bach's Well-Tempered Klavier and highlighting compositional techniques in the third movement. In chapter 10, **Jeffrey Perry** examines Arnold Schoenberg's compositional milieu in 1930s Los Angeles, discussing the influence on his American students. **Paulo Costa Lima**, in chapter 11, analyzes the cultural dialogue between Schoenberg's heritage and Bahia's Composition Movement through the works of composer Ernst Widmer and his students. Finally, in the twelfth chapter, **Sabine Feisst** highlights the global reception and legacy of Arnold Schoenberg's twelve-tone ideas, focusing on their influence beyond Western Europe and the United States.

This book provides readers with valuable insights into the complexities of Schoenberg's work and the enduring impact of his heritage on contemporary music theory, pedagogy, fine arts, and composition. We hope it will serve as a resource for anyone interested in understanding the depth and innovation of Arnold Schoenberg's contributions.

MusMat Research Group
May 2024
Rio de Janeiro, Brazil

1

Form and Process in Schoenberg's Piano Piece, Opus 23, Number 3

Robert Morris

The piano pieces in opus 23 have been described as transitional or pivotal in Schoenberg's evolution from his so-called atonal pieces to his twelve-tone works. Indeed, the first four pieces in the opus 23 set are based on ordered sets of various lengths and the last, entitled "Walzer", is among Schoenberg's first pieces based on a twelve-tone row. I show that the third piece represents this transition internally being itself a transition from implicit symmetries (as found in atonal works) to the explicit symmetries of the twelve-tone system. These symmetries are not only based on order relationships (i.e., permutations) but on the various and sundry ways the twelve-tone aggregate can be built up from smaller ordered and unordered sets.

As in all the pieces in opus 23, the progress of the third piece is complex and cannot be reduced to traditional forms, such as fugue, rondo, rounded binary, or sonata. While there are passages in the piece that can be heard to allude to such forms, I argue that the form of the piece is better understood as an exploration of the properties of the opening five-pc motive and its hexachordal extensions resulting in a series of episodes each displaying their implicit (partial) symmetries. The work is therefore sui generis, representing a generalization of the principle of developing variation in which the goal of the work is the stability of complete symmetry. This paper elaborates and extends observations and insights from the five papers in the references: in particular, Lewin (2008, pp. 197-221) studies many of these symmetries, but in a different formal setting.

The chronology of the pieces in opus numbers 23, 24, 25 and 26 is complicated and the pieces in opus 23 were written at different times from 1920 to 23 (while the movements in opus 24 (the serenade), opus 25 (the piano suite), and opus 26 (the wind quintet) were being formed and partially completed concurrently).

We will show that opus 23/3 is based on a five-pc motive called **P** that is immediately extended into a hexachord called **X** that is RI-invariant. It also is extended into another hexachord called **Y** that also has I-invariance. The composition is based on these two extended pentachords which through various expansions lead to symmetric closure in either the octatonic set or the twelve-tone aggregate. Nevertheless, these two hexachords X and Y cannot form aggregates with each other or alone, but Schoenberg finds a way to combine them at the end of the piece that we show as a K-net.

The first question to ask is about the form of the work. David Lewin has suggested the piece is a generalization of a fugue (in two voices). To support this, Lewin (2008, p. 218) shows that the opening five pc motive is a reordering and transposition of the head motive of the theme of Bach's four voice Fugue in B♭ minor, WTCII (Figure 1).

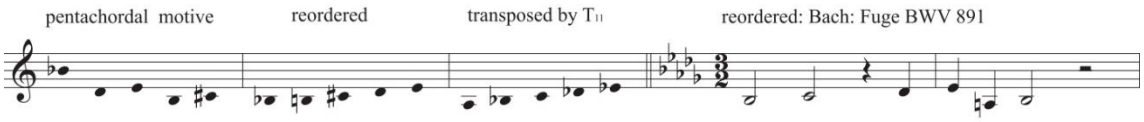

Figure 1: Pentachordal motive related to Bach Fugue (LEWIN, 2008, p. 218).

Figure 2 presents the beginning of the opus 23/3 labeled as a fugue exposition. While the opening measures of opus 23/3 can support such a traditional allusion, the various textures and multi-voiced gestures belie this fanciful interpretation. Nevertheless, the five-note motive occurs frequently in

many different guises and disguises in the work. See Perle (1991, pp. 46-47) and Bailey (2001, pp. 73-98).

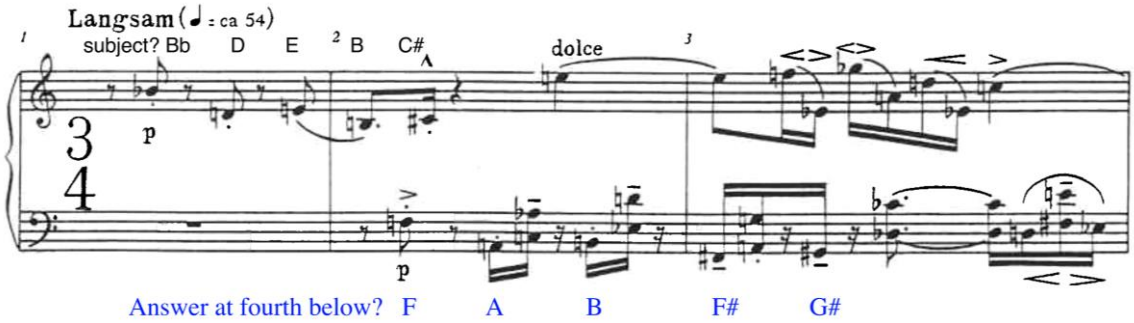

Figure 2: Schoenberg's Op. 23/3 (mm. 1-3).

If I divide the work into sections, the work falls into an ABABA' form in which the A-sections are a series of short passages each exploring the properties of the pentatonic motive in different ways; this suggest a variation form, or more pertinently, instances of developing variation. As these subsections within A progress, they become more complex and there is no returning to the simplicity of the opening of the piece. The B sections contain contrasting music only remotely based on the pentachordal motive. It is also notable that the two B sections are reordered variations of each other (Figure 3).

The last A' section has two parts. The first part stands out as having a new striking stable rhythmic character yet being the most complex passage based on the pentatonic motive.

While this ABABA' form suggests an arch symmetry to the whole, due to the ongoing character of the A sections and the change of character of the A' section, the piece is better heard as a type of teleological process. This might suggest an analogy to the processes of tonal music, but in this case the goal is not a return to the tonic, but a move from incomplete to complete pitch-class symmetry.

Now we take a closer look at measures 1 to 3 (Figure 3). Note, in the sequel, the pc 10 is given by the letter A and 11 is given by the letter B.

The first red pcs follow the pentachordal motive, called P. P is an ordered pcset. Adding the red F in the bass yields a hexachord derived from P. This hexachord, called X, is an ordered hexachord that has RI symmetry. It is not stated in temporal order, but it is in vertical order from high to low. The pc diagram on the lower right shows this symmetry since the RT3I transformation of P is embedded in X. The core of this symmetry is the middle four notes 24B1, which in order are RT3I invariant.

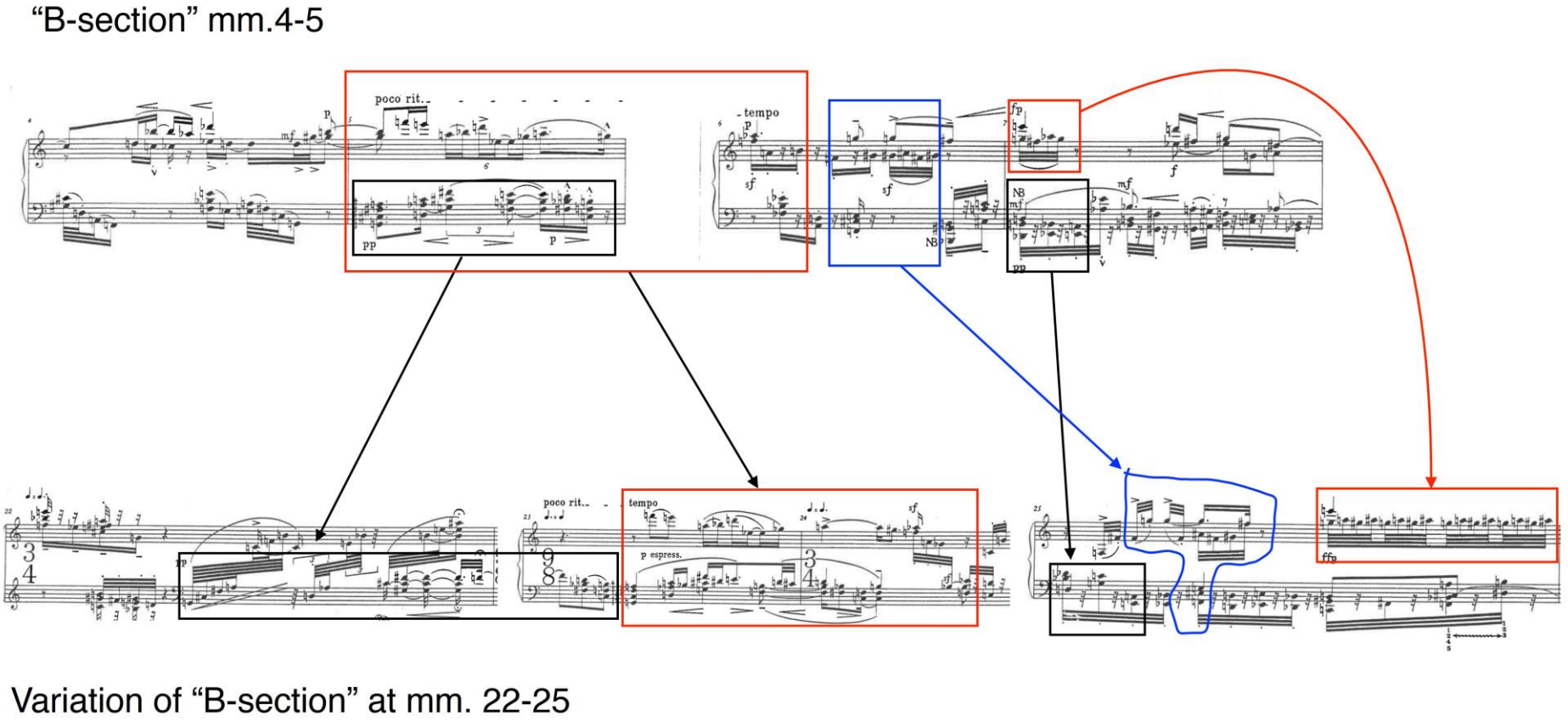

Figure 3: Excerpts of the B section in Schoenberg's Op. 23/3

The example shows other ordered pcsets related to X. First is T7X, colored blue, noted above in the fugal interpretation that gave this pcsegment as the answer to X. Two other transpositions of X follow: TAX and T5X, also related by T7, but these do not follow the contour of X and jump about in the texture. After that we have an inversion of X, T1IX. Various other analysts have found other more hidden and incomplete versions of P and or X in these measures, but I omit these as perhaps unintended. Some notes of the passage are not part of any transformations of X; these notes are printed in brown. I consider them chromatic embellishing pcs. Such "extra pcs" occur in the piece but are almost always chromatic therefore functioning as passing tones among more salient and basic pcs. The boxes show adjacent pcs that form members of the set-classes included in the unordered content of P (Figure 4). Thus, the local harmony is derived from P and X. A small but telling detail is the presentation of the SC 6-20 in measure 2 as mentioned in Babbitt (2003, p. 312).

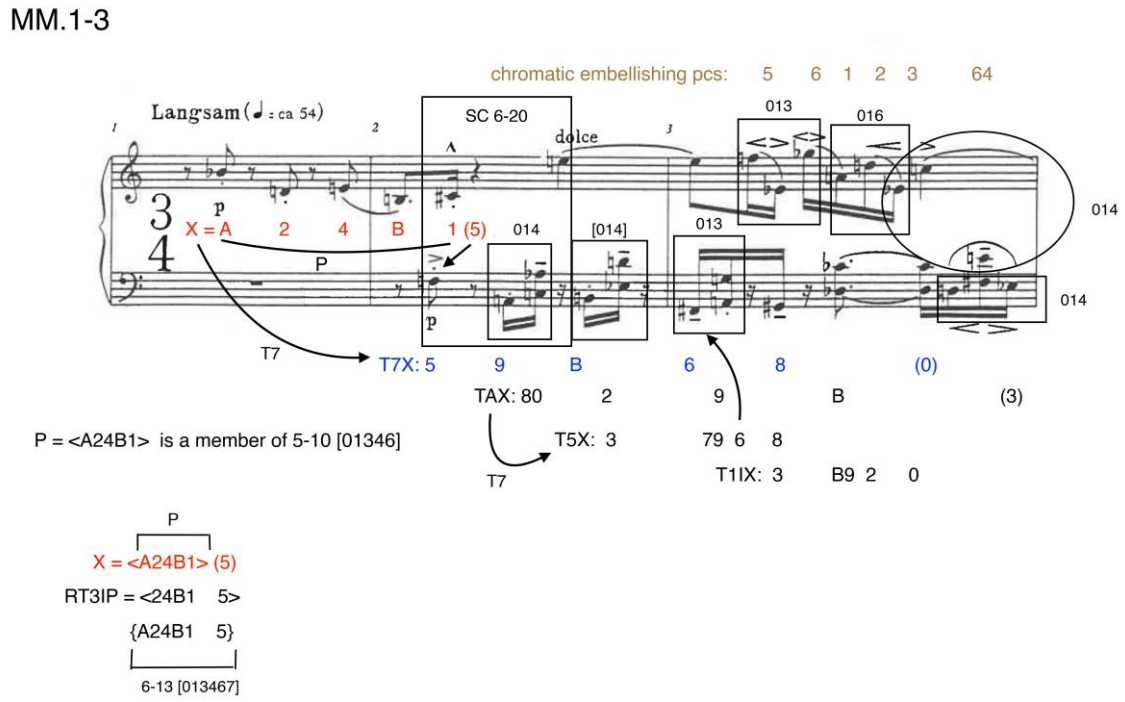

Figure 3: Transpositions of X Schoenberg's Op. 23/3 (mm. 1-3).

The internal inversional symmetry within hexachord X derived from P is not the only inversion symmetry possible. Figure 5 shows in music notation what was mentioned above: i.e., the relation of the content of P in union with the content of T3IP. The asterisk after the label "P" indicates that P is taken as an unordered set. So, P* and T3IP* intersect to form hexachord X. As shown, the intersecting pcset is a member of SC 4-10. X is a member of SC 6-13. Its Z-related complement is 6-42.

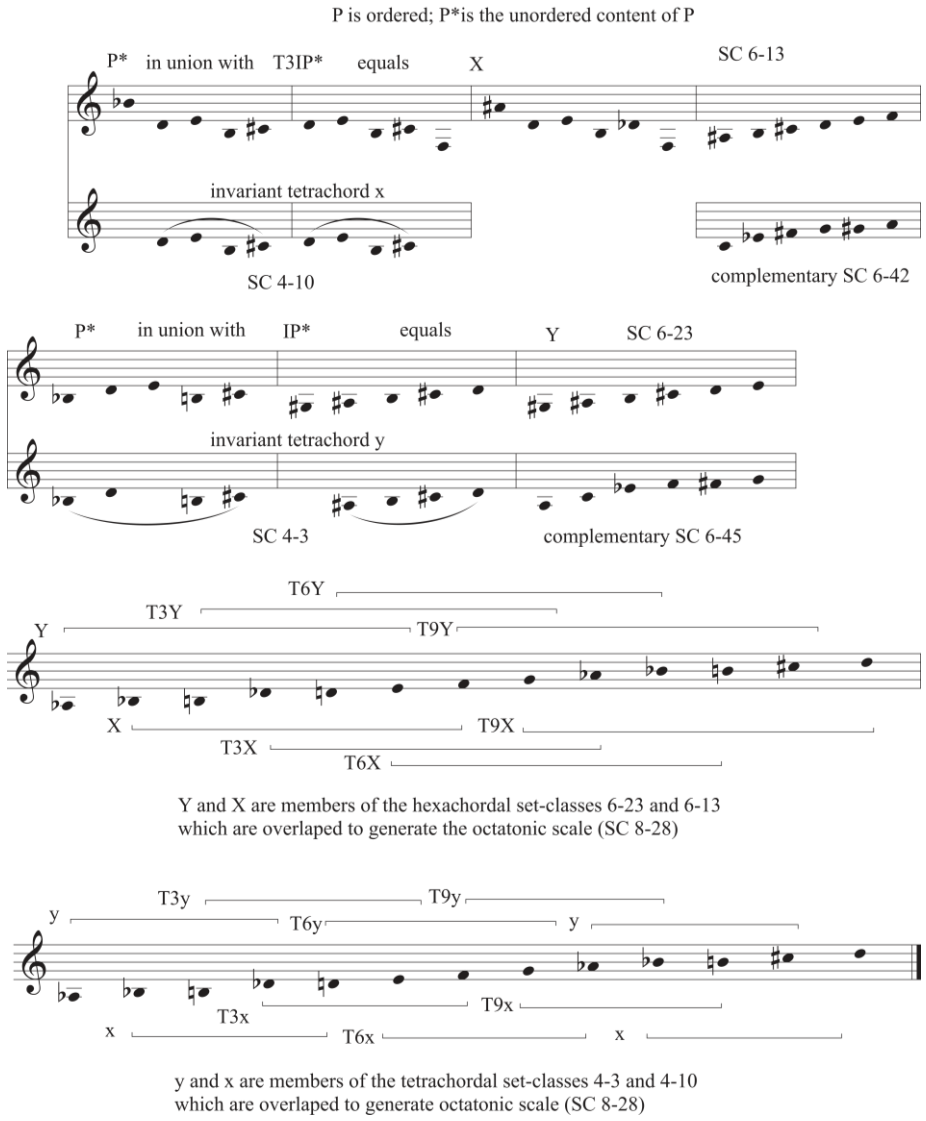

Figure 5: X and Y hexachords derived from P, the pentatonic motive.

The other symmetry derived from P* is more subtle. It is derived from the intersection P* with IP, which is the tetrachord SC 4-3. Thus, the union of P and IP yields another hexachord Y, a member of SC 6-23, whose complementary SC is 6-45.

These two hexachords X and Y have a significant relationship; they are the only two hexachords that can cover six successive notes of SC 8-28, commonly known as the octatonic scale. This is revealed near the bottom of the example. At the bottom, the octatonic scale similarly generated by the invariant tetrachords labeled (small) x and small) y. This octatonic relationship between X and Y and between x and y will figure importantly in opus 23/3.

Note also that the complementary set-classes 6-42 and 6-45 cannot be members of an octatonic scale. These hexachords frequently figure in the B section (Figure 6).

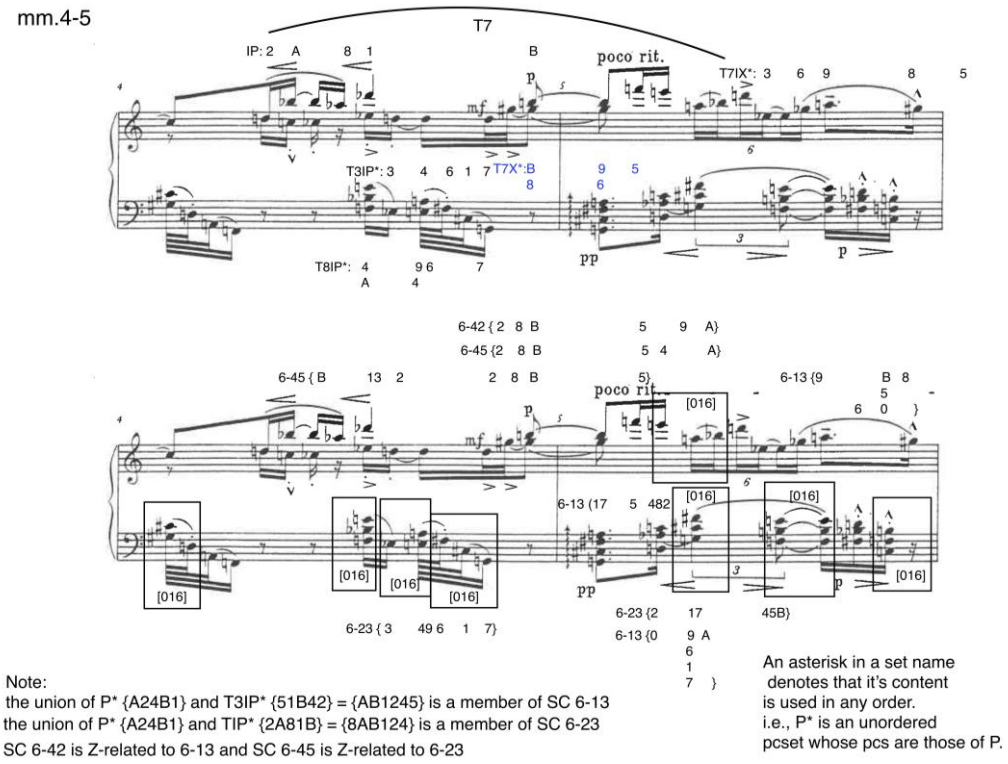

Figure 6: Hexachords 6-42 and 6-45 in the B section of Schoenberg's Op. 23/3.

Figure 6 shows the first B section of opus 23/3. The music is printed twice. The upper staves show how the use of P and X generate many of the notes. IP makes its appearance in m.4 Remember that union of P and IP generated the hexachord Y. Note that the upper staves shows that T7IY follows IY. This T7 move (recalling the opening relation between P and T7P) is important to what follows. Several other permutations of P and X accompany IP and T7IY.

The bottom part of Figure 6 shows the same music from the point of set-class membership. 6-13, the SC to which X belongs, occurs with many other hexachords, most saliently 6-23, the hexachordal set-class Y, which with 6-13 generates the octatonic scale. The note on the bottom left reiterates these hexachords' relations to P. The complements of 6-13 and 6-23, the set-classes 6-42 and 6-45, make their appearance in this passage. PCs in boxes show members of [016] a trichordal set class found in P, X and Y and their complements, but not in the tetrachordal generators (small) x and (small) y. In general, all the set-classes shown in the previous slide are implicated in this B section and they account for almost all the notes in the passage.

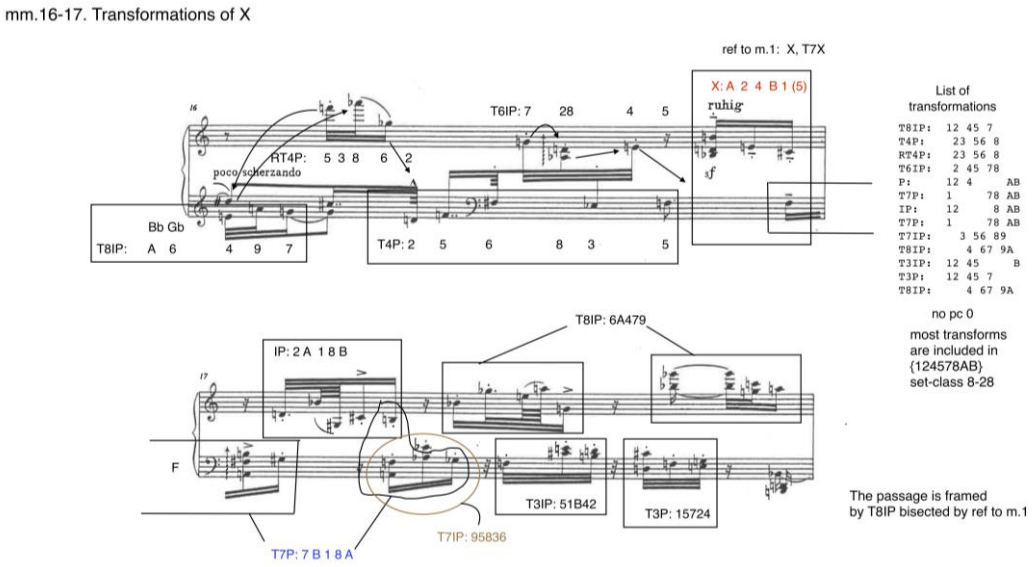

Figure 7: Pentachord P and its transforms in the middle A section of Schoenberg's Op. 23/3 (mm.16-17).

In contrast to the complexity of the B section, let us jump ahead to mm.16-17 in order to show a quite explicit usage of pentachord P and its transforms in the middle A section (Figure 7). The passage is framed by the T8I of P with an overt reference to P and X in the center of the passage. As shown, most of these transforms of P fit into a single octatonic scale and that the pc 0 is omitted from the passage. The colored members of P are important to the end of the piece, and they were presented at the opening of the composition.

The next set of Figures will show other relationships between P and its transforms.

Measures 12-13, feature P and its inversion (within T7IX) accompanied by a number of major and minor thirds (Figure 8). Transformations of P* are everywhere, but complexly intertwined. The presence of 6-23 is the only reference to hexachord Y. The texture of the left hand of m.12 will be extended later in the singular measures 26-29. Another forward reference is the presence of tetrachord x and its T4 transform as shown under the top system on the right. Three notes of the third tetrachord T8x are present, but the pc 7 is not. The fact that x and T4x and T8x can generate an aggregate implies that the set-class 4-10 is an all-combinatorial set-class.

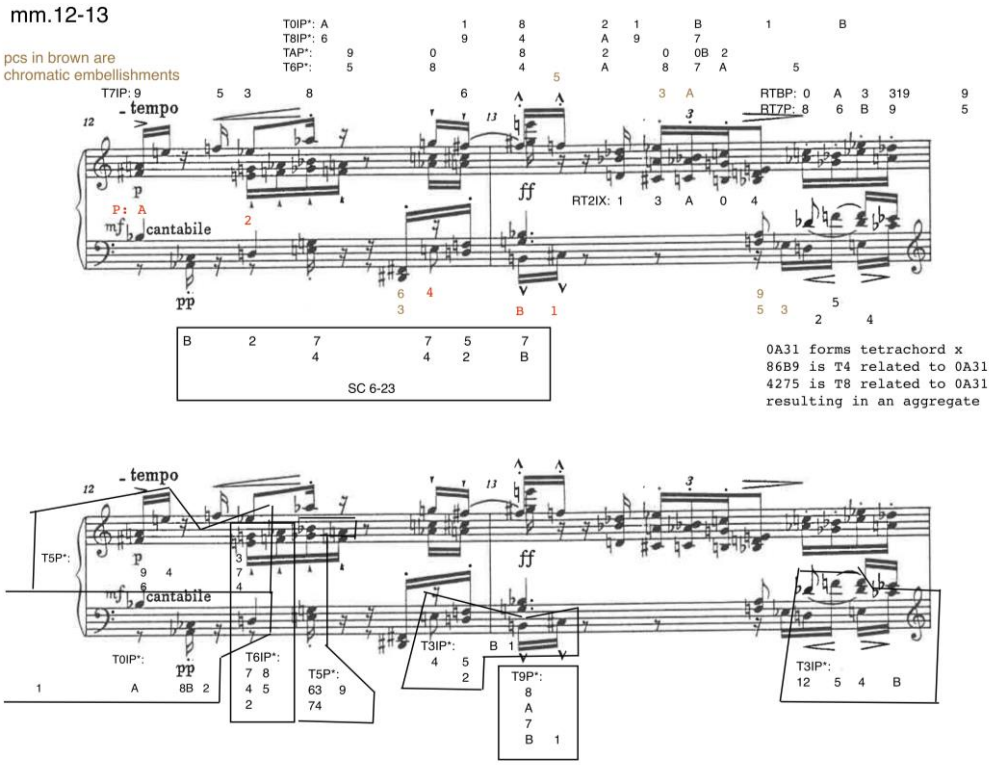

Figure 8: Pentachord P and its inversion (within T7IX) accompanied by a number of major and minor thirds of Schoenberg's Op. 23/3 (mm. 12-13).

The very next measures, 14 and 15, show how pairs of transforms of P* can generate chromatic sequences of thirds (Figure 9). This is given by the pcs above each of the two systems. Below the top system is a sequence of linked transforms of P constructing an RI-chain, the content of which is the octatonic scale 8-28. Other instances of P*'s transformations are also shown.

Continuing through the middle A section, we pass over measures 16 and 17, since we have already examined those measures. The end of measure 17 and measure 18 produce one of the pitch-class climaxes of the piece (Figure 10). Both the octatonic scale and the 12-tone aggregate are generated by P (with the help of X). First, the pcs in the left hand in m.17 and beginning of 18 produce a set of T3-related members of P that generates the octatonic scale; then, with the pcset {0369} in the right hand, an aggregate is obtained. Babbitt (2003, pp. 310-311) shows that an aggregate is generated in a different way in the left hand of m.18 by aligning three T4-related members of X (including P). The presence of tetrachord x in these pcsets forms this aggregate. We saw this possibility but not completely realized in measure 13. And the predecessor to the aggregate involving the octatonic scale was found in measure 15 where that scale was realized in the RI-chain in the left hand.

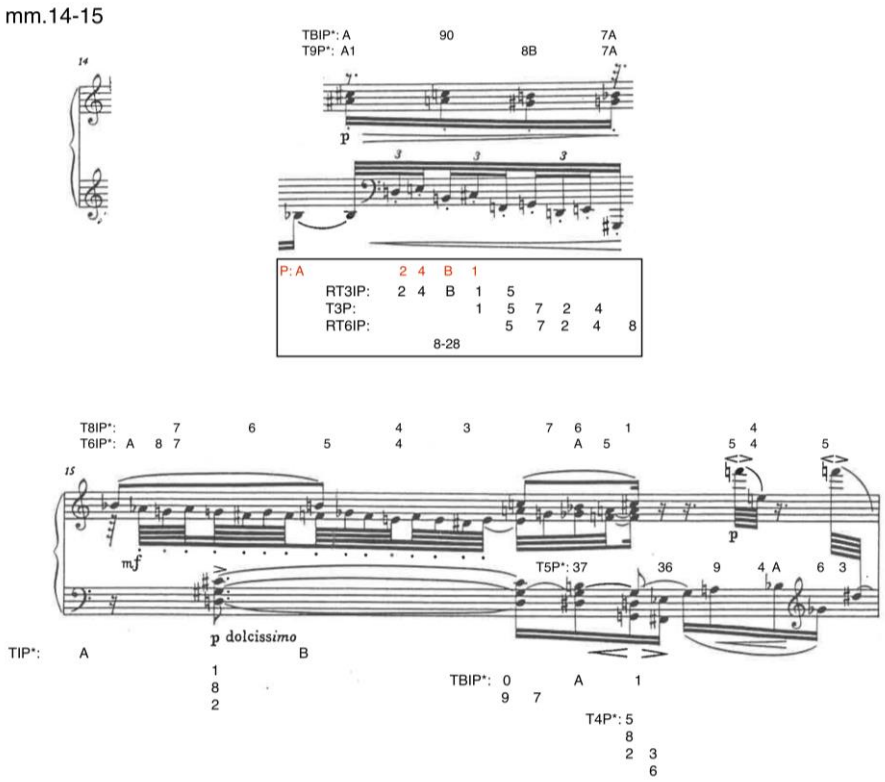

Figure 9: Pairs of transforms of P* generating chromatic sequences of thirds of Schoenberg's Op. 23/3 (mm. 14-15).

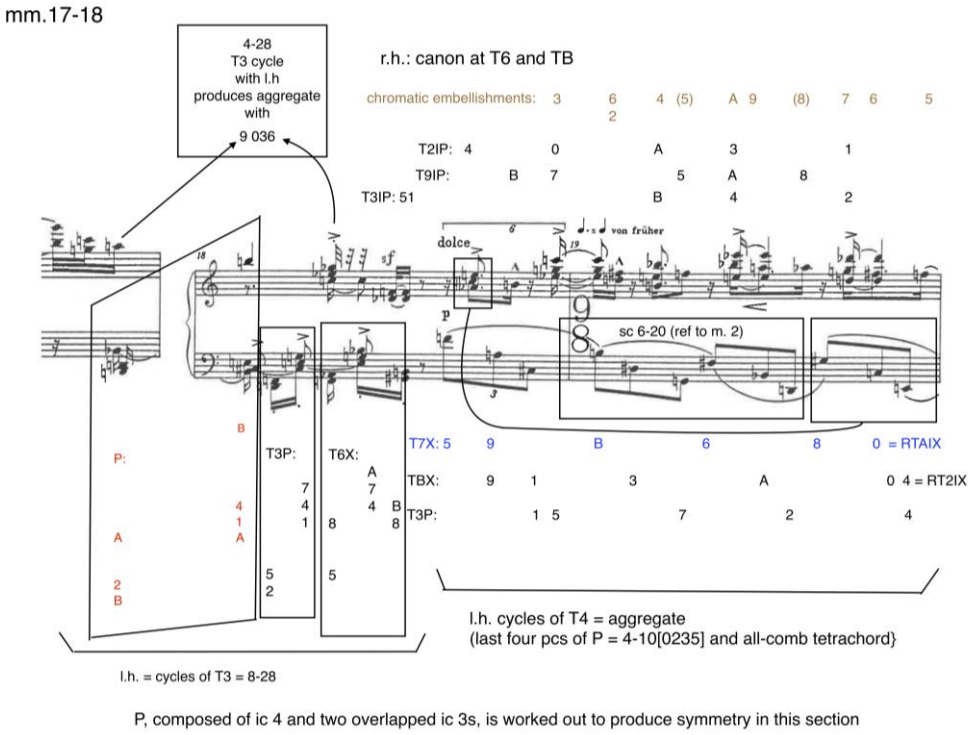

Figure 10: One of the pitch-class climaxes of the piece in which both the octatonic scale and the 12-tone aggregate are generated by P in Schoenberg's Op. 23/3 (mm. 17-19).

And above the aggregate formation in measure 18 we have a three-voice canon based on T3I, T9I, and T2I transforms of P. These three inversions of P allow the trichord [016] to dominate the texture but elaborated by chromatic embellishments. Finally, the presence of the set-class 6-20 references the same set at the opening of the composition. As amazing as this passage is, more examples of Schoenberg's compositional virtuosity are yet to come.

We pass over the second B section to the beginning of the A' section. At measure 26 it is clear that the sustained notes project a canon by inversion between P and T7IP; this is followed in measure 28 by a similar canon involving IP and T7P (Figure 11). These four transforms of P have occurred many times earlier in the composition and will form the basis for its concluding measures. The rest of the notes in the passage grouped into major and minor thirds have eluded many analysts but Roeder (1998, pp. 240-251) revealed that these thirds are derived from other members of P. Thus, this passage is a ten-voice canon with transpositions of P/IP in the upper staff and transpositions of T7IP/T7P in the lower staff. The reason that this ten-voice texture was not noticed is that many of the thirds, which should be all minor, are changed to major thirds. It is not clear if these changes are compositional errors or not. I think they are deliberate alterations for reasons of sonority and variety and were perhaps used to hide the all-too-systematic generation of the array of ten voices. At any rate, the end of the ten-voice array is composed out to produce an inversional symmetry around the pitches A and A♯. This is the relation between P and T7IP in measures 26 and 27. (A and A♯ are T7I related).

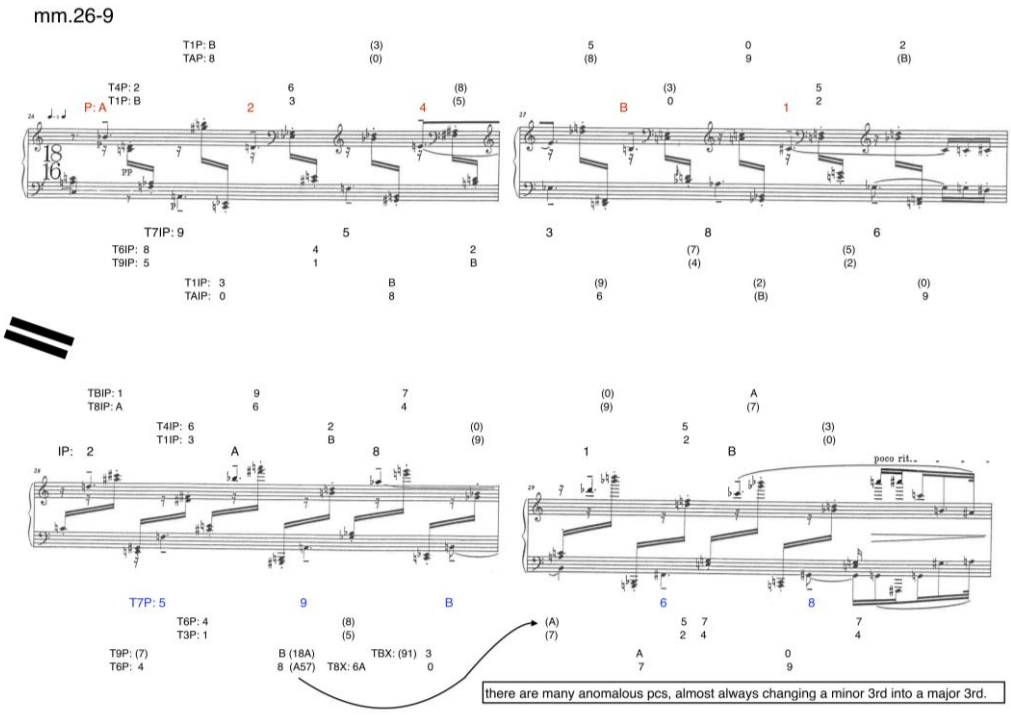

Figure 41: P-IP, T7IP-T7P inversional canon with 4 pairs of transformations of P, each pair at T3.

Now we come to the last measures of the piece (Figure 12). The first six notes are continuations of the 10-voice array symmetric around A/A♯. Note these notes are not only related under T7I but are pitch-mirrors around the pitches E♭4 and E4. The next four events are simultaneous presentations of the four versions of P used in the basic canon from which the 10-voice array was derived. However, these four chords are all accompanied by the pcs 0 and 7 and are mirrored in pairs around the E♭4 and E4 axis, or better, around C4 and G4. The red and blue chords are related to the first two versions of P at the opening of the piece.

As shown, the four chords without the 0 and 7 produce a symmetric arrangement implicating the transforms, T5, T7, T7I, I and T2I. The combination of P and IP produces X, the member of the octatonic hexachord 6-13. The combination of T7IP and T7P gives Y, the other octatonic hexachord, 6-23. The four chords in union produces a pcset that excludes the pcs 0 and 7, which together with the four chords produces a complete 12-pc aggregate. The last three measures of the piece oscillate arpeggiations of the four chords in a sort of swinging rhythm suggesting a scale coming to rest. Everything is pitch and pc symmetric, until the last three attacks in which the pitch symmetry is violated.

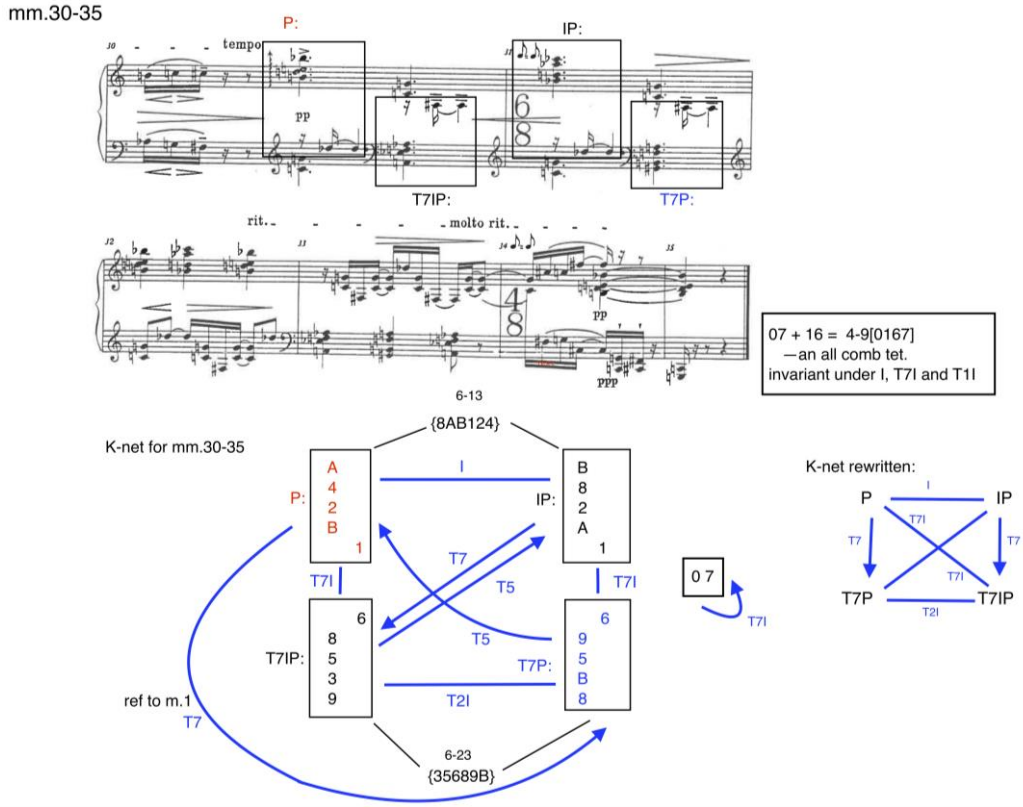

Figure 52: K-nets used in the last measures of Schoenberg's Op. 23/3.

In sum, opus 23/3 does not follow any fixed or traditional form. Rather, the piece begins as an exploration of the properties of P, the opening five-pc

motive, and its hexachordal extensions resulting in a series of episodes each displaying its implicit (partial) symmetries This continues until measure 18/19 at which point a concise presentation of the 12-pc aggregate occurs. Along the way the potential of the hexachords X and Y is realized in complete octatonic passages. The piece continues in its exploratory way, gaining rhythmic stability especially in the 10-voice array. After the array is completed, the music settles into complete pc and pitch inversional symmetry derived from the four most salient versions of the pentachord P. In this way I hear the work as formally unique, representing a generalization of the principle of developing variation in which the goal of this work is the stability of complete symmetry.

Thus opus 23/3's formal process is a microcosm of Schoenberg's compositional trajectory from "working with the tones of a motive" to his "method of composing with twelve tones which are related only with one another."

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2

Symmetry and the Musical Idea in Schoenberg's Op. 33a Piano Piece

Jack Boss

Schoenberg's Piano Piece Op. 33a has long presented a challenge to scholars, with its frequent reorderings of, additions to, and omissions from the tone row. My presentation will show how these row-counting deviations are not accidental, but motivated by an overarching narrative of conflict, elaboration, and resolution, akin to what Schoenberg himself called "musical idea." Many scholars agree that Op. 33a is framed as a sonata movement. The first theme of the exposition consists of a vertically- and horizontally-symmetrical interval pattern that necessarily obscures row order (the conflict), immediately abstracts it to form set-class symmetry, and then progressively deforms both the interval and set-class patterns (the elaboration of the conflict), by means of the note reorderings and additions about which writers on this piece have commented. The second theme then proposes another kind of symmetry, pitch symmetry, but that also is shown to be inadequate in the closing theme, as the pitch symmetries lead to incomplete rows through omission of notes. After a development section in which Schoenberg gradually builds the first theme's symmetrical interval pattern back up, he returns to a modified version of it in the recapitulation that allows some symmetry to persist, despite presenting the basic row in correct row order for the first time in the piece (the resolution of the initial conflict).

Back in 1992, I published an article on Schoenberg's music that applied concepts from his theoretical writings to the analysis of his music. ("Going back to the composer's own words" has always been sort of an overarching theme in my work.) The article was called "Schoenberg's Op. 22 Radio Talk and Developing Variation in Atonal Music" (BOSS, 1992). It took a number of his ideas about "developing variation" and adapted them for the motivic analysis of one of his atonal songs. But one concept, closely related to developing variation, that I was having a little bit of trouble figuring out was his concept of "musical idea" or *musikalische Gedanke*, as he called it in German. Many scholars who used the term "musical idea" back in the 1990s wanted to limit it to musical motives or themes, while others wanted to understand it as some vague expression of the "essence" of a piece of music. But I was seeking a broader, yet more specific meaning for musical idea: I wanted to use it as a large analytic framework to organize the details of a whole piece.

In 2014, Cambridge University Press published my book, titled *Schoenberg's Twelve-Tone Music: Symmetry and the Musical Idea* (BOSS, 2014). The book explains Schoenberg's "musical idea" as a three-part process—a musical problem of some kind is presented at the beginning, that problem is elaborated through the course of the piece, and then there is a solution of the problem at or near the end. Very often, the problem at the beginning of a piece involves some kind of symmetrical pitch or interval pattern that is either presented outright or suggested, and then gradually obscured (that is why I added the word "symmetry" to the title).

The elaboration of the problem involves Schoenberg replacing the first symmetrical pattern with a different one, a sort of "alternative fact" if you will. But, at or near the end of the piece, the first pattern returns, and shows how the forces that had obscured it, including the "alternative fact" pattern, can be used to support it (I should mention at this point that I had a very important model for this interpretation of "musical idea": the work Patricia Carpenter of Columbia University had done on the analysis of tonal music using Schoenberg's concept, together with her students, particularly Severine Neff and Murray Dineen.[1] Carpenter was herself a student of Schoenberg's, during his time in America in the 1930s and 40s).

Schoenberg's Piano Piece Op. 33a is a concise example of a musical idea that involves obscuring, replacing, and recapturing a symmetrical pattern; and as such, I'd like to use it to introduce my understanding of "musical idea" to you.

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[1] Some representative articles and chapters: CARPENTER (1983; 1984; 1988); CARPENTER; NEFF (1997); NEFF (1984; 1993; 2000); DINEEN (2005a; 2005b).

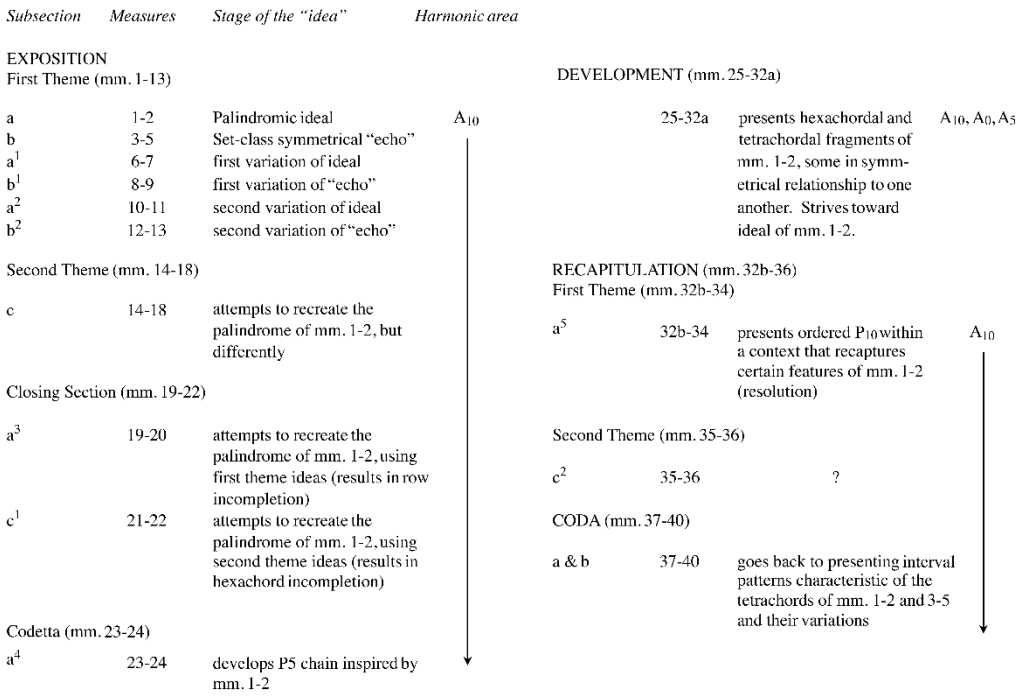

Figure 1: Form chart for Schoenberg, Piano Piece, Op. 33a.

My Figure 1 is a form chart for Op. 33a, with section letters, measure numbers, a description of each stage of the idea the piece goes through, and the "harmonic area" (a "harmonic area" consists of a tone row, its inversionally-combinatorial partner, and their retrogrades. This piece mostly uses harmonic area A_{10}, which includes P_{10}, I_3 and their retrogrades. But it does "modulate" to other harmonic areas in the middle).[2] As you can see, I understand Op. 33a as a sonata form—exposition, development and (condensed) recapitulation.[3] The first theme is where the initial symmetrical pattern is presented and obscured, the second theme and closing theme present the alternative pattern, the development strives step-by-step to get back to the original symmetry, and the recapitulation recaptures it and resolves a problem that the initial pattern had created at the beginning. Let us consider the details of how that happens.

We will start with the opening five measures, Figure 2. The first two measures present a symmetrical shape that I call the "palindromic ideal." What I mean by that is that the six chords create a series of intervals that is both horizontally and vertically symmetrical (represented by numbers in bold): from the bottom, 1-5-5, 4-2-3, 6-2-3, 3-2-6, 3-2-4, 5-5-1. The first and sixth chords mirror

[2] The concept of harmonic area was introduced by David Lewin in LEWIN (1967, pp. 18-19).
[3] This reading of the form is similar to that in PERLE (1991, p. 113).

each other vertically, as do the second and fifth, and third and fourth (a quick side note here: throughout the paper, I will be counting vertical intervals in half-steps with no plus or minus sign; horizontal intervals will be counted in half-steps with a plus or minus sign to indicate direction).

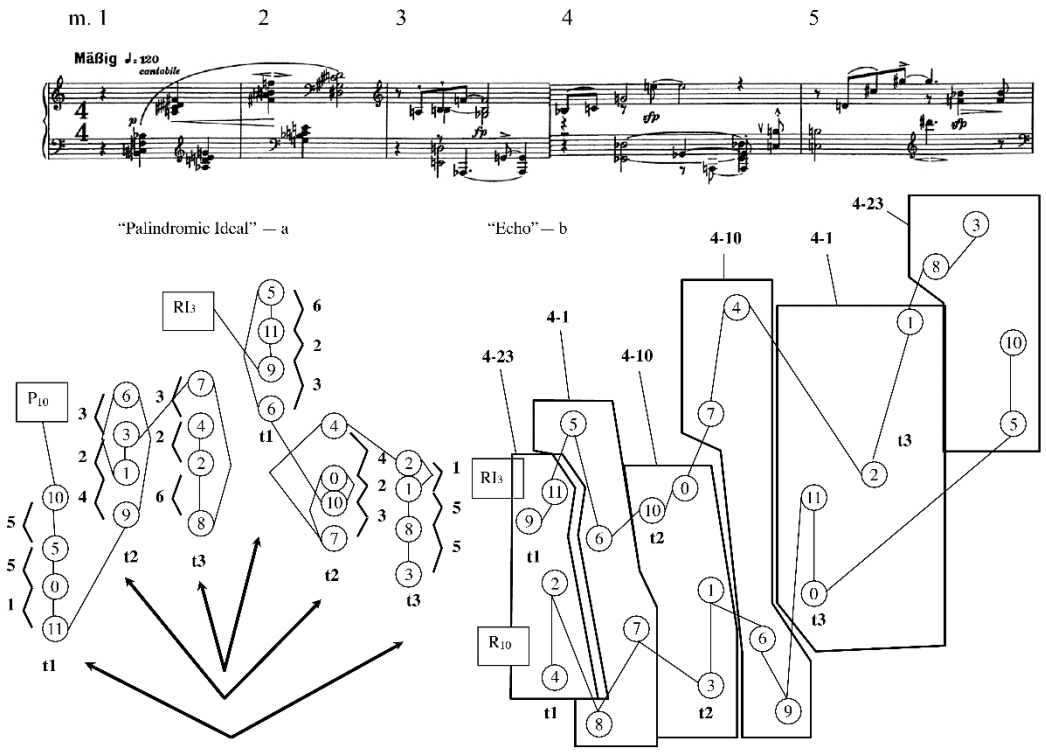

Figure 2: Schoenberg, Piano Piece, Op. 33a, mm. 1-5 (first theme, sections a and b).

The other thing you should notice about mm. 1-2 is that to create all this intervallic symmetry, it is necessary for Schoenberg to jumble the order of his rows. P_{10} takes the row out of order in the second and third chords, as you can see from the crossed lines in the pitch class map, and RI_3 changes row order in the first and second chords. Herein lies the problem—if we want symmetrical intervals, we have to change the order of the row. Or do we?

Measures 3-5 suggests that we do; that row order and symmetrical intervals are not compatible. In this passage, the rows go in order: RI_3 mostly in the right hand, and R_{10} in the left hand (in the score, they cross over one another at the end). And the intervals created vertically do not yield a symmetrical pattern like that of mm. 1-2. Instead, we have a weaker kind of symmetry: the *set-classes* in mm. 3-5 are horizontally symmetrical. I call this the "echo." 4-23 (0257) begins, then 4-1 (0123), 4-10 (0235), 4-10, 4-1, and 4-23. I think this symmetry is audible: it gives us a perfect fourth chord at the beginning and the

end of the passage, and chromatic sets as the second and fifth chords. But it certainly seems more abstract than the symmetries of mm. 1-2.[4]

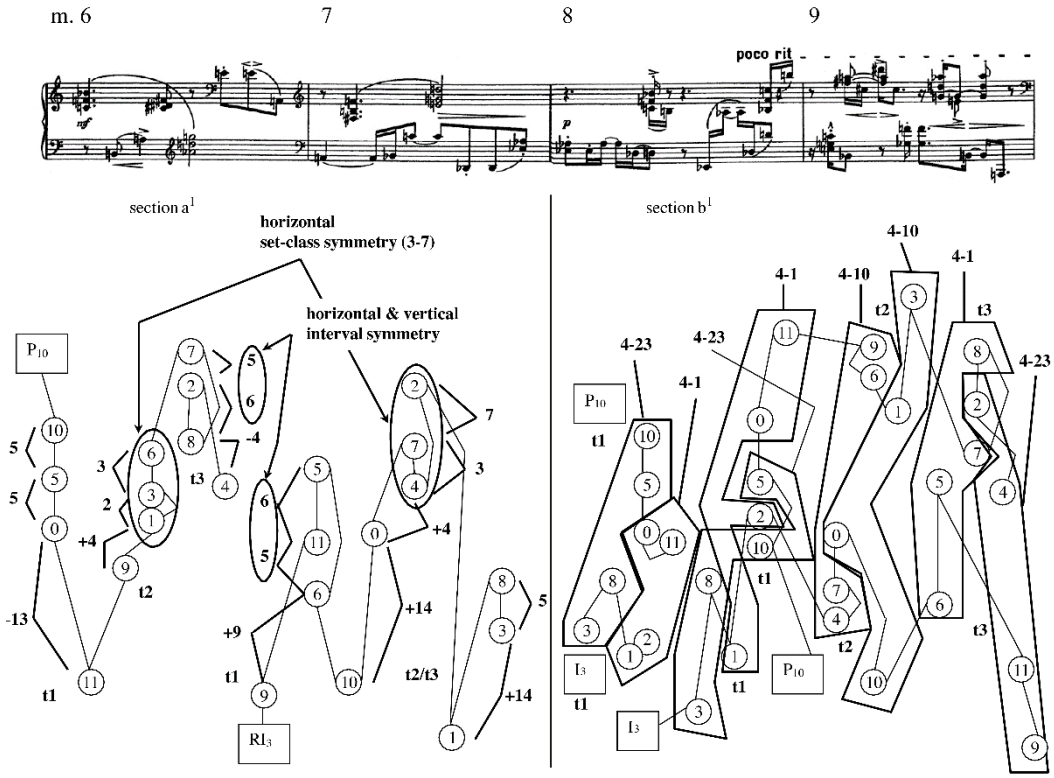

Figure 3: Schoenberg, Piano Piece, Op. 33a, mm. 6-9 (first theme, sections a^1 and b^1).

The measures following mm. 1-5, measures 6-13, contain two variations on the "palindromic ideal" and "echo," which increasingly disrupt their intervallic and set-class symmetries through rhythmic displacement—moving certain notes forward in time and others back. Let's look at the first variation, mm. 6-9. Figure 3 shows how these measures obscure the symmetries of mm. 1-5 through rhythmic displacement, as well as moving certain notes up or down by octave. In m. 6's variation of the first chord of m. 1, C3, F3 and B♭3 rise an octave and B2 is delayed an eighth note, forming unordered pitch-interval stack <5, 5> followed by -13 (this is in the place of m. 1's <1, 5, 5>). The second chord delays its top three notes by an eighth, changing <4, 2, 3> to <+4, 2, 3>. And the rhythmic displacements and octave transfers carry on through the remaining four chords, changing what had been six horizontally- and vertically symmetrical tetrachords into six conglomerations of chords and melodic intervals, all of which

[4] Previous writers who have taken note of and discussed the set-class symmetrical "echo" in mm. 3-5 are Graebner (1973-74, p. 134); Straus (2005, pp. 254-58); and Babbitt (1987, pp. 75-78). Babbitt's account of the first five measures is interesting in that it reverses the priority between what I call the "ideal" and the "echo," claiming that the set-class symmetry of mm. 3-5 is a "deep structural" idea that is "reflected" on the surface of the music in mm. 1-2.

have unique interval patterns, and some of which are not even tetrachords (at the end of m. 7).

Measures 8-9 perform the same kind of obscuring function, through rhythmic displacement of the set-class symmetrical "echo" of mm. 3-5. To create an analysis that looks something like the original passage's palindrome (4-23, 4-1, 4-10, 4-10, 4-1, 4-23), I had to group notes from different parts of the measure and from overlapping parts of the texture, creating a pitch-class segmentation that looks a lot like the "gerrymander" from American partisan politics (a "gerrymander" is the practice of drawing the lines of congressional districts so that the majority of the voters in the district belong to the same political party. This can result in some pretty tortured-looking shapes). Even with the gerrymanders, though, the sequence of set-classes in mm. 8-9 does not form a pure horizontal mirror: the initial pair (4-23 and 4-1) repeats. Through rhythmic and registral changes, but also through repeating parts of the row out of order, Schoenberg is obscuring the perfect and imperfect symmetries of his opening.

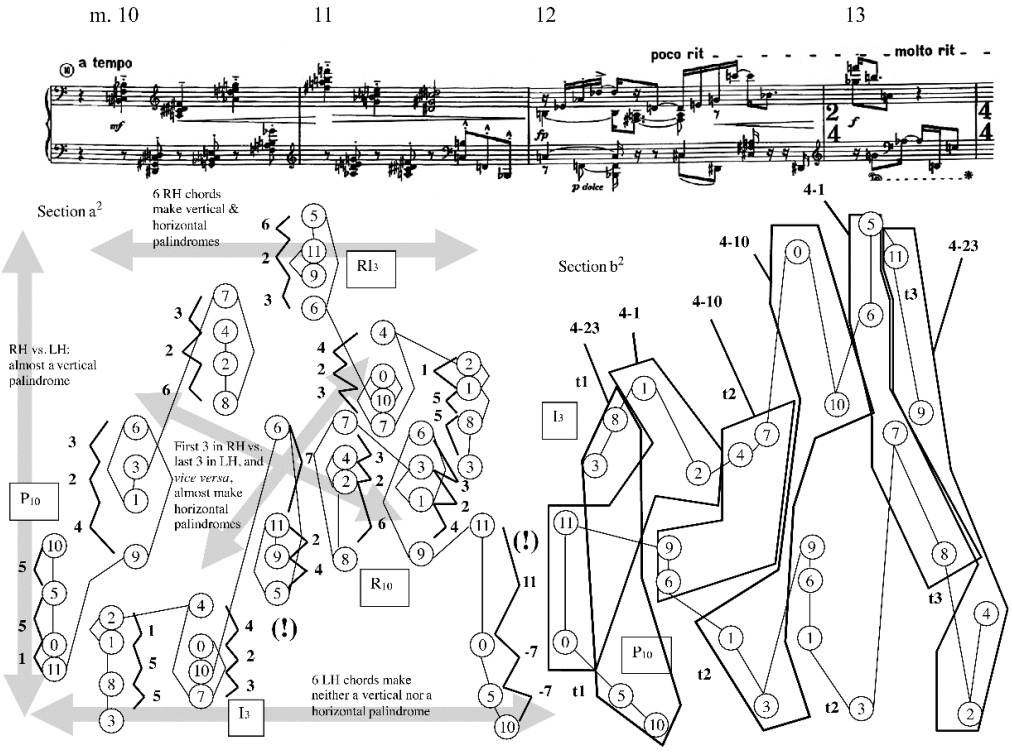

Figure 4: Schoenberg, Piano Piece, Op. 33a, mm. 10-13 (first theme, sections a² and b²).

The second variation of mm. 1-5, found in mm. 10-13, is illustrated in Figure 4. The first two measures of this passage take a step back toward the original "palindromic ideal" of mm. 1-2 in one way, but lead us further away from it in another way. Essentially, the right hand reproduces the six horizontally- and vertically symmetrical chords from mm. 1-2: 1-5-5, 4-2-3, 6-2-3,

3-2-6, 3-2-4, 5-5-1, while the left hand starts to mirror that intervallic pattern virtually, but goes astray on the third and sixth chords (the places where the vertical symmetry goes astray are marked with exclamation points). The left hand in m. 10 goes 5-5-1, 3-2-4, both vertical mirrors of the chords above; then 4-2-7, which is *not* a vertical mirror of 6-2-3. Likewise, the left hand in m. 11 starts with 6-2-3 and 4-2-3, again both vertical mirrors of the chords above, but ends with a vertical 11 followed by two ordered -7s, descending perfect fifths. This is not a vertical mirror of 5-5-1 in the right hand. The left hand almost seems to be teasing the listener, by nearly finishing a symmetrical shape and then turning away from it at the last minute.

This is followed in mm. 12-13 by more gerrymanders, some of them even more tortured than the ones in Figure 3. This makes the "echo's" palindrome of set classes harder and harder to hear. To summarize, then, over the first 13 measures, which I suggested earlier correspond to the first theme of a sonata form, the problem regarding symmetry and row order (and whether they can coexist) is presented, then steadily made worse.

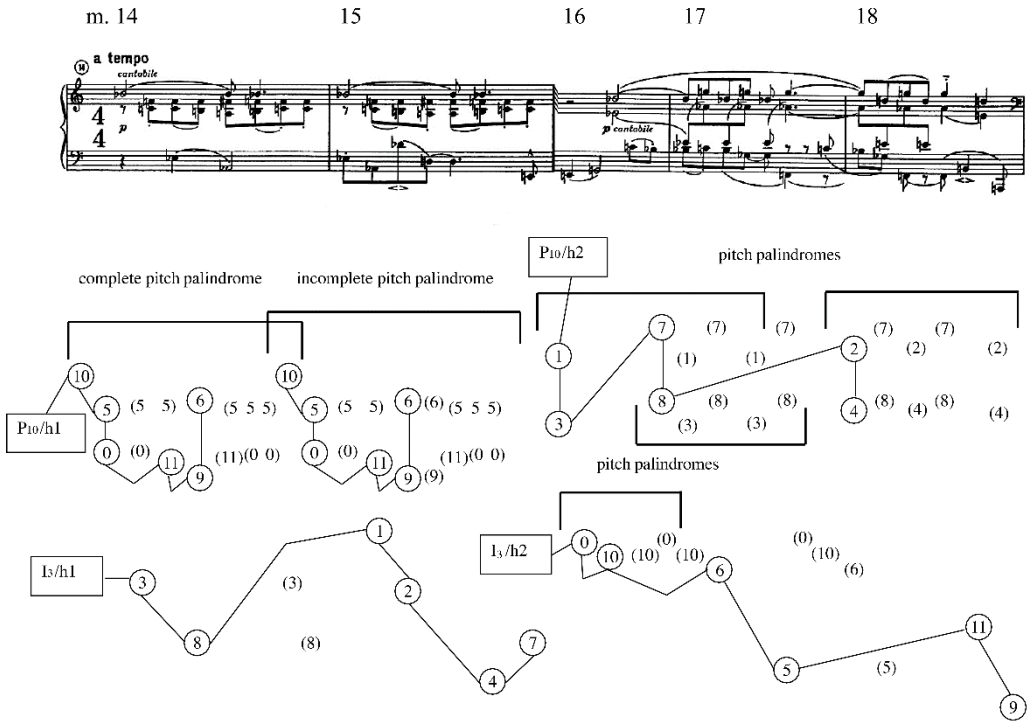

Figure 5: Schoenberg, Piano Piece, Op. 33a, mm. 14-18 (second theme; section c).

In response, Schoenberg suggests a different kind of symmetry in the second theme, perhaps as a potential solution to the first theme's problem. This is shown in Figure 5. In this example, the right hand provides several horizontally symmetrical *pitch* structures: first 10-5-0-11-6-9-11-0-5-10 in m. 14, which repeats

in m. 15, then 1-3-7-8-3-1 in mm. 16-17 and 2-4-7-8-4-2 in m. 18.[5] A smaller palindrome, 0-10-10-0, appears in the left hand of mm. 16-17. All of these palindromes are marked with brackets. But this type of symmetry doesn't solve the problem either; it doesn't promote the row order. Because of the very nature of pitch palindromes, the rows turn around in the middle and fail to complete themselves (that is, until later in the passage).

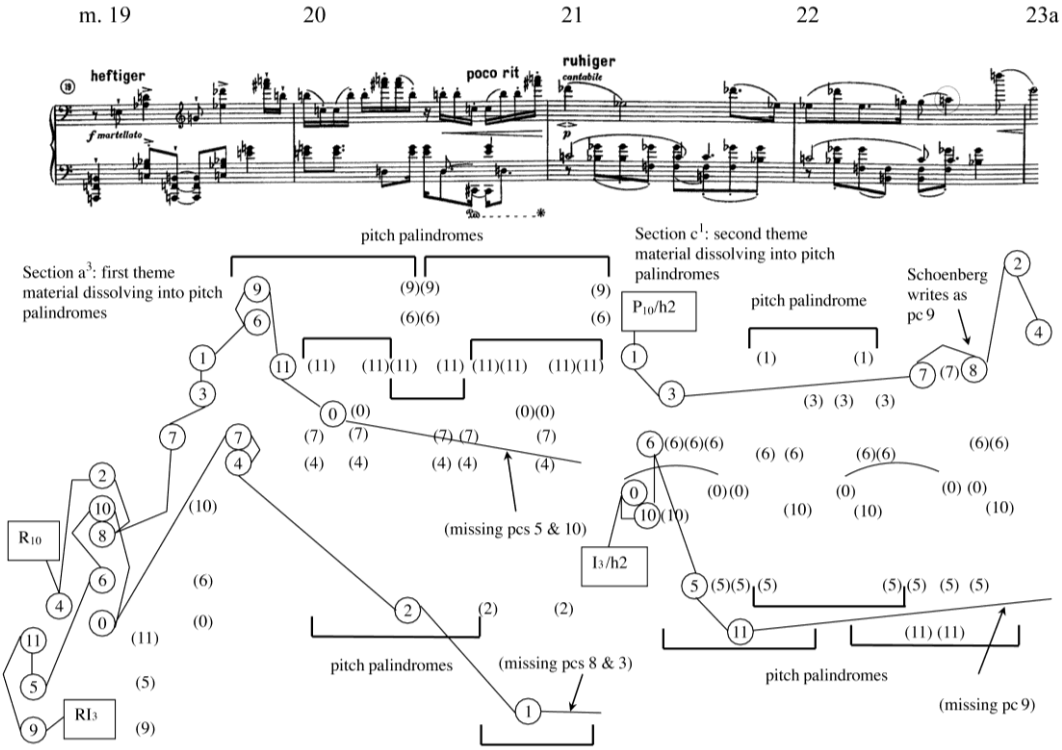

Figure 6: Schoenberg, Piano Piece, Op. 33a, mm. 19-23a (closing theme; sections a^3 and c^1).

In fact, this second type of disagreement between horizontal pitch symmetry and row order becomes even more destructive to the order of the row in what I call the closing theme, mm. 19-23. Let's review Figure 6. Here, especially if you look at mm. 19 and 20, you can see that neither R_{10} in the right hand nor RI_3 in the left hand can complete itself. R_{10} stops short before its last two notes, pitch classes 5 and 10, and RI_3 can't complete itself either: pitch classes 8 and 3 are missing. The reason these rows are incomplete is that the previous notes turned back on themselves to create pitch palindromes, 9-6-11-0-11-6-9 in the right hand and 7-4-2-4-7 in the left hand. This demonstrates that horizontal pitch symmetry is certainly not an acceptable solution to our problem, because it not only destroys the row order, but also makes the rows incomplete! The dynamics

[5] All of these contain a vertical at their center; <6, 9> in the first and <7, 8> in the second and third, and since these notes occur simultaneously, the three sequences can still be heard as palindromes.

and articulation markings in these measures seem to me to be reinforcing this point: *forte, martellato,* with accents and wedge accents in m. 19 and staccato marks in m. 20. It is almost as if the music is expressing exasperation, even anger, at its ongoing conflict between symmetry and row order.

I should pause my analytic tour at this point, to mention that this closing theme, mm. 19-23a, has caused a fair amount of disagreement among scholars who have attempted to row-count the piece in the past. David Lefkowitz (1997, pp. 75-78), John Glofcheskie (1976, pp. 95-97), Edward Cone (1972, p. 74), and Kathryn Bailey (1976, pp. 46 and 57) have suggested a number of different rationales for the palindromes and missing notes I have just described, and Bailey goes so far as to suggest that perhaps Schoenberg may not have understood the 12-tone system completely. But I believe that it makes more sense to interpret the row-counting anomalies in these measures in terms of the piece's "musical idea." At this point in the large narrative, he's demonstrating that horizontal pitch symmetry is inferior to horizontal and vertical intervallic symmetry, because it takes us farther away from the ideal of uniting symmetry with row order (he's also expressing some exasperation over that fact). Now, I can't say with complete certainty that it was Schoenberg's *intention* to demonstrate all this in the closing theme; but there are some interesting features of Schoenberg's sketches for Op. 33a that seem to show a cause-and-effect relationship between the pitch palindromes of the closing theme and its incomplete row forms.

The sketch reproduced as Figure 7a includes an earlier conception of mm. 19-23, which breaks off after 7 measures and 1 beat (I enclosed this part of the sketch in a box in my example).[6] Figure 7b transcribes the first 3 measures of this conception, which correspond to mm. 19 and 20 in the final version. You can see from the pitch-class map on the bottom of Figure 7b that, in the earlier version of this passage, both R_{10} in the right hand and RI_3 in the left hand complete themselves: pitch classes 10 and 5 appear on the second beat of the example's third measure, and pitch classes 8 and 3 appear on the downbeat of the same measure. The reason the rows can finish themselves in the sketch is that neither hand creates the more extensive kinds of pitch palindromes that we saw in the final version of the closing theme. The right hand has one five-interval sequence in the example's third measure that would have been symmetrical if pitch classes 6 and 9 were disposed vertically, and the left hand has a three-note configuration involving pitch classes 2 and 1 in the second measure. It may be dangerous to assert this, but I believe that since the idea of creating more extensive palindromes in the closing theme apparently occurred to Schoenberg at the same time as the notion of leaving R_{10} and RI_3 incomplete, I can argue (or at least

[6] The sketch can be viewed at the Arnold Schoenberg Center website, by going to www.schoenberg.at and searching the database of sketches under *Klavierstück* Op. 33a for "Skizzenblatt," MS37_28.jpg (Seite 1).

suggest) that he too may have thought of the increased pitch-palindromic activity as the cause for the rows' incomplete nature.

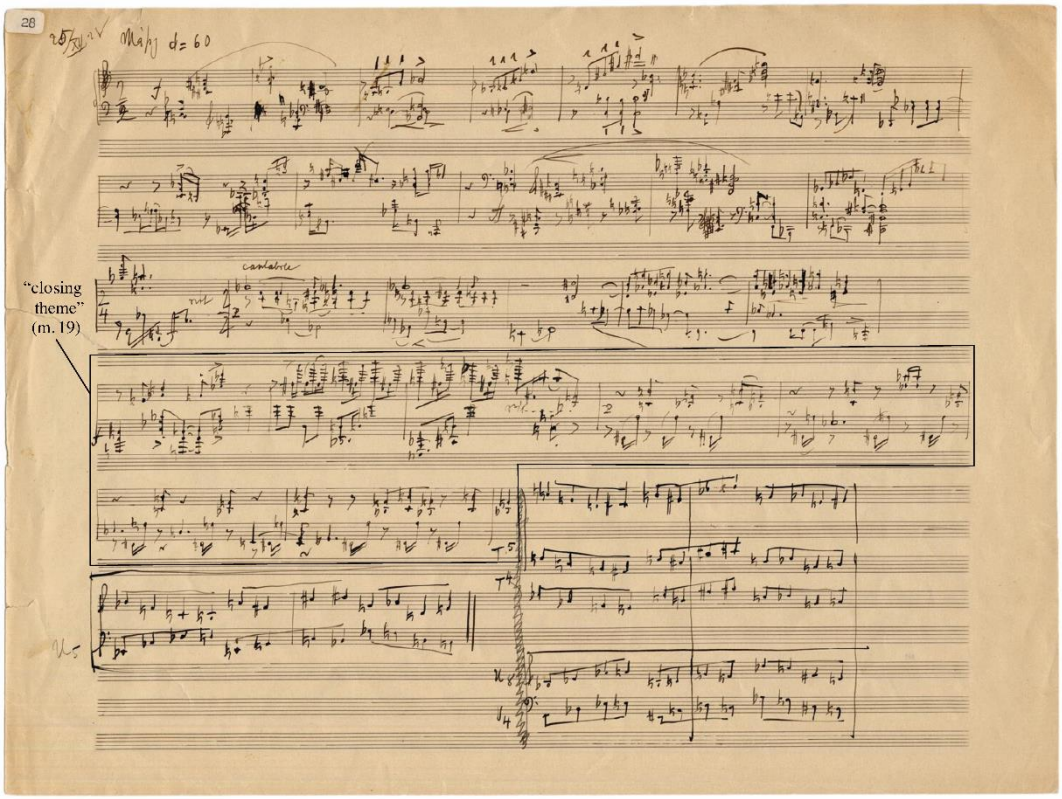

Figure 7a: Schoenberg's original sketch for the opening of Op. 33a.

So far the exposition of this sonata form has shown us that neither intervallic symmetry (in the first theme) nor pitch symmetry (in the second and closing themes) can be reconciled to the order of the row. Intervallic symmetry jumbles the order of the row, while pitch symmetry not only reorders it but leaves it incomplete. So what *is* the answer to Schoenberg's question—how can symmetry and row order coexist? In the recapitulation section at m. 32b, he will solve the problem using a variation of the original palindromic ideal from mm. 1-2. Before that happens, though, we have a development section from mm. 25b to 32a, shown in Figures 8 and 9. The main characteristic of the development is that it rebuilds the six intervallically-symmetrical chords of the "palindromic ideal" up from larger and larger fragments. In addition, it displays characteristics that mark it as parallel to development sections in classical sonata form. It "modulates" from harmonic area A_{10} to A_0 and ends on a prolongation of A_5, which could correspond to a progression in B♭ major from I to ii, with a "dominant pedal" or "dominant lock" on V. And the fragmentation I just mentioned recalls similar fragmentation techniques in developments of Mozart and Beethoven.

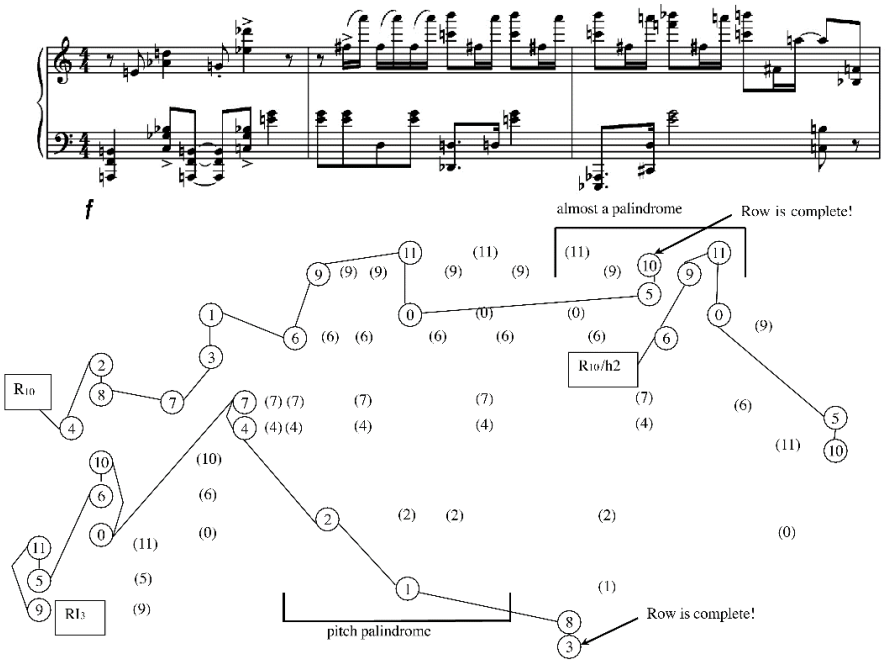

Figure 7b: Transcription of Schoenberg's original version of the closing theme, first part (original mm. 19-21, which correspond to mm. 19-20 in the final version).

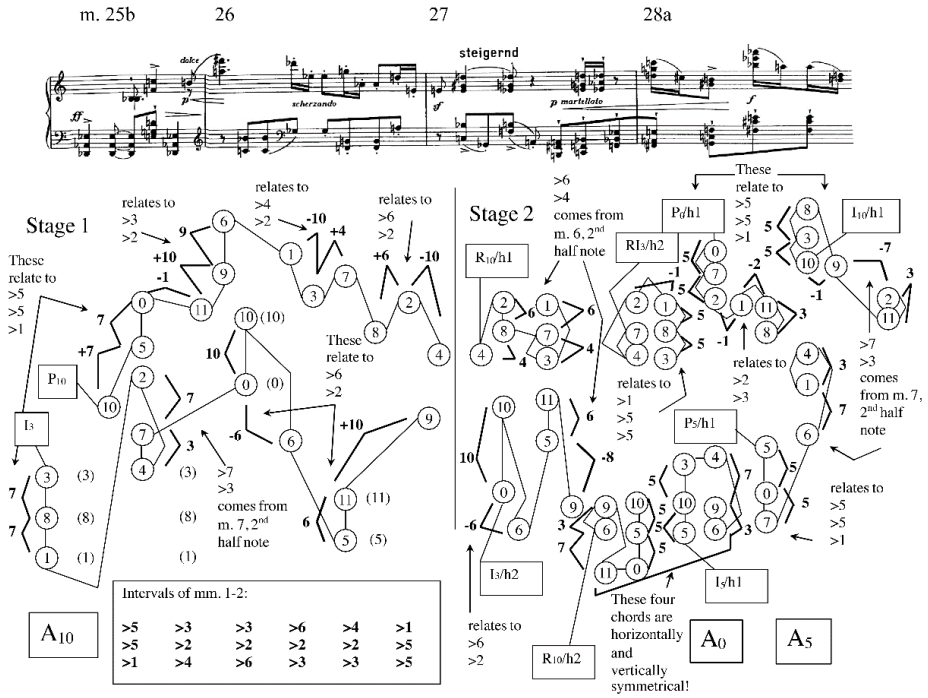

Figure 8: Schoenberg, Piano Piece Op. 33a, mm. 25b-28a (Development, first part).

I split the development into 4 stages: the first two are shown in Figure 8. In Stage 1, we have smaller fragments of the "palindromic ideal" that mostly consist of only two intervals, and mostly don't pair with one another to form symmetries. For example, the perfect-fifth stacks that begin both left and right

hands in m. 25b, 7-7 and +7, 7. As I indicated, these fragments recall the first of the six chords of the palindromic ideal, 1-5-5 from the bottom up; though if you add the subsequent descending half-step -1 to the second fragment, there is perhaps a suggestion of the sixth chord from mm. 1-2 as well. Other pairs of intervals later in the first two measures invert fragments of the original six chords: +10, 9 in the right hand inverts 2, 3 and the -10, +4 that follows it inverts 2, 4. The rest of measure 26 presents three renditions of a fragment from chord 4: 10, -6; 6, +10; and +6, -10 all relate back to 2, 6.

Stage 2 of the development begins to work with three-interval configurations, as well as some two-interval ones, and starts to place them in vertically- and horizontally-symmetrical pairs and groups of four, taking us one step closer to the original ideal. Most interesting in this regard are the four wedge-accented chords in the left hand at the end of m. 27 and beginning of m. 28. Their intervals go 7, 3; 5, 5; 5, 5; and 3, 7—a horizontally- and vertically-symmetrical group of four that uses parts of the original first and sixth chords, the 5, 5s; and combines those with another pair of intervals, (7, 3), that first appeared in m. 7. Directly above these four chords in the right hand, we see a descending -1 leading into a stack of two interval 5s, followed immediately at the beginning of m. 28 by a stack of two interval 5s followed by a descending -1. This pair of three-interval configurations exhibits vertical symmetry in the sense that the descending half step moves from above the two interval 5s to below them, and horizontal symmetry in the sense that the half step moves from before the interval 5s to after them. It also includes all three intervals of the first and sixth chords of the "palindromic ideal". The example ends in the middle of m. 28 with two interval pairs that exhibit vertical symmetry: -7 above leading into 3 below in the right hand, and 7 below with 3 above in the left.

Stages 3 and 4 are shown in Figure 9. In the right hand of mm. 28b and 29a there are four interval configurations that demonstrate horizontal symmetry and recall even more of the original palindromic ideal. First, we see a stack of two interval 5s followed by a horizontal -1 below its bottom note, followed by 10, +6; then +6 leading into a vertical 10, and finally the stack of two interval 5s leading into the horizontal -1 below again. Now, the third and fourth chords don't mirror the first and second ones vertically, but these chords still do take us a step closer to the palindromic ideal, by referencing more of the intervals in the original collection. The two 5, 5, -1s suggest the first chord of that six-chord sequence, and the chords in the middle with the vertical 10 and +6 relate to a fragment of the fourth chord: 2, 6.

The left hand in m. 28b and 29a also creates some vertical symmetries with the right hand above it, taking us back to the vertically-mirrored presentation of the palindromic ideal that couldn't complete itself in mm. 10-11. At the beginning of the example in the left hand, we hear a stack of two interval 5s with a horizontal +1 above the top note, vertically mirroring what the right hand is

playing at that moment. And, the downbeat of m. 29 in the left hand creates a vertical mirror, 6-10, with the 10, +6 straddling the same barline in the right hand. However, this increase in horizontal and vertical interval symmetries in Stage 3 is interrupted by a *pitch* palindrome at the end of the stage, the beginning of m. 30 in the right hand. A stack of pitch classes 0-6-1 leads to pitch class 4, then a second 4 leads back to 0-6-1. This briefly recalls the disruptive influence of the second theme and closing theme sections.

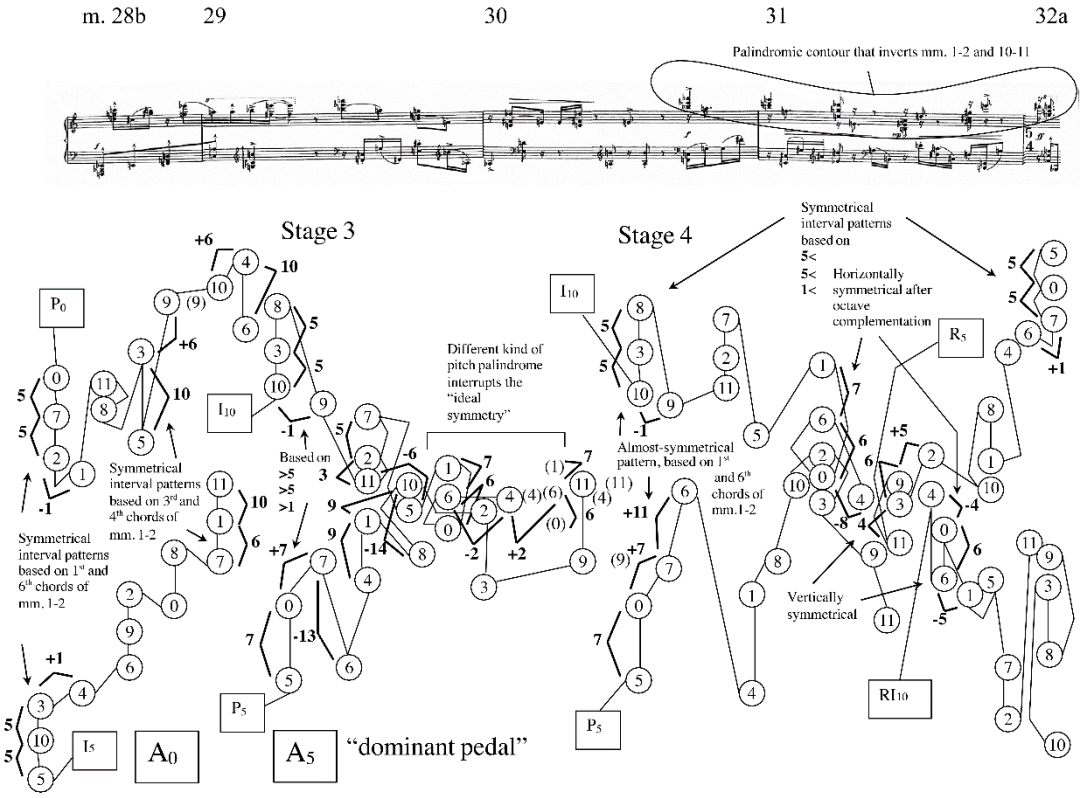

Figure 9: Schoenberg, Piano Piece Op. 33a, mm. 28b-32a (Development, second part).

With Stage 4, the piece returns to creating larger conglomerations of horizontally- and vertically-symmetrical intervals, within and underneath a palindromic contour that recalls the whole arch shape formed by measures 1-2 by inverting it (circled in the score part of Figure 9). At the ends of the contour pattern I just traced, we have interval patterns that mirror each other horizontally: the first begins with a stack of two interval 5s followed by a descending half-step -1 below, while the final one features an ascending half step +1 below that leads into a stack of two interval 5s. These both correspond to the first chord of the palindromic ideal, but in a way that suggests the full six-chord pattern. Within the boundary created by these two 1-5-5s, we also hear a vertically-symmetrical pair in both hands of the piano on the 3rd beat of m. 31. The interval stack 4-6, with +5 above leads to -4, 6, with -5 below. Thus, in mm. 30b-32a, we have a sequence of 4 three-interval chords that demonstrate

horizontal and vertical symmetry of different kinds. But, in addition, the left hand on the 4th beat of m. 30 produces the intervals 7, +7, +11, a vertical mirror of the 5, 5, -1 in the right hand above it, with all three intervals inverted. This again suggests an attempt to recapture the double "palindromic ideal" which almost came to fulfillment in mm. 10 and 11.

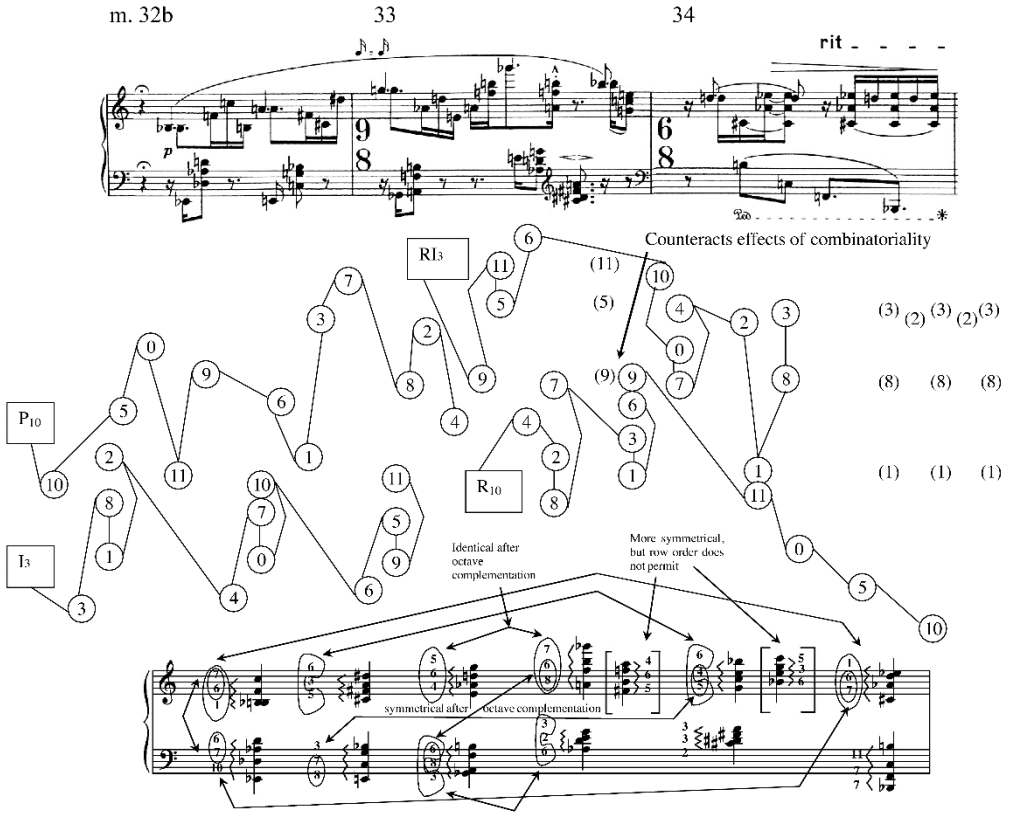

Figure 10: Schoenberg, Piano Piece Op. 33a, mm. 32b-34 (Recapitulation, first theme; section a[5]).

This gradual process of building up horizontally- and vertically-symmetrical interval patterns from smaller fragments (with one interruption) reaches its culmination in the recapitulation, which is illustrated in Figure 10. Here, in mm. 32b and 33a, for the first time in the piece, we have the basic row form, P_{10}, *in order* in the right hand (when I taught this piece to my undergraduates, my first advice to them was "go to m. 32 to start your row count"). Below P_{10} we hear I_3, mostly in order. They are followed by RI_3 in the right hand in mm. 33b and 34, and R_{10} in the left. At the same time, however, the verticals that are created by taking each four-note group in these four rows and listing the notes from lowest to highest in registral order display some interesting symmetrical patterns. I created verticals in this way at the bottom of Example 10; you can see that that the first chord in the right hand (the first four notes of P_{10}) has the intervals 1, 6, and 7 from the bottom, while the last chord in the right hand (formed from the last four notes of RI_3) has the intervals 7, 6 and 1. This is

the same relationship we saw between first and sixth chords in mm. 1-2, vertically and horizontally symmetrical, but now within a context where the rows are taken in order. The question about whether row order and symmetry can coexist has been answered, but not in a perfect way, since only the first and sixth chords in the right hand are completely symmetrical. The other chords have some symmetrical intervals (these are circled in the example) but not enough to duplicate the complete pattern of mm. 1-2. For example, intervals 5-3-6 in the second chord come back with a different middle interval in the fifth chord, 5-4-6: almost a horizontal symmetry but not a vertical one. And the third and fourth chords have the intervals 4-6-5 and 8-6-7, which again create horizontal symmetry (but not vertical) if each interval is inverted.

I will end our tour through Schoenberg's Piano Piece Op. 33a at this point. We have analyzed enough music to see that he follows quite faithfully the large narrative of presenting a symmetrical interval structure in the exposition's first theme, opposing it to the ordered tone rows of the piece, obscuring that symmetrical structure gradually, replacing it with an alternative, horizontal pitch symmetry in the second and closing themes, striving back toward the original symmetrical pattern (in the development), and finally recapturing part of the palindromic ideal in the recapitulation (and showing how it might be reconciled to the ordered row). I understand that narrative as a twelve-tone manifestation of Schoenberg's "musical idea".

I believe that Schoenberg projects the musical idea through much (not all) of his atonal and twelve-tone music, but does it in different ways for different pieces. And the musical idea explains many things about his twelve-tone music that seem strange, like the fact that he takes rows out of order or doesn't even finish the row in some cases (we just saw this in the "closing theme section" of Op. 33a). My main point is that the musical idea should *not* be understood as a vague, ephemeral, expression of the essence of a piece of music. Instead, it is a concrete framework that we as analysts can use to organize the analytic details of a complex piece. Hopefully it will be useful to you too, as you do your own investigations of Schoenberg's music in the future.

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3

From *Kaleidocycles* to Tiling Canons

Luigi Verdi

Many composers of the 20th century used cyclic harmonic patterns, although they did not develop a unitary theory to systematize their use. An analysis of these patterns is particularly interesting as regards composition: the purpose of this paper is to show how a reset of cyclic elements inherited from the musical tradition could be a starting point for a "kaleidocyclic" technique in musical composition, that is the transformation of chords following a transposition cycle. Starting from the analysis of the features of traditional temperament, the theory of musical kaleidocycles (name derived after some Maurits Escher's graphic techniques) points out the fundamental numerical rules governing a chord structure, which may have analytical and compositional implications, like when processing kaleidocycles into rhythmic-melodic canons. In all these approaches the numerical translation of some events leads to results which can be differently applied both in the vertical space of the pitch arrangement and in the horizontal-one as regards the rhythmic-melodic patterns. The kaleidocycle could be the effect of a transformation of space into time, that is a vertical structure which changes into a horizontal one. The complementarity relation towards a reference set is a key principle of the kaleidocyclical system, which may have other compositional implications, as when processing kaleidocycles into tiling canons. In this perspectives, I will also review briefly different kinds of canons, as for example the *Tiling six-part double canons on trichords*, intended as pairs of three-note series that combine by simple, inverse, and retrograde motions without doubling: this almost alchemichal mixture make possible a state of aggregations that we could call "chemusical", in a sort of synthesis between chemistry and music (Chemusic) (VERDI, 2019b, p. 500).

Common-notes numerical vector

Every pitch class set in 12-tempered space can be transposed many times and has a close relation with its own transpositions, depending on the common notes it shares with them. The number of notes shared in the various transpositions gives rise to the *common-notes numerical vector*. It is a similar concept to David Lewin's *interval function* (LEWIN, 1960). The vector can be identified by adopting some reference values. Thus, if a pitch of the chord is given as = 0, a series of numbers indicates all pitches, considering a semitone as corresponding to 1; so, the Scriabin's *Prometheus* chord, as example, corresponds to (013579) and its *interval structure* — the series of semitones separating every note of the chord — is 1-2-2-2-2-(3) while the common-notes numerical vector is (614242424241). The vector is fundamental to check some properties of a chord and stands as a sort of "genetic code" containing all necessary indications to identify particular relations. It can be calculated empirically by means of a simple system of Cartesian axis reporting all 12 transpositions arranged one on the other. The arrangement of common notes on various transposition levels — which can be compared to pieces on a draughtboard (on the abscissa the chord, on the ordinate the common notes on 12 transposition levels) — is shown in the following figure, based on (013579) chord (VERDI, 2008-9, p. 71).

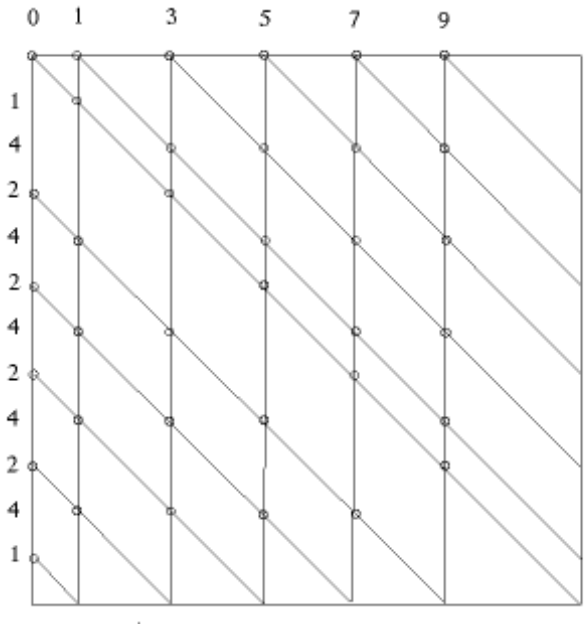

Figure 1: Transposition diagram of (13579) *Prometheus* chord

The previous "transposition diagram" can also be represented by the following Table 1, where common notes shared by transpositions are highlighted:

Table 1: Transposition chart of (013579) *Prometheus* chord.

Chromatic scale	0	1	2	3	4	5	6	7	8	9	10	11	Vector
Chord	o	o		o		o		o		o			
0	o	o		o		o		o		o			6
1		o	o		o		o		o		o		1
2			o	o		o		o		o		o	4
3	o			o	o		o		o		o		2
4		o			o	o		o		o		o	4
5	o		o			o	o		o		o		2
6		o		o			o	o		o		o	4
7	o		o		o			o	o		o		2
8		o		o		o			o	o		o	4
9	o		o		o		o			o	o		2
10		o		o		o		o			o	o	4
11	o		o		o		o		o			o	1
Transp. level													

In surveying the various possibilities to link the transpositions of a chord, it is possible to identify modal groups given by the number of common notes. A graphical representation allows to immediately check the relation (Figure 2): numbers corresponding to common-notes have been inserted at the vertices of a dodecagon, representing 12-tempered space. In this way two transpositions with one common note give rise to an isosceles triangle; four transpositions with two common notes give rise to an irregular pentagon; five transpositions with four common notes give rise to a regular hexagon.

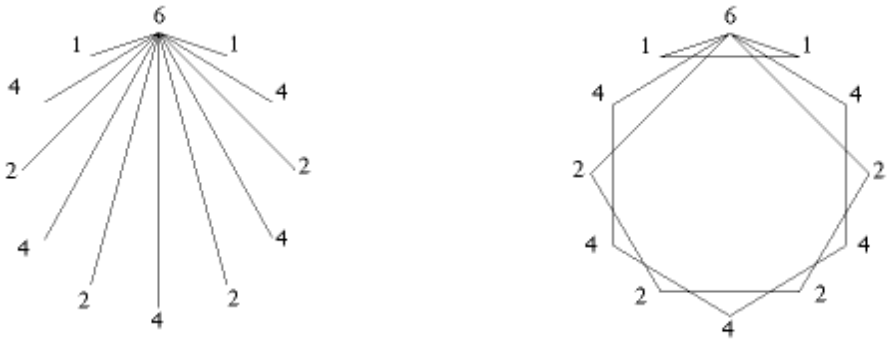

Figure 2: Relationship among *Prometheus* chord transpositions.

The transposition diagrams of the common-notes numerical vector have interesting geometrical arrangement properties. By matching each pitch to a colour (from *c*-0 red to *b*-11 violet), coloured graphs can derive. Those shown here (Fig. 3), originally watercolors on cardboard or chalk, are from a series of works developed in the 80s of twentieth century (VERDI, 2010, pp. 23–25).

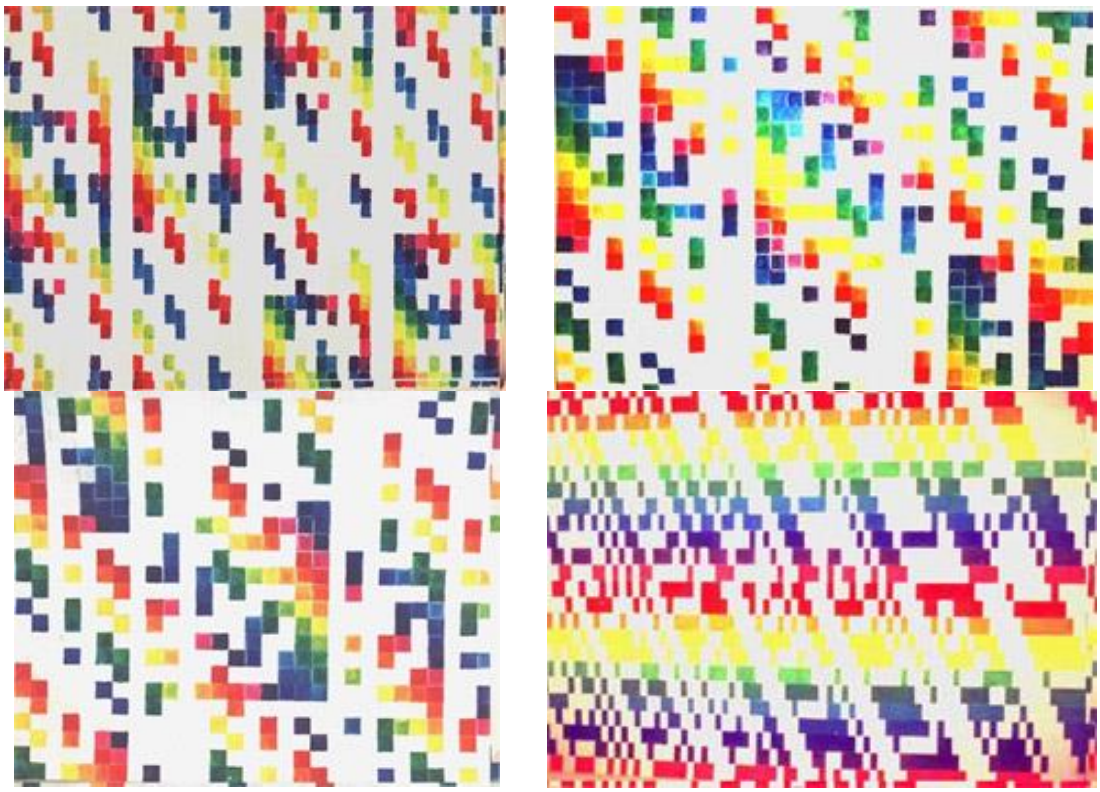

Figure 3: Some coloured graphs of transposition diagrams.

Supplementary vector

A chord transposed on its pitches forms a sort of meter-harmonic group, where horizontal and vertical sequences are regulated by the same rule. In the Tab.2, the Scriabin's *Prometheus* chord (013579) is placed on grades 0,1,3,5,7,9, corresponding to its pitches at 1-2-2-2-2-3 semitone intervals: some notes are repeated more frequently than others. The series of number indicating the repeating notes of a chord transposed on its pitches — in this case 524242424250 — is named *supplementary vector* and has some interesting properties. In fact, every entry of the supplementary vector summed to the corresponding of the common-note numerical vector gives 6 as a result (VERDI, 2010, pp. 26–28).

Table 2: Supplementary vector chart of (013579) chord.

Chromatic scale	0	1	2	3	4	5	6	7	8	9	10	11	
Chord	o	o		o		o		o		o			
0	o	o		o		o		o		o			
1		o	o		o		o		o		o		
2													
3	o			o	o		o		o		o		
4													
5	o		o			o	o		o		o		
6													
7	o		o		o			o	o		o		
8													
9	o		o		o		o			o	o		
10													
11													
Transp. levels	5	2	4	2	4	2	4	2	4	2	5	0	Suppl. vector

Consequently, within the dodecagon representing the 12-tempered space, the irregular hexagon generated by chord (013579) transposed on 0,1,3,5,7,9 levels, creates a figure of fractal type, as the chord generates levels of transpositions that generate the same chord, and so on (Figure 4).

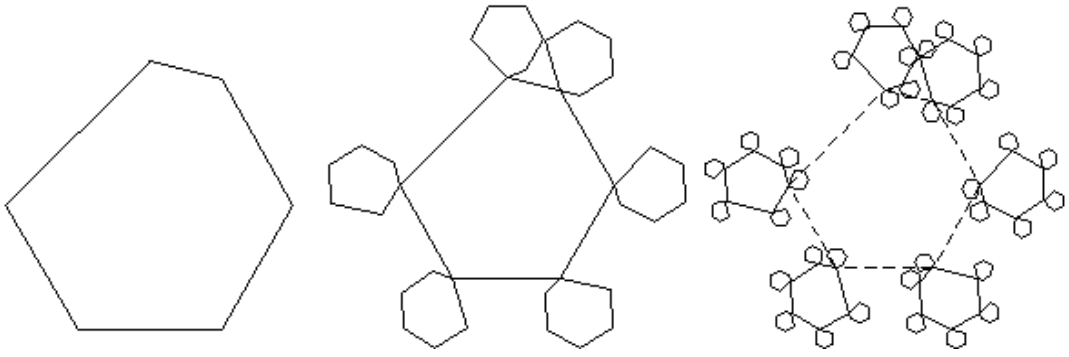

Figure 4: Fractal representation of *Prometheus* chord transposed on itself.

The previous chart has an immediate and effective practical application. The following musical example (Figure 5) is built on Prometheus chord transposed on itself.

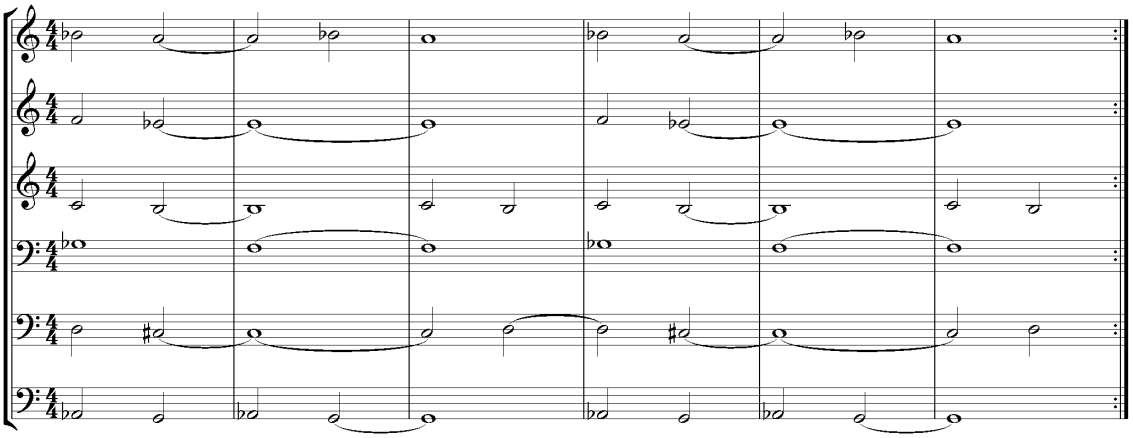

Figure 5: *Prometheus* chord transposed on itself.

Theory of cycles

In the twelve-tone system, every group of intervals can be graphed by a set of segments inscribed in a regular dodecagon. The set of all possible intervals may be represented by a figure composed of 66 segments (Figure 6).

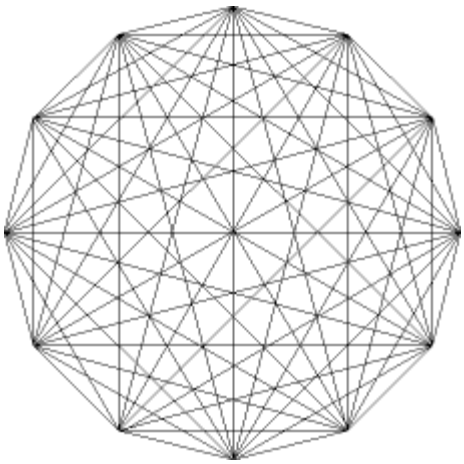

Figure 6: All possible intervals represented into a regular dodecagon.

A link based on groups of intervals periodically repeating themselves originates a *cycle*. The repeating group of intervals is the *module* of the cycle. The module repeats itself on an interval named *base*, given by the sum of the module intervals. In the Fig. 6 is shown a 4-3 module: the base amounts to 7 (as 4 + 3 = 7), because the 4-3 module repetitions are placed on interval 7. The amount of intervals of a module is named *meter*; in the previous case, the 4-3 module is meter 2 (VERDI, 2007b, pp. 40–41).

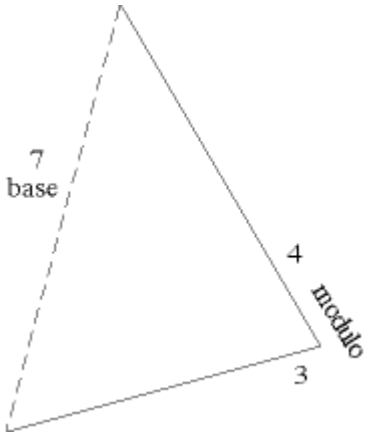

Figure 7: Cycle on the 4-3 module (base 7, meter 2).

Application of chords

Any chord can be applied to a module. An issue for this system to be consistent is defining the relation between the module and the applied chord. The module can be random chosen among the many possible, but we can set analogies between a cycle-module and a chord-interval (VERDI, 2008-9, pp. 80–81). Considering now a 2-3 module spreading into a 24-interval cycle, his development gives rise to the following chart (Table 3 and Figure 8).

Table 3: Development of a 2-3 module.

module	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3	2 - 3
base	5	5	5	5	5	5	5	5	5	5	5	5

base module	(0, 2)	(5, 7)	(10, 0)	(3, 5)	(8, 10)	(1, 3)	(6, 8)	(11, 1)	(4, 6)	(9, 11)	(2, 4)	(7, 9)
base class	(0)	(5)	(10)	(3)	(8)	(1)	(6)	(11)	(4)	(9)	(2)	(7)

It is possible to adopt a methodical approach arranging modules deriving from the common-notes numerical vector of the chord used. If we want to define a meter-2-module linking the transposition of (013579) chord alternatively on its transpositions with 4 and 2 common, this is possible on various ways, as in the following transposing grid (Table 4):

If a *Prometheus* (013579) chord is applied to a 2-3 module, this means that the chord is cyclically transposed alternatively at 2 and 3 semitones. For the musical realization, it is useful to develop a chart, by transcribing every transposed chord vertically, according to a horizontal axis representing the base class. In the musical development of applied chords, every module repetition gives rise to a canon entry on the transposition levels generated by the base (in this case, 2+3=5). Since the base 5 rules the entrance of the different canon entries, it is possible to infer that its period corresponds to the number of entries (12),

because the base repeats 12 times before ending the cycle and returning to initial situation. Entries follow one another along the time axis at a distance corresponding to the module's meter (in this case, 2). In the following Tab. 6, "o" represents pitches, the base class along the time axis is marked red and the entries are marked green.

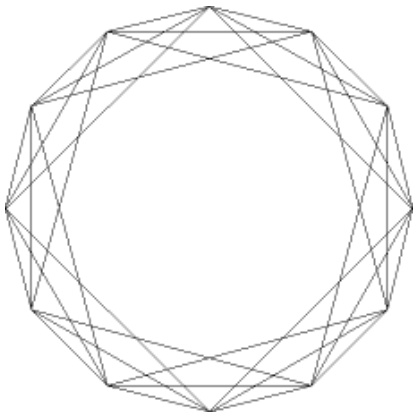

Figure 8: Development of a 2-3 module into a dodecagon.

Table 4: Transposing grid of Prometheus (013579) chord.

transp. level	0	1	2	3	4	5	6	7	8	9	10	11
cn vector	6	1	4	2	4	2	4	2	4	2	4	1

You will get the cycle shown in Table 5.

Table 5: Development of a 2-3 module, based on transpositions with 4-2 common notes of Prometheus (013579) chord.

module common notes	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2	2 - 3 4-2
base	5	5	5	5	5	5	5	5	5	5	5	5

base module	(0, 2)	(5, 7)	(10, 0)	(3, 5)	(8, 10)	(1, 3)	(6, 8)	(11, 1)	(4, 6)	(9, 11)	(2, 4)	(7, 9)
base class	(0)	(5)	(10)	(3)	(8)	(1)	(6)	(11)	(4)	(9)	(2)	(7)

If (013579) is applied to 2-3 module, a particularly consistent structure derives, since the module is drawn from the chord. In the following musical realization, the previous chord generates a six-part canon, with entries placed at ascending perfect fourth (VERDI, 2007b, p. 48).

Table 6: Development chart of a (013569) *Prometheus* chord into a 2-3 module.

Transpn. levels (vertical)																								
5	o	o	o.		o	o		o	o	o				o			o				o	o		
4				o			o				o	o			o	o	o.		o	o		o	o	o
3	o	o			o	o	o.		o	o		o	o	o				o			o			
2		o	o	o				o			o				o	o			o	o	o.		o	o
1	o				o	o			o	o	o.		o	o		o	o	o						
0	o.		o	o		o	o	o				o			o				o	o			o	o
11		o			o				o	o			o	o	o.		o	o		o	o	o		
10			o	o	o.		o	o		o	o	o				o			o				o	o
9	o	o				o			o				o	o			o	o	o.		o	o		o
8			o	o			o	o	o.		o	o		o	o	o				o			o	
7	o	o		o	o	o				o			o				o	o			o	o	o.	
6			o				o	o			o	o	o.		o	o		o	o	o				o
Base class (horizontal)	0	2	5	7	10	0	3	5	8	10	1	3	6	8	11	1	4	6	9	11	2	4	7	9
Common notes		4	2	4	2	4	2	4	2	4	2	4	2	4	2	4	2	4	2	4	2	4	2	4

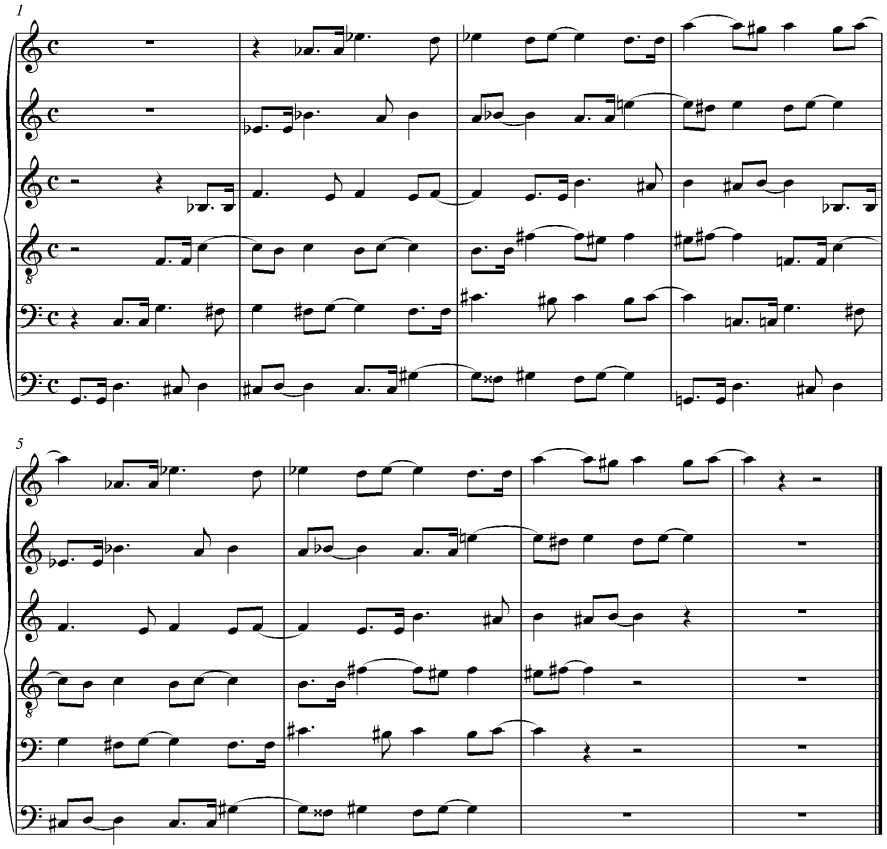

Figure 9: Musical realization of a six-part canon on a 2-3 module, based on transpositions with 4,2 common notes of *Prometheus* chord.

In the canon the single entries are placed at meter-2-horizontal-distance and at base-5-vertical distance. The canon is infinite, the level on which it starts

is merely conventional. Thus, a *cyclical rhythmical scheme* is generated by the vertical development of the chord on its base, placed on the horizontal axis of the meter. The application of a chord to a module gives rise to this original structure, which I have named "kaleidocycle", after some Escher's graphic techniques. The kaleidocycle is a transformation of space into time, that is a vertical structure which changes into a horizontal one (VERDI, 2010, pp. 28–30).

Otherwise, the *Prometheus* chord (013579) transposed on levels with alternatively 4 and 2 common-notes, but based on 4-3 module, can originate a double canon, at four and two-part (AB), with entries on ascending perfect fifth.

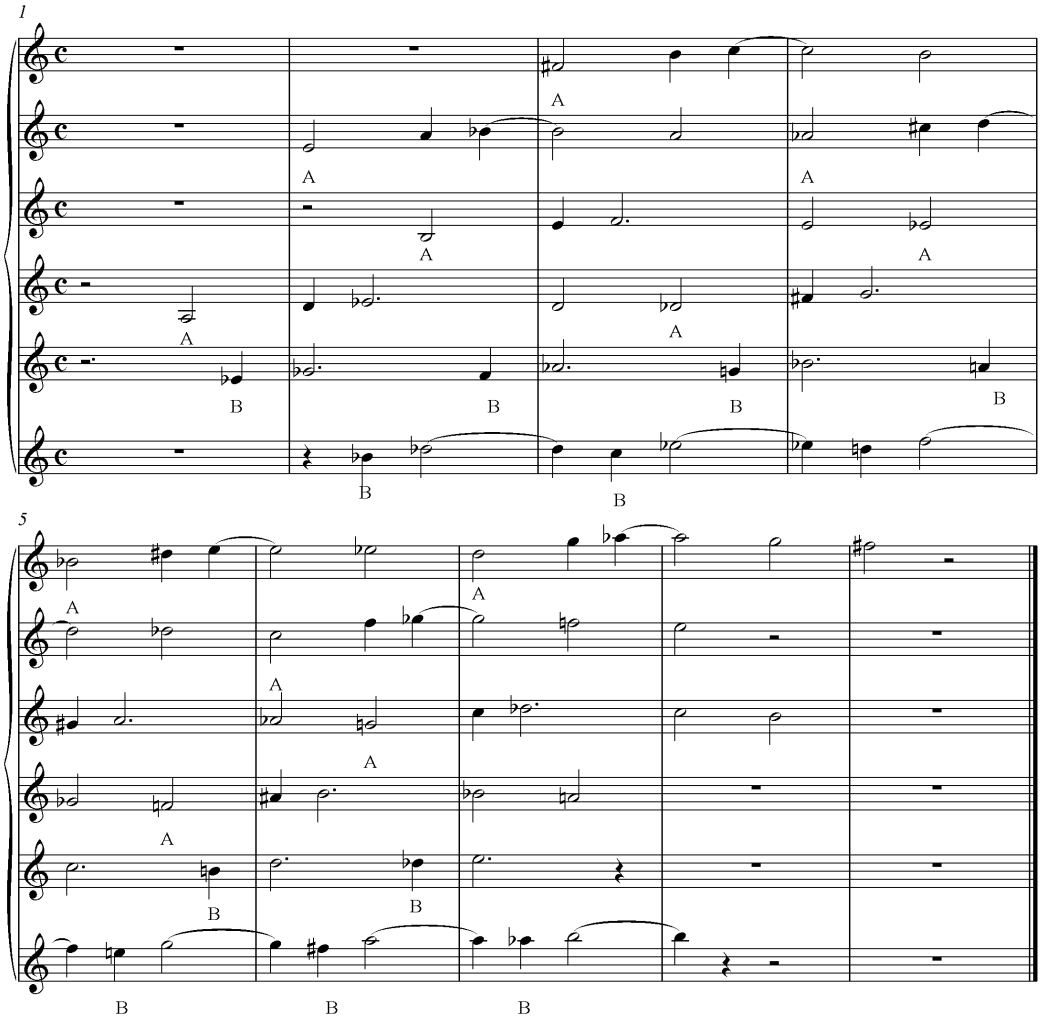

Figure 10: Musical realization of a kaleidocycle-double-canon on a 4-3 module based on transpositions with 4-2 common notes of *Prometheus* chord.

The symmetry of the kaleidocycle allows to reverse all intervals without any change of the structure (Figure 11).

You can also superimpose a kaleidocycle to its inverse. In this case, you will notice that the beginning of the inversion is at the tritone, the end at the

octave and there are never unisons (VERDI, 2010, p. 36). In the following figure, there is the graphic transposition of the original kaleidocycle, of the reversed one and their superimposition.

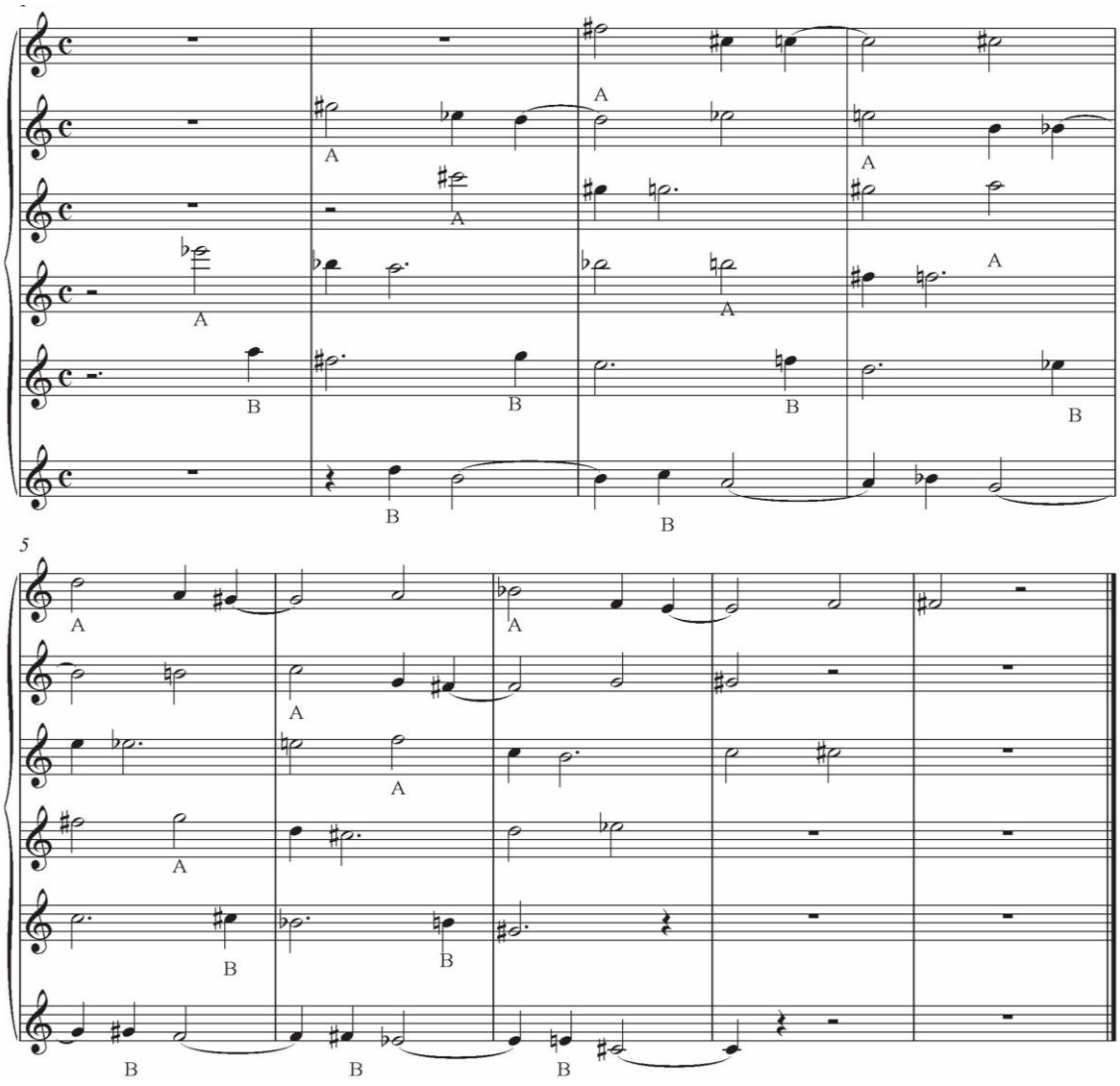

Figure 11: Inversion of the kaleidocycle-double-canon as shown in Figure 10.

A kaleidocycle based on a chord can therefore be superimposed to another derived from a different but related chord. There will be two complementary kaleidocycles related to a super-chord that includes both, so that there are never repeats the same pitch simultaneously. Maybe also the case for three or even four related kaleidocycles, according to each applied chord. The relations among all components of a kaleidocycle may also become extremely complex, therefore the rigor of the development may have extreme outcomes, to lead every element of a musical construction back to one only matrix. The possibilities to be explored in this field are infinite and it may be puzzling with many relations to be ruled. On the other hand, a good musical outcome is not taken for granted.

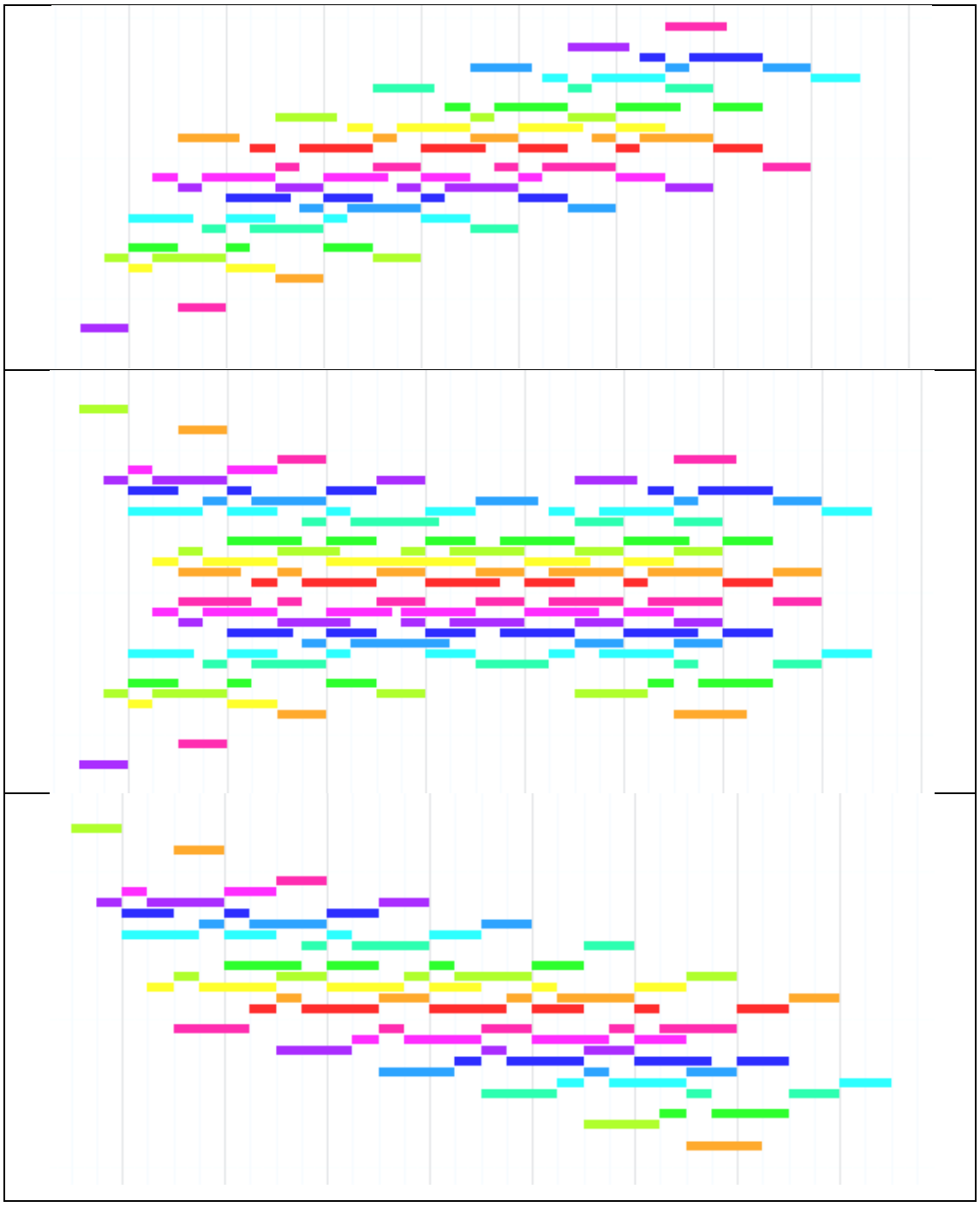

Figure 12: Graphic transposition of the original kaleidocycle-double-canon, of the reversed one and their superimposition.

Composing with kaleidocycles. Some advanced samples

Let me show some kaleidocycle samples from my own compositions. In *Fughetta a 6 su un solo accordo*, for strings (1986), the *Prometheus* chord (013579) develops vertically, so as to create horizontal canon lines. The module, on which the *Fughetta* chord revolves, form a series of 10 figures (alternating 3 with pair

numbers, i.e 2 and 4 common notes, see Tab. 4): 3-2-3-4-3-6-3-8-3-10; in the deriving six-part canon, the entries are placed along the axis of a descending diminished seventh (3,0,9,6). The piece is divided into two sections: in the second section, the module is inverse so as to develop another canon along the axis of a rising diminished seventh (7,10,1,4) (VERDI, 2010, pp. 93-95).

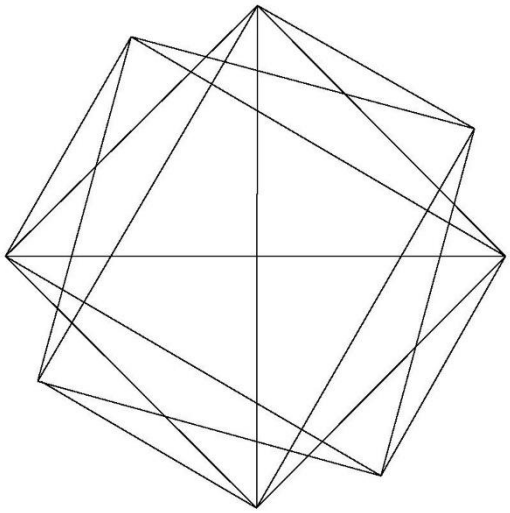

Figure 13: Development of the 3-2-3-4-3-6-3-8-3-10 *Fughetta* module into a dodecagon.

In *Organum* (1990), for twelve saxophones, the *Prometheus* chord (013579) develops canonically together with its inverse. The module of *Organum* forms a series of 40 items, so that the various entries of the canon are placed at five 4/4 bars distance. The work can be performed in two different versions, since it can start indifferently by the development of the chord original then superimposing on its inverse, or by its inverse then superimposing on the original. The module is divided into five parts: the first built by alternating the common notes on the odd entries of the vector 3,5,7,9 (with 4 common notes), with the even entry 2 (2 common notes); the second alternating the common notes on the odd entries, with the even 4 (2 common notes); the third alternating the common notes on the odd entries, with the even 6 (2 common notes); the fourth alternating the common notes on odd entries, with the even 8 (2 common notes); the fifth alternating the common notes on the odd entries, with the even 10 (2 common notes) (VERDI, 2010, pp. 99-102).

In the *Three Kaleidocycles* for instrumental group (1991), the n.1 *Ottotonico* for eight instruments is harmonically built on the *Prometheus* minor chord (013479); hence there develops a five-voice canon with tritone entries, while one independent voice develops the tone-semitone (octatonic) scale. The module forms a three-figure series: 4-2-3 (VERDI, 2010, p. 59-61).

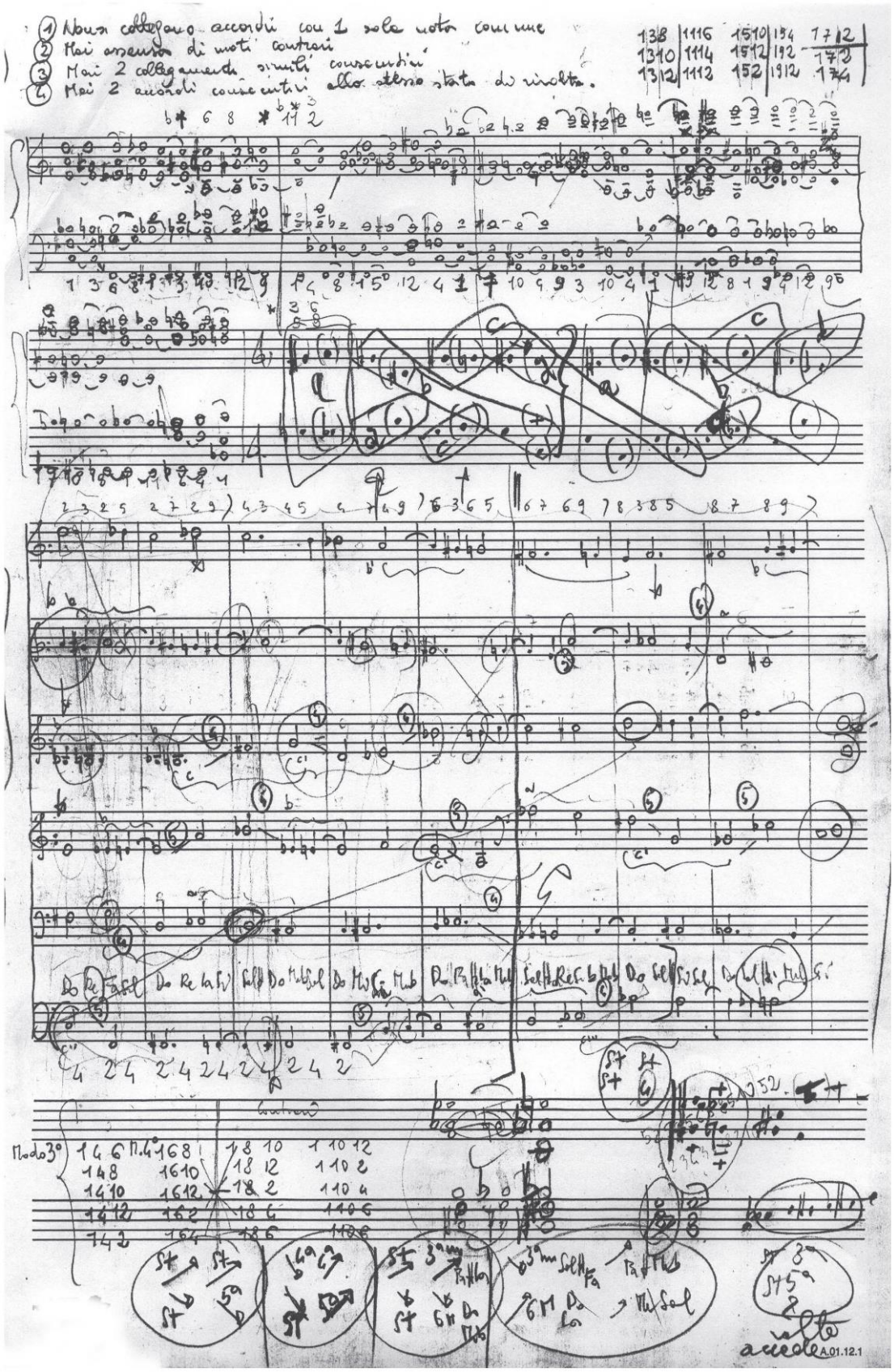

Figure 14: Sketches for *Organum* (1990).

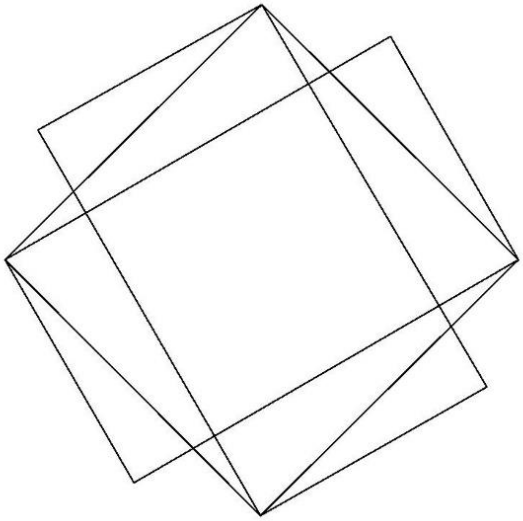

Figure 15: Development of the 4-2-3 *Ottotonico* module into a dodecagon.

The n. 3 *Sintetico*, for two six-instrument groups, is built on two different hexachords: *Prometheus* minor (013479) and major (013579). The six parts of the first group unfold an inverse canon to the six of the second group; the canon entries are placed inversely along the axis of an augmented triad. The module forms a series of six figures (6-3-1-7-11-4), deriving from the common-notes 4,2,1,2,1,4 among chord transpositions (VERDI, 2010, pp. 68-74).

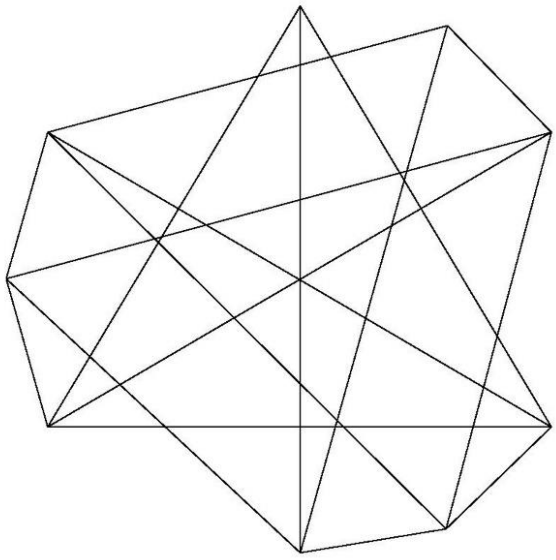

Figure 16: Development of the 6-3-1-7-11-4 *Sintetico* module into a dodecagon.

The following is an example of orchestral elaboration of previous kaleidocycle.

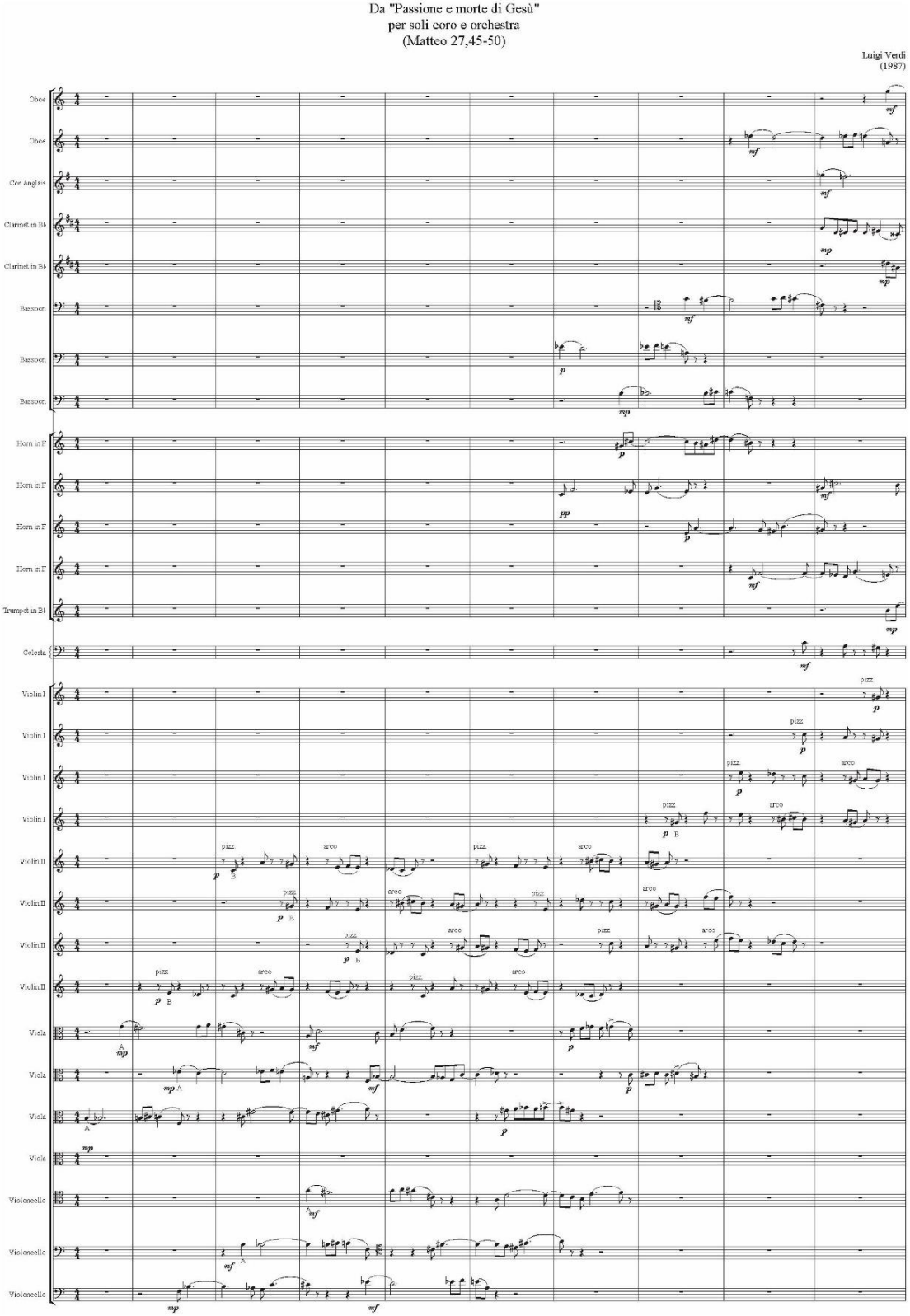

Figure 17: Orchestral elaboration of *Sintetico* kaleidocycle on both *Prometheus* minor and major chords.

In *Rodion* for strings (1990), the (012456810) "glass-window" chord develops freely on module forming a (1-2-1-4-1-6-1-8-1-10) series of 10 figures. In *Flussi ermetici*, for wind quartet, the (37911) chord, complementary to the previous one, is developed on the same module. It is possible to superimpose the two pieces and to perform them simultaneously (VERDI, 2010, pp. 96-98).

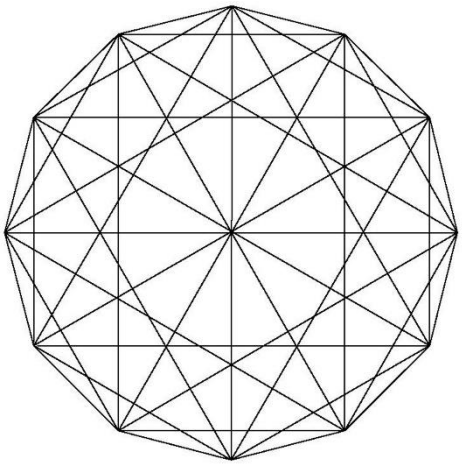

Figure 18: Development of the 1-2-1-4-1-6-1-8-1-10 *Rodion* module into a dodecagon.

In *Notturno* (1991), the material used is based exclusively on three tetrachords that totally fill the 12-tempered space in canonic form. Every chord is given to a four-instrument group; the first group (3 clarinets and viola) develops the (15610) chord, which originates a three-part canon and a free part. The second group (bassoon, 2 horns, cello1) develops the (23711) chord, which originates a double canon, so the third group (flute, oboe, 2 violins) develops the (0489) chord, developing a double canon. The module forms a series of 20 figures (1-2-3-4-5-6-7-8-9-10-11-10-9-8-7-6-5-4-3-2). The three groups fit together without doublings rotating kaleidocyclically (VERDI, 2010, pp. 168-173).

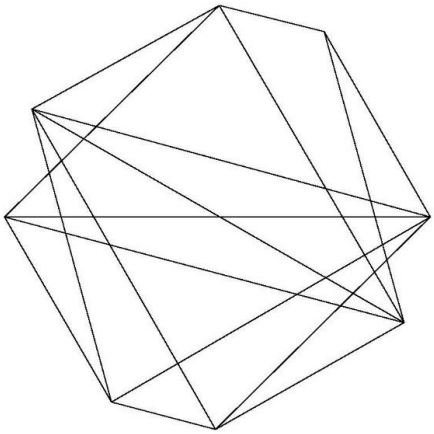

Figure 19: Development of *Notturno* 20 figures module into a dodecagon.

In *New Kaleidocycles* for instrumental group (1992), the n. 1, for 12 instruments, is harmonically built on the (0126810) chord and on its complementary one. The module forms a series of 11 figures (1-2-3-2-5-2-7-2-9-2-11); melodically, each of the two chords develops one six-part-canon, with entries placed along the axis of an ascending hexatonal scale.

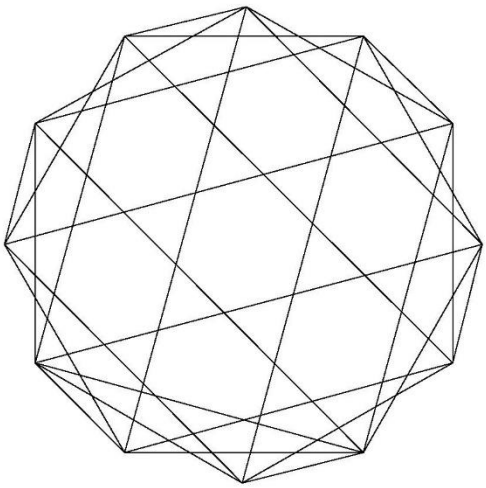

Figure 20: Development of n.1 *New Kaleidocycles* 11 figures module into a dodecagon.

The n. 3, for 12 instruments, is generated by a chromatic tetrachord (0123) on a module of 24 figures given by the intersection of all ascendent intervals and all descendent ones, so that the second part is the inversion of the first (1-12-2-11-3-10-4-9-5-8-6-7-7-6-8-5-9-4-10-3-11-2-12-1). The tetrachord develops one four-part double canon with unison entries at six 4/4. Every two bars, the tetrachord overlap to itself, until fil the 12-tempered space, giving rise to one twelve-part double canon, as above.

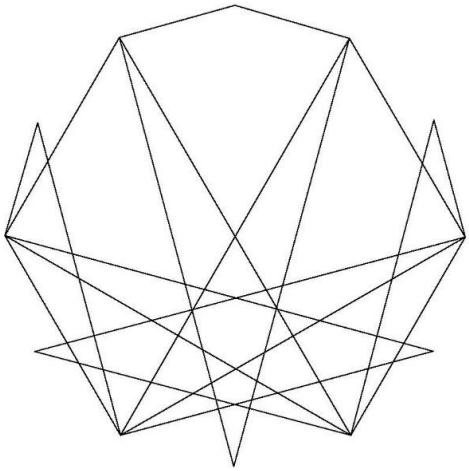

Figure 21: Development of n.3 *New Kaleidocycles* 24 figures module into a dodecagon.

But how it is possible that every two 4/4 bars the (0123) tetrachord overlaps itself without doublings on the given module? Because the deriving base-module is divisible into three parts, on the axis of an augmented triad (0-4-8), as follows.

Table 7: Development of the given module and deriving base-module on (0-4-8).

module	1	12	1	11	3	10	4	9	5	8	6	7	7	6	8	5	9	4	10	3	11	2	12	1
base module	**0**	1	1	3	2	5	3	7	**4**	9	5	11	6	1	7	3	**8**	5	9	7	10	9	11	11

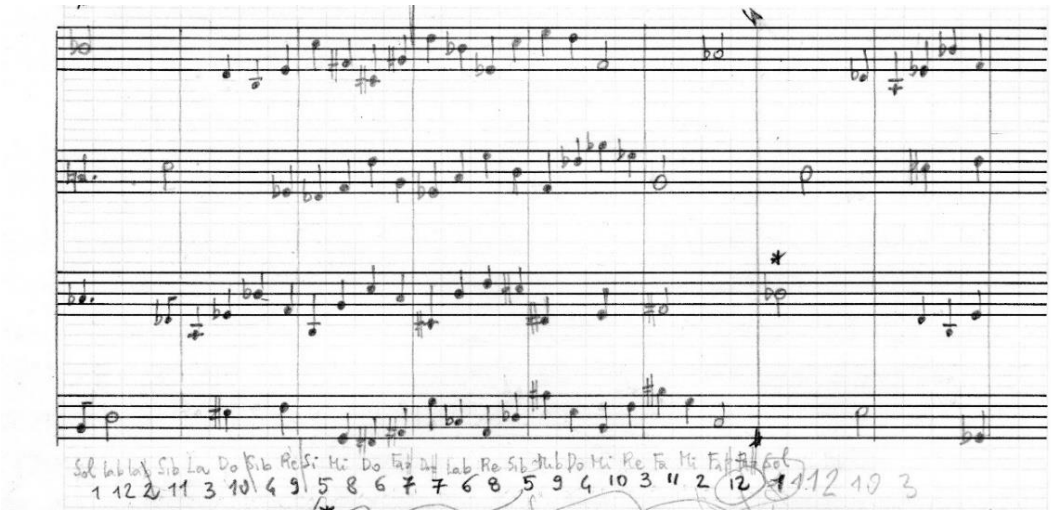

Figure 22: Sketches for *New Kaleidocycles* n.3.

In *Landscape* for double string ensemble of 14 players (1992), the seven note (0124789) chord (Messiaen's dominant chord) develops canonically. The module on which the chord is placed always follows a 7 rising interval (corresponding to perfect fifth) for one string ensemble, and a 7 descending interval for the other ensemble. Horizontally, opposite fragments of chromatic and hexatonal scales derive.

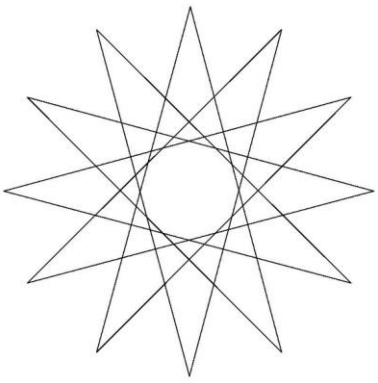

Figure 23: Development of Landscape module into a dodecagon.

Three-part canons on twelve trichords

We have seen that the kaleidocycle technique leads to the formulation of various types of canons. The second part of this essay focuses on Tiling six-part double canons on trichords, intended as pairs of three-note series that combine by simple, inverse, and retrograde motions without doubling.

Every trichord can be graphically represented and labeled based on the pitch classes of which it consists. In its original primary form, the trichords are set in ascending order, while their inversion in descending one. In 12-tempered space, there exist five trichords that are inversionally symmetrical (I) and seven plus seven mutually inverse (AB). In Table 9 the trichords are set in a graphic representation as "Christmas tree". Just as an example, every trichord could be paired with the emblematic name of a composer who made special or significant use of it (VERDI, 2019a, pp. 315-316).

Table 8: Summary chart of trichords and "Christmas tree".

3-1	ooo	(012)	I	*Half Steps*	Varese
3-2	oo o	(013)	AB	*Half/Whole Steps*	Messiaen
3-3	oo o	(014)	AB	*Minor/Major Third*	Berg
3-4	oo o	(015)	AB	*Half/Fourth*	Schoenberg
3-5	oo o	(016)	AB	*Tritone/Fourth*	Webern
3-6	o o o	(024)	I	*Whole Steps*	Ravel
3-7	o o o	(025)	AB	*Whole/Fourth*	Respighi
3-8	o o o	(026)	AB	*Whole/Tritone*	Scriabin
3-9	o o o	(027)	I	*Fourths*	Hindemith
3-10	o o o	(036)	I	*Diminished*	Stravinsky
3-11	o o o	(037)	AB	*Diatonic*	Bartók
3-12	o o o	(048)	I	*Augmented*	Debussy

						o	o	o		o	o	o						
					o		o	o		o	o		o					
				o			o	o		o	o			o				
			o				o	o		o	o				o			
		o					o	o		o	o					o		
				o		o		o		o		o		o				
			o			o		o		o		o			o			
		o				o		o		o		o				o		
	o					o		o		o		o					o	
		o			o			o		o			o			o		
	o				o			o		o			o				o	
o				o				o		o				o				o

My first objective is to construct twelve three-part canons, each of which is based on various vertical forms of a single trichord. To this end, I have established a module (series of intervals by which the trichords are transposed),

so as to create canons derived by a periodic repetition of transposition. I chose an *all-interval series* module (1-2-3-4-5-6-7-8-9-10-11) (N.B: this is a listing of intervals rather than pitch-class numbers and is thus not a 12-tone series). (Figure 24).

We have already seen that the sum of the *module* intervals is named *base*. In this case, 1+2+3+4+5+6+7+8+9+10+11 =66 ≡ 6 (mod 12), with *base class* (0,6) and *period* 2, because the base repeat 2 time before ending the cycle and returning to initial situation.

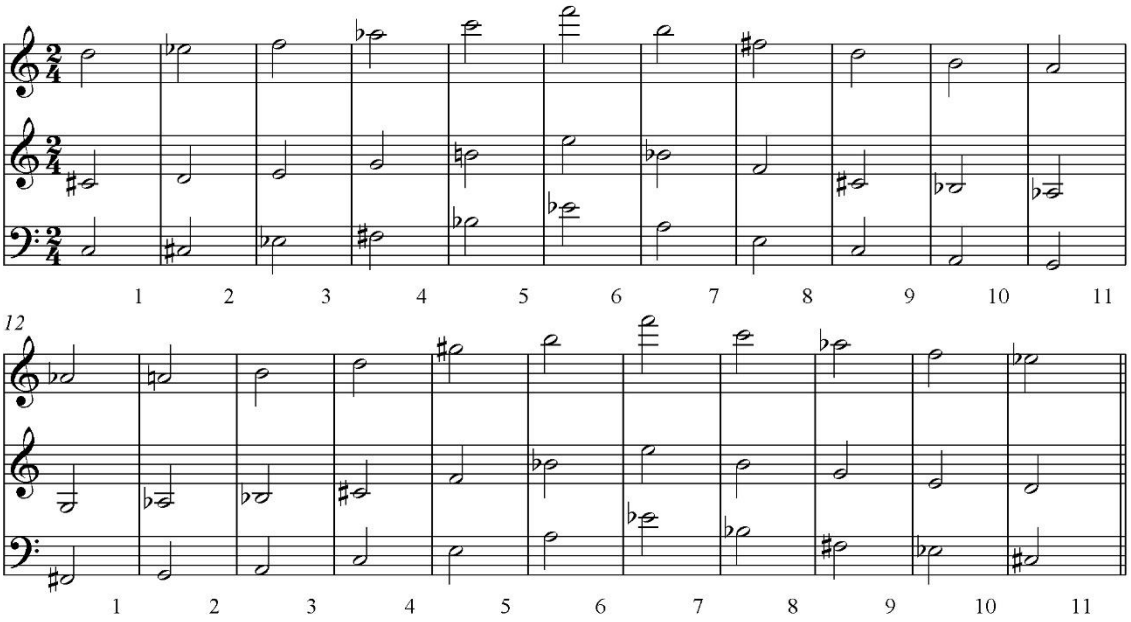

Figure 24: Development of the given module.

About contrapuntal movements connecting the chords, the common notes are normally held, while the others move toward the nearest components, while avoiding parallel motion, and usually giving preference to contrary over direct motion. Melodic leaps do not exceed a fifth, with preference given to thirds over sixths, and seconds over sevenths. So, the rules could be as follows:

T_1: There are two common tones, and both are held.
T_2: There is only one common tone, and it is held; the other two move to the nearest notes, but not by parallel motion: they proceed exchanging their functions by direct motion.
T_3: One note can move by semitone, the others move to the nearest notes, but not by direct motion: they proceed by contrary motion, exchanging their functions.
T_4: One note can move by a tone, the other two proceed by contrary motion.

T_5: One note can move by a minor third, the other two proceed by contrary motion.
T_6: Two notes can move by major third in contrary motion (VERDI, 2019b, pp. 501–502).

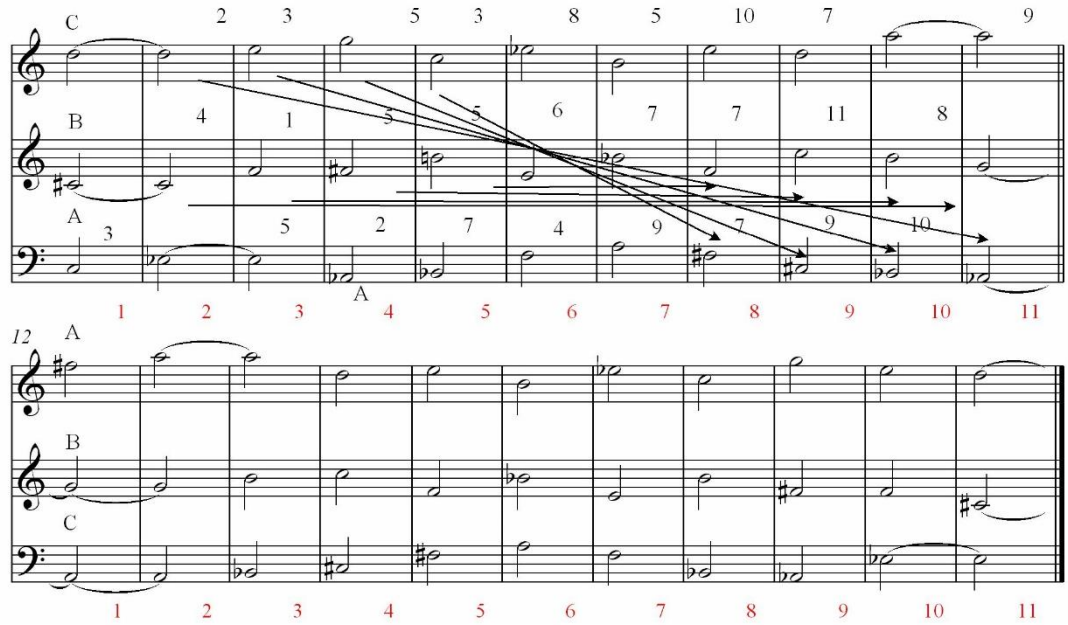

Figure 25*:* Assembling a (012) Three-part canon into the given module.

As an example, I have decided to realize the canon for a string trio. The total numerical elements of the module define the *meter*. In this case, the *module* 1-2-3-4-5-6-7-8-9-10-11 consists of eleven intervals, so it will be realized musically in an eleven-count meter such as 11/8. The canon will be fourteen measures in length; each of the three instruments will play the theme four times, with entrances on C, F-sharp, C (in the order: cello, violin 2, violin 1). Therefore, the canon in contrary motion will be played with inverted entrance order (violin 1, violin 2, cello) for another fourteen measures. In the second parts of the canon, the entrances will be at intervals inverted around a different axis for each trichord; in fact, it is the same intervallic structure of the trichord that defines the axis of inversion (VERDI, 2019b, p. 505). In Figure 26 there is a draft of the musical realization.

The procedure that we have examined for trichord (012) can be extended to all the trichords. The inversion for each trichord will be transposed to the levels dependent on the intervallic formation of the chosen trichord, following a schema in which the first note of the original (C = 0) corresponds to the last note of the inversion (F-sharp = 6), as shown in Figure 27.

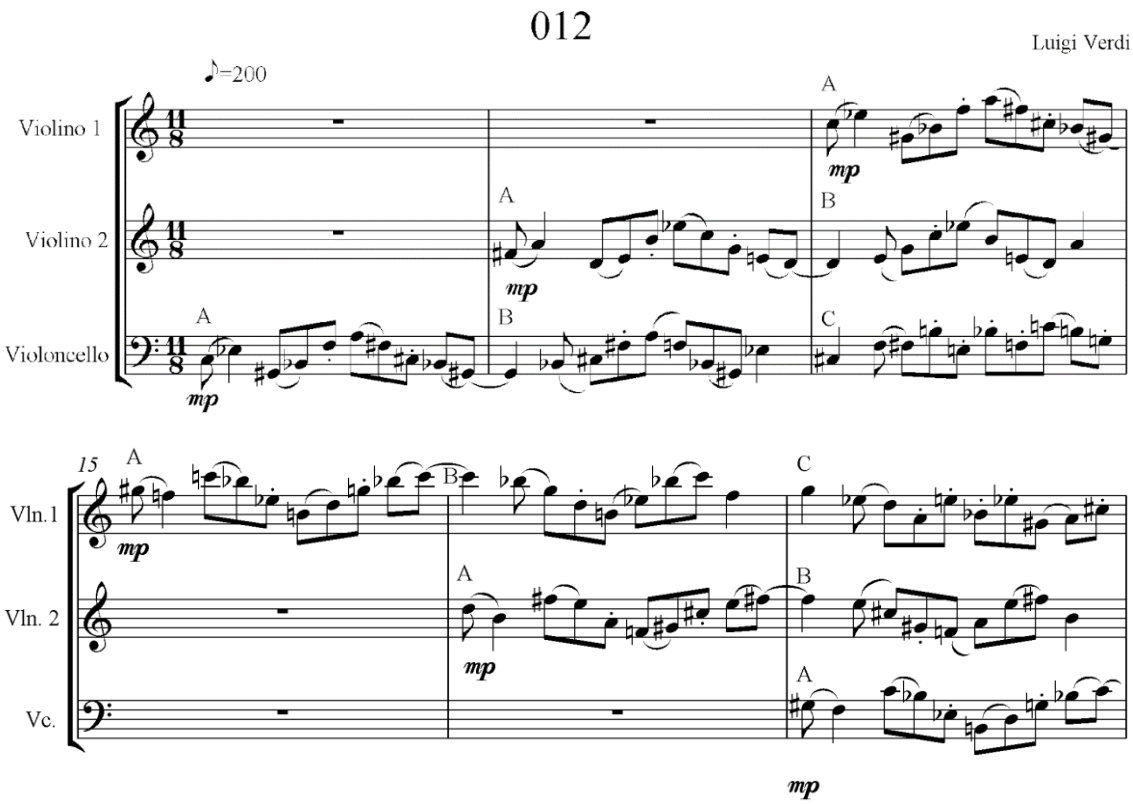

Figure 22: Opening of canon on (012), original (top) and inverted (bottom).

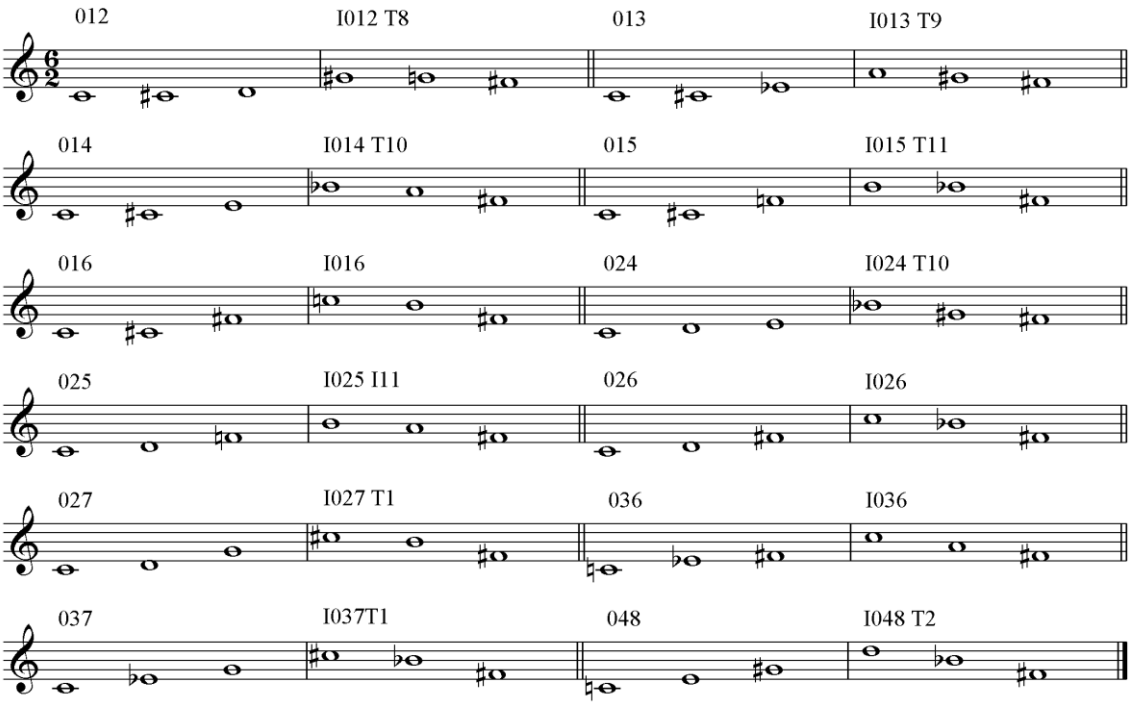

Figure 27: Setting the level of inversional transposition.

And here in Figure 28, the draft of the musical realization on trichord (037), the minor triad. The inversion of the entire structure, based on the major chord, can be automatically deduced.

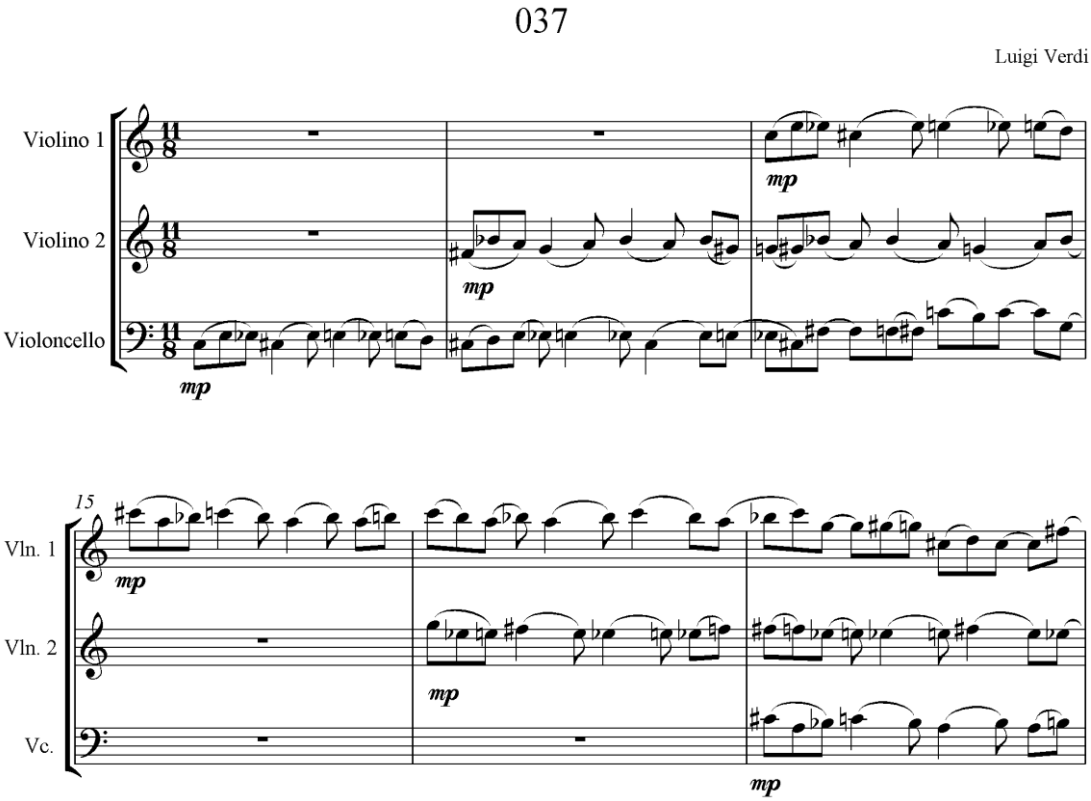

Figure 28: Opening of canon on (037), original (top) and inverted (bottom).

It is possible to combine right and inversion chords without ever overlapping notes, so that the major and minor chords give rise to a super chord that includes both of them (VERDI, 2010, p. 155).

Coming into contact with the same module, each trichord triggers different aggregative reactions, analogous in certain ways to true and proper *chemusical reactions*. We will see in more detail a few surprising results of these reactions and some implications of the idea of *chemusic*.

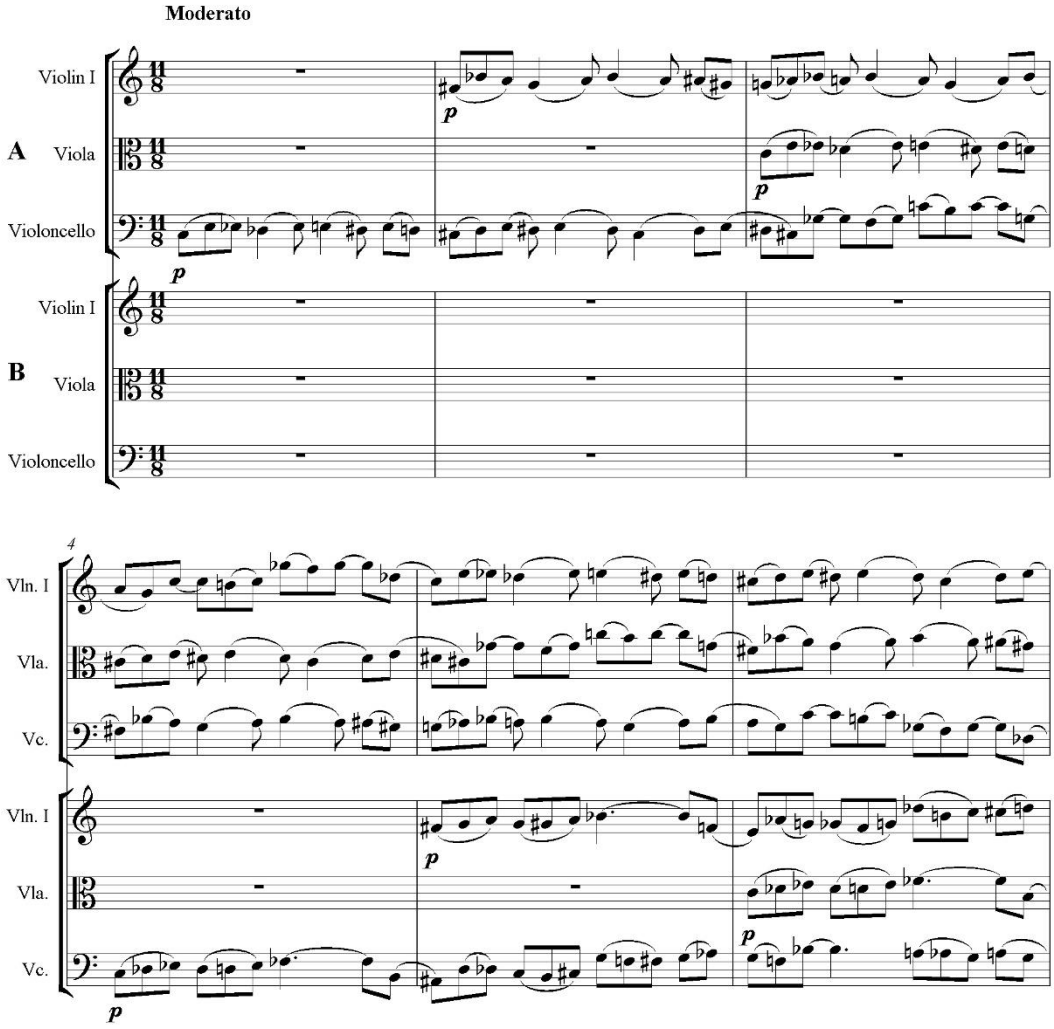

Figure 29: Fragment of a musical realization of the two complementary minor- major chords kaleidocycle-canons.

Tiling six-part double canons on trichords

The preceding twelve three-part canons can evolve into six-part canons, but by virtue of the reaction of the single trichords in contact with the given module, under certain conditions the three-part canons can give rise to six-part canons without doubling, assembling the hexachordal partition in non-overlapping segments. Then, one will have Tiling six-part double canons on trichords. These canons are relatively easy to realize on one trichord only at Rt_0 (zero rotation), with simultaneous entries. In fact, it is sufficient to transport the three overlapping canonic parts to an adequate transposition level. Further, it is possible to have a redoubling of the whole structure, thus creating twelve-part canons by superimposing one, two, or even four different trichords, as shown in Table 9 and Figure 30.

Table 9: Mixture of (012)+(014)T_4+(013)T_6+(015)T_{10} and (012)+(027)T_3+(025)T_4+(014)T_7.

(012)	o	o	o								
(014)T_4					o	o			o		
(013) T_6							o	o		o	
(015) T_{10}				o							o	o
(012)	o	o	o								
(027)T_3				o		o					o	
(025) T_4					o		o			o		
(014) T_7								o	o			o

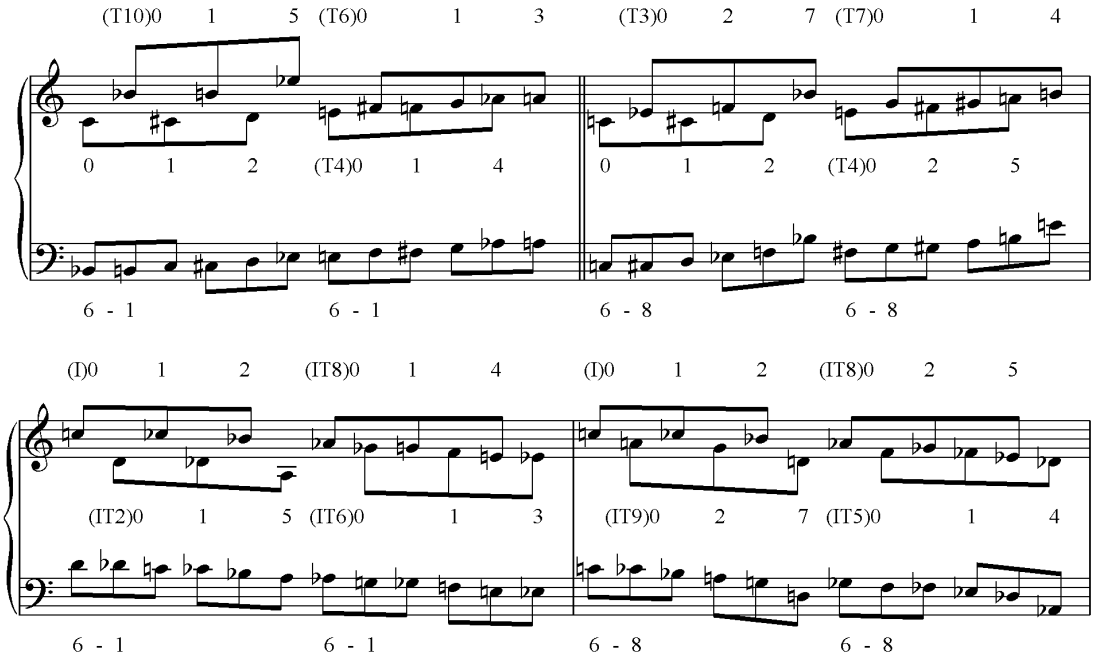

Figure 30: Mixture of (012)+(014)T_4+(013)T_6+(015)T_{10} and (012)+(027)T_3+(025)T_4+(014)T_7.

Figure 31 shows the opening of a Tiling twelve-part quadruple canon on four trichords.

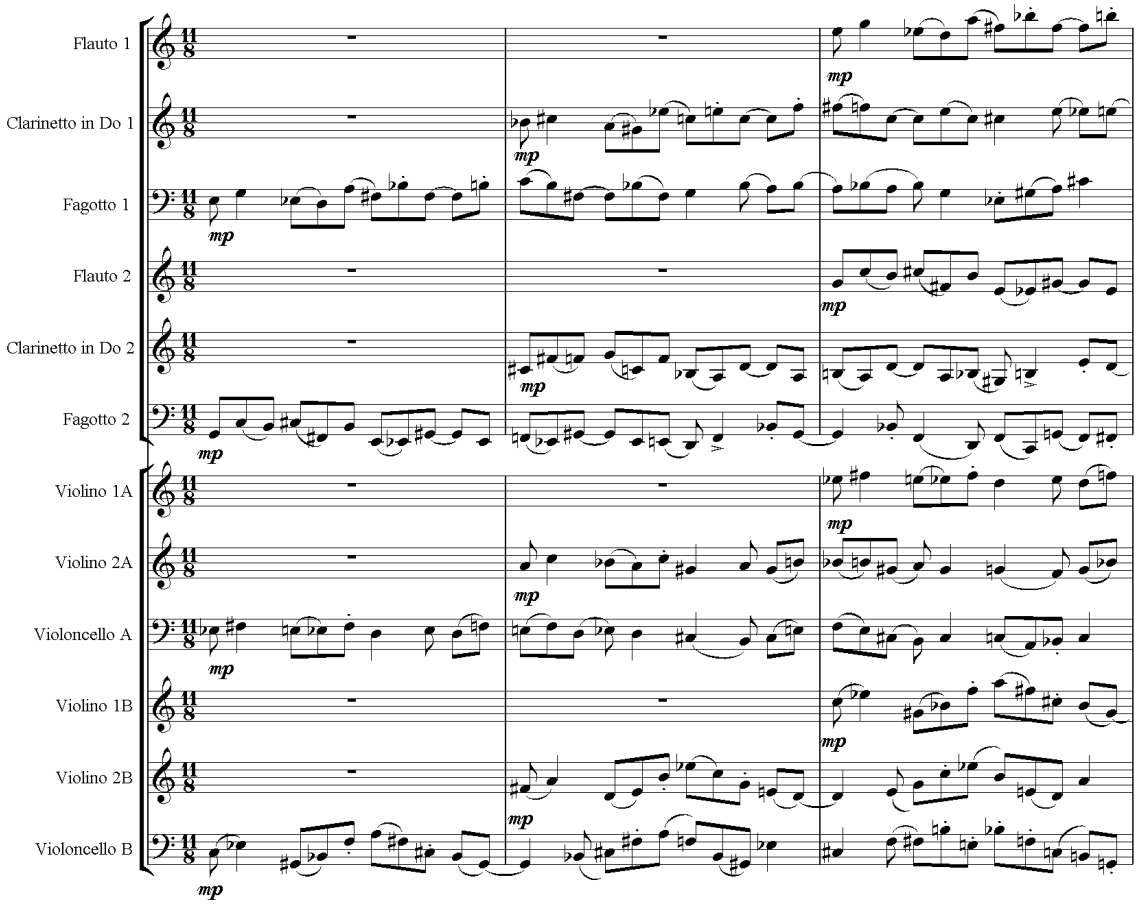

Figure 31: Opening of Tiling twelve-part quadruple canon on four trichords. (012)+(027)T_3+(025)T_4+(014)T_7.

Tiling retrograde six-part double canon on trichords

A variant of the preceding six-part canons is the Tiling retrograde six-part double canon on trichords. By virtue of the module's structure, the retrograde corresponds to the transposition at the tritone: transpositional symmetry is directly connected to the module on which the canons are based, where the second part is also a transposition of the first part at the tritone. Figure 32 provides examples of this kind of canons.

A topic to be explored is concerning resultant hexachords; in Table 10 is shown the succession of the resultant deriving from (012)+R(012) on the given module; the hexachords will always be equal to (012678), or 6-7 adopting the Allen Forte number (Forte 1973), placed on transposition levels following each other symmetrically.

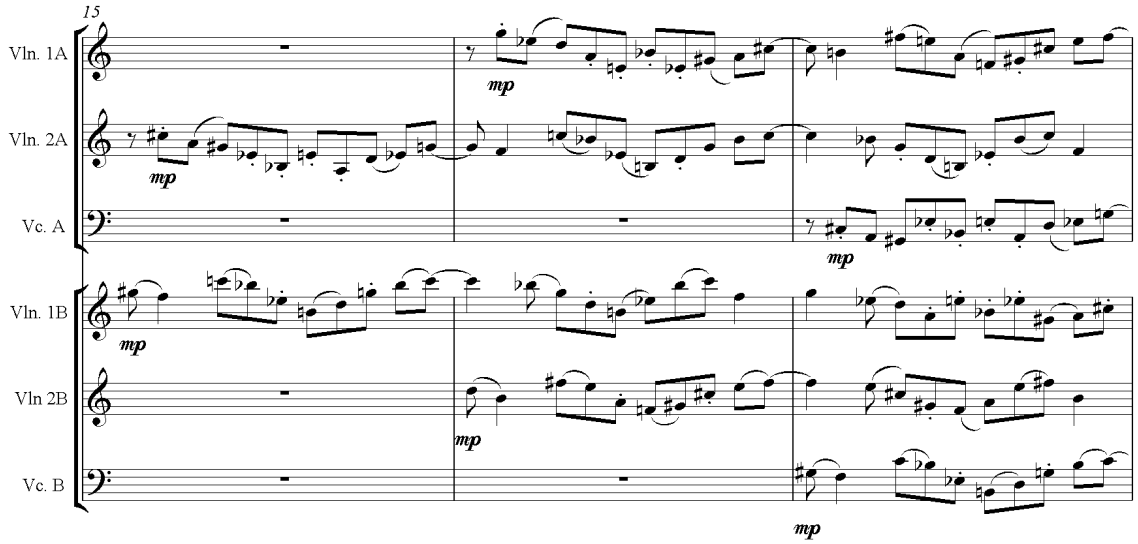

Figure 32: Opening of Tiling retrograde six-part double canon on (012)+R(012).

Table 10: Resultant hexachords of (012)+R(012) on the given module.

Base module	6(F#)	5(F)	3(Eb)	0(C)	8(Ab)	3(Eb)	9(A)	2(D)	6(F#)	9(A)	11(B)	0(C)
Pitch												
0 (C)	o	o		**o**					o		**o**	**o**
1 (C#)	o	o		**o**					o		**o**	**o**
2 (D)	o			**o**	o			**o**	o			**o**
3 (Eb)			**o**		o	**o**	o	**o**		o		
4 (E)			**o**		o	**o**	o	**o**		o		
5 (F)		**o**	**o**			**o**	o			o	o	
6 (F#)	**o**	**o**		o					**o**		o	o
7 (G)	**o**	**o**		o					**o**		o	o
8 (Ab)	**o**			o	**o**			o	**o**			o
9 (A)			o		**o**	o	**o**	o		**o**		
10(Bb)			o		**o**	o	**o**	o		**o**		
11(B)		o	o			o	**o**			**o**	**o**	
Transpositions 6-7 (012678)	0, 6	5, 11	3, 9	0, 6	2, 8	3, 9	3, 9	2, 8	0, 6	3, 9	5, 11	0, 6

If you decide to transform this six-part canon (012)+R(012) into a fewer parts one, the vertical result will always be a subset of (012678). Theoretically there would be 6 five-part subsets, 15 four-part, 10 three-part and 15 two-part but, due the limited transposition, they will be reducible to few cases; for

example, five-part subsets will be reducible from 6 to 3: one inversional symmetrical, I (5-15: 01268) and two mutually inverse, AB (5-7: 01267 and 01278), as shown in the Table 11.

Table 11: Six five-part subsets of 6-7, reduced to three.

o	o	o				o	o	o				6-7
	o	o				o	o	o				5-7B
o		o				o	o	o				5-15
o	o					o	o	o				5-7A
o	o	o					o	o				5-7B
o	o	o				o		o				5-15
o	o	o				o	o					5-7B

Another example of Tiling retrograde six-part double canon, on (025)+R(025), can be seen in Figure 33.

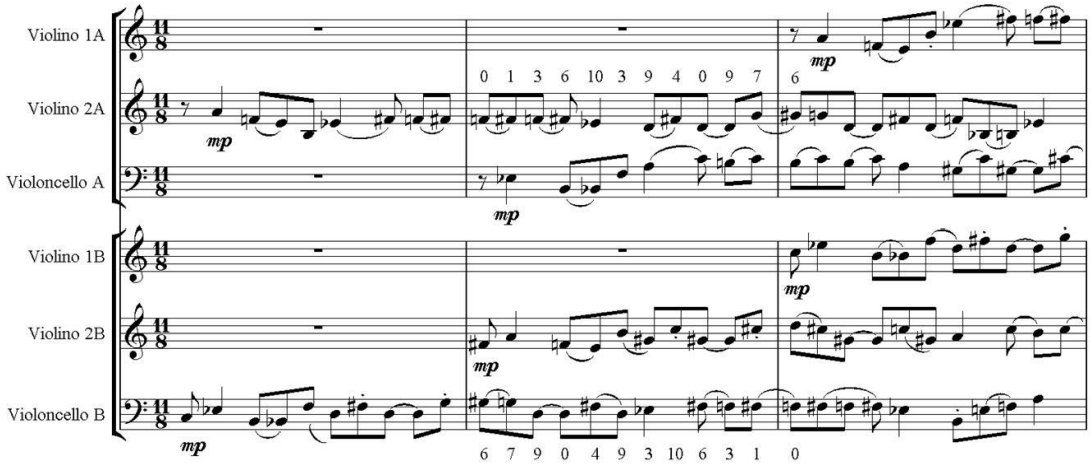

Figure 33: Opening of Tiling retrograde six-part double canon on (025)+R(025).

The trichord (025) of Figure 33 has different properties confronting to (012); in fact, it is not inversional symmetrical but has two mutually inverse forms. Table 12 shows the succession of resultant deriving from (025)+R(025), on the given module: hexachords will always be equal to (013679), that is 6-30.

But how is it possible that a such vertical interlocking of retrograde canons always gives the same hexachord? We leave the solution of this puzzle to the patient and curious reader.

It is therefore possible to create quadruple tiling retrograde canons in twelve parts, based on various combinations of one or two trichords, giving rise to *mixtures* (Table 13).

Table 12: Resultant hexachords of (025)+R(025) on the given module.

Base module	6(F#)	5(F)	3(E♭)	0(C)	8(A♭)	3(E♭)	9(A)	2(D)	6(F#)	9(A)	11(B)	0(C)
Pitch												
0 (C)	**o**	o		o	**o**			o	**o**		**o**	o
1 (C#)		**o**									o	
2 (D)	**o**		o	o		o	**o**		**o**	**o**		o
3 (E♭)		**o**	**o**		**o**	**o**	o	o		o	o	
4 (E)					o			**o**				
5 (F)	**o**		**o**	o		**o**	o		**o**	o		o
6 (F#)	o	**o**		**o**	o			**o**	o		o	**o**
7 (G)		o									**o**	
8 (A♭)	o		**o**	**o**		**o**	o		o	o		**o**
9 (A)		o	o		o	o	**o**	**o**		**o**	**o**	
10(B♭)					**o**			o				
11(B)	o		o	**o**		o	**o**		o	**o**		**o**
Transpositions 6-30 (013679)	5, 11	0, 6	2, 8	5, 11	3, 9	2, 8	2, 8	3, 9	5, 11	2, 8	0, 6	5, 11

Figure 34 shows the opening of a Tiling retrograde twelve-part quadruple canon on one trichord only (012)+R(012)+(012)T_3+R(012)T_3, realized for instrumental ensemble of six strings and six winds.

Figure 35 show the opening of a Tiling twelve-part retrograde quadruple canon on two trichords, (014)+R(014) +(013)T_2+R(013)T_2.

Table 13: Some Tiling retrograde quadruple formulas on trichords.

(012)+R(012) +(012)T_3+R(012)T_3
(012)+R(012) +(027)T_3+R(027)T_3
(014)+R(014) +(013)T_2+R(013)T_2
(014)+R(014) +(025)T_3+R(025)T_3
(014)+R(014) +(037)T_2+R(037)T_2
(015)+R(015) +(015)T_3+R(015)T_3
(015)+R(015) +(012)T_2+R(012)T_2
(015)+R(015) +(027)T_2+R(027)T_2
(024)+R(024) +(024)T_5+R(024)T_5
(027)+R(027) +(027)T_9+R(027)T_9
(048)+R(048) +(048)T_5+R(048)T_5
(048)+R(048) +(024)T_1+R(024)T_1

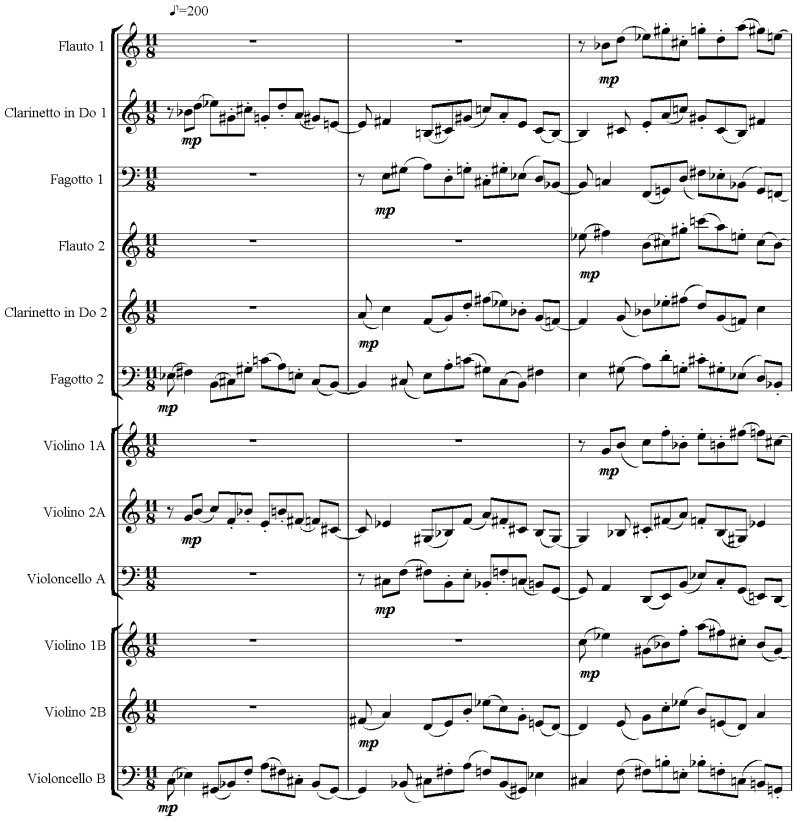

Figure 34: Tiling retrograde twelve-part quadruple canon on (012)+R(012) +(012)T_3+R(012)T_3.

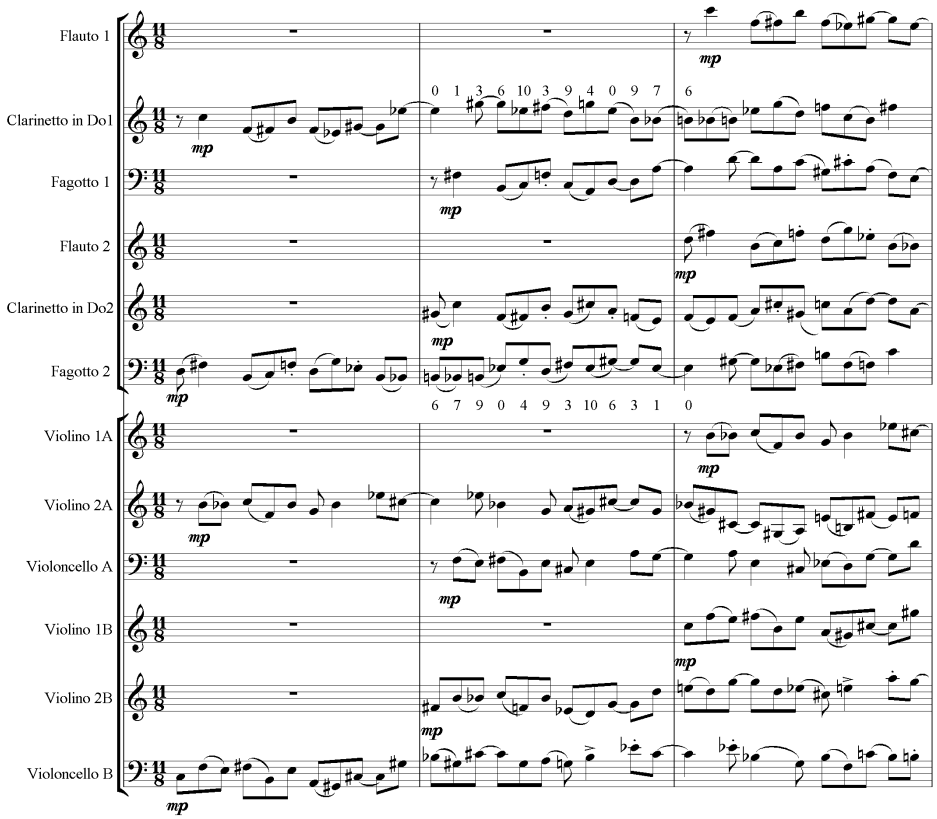

Figure 35: Opening of Tiling retrograde twelve-part quadruple canon on (014)+R(014) +(013)T_2+R(013)T_2.

Resultant hexachords by trichordal inversion

All the trichords have various levels at which one can transpose their inversion without doubling, giving rise to hexachords. The levels of transposition of *clean inversion* — that is, without doubling — is deduced from the inverse common-tone vector: the entries of the vector corresponding to 0 are good for inversion (*good inversion entries*) and their total constitutes the *inverse auto-similarity index* (VERDI, 2019, p. 513).

One will note that the character of the auto-inversion is not determined by the index of inverse auto-similarity, even though a higher index gives rise to a larger number of combinations, while a lower index produces a smaller number. Nevertheless, in the case of a very high index, as in trichord (048), which is also inversionally symmetrical, many combinations are really replicas, so that, e.g., index 9 can be reduced to 3. The total of all the possible hexachords thus generated for all the trichords is equal to 82. The following considerations provide an explanation: adopting the Allen Forte number (Forte 1973) there are sixteen inversionally symmetrical hexachords, six all-combinatorial (6-1, 6-7, 6-8, 6-20, 6-32, 6-35) and ten Z-related, of which three are pairs (6-Z6 and 6-Z38, 6-Z13 and 6Z42, 6-Z29 and 6-Z50); of the fourteen Z-related, four are omitted (6-Z28, 6-Z37, 6-Z45, 6-Z48). Therefore, there is no direct rapport with Z-relationship. Naturally, the level of embedding will profoundly influence the resultant hexachord. (Tab. 14 and Figure 36).

Table 14: Recurrences of hexachords by clean trichordal inversion.

Recurrences	Hexachord
9	6-20
6	6-1
6	6-32
5	6-8
5	6-35
5	6-4Z37
5	6-49Z28
5	6-23Z45
5	6-26Z48
5	6-6Z38
5	6-38Z6
5	6-13Z42
5	6-50Z29
4	6-42Z13
4	6-29Z50
3	6-7
Total = 82	

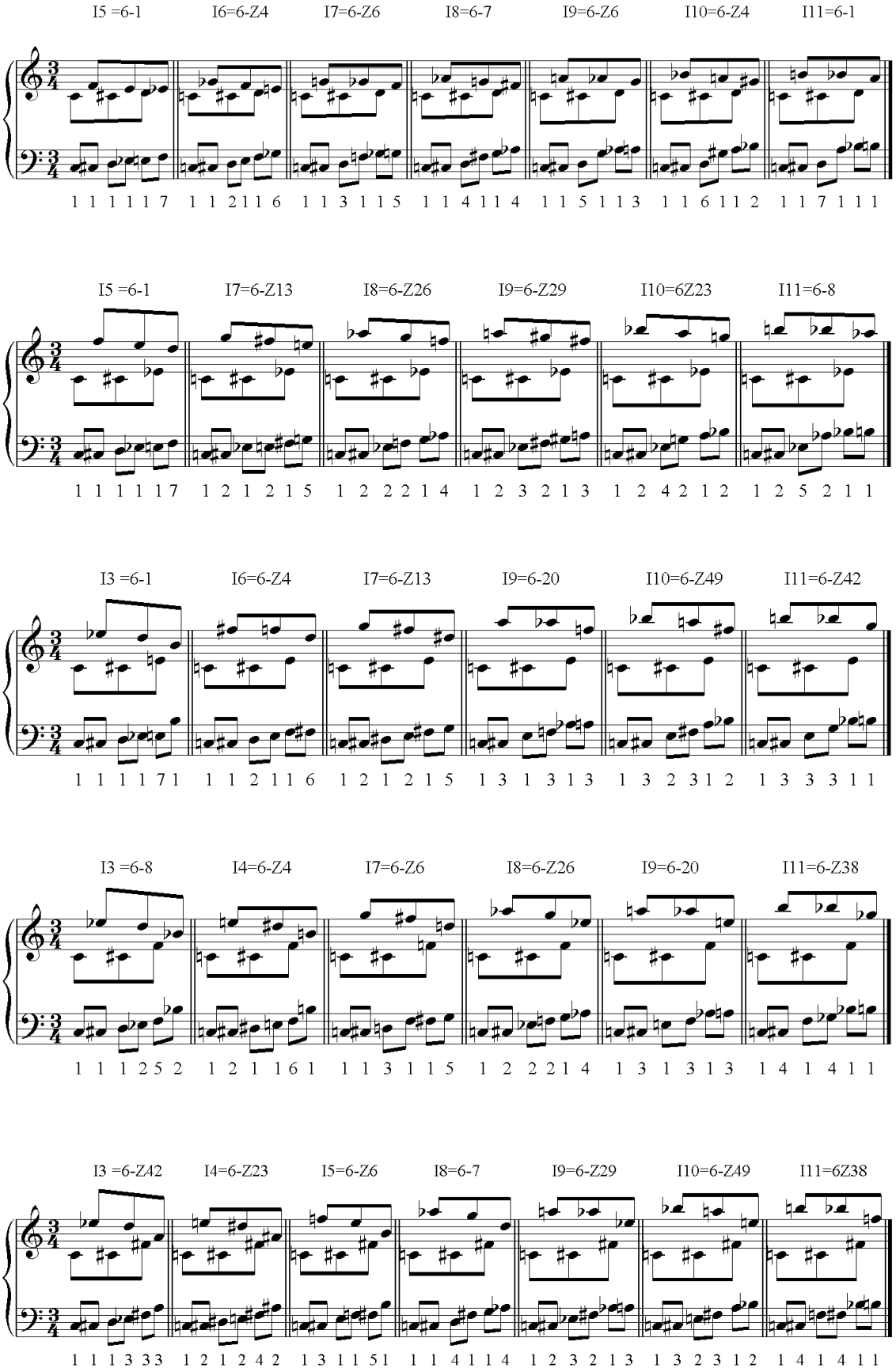

Figure 36: The 82 occurrences of clean trichordal inversions.

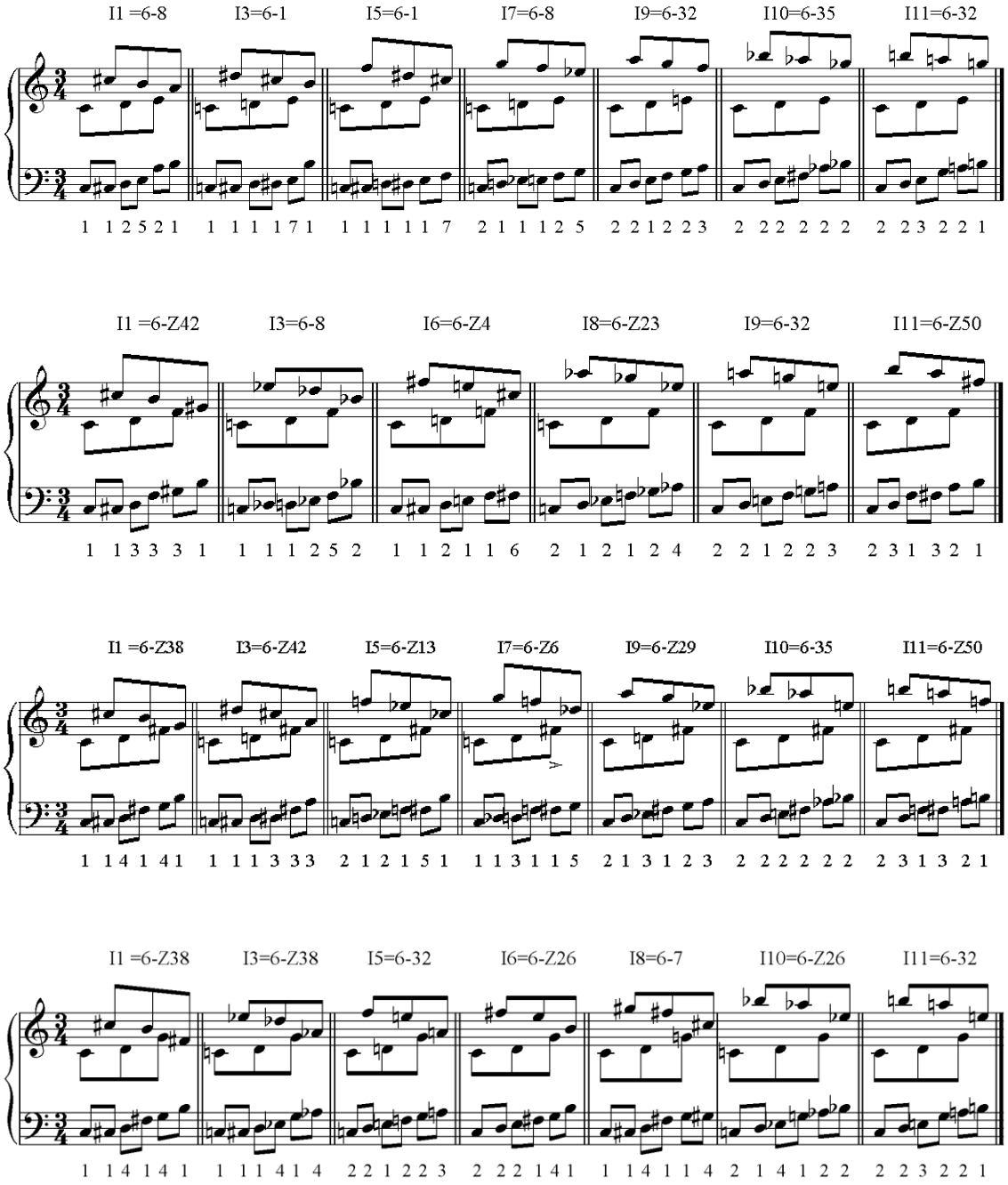

Figure 36 (continuation): The 82 occurrences of clean trichordal inversions.

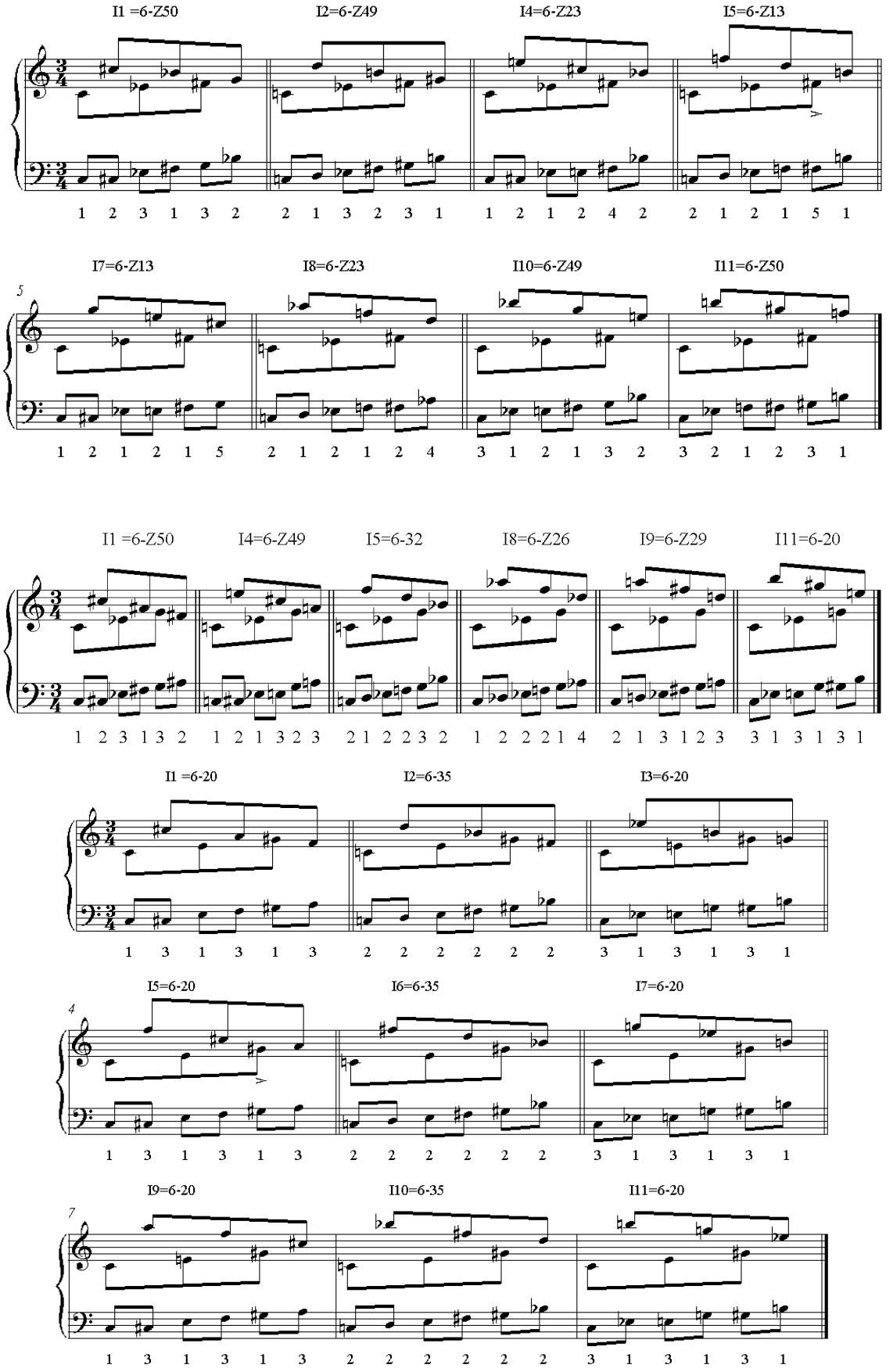

Figure 36 (continuation): The 82 occurrences of clean trichordal inversions.

Special issues

The trichord permitting the greatest possibility of inverse canons, also rarely generating a single resultant hexachord, is (048), which generates 6-20 (014589) on transposition levels 1,3,5,7,9,11 and 6-35 (0246810) on 2,6,10.

Table 15: Resultant hexachords by clean inversion of (048).

Chromatic scale	0	1	2	3	4	5	6	7	8	9	10	11	common notes	
(048)	o				o				o					
I (048)	o				o				o					
1	o	o			o	o			o	o			0	6-20
2	o		o		o		o		o		o		0	6-35
3	o			o	o			o	o			o	0	6-20
5	o	o			o	o			o	o			0	6-20
6	o		o		o		o		o		o		0	6-35
7	o			o	o			o	o			o	0	6-20
9	o	o			o	o			o	o			0	6-20
10	o		o		o		o		o		o		0	6-35
11	o			o	o			o	o			o	0	6-20
Transp. level														

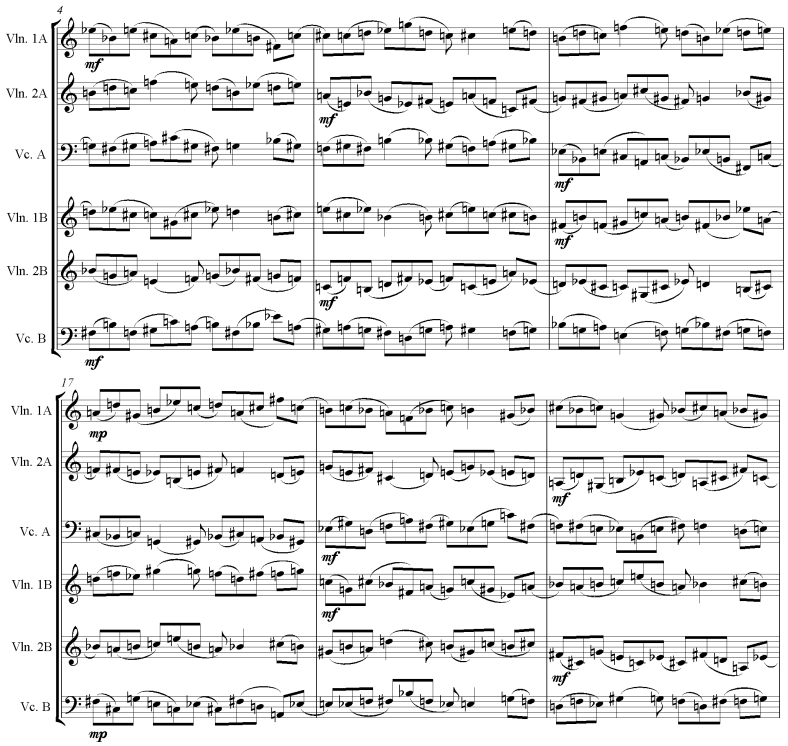

Figure 37: Fragment of (048)+I (048)T_5 of Tiling inverse six-part double canon, with vertical sum on 6-20 hexachord, original (top) and inverted (bottom).

Figure 37 shows a rare case of Tiling inverse six-part double canon.

One can see (036)+I(036)T_{10} as a final example. The resultant hexachords can be listed, as shown in Figure 38. One will note that the sum of (036)+I(036)T_{10}, on the given module, is always and only of two hexachords 6-Z49 (013479) and 6-Z23 (013461<u>0</u>): this will give extraordinary harmonic unity to the entire derived canon.

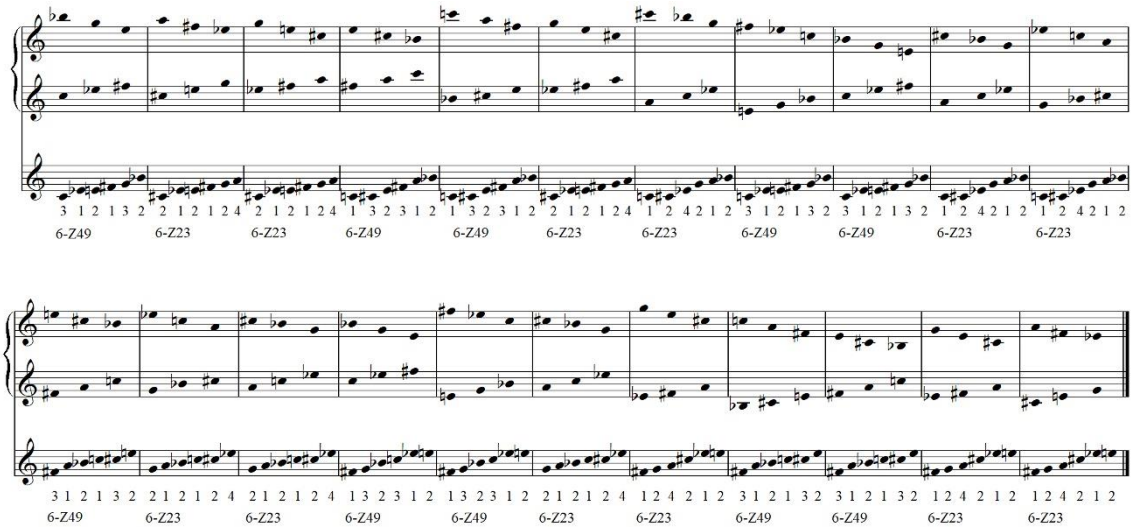

Figure 38: Resultant hexachords 036+I036 T_{10} on the given module.

In (036)+I(036)T_7, the sum is always and only of its hexachords 6-Z13 (013467) and 6-Z50 (013671<u>0</u>), etc.

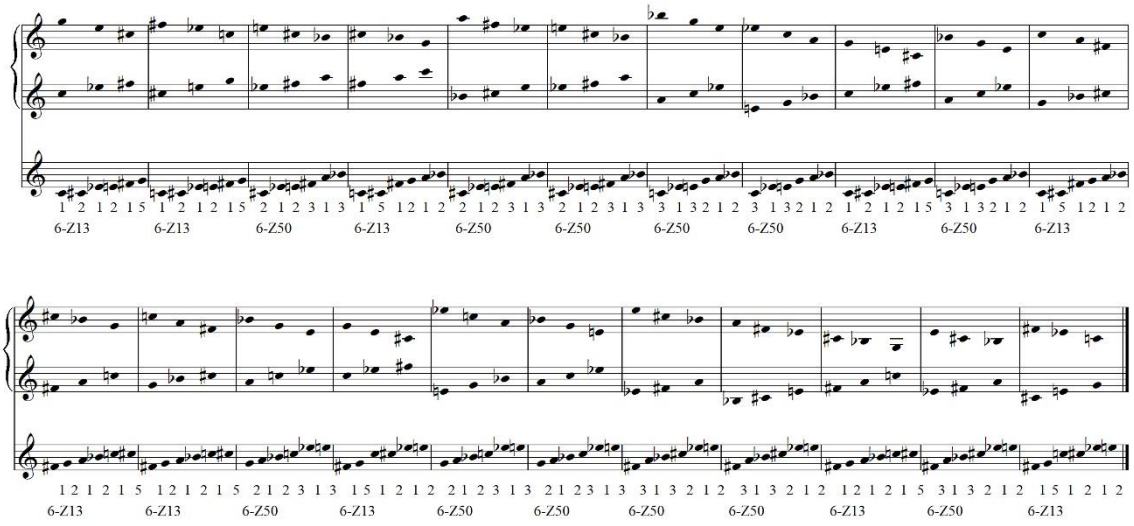

Figure 39: Resultant hexachords 036+I036 T_7 on the given module.

The second part of the canon (036)+I(036)T_7, in which the parts invert, in reality sounds a fourth above, because I(7) = 5. It is the same in the versions T_1 (I(1) = 11, a semitone below), T_2 (I(2) = 10, a tone below), T_4 (I(4) = 8, a major third below), T_5 (I(5) = 7, a fifth above), T_{10} (I(10) = 2, a tone above), T_{11} (I(11) = 1, a semitone above).

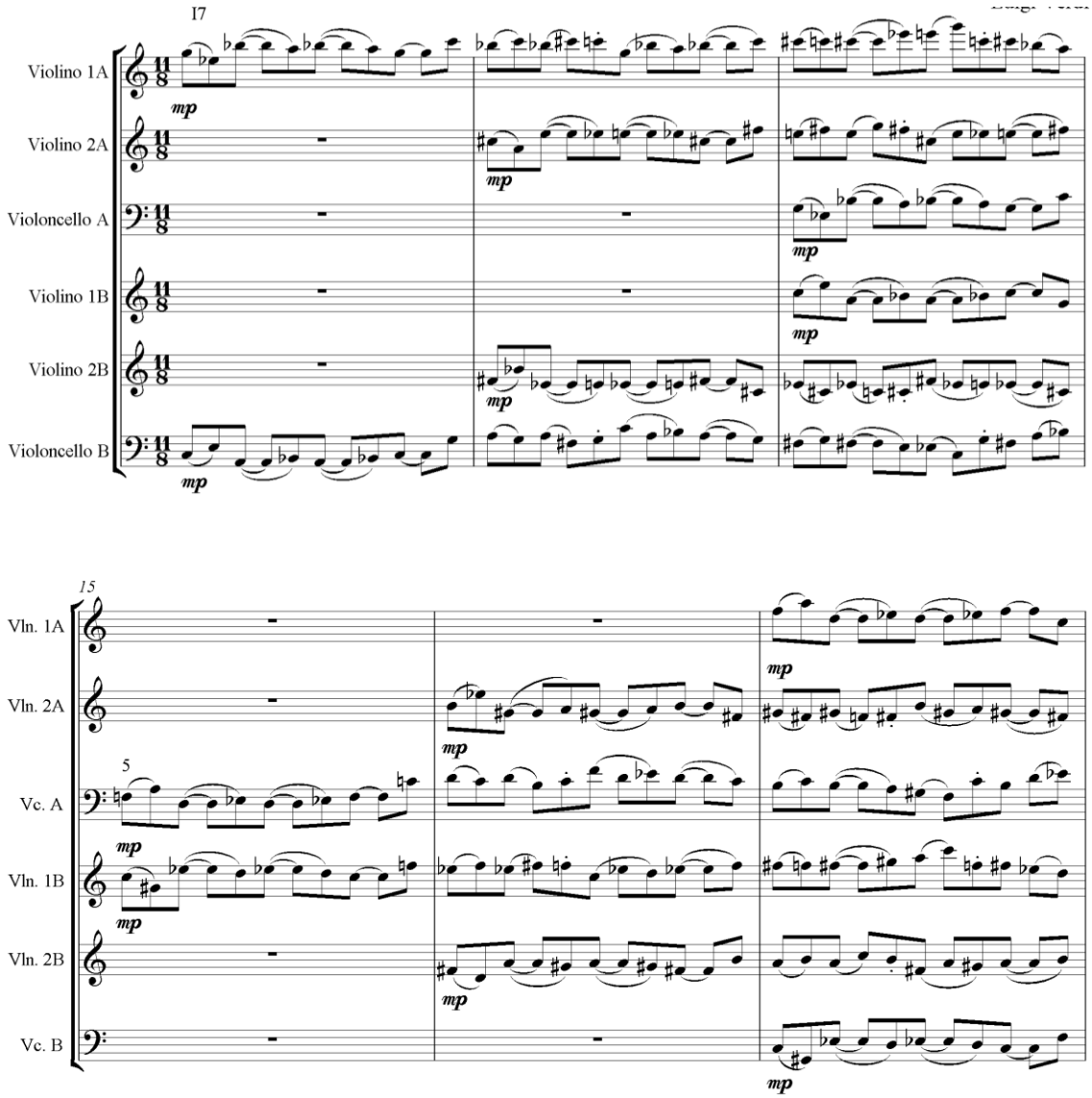

Figure 40: Opening of (036)+I(036)T_7 Tiling inverse six-part double canon, original (top) and inverted (bottom).

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4

Schoenberg's I-Combinatorial Space

Robert Peck

Inversional combinatoriality (I-combinatoriality) characterizes much of the serial music of Arnold Schoenberg. Approximately 85% of his serial works use hexachords that display this property. In this study, we investigate the space of I-combinatorial hexachords, constructing a graph-theoretical model of the space. This model assists us in enumerating the members of the set of these hexachords and the set classes to which they belong. In determining the latter, we incorporate Burnside's Lemma to count the numbers of orbits of the transposition and inversion group's action on the set. Throughout, we draw examples from Schoenberg's serial composition to illustrate various aspects of the theory. We conclude with a generalization of the theory to enumerate I-combinatorial n-chords and their set classes in modular pitch-class spaces of size 2n.

Introduction

Hexachordal combinatoriality—the property that obtains when unions of respective discrete hexachords within two twelve-tone rows constitute aggregates—characterizes much of Arnold Schoenberg's serial music. Specifically, Schoenberg's compositions incorporate combinatoriality under the operation of inversion, or *I* combinatoriality. Regarding the basic set of his *Variations for Orchestra*, op. 31, Schoenberg (1950, p. 116) writes: "the inversion a fifth below of the first six tones, the antecedent, should not reproduce a repetition of one of these six tones, but should bring forth the hitherto unused six tones of the chromatic scale. Thus, the consequent of the basic set...comprises the tones of this inversion, but, of course, in a different order," shown here in Figure 1.

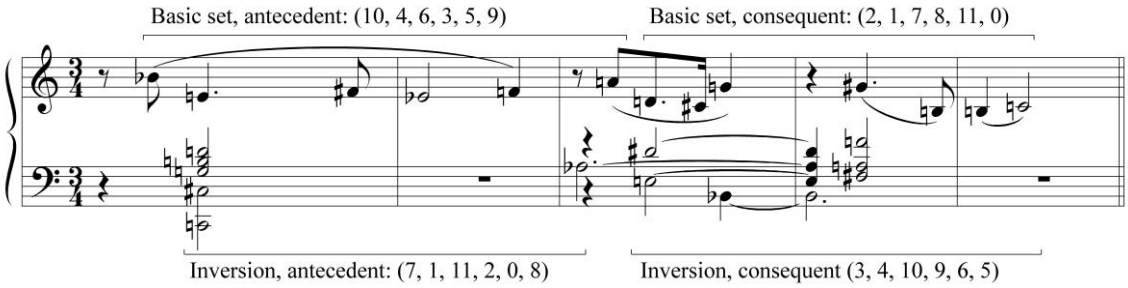

Figure 1: Hexachordal I combinatoriality in the Thema to Schoenberg's *Variations for Orchestra*, op. 31, mm. 34-38.

Table 1 provides a list of Schoenberg's twelve-tone rows, including the Ix operations under which they display combinatoriality. We note that, in addition to the operation I5, which maps the first hexachord of the basic set from op. 31 to its complement, all the other Ix operations with odd indices appear at least once in the list. Further, some tone rows are I-combinatorial under multiple Ix operations. The tone row from the fifth movement, "Tanzscene," of the Serenade, op. 24, is combinatorial under two inversion operators: I5 and I11. Similarly, the tone row of his Psalm 130 (De Profundis), op. 50B, is combinatorial under I3 and I9. Opus numbers 29, 41, and 50C each contain tone rows that are combinatorial under three inversion operators: I3, I7, and I11 for the Suite, op. 29, and for the Ode to Napoleon Buonaparte, op. 41; and I1, I5, and I9 for the Moderner Psalm, op. 50C. Remarkably, the tone row of the third song, "Mädchenlied," from Drei Lieder, op. 48, is combinatorial under all six inversion operators with odd indices. Even for compositions in which he did not use the full combinatorial potential of their tone rows' hexachords, their inversional complement relation appears to have been an important desideratum in his choice of row structure. Among the tone rows in his forty-two serial compositions, thirty-six, or 85.7%, use

hexachords that produce I combinatoriality, whereas only 348, or 36.7%, of all 924 hexachords display this property.

Table 1: Schoenberg's twelve-tone rows and the I_x operators under which they are combinatorial (* denotes unpublished fragment).

Composition	Hexachord A	Hexachord B	Set class(es)	I_x combinatorial
op. 23, no. 5	1 9 11 7 8 6	10 2 4 3 0 5	6-9	I_{11}
op. 24, mov. 4	4 2 3 11 0 1	8 6 9 5 7 10	6-1	I_9
op. 24, mov. 5	9 10 0 3 4 6	5 7 8 11 1 2	6-30	I_5, I_{11}
op. 25	4 5 7 1 6 3	8 2 11 0 9 10	6-2	I_3
op. 26	3 7 9 11 1 0	10 2 4 6 8 5	6-21	I_5
op. 27, no. 1	6 5 2 8 7 1	3 4 10 9 11 0	6-5	I_5
op. 27, no. 2	0 11 4 10 2 8	3 7 6 5 9 1	6-21	I_5
op. 27, no. 3	7 6 2 4 5 3	11 0 8 10 9 1	6-1	I_3
op. 27, no. 4	1 3 10 6 8 4	11 0 2 9 5 7	6-33	I_3
op. 28, no. 1	0 4 7 1 9 11	5 3 2 6 8 10	6-Z46/Z24	N/A
op. 28, no. 3	5 6 4 8 2 10	7 9 3 11 1 0	6-21	I_5
op. 29	3 7 6 10 2 11	0 9 8 4 5 1	6-20	I_3, I_7, I_{11}
op. 30	7 4 3 9 0 5	6 11 10 1 8 2	6-Z46/Z24	N/A
op. 31	10 4 6 3 5 9	2 1 7 8 11 0	6-5	I_5
op. 32	2 3 9 1 11 5	8 7 4 0 10 6	6-21	I_9
op. 33A, no. 1	10 5 0 11 9 6	1 3 7 8 2 4	6-5	I_1
op. 33B, no. 2	11 1 5 3 9 8	6 10 7 4 0 2	6-34	I_3
op. 34	3 6 2 4 1 0	9 11 10 8 5 7	6-2	I_{11}
op. 35, no. 1	2 11 3 5 4 1	8 10 9 6 0 7	6-2	I_{11}
op. 35, no. 2	6 9 7 1 0 2	5 11 10 3 4 8	6-18	I_5
op. 35, no. 3	3 6 7 8 5 0	9 10 4 11 2 1	6-Z40/Z11	N/A
op. 35, no. 5	1 7 10 2 3 11	8 4 0 6 5 9	6-15	I_7
op. 36	9 10 3 11 4 6	0 1 7 8 2 5	6-18	I_{11}
op. 37	2 1 9 10 5 3	4 0 8 7 6 11	6-16	I_7
*Phantasia**	1 5 3 6 4 8	0 11 2 9 10 7	6-8	I_3
Sonata for Organ*	1 7 11 3 9 2	8 6 10 5 0 4	6-22	I_7
Piano 4 Hands*	6 9 0 7 1 2	8 11 5 10 4 3	6-18	I_5
op. 41	1 0 4 5 9 8	3 2 6 7 11 10	6-20	I_3, I_7, I_{11}
op. 42	3 10 2 5 4 0	6 8 1 9 11 7	6-9	I_{11}
Die Jakobsleiter	1 2 5 4 8 7	0 3 11 10 6 9	6-Z13/Z42	N/A
op. 44	10 6 2 5 4 0	11 8 1 3 9 7	6-22	I_1
op. 45	2 10 3 9 4 1	11 8 6 7 5 0	6-5	I_9
op. 46	6 7 0 8 4 3	11 10 5 9 1 2	6-15	I_5
op. 47	10 9 1 11 5 7	3 4 0 2 8 6	6-21	I_1
op. 48, no. 1	1 2 0 6 3 5	4 10 11 7 9 8	6-Z3/Z36	N/A
op. 48, no. 2	2 3 9 1 10 4	8 7 0 11 5 6	6-5	I_9
op. 48, no. 3	1 7 9 11 3 5	10 6 4 0 8 2	6-35	I_1, I_3, I_5, I_7, I_9, I_{11}
Israel Exists Again	0 3 4 9 11 5	2 1 10 8 6 7	6-Z43/Z17	N/A
op. 50A	7 9 6 4 5 11	10 2 0 1 3 8	6-9	I_7
op. 50B	3 9 8 4 2 10	7 11 0 6 5 1	6-7	I_3, I_9
op. 50C	4 3 0 8 11 7	5 9 6 10 1 2	6-20	I_1, I_5, I_9
Moses und Aaron	9 10 4 2 3 1	7 5 6 8 11 0	6-5	I_9

Below, we investigate how the set of *I*-combinatorial hexachords forms a space, situating examples from Schoenberg's works within that space. Further, we generalize the space, creating a model that allows us to determine the set of all *I*-combinatorial *n*-chords, to observe the action of the musical transposition-and-inversion group on this set, and to enumerate the set classes that this action generates. We conclude with some additional potential musical applications of the general theory.

Hexachordal I Combinatoriality: Combinatoriality for a specific I_x

Let us consider again the tone row from Schoenberg's op. 31, which is combinatorial under I_5.

To maintain the complement relation between the hexachords, no two pitch classes that relate by I_5 can be present in the same hexachord, as those pitch classes map onto one another under that operation. Figure 2 depicts the members of the row's two hexachords as beads in a binary necklace; the white beads represent the pitch classes of the first hexachord and the black beads represent those of the second. We note that the necklace balances across the I_5 axis: for each pitch class c of one hexachord, a corresponding pitch class $d = 5 - c \pmod{12}$ from the other hexachord appears directly across the axis. Any partition of the twelve-tone aggregate into I_5-combinatorial hexachords can be represented in this way. Therefore, as we find two possible positions relative to the I_5 axis for any one of the six $\{c, d\}$ pairs, we note that there exist $2^6 = 64$ hexachords that are I_5-combinatorial.

To determine the transpositional symmetry of an I_x-combinatorial hexachord, we derive a binary representation of the hexachord by placing its pitch classes first into a particular order: we use ascending order for even pitch classes, beginning with 0; and descending (cyclic) order, beginning with an odd x, for odd pitch classes. Hence, the ordering of the even whole-tone hexachord is $(0, 2, 4, 6, 8, 10)$ and that of the odd whole-tone hexachord is $(x - 0, x - 2, x - 4, x - 6, x - 8, x - 10) \pmod{12}$. If an I_x-combinatorial hexachord incorporates a mixture of even and odd pitch classes, each pitch class occupies the same coordinate as above. In other words, the first coordinate is either 0 or $x - 0$, the second coordinate is 2 or $x - 2$, and so forth. Figure 3 shows the reordering of the first hexachord of the op. 31 row, using $x = 5$, which yields $(5, 3, 4, 6, 9, 10)$.

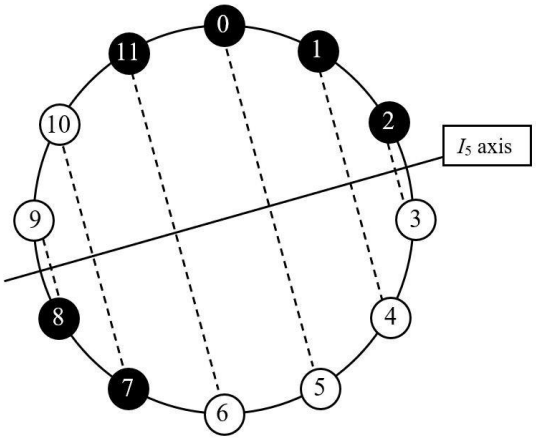

Figure 2: The tone row of Schoenberg's *Variations for Orchestra*, op. 31, as a balanced binary necklace (first hexachord in white, second hexachord in black).

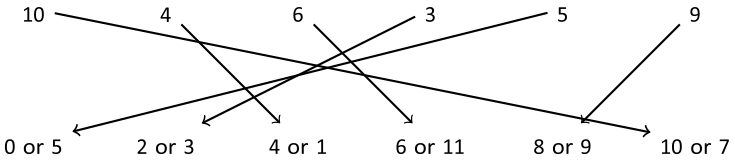

Figure 3: Reordering the first hexachord of the op. 31 tone row, using x = 5.

To derive the representation, we construct a six-element binary tuple by mapping all even pitch classes in the reordered set to 0s and all odd pitch classes to 1s. For purposes of illustration, we reorder the pitch classes of the first and second hexachords of various of Schoenberg's tone rows that are combinatorial under I_5, yielding the corresponding binary representations. The first two of the following examples show binary representations of reordered hexachords from the tone row of op. 48, no. 3; the third and fourth show those of the op. 50C row; the fifth and sixth show those from the row of op. 24, mov. 5, and the final two show those from op. 31.

op. 48, no. 3:	(5, 3, 1, 11, 9, 7) → (1, 1, 1, 1, 1, 1)
	(0, 2, 4, 6, 8, 10) → (0, 0, 0, 0, 0, 0)
op. 50C:	(0, 11, 4, 7, 8, 3) → (0, 1, 0, 1, 0, 1)
	(5, 2, 1, 6, 9, 10) → (1, 0, 1, 0, 1, 0)
op. 24, mv. 5:	(0, 3, 4, 6, 9, 10) → (0, 1, 0, 0, 1, 0)
	(5, 2, 1, 11, 8, 7) → (1, 0, 1, 1, 0, 1)
op. 31:	(5, 3, 4, 6, 9, 10) → (1, 1, 0, 0, 1, 0)
	(0, 2, 1, 11, 8, 7) → (0, 0, 1, 1, 0, 1)

Because of the cyclic reorderings of even and odd pitch classes that go into determining a binary representation, the periodicity of a representation reflects the transpositional symmetry of its corresponding hexachord. Given a binary representation with a period of length p, the hexachord maps onto itself under the action of the members of the transposition subgroup $\langle T_{2p} \rangle$; accordingly, the hexachord has transpositional symmetry of degree $d = 6/p$. We note that the binary representations of the first two examples above have a periodicity of length $p = 1$, the third and fourth of $p = 2$, the fifth and sixth of $p = 3$, and the seventh and eighth of $p = 6$. Hence, the hexachords of the first two examples are symmetrical under the members of the transposition subgroup $\langle T_2 \rangle$, those of the next two examples are symmetrical under $\langle T_4 \rangle$, those of the fifth and sixth examples under $\langle T_6 \rangle$, and the final two examples under those of the trivial subgroup $\langle T_{12} = T_0 \rangle$.

We may also use the periodicity p of binary representations to determine the numbers of I_x-combinatorial hexachords that have transpositional symmetry of degree d. As a result of the periodic repetition of binary coordinates in a representation and the fact that each coordinate within a period may equal 0 or 1, there exist 2^p hexachords for each value of p. We find

- $2^1 = 2$ hexachords with $p = 1$ (and that are symmetrical under six transposition operators),
- $2^2 = 4$ hexachords with $p = 2$ (and that are symmetrical under three transposition operators), and
- $2^3 = 8$ hexachords with $p = 3$ (and that are symmetrical under two transposition operators).

(As we have noted above, the trivial case with $p = 6$ gives $2^6 = 64$ hexachords that are symmetrical under one transposition operator.)

Hexachordal I Combinatoriality: The set *S* of all *I*-combinatorial hexachords

Whereas we find sixty-four I_x-combinatorial hexachords for each one of the six odd values of x, we note that there exist fewer than $64 \cdot 6 = 384$ I-combinatorial hexachords in total. Counting the members of the nineteen set classes of I-combinatorial hexachords reveals 348 pitch-class sets that have this property (see Table 2). The reason for this difference is that certain hexachords are combinatorial for more than one I_x operator. For instance, as we saw above, the tone row from the fifth movement of Schoenberg's *Serenade*, op. 24, is combinatorial under both I_5 and I_{11}. Figure 4 displays the members of the two hexachords of this tone row respectively as white and black beads in a binary

necklace that balances across two axes; (a) shows the necklace balanced across the I_5 axis and (b) shows it balanced across the I_{11} axis.

Table 2: Set classes of I-combinatorial hexachords, their size, and the degree of T_x symmetry of their members.

Prime form	Forte label	Size of set class	Degree of T_x symmetry
0, 1, 2, 3, 4, 5	6-1	12	1
0, 1, 2, 3, 4, 6	6-2	24	1
0, 1, 2, 3, 6, 7	6-5	24	1
0, 1, 2, 6, 7, 8	6-7	6	2
0, 2, 3, 4, 5, 7	6-8	12	1
0, 1, 2, 3, 5, 7	6-9	24	1
0, 1, 2, 4, 5, 8	6-15	24	1
0, 1, 4, 5, 6, 8	6-16	24	1
0, 1, 2, 5, 7, 8	6-18	24	1
0, 1, 4, 5, 8, 9	6-20	4	3
0, 2, 3, 4, 6, 8	6-21	24	1
0, 1, 2, 4, 6, 8	6-22	24	1
0, 1, 3, 4, 6, 9	6-27	24	1
0, 1, 3, 6, 7, 9	6-30	12	2
0, 1, 3, 5, 8, 9	6-31	24	1
0, 2, 4, 5, 7, 9	6-32	12	1
0, 2, 3, 5, 7, 9	6-33	24	1
0, 1, 3, 5, 7, 9	6-34	24	1
0, 2, 4, 6, 8, 10	6-35	2	6
		Total: 348	

Inversional axes translate along with pitch-class content under transposition. Hence, *I*-combinatorial hexachords with transpositional symmetry of degree *d* are also combinatorial under *d* inversion operators, where that set of inversion operators is a coset of $\langle T_{2p} \rangle$, multiplying the transpositional subgroup (on the left or the right) by any inversion operator under which the hexachord is combinatorial. (It does not matter if we use left or right cosets, as any rotational subgroup of a dihedral group is a normal subgroup.) For instance, the members of set class 6-30, to which the hexachords in the op. 24 tone row belong, are invariant under two transposition operators: T_0 and T_6 (i.e., the set class has transpositional symmetry of $d = 2$). The op. 24, mov. 5 row is combinatorial under I_5 and I_{11}, which constitute a coset of the subgroup consisting of T_0 and T_6, formed by multiplying the subgroup by either I_5 or I_{11}.

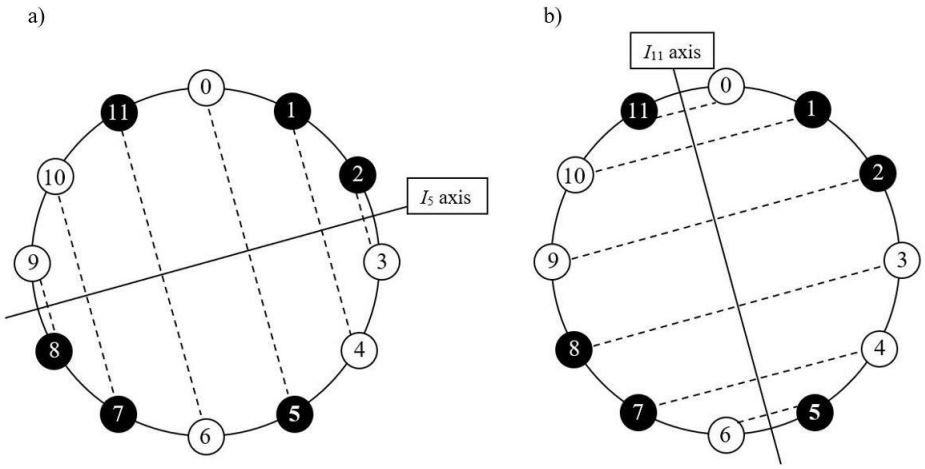

Figure 4: The tone row of the fifth movement, "Tanzscene," of Schoenberg's *Serenade*, op. 24, as a binary necklace; (a) shows the necklace balanced across the I_5 axis and (b) across the I_{11} axis.

We determine the size of the set S of all hexachords with inversional combinatoriality via the formula below in Equation 1. It incorporates the Möbius μ-function, which returns the following values for any positive integer n:

- $\mu(n) = 1$, if $n = 1$;
- $\mu(n) = 0$, if n has a square prime factor; and
- $\mu(n) = (-1^r)$, if n has r distinct prime factors.

$$|S| = \sum_{z \mid 6} \sum_{v \mid z} \mu(v) 2^{\frac{z}{v}} z \tag{1}$$

Calculating for each divisor z of 6 yields the following values, which correspond to the total numbers of hexachords that are I-combinatorial under exactly $6/z$ different inversion operators.

$z = 6$:

$v = 1$: $(1 \cdot 2^6) \cdot 6 = 348$
$v = 2$: $(-1 \cdot 2^3) \cdot 6 = -48$
$v = 3$: $(-1 \cdot 2^2) \cdot 6 = -24$
$v = 6$: $(1 \cdot 2^1) \cdot 6 = \underline{12}$
324

$z = 3$:

$v = 1$: $(1 \cdot 2^3) \cdot 3 = 24$
$v = 3$: $(-1 \cdot 2^1) \cdot 3 = \underline{-6}$
18

$z = 2$:

$v = 1$:	$(1 \cdot 2^2) \cdot 3$	=	8
$v = 2$:	$(-1 \cdot 2^1) \cdot 3$	=	-4
			4

$z = 1$:

$v = 1$:	$(1 \cdot 2^1) \cdot 1$	=	2

For $z = 6$, we find 324 hexachords that are combinatorial under only one inversion operator; for $z = 3$, eighteen are combinatorial under precisely two. Four hexachords are combinatorial under three inversion operators, and two under all six odd-indexed inversion operators—all of which sum to 348, the size of S.

Hexachordal I Combinatoriality: Partitioning *S* into set classes

Table 2 above indicated that the 348 *I*-combinatorial hexachords belong to nineteen set classes. This number, which is prime and does not divide 348 evenly, may seem counterintuitive. The process for determining the number of set classes to which the *I*-combinatorial hexachords belong is tantamount to counting the number of orbits that exist in the action of the transposition-and-inversion group's action on the set S. For this task, we use Burnside's Lemma,

$$|S/G| = \frac{1}{|G|}\sum_{g \in G}|S^g|, \tag{2}$$

which calculates the average number of elements of a set S that are stabilized by the members of a group G. In this case, G is the transposition-and-inversion group and S is the set of *I*-combinatorial hexachords.

Equation 3 is a variant of the formula in Equation 1; it determines the numbers of *I*-combinatorial hexachords that are stabilized by the various transposition operators with even indices, the various operations T_{2x}. In essence, the formula in Equation 1 enumerated the number of hexachords that are stabilized under $T_{2\cdot 0} = T_0$. We note as a corollary that no hexachords are stabilized under any transposition operators with odd indices.

$$|S^{T_{2x}}| = \sum_{z \mid \gcd(x,6)} \sum_{v \mid z} \mu(v) 2^{\frac{z}{v}} z \tag{3}$$

We find 348 *I*-combinatorial hexachords that are stabilized under T_0, two that are stabilized under both T_2 and T_{10}, six that are stabilized under both T_4 and T_8, and twenty that are stabilized under T_6.

Next, we determine the numbers of *I*-combinatorial hexachords that are stabilized under inversion operators with even and odd indices, respectively. To calculate the former, we require the 2-adic order of the integer n, which we call q; it is the largest power of two that divides n. In the case of $n = 6$, $q = 2^1$, because 2 itself is the greatest power of 2 that divides 6.

$$|S^{I_{2x}}| = 2^{((6/q)+1)/2} = 4 \tag{4}$$

As Equation 4 shows, four *I*-combinatorial hexachords are stabilized by I_{2x}. Multiplying this number by 6—the number of even-indexed inversion operators—yields a total of twenty-four.

The formula in Equation 5, then, determines the number of *I*-combinatorial hexachords that are stabilized by an inversion operator with an odd index. It incorporates a function, $\alpha(n)$, which counts the total number of balanced binary necklaces with a given even number $2n$ of beads—in our case, $n = 6$, so $\alpha(6)$ counts the number of balanced binary necklaces with twelve beads—from which we subtract the number of *I*-combinatorial hexachords that are stabilized by an even-indexed inversion, I_{2x}.

$$|S^{I_{2x+1}}| = 2\alpha(6) - |S^{I_{2x}}|, where \begin{cases} \alpha(0) = 1 \\ \alpha(2n) = \alpha(n) + 2^{n-1} \\ \alpha(2n+1) = 2^n \end{cases} \tag{5}$$

As $\alpha(6) = 6$, the number of *I*-combinatorial hexachords that are stabilized by I_{2x+1} is twice that number, twelve, minus the four hexachords that are stabilized by I_{2x}, yielding eight. Hence, there exist $8 \cdot 6 = 48$ *I*-combinatorial hexachords that are stabilized by odd-indexed inversions.

Table 3 presents a summary of all the above values. Thus, by Burnside's Lemma, the number of set classes to which the members of the set *S* belong is the average number of hexachords stabilized by the twenty-four members of the transposition and inversion group, or nineteen.

Table 3: Sizes of S^g for each member g of the transposition-and-inversion group.

$\|S^{T_0}\| = 348$	$\|S^{T_1}\| = 0$	$\|S^{I_0}\| = 4$	$\|S^{I_1}\| = 8$
$\|S^{T_2}\| = 2$	$\|S^{T_3}\| = 0$	$\|S^{I_2}\| = 4$	$\|S^{I_3}\| = 8$
$\|S^{T_4}\| = 6$	$\|S^{T_5}\| = 0$	$\|S^{I_4}\| = 4$	$\|S^{I_5}\| = 8$
$\|S^{T_6}\| = 20$	$\|S^{T_7}\| = 0$	$\|S^{I_6}\| = 4$	$\|S^{I_7}\| = 8$
$\|S^{T_8}\| = 6$	$\|S^{T_9}\| = 0$	$\|S^{I_8}\| = 4$	$\|S^{I_9}\| = 8$
$\|S^{T_{10}}\| = 2$	$\|S^{T_{11}}\| = 0$	$\|S^{I_{10}}\| = 4$	$\|S^{I_{11}}\| = 8$

A General Model

In general, *I* combinatoriality is a means of filling a space with an object and its negative. In the case of *I* hexachordal combinatoriality, the space is the usual chromatic pitch-class space, the object is a hexachord, and its negative is its inverse. However, we find other musical applications of this concept, so it is useful to generalize the theory to other spaces.

We begin by observing that *I* combinatoriality can obtain only in even-sized spaces—hence, of size $2n$ for some integer n—the reason's being that the object such as an n-chord and its complement must (a) cover the space and (b) not intersect in any points. Accordingly, we will model *I*-combinatorial n-chords in the integers modulo $2n$. Then, as above, *I* combinatoriality occurs only under inversion operators with odd indices, as even-indexed inversions in spaces of size $2n$ always produce two invariants.

We may adapt several of the equations we investigated above to accommodate the general space. First, we observe that there exist 2^n I_x-combinatorial n-chords for any particular odd-indexed I_x operator. Some of these n-chords are combinatorial under additional values of I_x, however, so we replace the integer 6 in Equation 1 with the variable n to determine the number of all *I*-combinatorial n-chords in a given space.

$$|S| = \sum_{z \mid n} \sum_{v \mid z} \mu(v) 2^{\frac{z}{v}} z \tag{6}$$

Again, we use Burnside's Lemma to enumerate the numbers of set classes to which these n-chords belong. To do so, we need to determine the number of n-chords that are stabilized by the various *T* and *I* operators. For the former, we modify Equation 3 (again, showing the substitution of 6 with n), and, for the latter, Equations 4 and 5. We may now determine the numbers of *I*-combinatorial n-chords and of their set-classes in any even-sized pitch-class space. Table 4 shows these values through $n = 12$.

Table 4: Numbers of I-combinatorial n-chords and of their set-classes in pitch-class spaces $\mathbb{Z}_{2n}$.

Space	$\mathbb{Z}_2$	$\mathbb{Z}_4$	$\mathbb{Z}_6$	$\mathbb{Z}_8$	$\mathbb{Z}_{10}$	$\mathbb{Z}_{12}$	$\mathbb{Z}_{14}$	$\mathbb{Z}_{16}$	$\mathbb{Z}_{18}$	$\mathbb{Z}_{20}$	$\mathbb{Z}_{22}$	$\mathbb{Z}_{24}$
n	1	2	3	4	5	**6**	7	8	9	10	11	12
n- chords	2	6	20	54	152	**348**	884	1974	4556	10056	22508	48636
Set classes	1	2	3	6	10	**19**	36	70	136	266	528	1043

Conclusions

One potential further application of the theory would be to rhythmic structures. For example, how many ways might we tessellate a measure in $\frac{4}{4}$ with an eighth-note-level beat-class set and its retrograde? Figure 5 gives one such example, using the beat-class set {0, 3, 5, 6} and its retrograde {1, 2, 4, 7}. We know from the discussion above that there exist $2^4 = 16$ such rhythms for each of the four odd axes around which the rhythm may potentially retrograde (the example in Figure 5 reflects around the midpoint of the measure). But we also know that some of these rhythms will be identical under rotation, so fewer than $4 \cdot 16 = 64$ such rhythms exist. We may use the formula in Equation 1 for determining the size of S to discover that there are 54 such rhythms in total, and we apply Burnside's Lemma to ascertain that those rhythms belong to six set classes, and so forth.

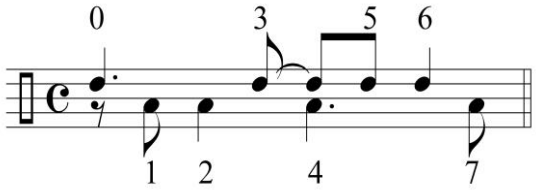

Figure 5: Beat-class set {0,3,5,6} and its retrograde {1,2,4,7}, tiling a measure of $\frac{4}{4}$.

We might also adapt the methodologies here to study other types of combinatoriality: transpositional, retrograde, retrograde-inversion, and Babbitt's all-combinatoriality. Such investigations will help us reveal further aspects of these compositional spaces.

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5

Schoenberg and Group Theory: An Intervallic Approach to Tone Rows, Symmetry and Combinatoriality

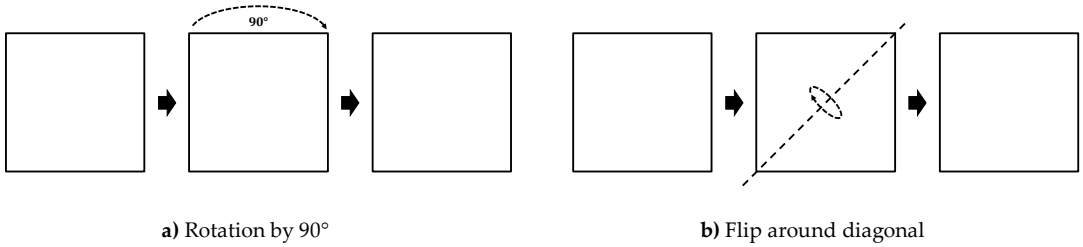
Marco Feitosa

In music, it is always useful to observe already known musical objects from different perspectives in order to reinforce or deepen previous knowledge, or even to open the way for new theoretical possibilities. That is the case with tone rows. In this work, we make a brief theoretical review of group theory's applications in twelve-tone music. Then we show, through some examples of Schoenberg and his circle, how tone rows behave from the point of view of their interval structures and how twelve-tone operations, or rather, group actions act on such structures, generating different row classes (orbits). Finally, we present the advantages of such an intervallic approach and how we can build symmetric or combinatorial tone rows exclusively from intervals, or more precisely, from *interval compositions*.

Group Theory and Twelve-Tone Music

In mathematics, particularly in abstract (or modern) algebra, group theory studies the algebraic structures known as groups, which are sets equipped with a binary operation (usually called multiplication) that satisfies certain basic properties or axioms (associativity, identity element, and inverse element).[1] According to Weisstein (2021), group theory is "a powerful formal method for analyzing abstract and physical systems in which symmetry is present." In short, group theory is the study of symmetry: "Wherever symmetry occurs groups describe it." (BUDDEN, 1972, p. 542).

Symmetry, in turn, is "a vast subject, significant in art and nature. Mathematics lies at its root, and it would be hard to find a better one on which to demonstrate the working of the mathematical intellect" (WEYL, 2016, p. 145). Mathematically, the term symmetry is generally used to refer to an object that is invariant under some transformation, like translation, reflection, rotation, scaling, and so on. For example, if we rotate a square 90° clockwise (Figure 1a) or flip it around its diagonal (Figure 1b), then we get the same picture. In both cases, we cannot distinguish the resulting square from its original configuration. Since that object is invariant under those (and other) actions, it means that it is symmetric.



a) Rotation by 90° **b)** Flip around diagonal

Figure 1: Symmetric transformations on a square.

At the beginning of twelve-tone music, the same notion of transformations and symmetries (invariances) also took place in a very intuitive way. Let us consider Schoenberg's *Wind Quintet*, *Op. 26*, one of his first dodecaphonic compositions, written between 1923 and 1924 (Figure 2).

To compose that work, Schoenberg used a paper *twelve-tone selection dial* or *wheel chart* crafted by himself (Figure 3).[2] In order to see how it works, we can rearrange the shuffled chart (Figure 3a) to its original configuration (Figure 3b) so that it starts with the first pitch class of the row, which is the E♭ (or D♯) played by the flute. Now, as we move from 1 up to 12 (Figure 3c), we get the *original* row (O) on the inner staff and its *inversion* (I) on the outer staff. Conversely, when we

[1] For an introduction to group theory, see GRIMALDI (2003) and GALLIAN (2013).

[2] The original chart is available at the *Arnold Schönberg Center*, in Vienna, Austria. For more information, visit: https://www.schoenberg.at.

move from 12 down to 1, we get respectively its *retrograde* (*R*) and *retrograde inversion* (*RI*). All those constitute the 4 basic row forms (Table 1).

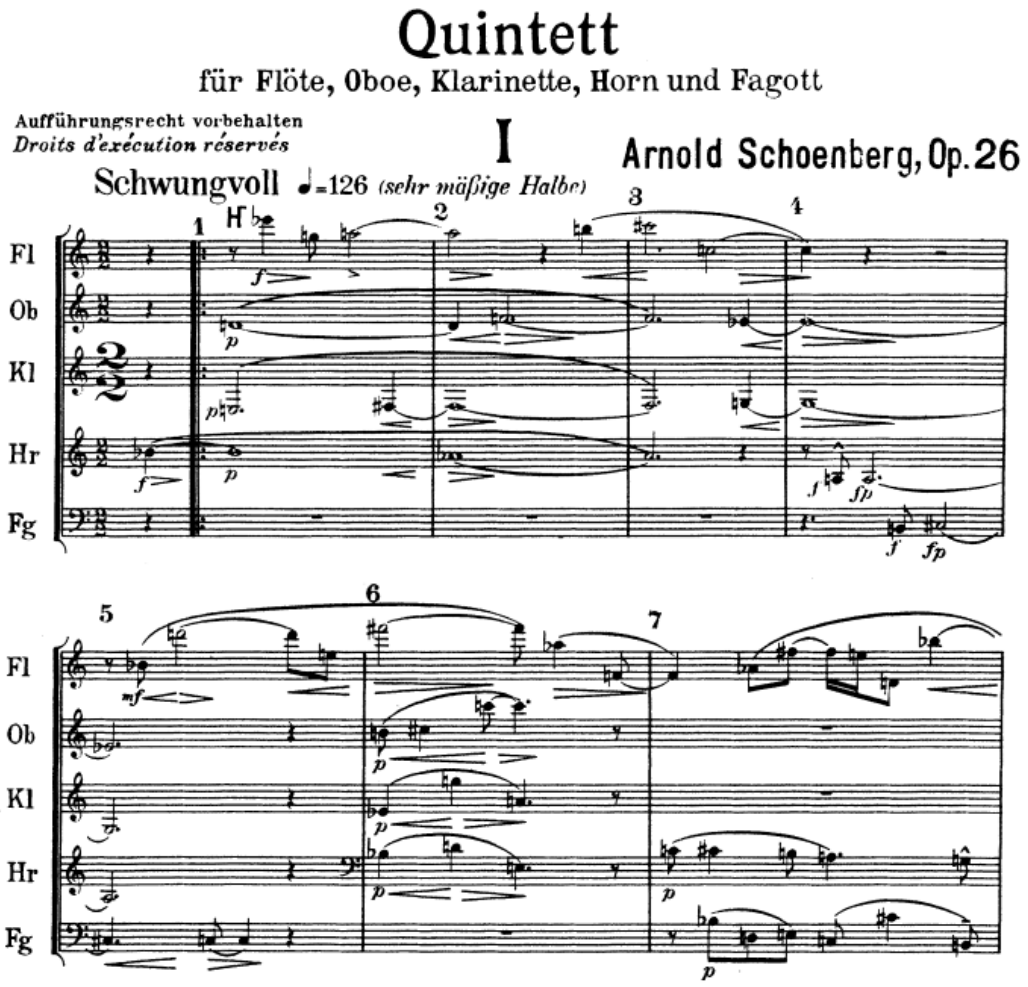
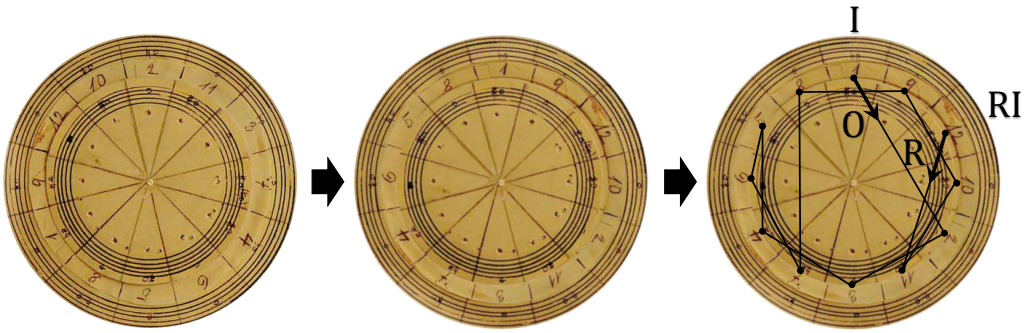


Figure 2: Schoenberg's *Wind Quintet, Op. 26* (1923-24). Copyright 1925 by Universal Edition. Copyright renewed 1952 by Gertrud Schoenberg. Public Domain.



a) Shuffled chart b) Original (*O*) configuration c) Basic row forms (*O, I, R, RI*)

Figure 3: Schoenberg's twelve-tone selection dial for the *Wind Quintet, Op. 26*. Adapted from Lux (2013, p. 89).

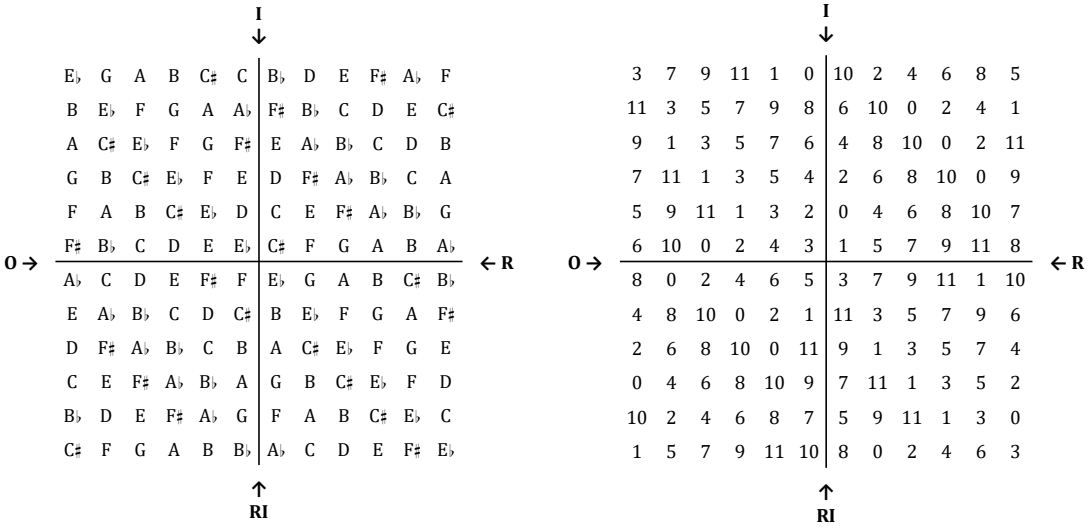
Table 1: The four basic row forms (***O***, ***R***, ***I***, ***RI***) of Schoenberg's *Wind Quintet, Op. 26*.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	
O →	E♭	G	A	B	C♯	C	B♭	D	E	F♯	A♭	F	← ***R***
I →	E♭	B	A	G	F	F♯	A♭	E	D	C	B♭	C♯	← ***RI***

By rotating the chart, Schoenberg was able to get different tone combinations in addition to all 48 row forms, which include the four basic ones (O, I, R, RI) and their 12 transpositions. Therefore, insofar as the wheel chart is somehow a concrete representation of a group, in the mathematical sense of the term, we can say that Schoenberg was in some way composing through group theory, even though he was not fully aware of it.

Years later, that rudimentary chart would be replaced by the *twelve-tone matrix*[3], which represents all those tone combinations in a simpler and more effective way (Figure 4a). Then, the notes, or rather, the pitch classes would be replaced by the pitch-class integers (Figure 4b).

And thus, an analytic theory began to flourish in the course of history, making analyzing and listening to that hermetic *"atonal music"* something more feasible and intelligible. To illustrate that, we can observe the well-known use of the *hexachordal combinatoriality* technique by Schoenberg, in which he combines the two complementary *hexachords* (two halves) of the row in order to obtain the *aggregate* (the total chromatic) (Figure 5).[4]



a) Representation with pitch classes b) Representation with pitch-class integers

Figure 4: Twelve-tone matrix of Schoenberg's *Wind Quintet, Op. 26*.

[3] For details on twelve-tone matrices, see STRAUS (2016, p. 301-302).

[4] For details on hexachordal combinatoriality, see STRAUS (2016, pp. 322-328).

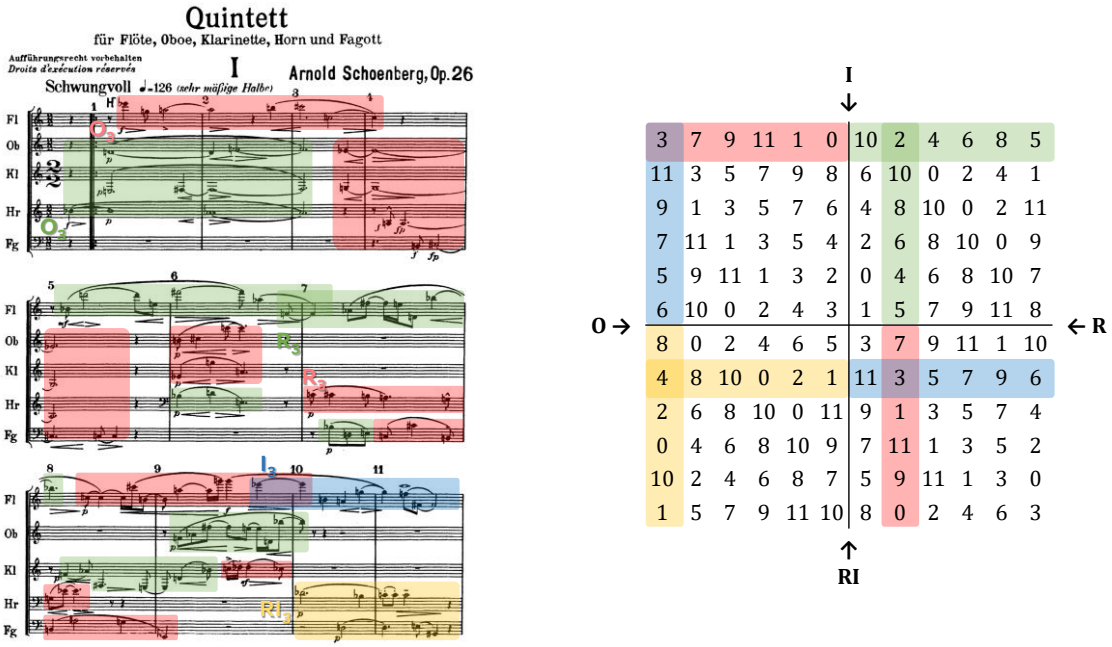
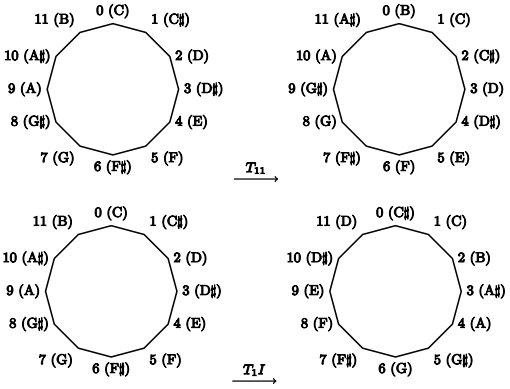


Figure 5: Hexachordal combinatoriality in Schoenberg's *Wind Quintet, Op. 26*.

Group theory became very useful in music as many of those musical objects and structures had morphisms in relation to specific mathematical groups (Figure 6). Since the set of pitch classes associated with those and other operations were isomorphic to many of the finite mathematical groups (cyclic, symmetric, dihedral, etc.), not only twelve-tone music but post-tonal music in general came to be studied in a group theoretical perspective.

O	*R*	*I*	*RI*
R	*O*	*RI*	*I*
I	*RI*	*O*	*R*
RI	*I*	*R*	*O*

a) Klein four-group (K_4)



b) Dihedral group (D_{12})

Figure 6: Morphisms between musical elements and mathematical groups.

In this sense, the development of music theory associated with group theory was considerably broad, both in chronological and geographical terms. We present here a brief theoretical chronology with some of the main theorists

who contributed to the development of the twelve-tone and post-tonal music theory in direct or indirect relation with group theory (Table 2).

Table 2: Theoretical chronology.

[…]-1960	Schoenberg; Hauer; Messiaen; Webern; Jelinek; Eimert; Fokker; [...]
1960-1970	Babbitt; Martino; Gamer; Hanson; Howe; Xenakis; [...]
1970-1980	Forte; Lewin; Perle; Halsey and Hewitt; Bazelow and Brickle; Solomon; Riotte; [...]
1980-1990	Lewin; Rahn; Morris; Mead; Starr; Alegant; Xenakis; Cohn; Clough; Reiner; Haimo; Vieru; Vuza; [...]
1990-[…]	Morris; Alegant; Clough; Xenakis; Mazzola; Noll; Hook; Andreatta; Amiot; Jedrzejewski; Vuza; Fripertinger; Tymoczko; Boss; Peck; Hunter and Hippel; Papadopoulos; Mayer; Zhang; [...]

Now let us turn our attention to an intervallic approach to twelve-tone rows, symmetry and combinatoriality.

An Intervallic Approach to Twelve-Tone Rows, Symmetry and Combinatoriality

Serial music or, more specifically, dodecaphonic serialism plays an important role in twentieth-century music. Developed by Arnold Schoenberg (1874-1951) from early 1920s onwards, the method of composing with twelve tones[5] is a compositional procedure based on his theory of "the emancipation of the dissonance" (SCHOENEBRG, 1950 and 1969), where dissonances are considered merely more remote consonances in the series of overtones, so that it does not aim at the establishment of a tonality (yet does not exclude it entirely), deriving a total musical structure from a complex of pitch classes not functionally differentiated.

The basis of the twelve-tone technique is what Schoenberg used to refer as the *basic set* (BS) and today is often called a *tone row* or *series*,[6] which consists of an ordered arrangement or succession, i.e., a permutation without repetition of the twelve pitch classes from the equal-tempered chromatic scale to be used as basic (structural) material in a musical composition.

As a general rule, the pitch classes within a tone row must be used according to their fixed order. Roughly speaking, one cannot be repeated until all others are played. In turn, a tone row may be combined with its various transformations which include the transpositions of its *original* or *prime form* (denoted O or P)[7] and its derivatives — *inversion* (I), *retrograde* (R) and *retrograde*

[5] For more details and historical overview, see HEADLAM *et al.* (2001) and GRIFFITHS (2001).
[6] Other names are *pitch row*, *note row*, or simply *row*, also *(ordered) set*, *sequence*, and so forth.
[7] We will adopt O here.

inversion (*RI*). As Schoenberg points out, all of these possibilities may appear in a wide variety of ways within a composition:

> For the sake of a more profound logic, the Method of Composing with Twelve Tones derives all configurations from a basic set (Grundgestalt) [tone row]. The order in this basic set and its three derivatives — contrary motion [inversion], retrograde, and retrograde inversion respectively — is, like the motive, obligatory for a whole piece. Deviation from this order of tones should normally not occur, in contrast to the treatment of the motive, where variation is indispensable. Nevertheless, variety is not precluded. The tones in the right order may appear either successively in a melody, theme or independent voice, or as an accompaniment consisting of simultaneous sounds (like harmonies) (SCHOENBERG, 1969, p. 193-194).

For Schoenberg (1950, p. 103-108), "composition with twelve tones has no other aim than comprehensibility." To ensure that, the tone row "functions in the manner of a motive," and this explains why it has to be "invented anew for every piece." For him, it has to be "the first creative thought" of a twelve-tone composition. In this sense, a tone row is not merely another type of pre-compositional material, but a compositional (musical) idea in itself, and its construction, more than just a previous stage of the compositional process, is instead already a fundamental part of it.

Building a tone row is much more than randomly picking a sequence of pitch classes (although it may also be just that). It is actually a creative decision-making process with countless musical implications, and almost all of these basically stem from the choice of the musical intervals within the row. In a nutshell, the "sonic identity" or "profile" of a tone row depends directly on its interval structure:

> Each row's "sound" is determined not so much by its sequence of pcs [pitch classes] (unless one has absolute pitch) as by the ordered intervals between its successive pcs. […] Rows can be created to maximize certain intervals, to omit others or to provide as much diversity as possible — as in all-interval rows (MORRIS, 2015, p. 182).

As we can notice, intervals — even more than the pitch classes themselves — are essential for the construction of a tone row and its sonic identity. But how can one use intervals to build a row? Rather, how can one build a row exclusively from intervals? At first glance, that seems like an easy question. However, as we will show, the answer is not as simple and trivial as one might think.

Thus, bearing that question in mind and endeavoring to answer it, we propose in this work an intervallic approach to tone rows, making use of mathematical concepts from the theory of (integer) partitions and original musical ones, in order to generate tone rows and row classes based solely on intervals and interval structures, or more precisely, on *interval compositions*, as will be defined.[8] So let us regard some concepts and definitions useful for our present purposes.

Concepts and definitions

Before introducing new musical concepts and definitions, let us take a look at some basic mathematical concepts. According to Andrews (1994, p. 149), "the theory of partitions is an area of additive number theory, a subject concerning the representation of integers as sums of other integers". Briefly, a *partition* is "a way of splitting a number into integer parts" (ANDREWS; ERIKSSON, 2004, p. 3). Let us consider the following:

$$4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1.$$

As we see, there are 5 partitions of the number 4, since there are 5 ways of splitting it into integer parts. Thus, a *partition* of a positive integer s is a representation of s as a sum of positive integers, called *summands* or *parts* of the partition, the order of which is irrelevant. Since order is irrelevant, we shall henceforth write partitions with non-increasing order of parts. In this sense, a partition of a positive integer s is a finite non-increasing sequence of positive integers $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_k)$, such that $\sum_{i=1}^{k} \lambda_i = s$ (ANDREWS, 1998, p. 1). Each λ_i corresponds to a part of the partition and if λ is a partition of s, we write $\lambda \vdash s$. The number of partitions of s is denoted by $p(s)$, therefore $p(4) = 5$.

By definition, the order of the parts of a partition is irrelevant, which technically characterizes an *unordered partition*. However, when the order of the parts is considered, then we have what is called an *ordered partition* or, simply, a *composition*. Hence, a composition of s can be thought of as an expression of s as an ordered sum of integers (STANLEY, 2012, p. 17). Formally, a composition of s is a finite (ordered) sequence of positive integers $\alpha = \langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle$,[9] such that $\sum_{i=1}^{k} \alpha_i = s$. For example, for $s = 4$, besides the 5 partitions mentioned before: (4), (3,1), (2,2), (2,1,1), (1,1,1,1); there are 8 distinct compositions: ⟨4⟩, ⟨3,1⟩, ⟨1,3⟩, ⟨2,2⟩, ⟨2,1,1⟩, ⟨1,2,1⟩, ⟨1,1,2⟩, ⟨1,1,1,1⟩. Again, each α_i corresponds to a part of the

[8] The theoretical bases of the concepts and definitions presented here are found in FEITOSA (2020).

[9] We will differentiate partitions from compositions by using round and angle brackets, respectively.

composition and the number of compositions of s is denoted by $c(s)$, in the present case, $c(4) = 8$.

Now that we are already familiar with the elementary concepts of partition and composition, let us take as a starting point the tone row B♭, A, C♯, B, F, G, E, C, A♭, E♭, F♯, D, from Schoenberg's *Fantasy for Violin and Piano, Op. 47* (1949),[10] presented in both musical and integer notation (Figure 7).[11]

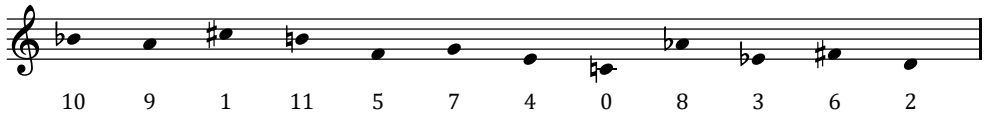


Figure 7: Tone row from Schoenberg's *Fantasy for Violin and Piano, Op. 47* (1949).

By replacing each pitch class with its corresponding numerical label, the row may be represented as the 12-tuple $(10, 9, 1, 11, 5, 7, 4, 0, 8, 3, 6, 2)$. In addition, since we are dealing with pitch classes within a modular space, an alternative representation of that row is as a *clock diagram*, i.e., a directed graph in the form of a regular dodecagon whose vertices correspond to the respective pitch classes being successively connected by directed edges (Figure 8).

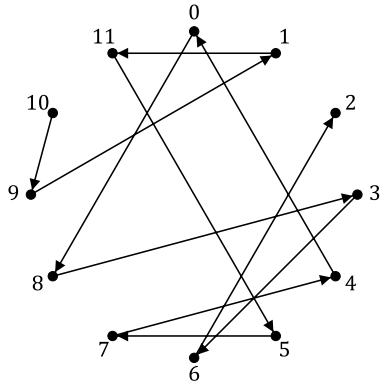
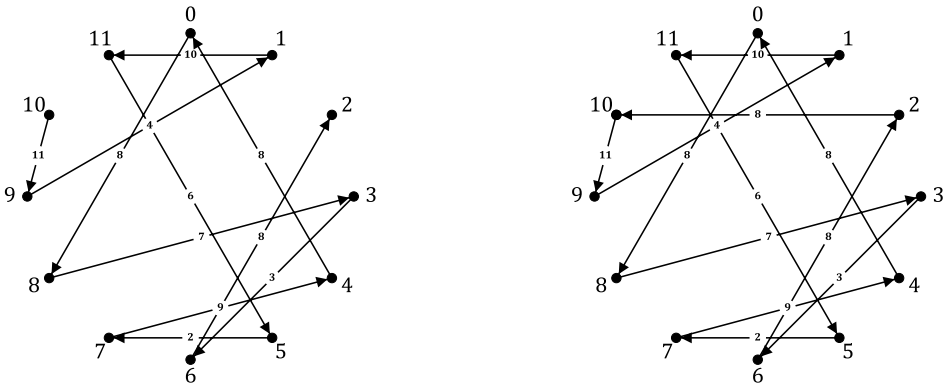


Figure 8: Clock diagram of the tone row $(10, 9, 1, 11, 5, 7, 4, 0, 8, 3, 6, 2)$.

Insofar as the directed edges may be associated with the directed intervals between the successive pitch classes (ordered pitch-class intervals[12]), we can also represent the row in terms of those intervals only. For that, we need to compute the number of clockwise steps necessary to get from one pitch class to the next, starting from the first to the second, then from the second to the third, and so on, obtaining the interval succession $11, 4, 10, 6, 2, 9, 8, 8, 7, 3, 8$ (Figure 9a). However, since the row is within a modular space, we should think of it cyclically and then,

[10] For surveys of that piece, see LEWIN (1967) and TIPTON (2017).
[11] For details on integer notation, see STRAUS (2016, p. 5-6).
[12] For details on ordered pitch-class intervals, see STRAUS (2016, p. 9-11).

in order to derive its complete interval structure, we should also include the interval between the last and the first pitch class, getting the sequence $11, 4, 10, 6, 2, 9, 8, 8, 7, 3, 8, 8$ (Figure 9b).



a) Intervals: 11, 4, 10, 6, 2, 9, 8, 8, 7, 3, 8 b) Intervals: 11, 4, 10, 6, 2, 9, 8, 8, 7, 3, 8, 8

Figure 9: Clock diagrams with directed intervals.

Mathematically, the directed interval from pitch class x to pitch class y is given by $(y - x)$ mod 12. The modular arithmetic gives us a more practical method for calculating directed intervals between pitch classes than counting clockwise steps. Hence, when we go from pitch class 10 to pitch class 9, we have the directed interval $(9 - 10) \bmod 12 = -1 \bmod 12 = 11$. By repeating this process for all successive pitch classes within the row (last and first inclusive), we will obtain the same sequence given in Figure 9b. If we finally associate those directed intervals with the parts of a partition, or rather, with the parts of a composition, and sum them up, we will get:

$$11 + 4 + 10 + 6 + 2 + 9 + 8 + 8 + 7 + 3 + 8 + 8 = 84.$$

From this we may verify that the present sequence of directed intervals is actually a composition of 84. Thus, by generalizing all the reasoning done so far, we finally arrive at the following definition.

DEFINITION 1 (INTERVAL COMPOSITION). Let $\mathbb{Z}_n = \{0, 1, 2, \ldots, n-1\}$, $n \in \mathbb{Z}^+$, be the set of n pitch classes labeled successively from 0 to $n - 1$, an *interval composition* α is generically defined by:

$$\alpha = \langle \alpha_1, \alpha_2, \ldots, \alpha_k \rangle, \quad \text{such that} \sum_{i=1}^{k} \alpha_i = n \cdot d, \tag{1}$$

where $\alpha_i \in \mathbb{Z}^+$ is called a *part* of α; $k \in \mathbb{Z}^+$, $k \leq n$, is the *length* of α; $d \in \mathbb{Z}^+$ is the *dimensional factor* of α; and $s = n \cdot d$ is the *span* of α. Then, given a tone row $(p_1, p_2, \ldots, p_k)$, $p_i \in \mathbb{Z}_n$, its corresponding interval composition α is defined by:

$$\alpha = \left\langle \left(p_{(i+1) \bmod k} - p_{i \bmod k}\right) \bmod n \right\rangle_{i=1}^{k}, \quad \text{where } p_0 = p_k. \tag{2}$$

For our present purposes, since we are dealing with pitch classes and twelve-tone rows within a modular space derived from twelve-tone equal temperament, it is clear that $n = 12$ and $k = 12$ always. Consequently, we may infer that $\alpha_i \leq 11$, which implies that $\sum_{i=1}^{12} \alpha_i \leq 12 \cdot 11 = 132$ (i.e., $s = n \cdot d \leq 132$) and, therefore, $d \leq 11$.

Now, returning to our particular example, the corresponding interval composition of the row $(10, 9, 1, 11, 5, 7, 4, 0, 8, 3, 6, 2)$ is $\langle 11, 4, 10, 6, 2, 9, 8, 8, 7, 3, 8, 8 \rangle$. In this case, as we mentioned before, the span $s = 84 = 12 \cdot 7$, so the dimensional factor $d = 7$. We may alternatively say that such row has *dimension* 7. It is worth mentioning that the dimensional factor corresponds to the number of overlapping clockwise turns around the clock diagram, which musically represents the "number of octaves" that the row theoretically comprises (thinking of its pitch realization only in ascending direction). Thus, in an abstract sense, the dimensional factor is a measure of *compression* or *dispersion* of a row in terms of its directed intervals.

As we can observe so far, an interval composition indicates only the interval structure of the row, not providing any information about its pitch classes. For that, it is necessary that we associate the *pitch* operator P_x, $x \in \mathbb{Z}_n$, with the interval composition, obtaining a *pitched interval composition* $P_x\alpha$, where the *pitch index* x is the first pitch class of the corresponding row. For the mentioned example, we have: $(10, 9, 1, 11, 5, 7, 4, 0, 8, 3, 6, 2) = P_{10}\langle 11, 4, 10, 6, 2, 9, 8, 8, 7, 3, 8, 8 \rangle$. Conversely, given a pitched interval composition $P_x\langle \alpha_1, \alpha_2, \ldots, \alpha_k \rangle$, $\alpha_i \in \mathbb{Z}^+$, we can obtain its corresponding tone row $(p_1, p_2, \ldots, p_k)$, $p_i \in \mathbb{Z}_n$, as follows:

$$P_x\langle \alpha_1, \alpha_2, \ldots, \alpha_k \rangle = \left(\left(x + \sum_{i=1}^{j} \alpha_{i-1} \right) \bmod n \right)_{j=1}^{k}, \tag{3}$$

where $\alpha = \langle \alpha_1, \alpha_2, \ldots, \alpha_k \rangle$ is a *restricted interval composition* whose parts satisfy

$$\left(\sum_{i}^{j} \alpha_i \right) \bmod n \neq 0, \quad \text{for every } i, j \in \mathbb{Z}^+, \text{ such that } i < j \leq k - 1. \tag{4}$$

Simply put, this last condition means that the sum of the successive (contiguous) parts of α, taken two by two, three by three, and so on, from the first up to the penultimate part, should not be a multiple of n, otherwise we would get repeated pitch classes. And again, since we are dealing with twelve-tone rows, $n = 12$ and $k = 12$.

At this point, we can already answer our motivation question — how can one build a row exclusively from intervals? And the answer is basically — through a restricted interval composition whose parts satisfy Equation 4. However, that is not an easy task to perform manually[13] and here computer assistance is welcome.[14] But now let us delve a little deeper into some properties of interval compositions and introduce other useful concepts and definitions.

First, let us consider the *twelve-tone matrix* of our sample row (Figure 10). We notice that there are 48 forms of the row, i.e., 12 transpositions for each of its 4 basic forms — original (O), inversion (I), retrograde (R), and retrograde inversion (RI).

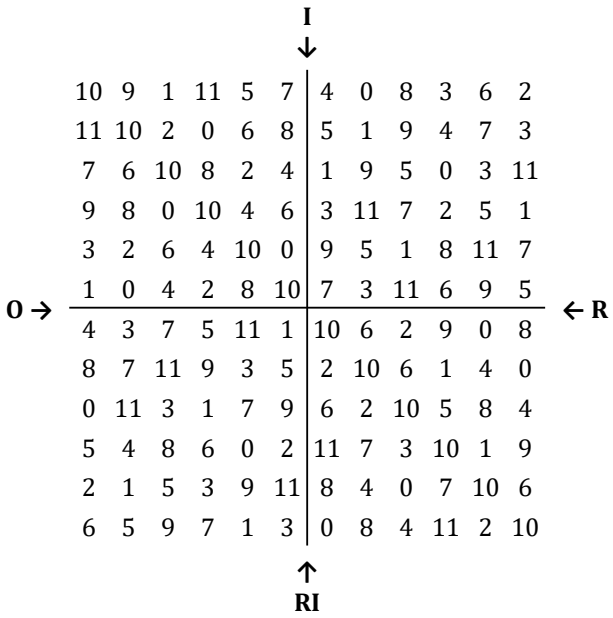


Figure 10: Twelve-tone matrix of (10, 9, 1, 11, 5, 7, 4, 0, 8, 3, 6, 2).

Since all transpositions of those basic forms have the same interval composition, we may conceive them as equivalence classes of rows related by transposition and represent them by their corresponding interval compositions, then we have:

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[13] It would be necessary to check $\binom{k-1}{2}$ successive sums manually. For $k = 12$, then the total is 55.

[14] For that task, we have developed a software called *SerialGen*. For details, see FEITOSA *et al* (2022).

$$\begin{aligned}
&\langle 11,4,10,6,2,9,8,8,7,3,8,8\rangle && \text{for } O,\\
&\langle 1,8,2,6,10,3,4,4,5,9,4,4\rangle && \text{for } I,\\
&\langle 4,9,5,4,4,3,10,6,2,8,1,4\rangle && \text{for } R,\\
&\langle 8,3,7,8,8,9,2,6,10,4,11,8\rangle && \text{for } RI.
\end{aligned}$$

According to Schoenberg's method, tone rows are equivalent if they can be determined by transposition, inversion, and/or retrograde from a single tone row. In this sense, those four interval compositions satisfy the Schoenbergian notion of equivalence insofar as they comprise all the rows related by such operations.

Finally, we may define those basic operations and more elaborate ones, such as *cyclic shift* (S), *multiplication* (M), and *five-step permutation* (F), in terms of interval compositions only, in order to obtain different equivalence classes, or rather, *row classes* from their various combinations, as follows.

DEFINITION 2 (OPERATIONS). Let $\alpha = \langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle$, $\alpha_i \in \mathbb{Z}^+$, be an interval composition, the operations of *inversion* (I), *retrograde* (R), *retrograde inversion* (RI), *cyclic shift* (S), *multiplication* (M), and *five-step permutation* (F) are respectively defined by:

$$I\langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle = \langle (-\alpha_i) \bmod n \rangle_{i=1}^{k}, \tag{5}$$

$$R\langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle = \left\langle \left(-\alpha_{(k-i) \bmod n}\right) \bmod n \right\rangle_{i=1}^{k}, \quad \text{where } \alpha_0 = \alpha_k, \tag{6}$$

$$RI\langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle = \left\langle \alpha_{(k-i) \bmod n} \right\rangle_{i=1}^{k}, \quad \text{where } \alpha_0 = \alpha_k, \tag{7}$$

$$S\langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle = \left\langle \alpha_{(i+1) \bmod n} \right\rangle_{i=1}^{k}, \quad \text{where } \alpha_0 = \alpha_k, \tag{8}$$

$$M\langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle = \langle (5 \cdot \alpha_i) \bmod n \rangle_{i=1}^{k}, \tag{9}$$

$$F\langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle = \left\langle \sum_{i=1}^{5} \alpha_{(i+5 \cdot j) \bmod n} \right\rangle_{j=0}^{k-1}, \quad \text{where } \alpha_0 = \alpha_k. \tag{10}$$

Therefore, taking our example:

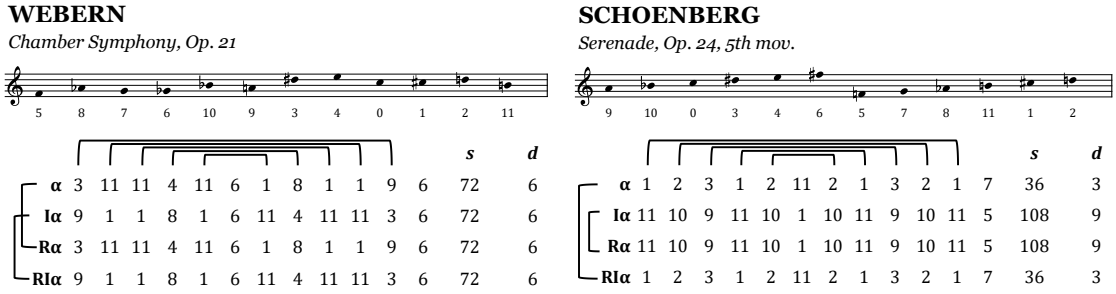
$$\begin{aligned}
&\alpha = \langle 11,4,10,6,2,9,8,8,7,3,8,8\rangle,\ s = 84 \text{ and } d = 7,\\
&I\alpha = \langle 1,8,2,6,10,3,4,4,5,9,4,4\rangle,\ s = 60 \text{ and } d = 5,\\
&R\alpha = \langle 4,9,5,4,4,3,10,6,2,8,1,4\rangle,\ s = 60 \text{ and } d = 5,\\
&RI\alpha = \langle 8,3,7,8,8,9,2,6,10,4,11,8\rangle,\ s = 84 \text{ and } d = 7,
\end{aligned}$$

$S\alpha = \langle 4, 10, 6, 2, 9, 8, 8, 7, 3, 8, 8, 11\rangle$, $s = 84$ and $d = 7$,
$M\alpha = \langle 7, 8, 2, 6, 10, 9, 4, 4, 11, 3, 4, 4\rangle$, $s = 72$ and $d = 6$,
$F\alpha = \langle 9, 11, 5, 9, 1, 7, 10, 3, 10, 10, 11, 10\rangle$, $s = 96$ and $d = 8$.

Here we verify that the dimensional factor is the same for the original interval composition α and $RI\alpha$ (also for $S\alpha$), as well as for $I\alpha$ and $R\alpha$, being different for $M\alpha$ and $F\alpha$. This means that $I\alpha$ and $R\alpha$ are the "most compact" in terms of directed intervals ($d = 5$) and, between them, $I\alpha$ would be the class representative (*normal form*[15]) of the row classes comprising such operations, since it is even more compact.

Symmetry and combinatoriality

Regarding symmetric tone rows, there are basically two types of symmetry:[16] *retrograde symmetry (R-symmetry)* and *retrograde-inverse symmetry (RI-symmetry)*. In the first case (Figure 11a), the interval composition of the original row (α) is the same as that of its retrograde ($R\alpha$). In the second case (Figure 11b), the interval composition of the original row (α) is the same as that of its retrograde inversion ($RI\alpha$).



a) *R-symmetry* (mirrored complementary intervals) b) *RI-symmetry* (mirrored identical intervals)

Figure 11: Interval compositions of the basic forms (O, I, R, RI) of symmetric tone rows.

We can observe that in the case of *R-symmetry*, the interval compositions have mirrored *complementary intervals* (i.e., which sum up to 0 mod n) around α_6 and the *dimensional factor* is always $d = 6$. In the case of *RI-symmetry*, the interval compositions also have mirrored *identical intervals* around α_6, but the dimensional factor is always $d \neq 6$. Thus, we may define symmetry as follows.

[15] The *normal form* is the interval composition with the smallest d and greatest α_k, α_{k-1}, and so on.
[16] There are other types if we consider the operations S, M, and F, but we will not deal with them here.

DEFINITION 3 (SYMMETRY). Let $\alpha = \langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle$, $\alpha_i \in \mathbb{Z}^+$, be an interval composition, α is said to have *R-symmetry* or *RI-symmetry*, if and only if α is a *restricted interval composition* which satisfies:

$$\begin{cases} \alpha = R\alpha \Leftrightarrow (\alpha_i = n - \alpha_{k-i})_{i=1}^{k}, & \text{where } \alpha_0 = \alpha_k, \text{ for } R\text{-symmetry}, \\ \alpha = RI\alpha \Leftrightarrow (\alpha_i = \alpha_{k-i})_{i=1}^{k}, & \text{where } \alpha_0 = \alpha_k, \text{ for } RI\text{-symmetry}. \end{cases} \quad (12)$$

Concerning combinatorial tone rows, there are five types of combinatoriality[17]: *O-combinatoriality*, *I-combinatoriality*, *R-combinatoriality*, *RI-combinatoriality*, and *ALL-combinatoriality*. The first (O) and second (I) occur when the hexachord that constitutes the first half of the row can be mapped to its *complement* respectively under some transposition or some inversion. The third (R) and fourth (RI) occur when the same hexachord can be mapped to *itself* respectively under some transposition or some inversion. And the fifth (ALL), when it has all these four properties at the same time.

In order to verify these properties, we first reduce the interval composition α of the entire row to the interval composition $\mathcal{H}$ of just its first hexachord:

$$\mathcal{H} = \langle \mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, \mathcal{H}_4, \mathcal{H}_5, \mathcal{H}_6 \rangle = \langle \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \left(\sum_{i=6}^{k} \alpha_i \right) \bmod n \rangle. \quad (13)$$

Then, we obtain the *T-matrix* ($\mathcal{M}_T$) and the *I-matrix* ($\mathcal{M}_I$)[18] of $\mathcal{H}$, which are square matrices of order $m = 6$, respectively defined by:

$$\mathcal{M}_T = \left[\mathcal{M}_{T_{i,j}} \right]_{m \times m}, \text{ such that } \mathcal{M}_{T_{i,j}} = \left(\sum_{a=1}^{j} \mathcal{H}_{a-1} - \sum_{b=1}^{i} \mathcal{H}_{b-1} \right) \bmod n, \text{ where } \mathcal{H}_0 = 0, \quad (14)$$

$$\mathcal{M}_I = \left[\mathcal{M}_{I_{i,j}} \right]_{m \times m}, \text{ such that } \mathcal{M}_{I_{i,j}} = \left(\sum_{a=1}^{i} \mathcal{H}_{a-1} + \sum_{b=1}^{j} \mathcal{H}_{b-1} \right) \bmod n, \text{ where } \mathcal{H}_0 = 0. \quad (15)$$

Now, let $\#\mathcal{M}_T(x)$ and $\#\mathcal{M}_I(x)$ denote the respective number of occurrences of $x \in \mathbb{Z}_n$ in $\mathcal{M}_T$ and $\mathcal{M}_I$, the *T-vector* ($\mathcal{V}_T$) and the *I-vector* ($\mathcal{V}_I$) of $\mathcal{H}$[19] are respectively defined by:

$$\mathcal{V}_T = \left[\mathcal{V}_{T_1}, \mathcal{V}_{T_2}, \dots, \mathcal{V}_{T_n} \right] = [\#\mathcal{M}_T(0), \#\mathcal{M}_T(1), \dots, \#\mathcal{M}_T(n-1)], \quad (16)$$

---

[17] We will deal here only with hexachordal combinatoriality.

[18] For details on *T-matrix* and *I-matrix*, see STRAUS (2016, pp. 104-105) and MORRIS (1987, ch. 3).

[19] These vectors are also known respectively as *interval* and *index vectors* in musical literature.

$$\mathcal{V}_I = \left[\mathcal{V}_{I_1}, \mathcal{V}_{I_2}, \dots, \mathcal{V}_{I_n}\right] = [\#\mathcal{M}_I(0), \#\mathcal{M}_I(1), \dots, \#\mathcal{M}_I(n-1)]. \quad (17)$$

Finally, from all that was settled before we may define combinatoriality as follows.

DEFINITION 4 (COMBINATORIALITY). Let $\alpha = \langle \alpha_1, \alpha_2, \dots, \alpha_k \rangle$, $\alpha_i \in \mathbb{Z}^+$, be an interval composition, α is said to have *O-combinatoriality*, *R-combinatoriality*, *I-combinatoriality*, *RI-combinatoriality*, or *ALL-combinatoriality*, if and only if α can be reduced to a *restricted interval composition* $\mathcal{H}$ whose entries of its *T-vector* and *I-vector* respectively satisfy:

$$\exists\, i \in \{1,2,\dots,n\} \text{ such that } \begin{cases} \mathcal{V}_{T_i} = 0, & \text{for } \textit{O-combinatoriality}, \\ \mathcal{V}_{T_i} = 6, & \text{for } \textit{R-combinatoriality}, \\ \mathcal{V}_{I_i} = 0, & \text{for } \textit{I-combinatoriality}, \\ \mathcal{V}_{I_i} = 6, & \text{for } \textit{RI-combinatoriality}, \\ \text{all above are true}, & \text{for } \textit{ALL-combinatoriality}. \end{cases} \quad (18)^{20}$$

Taking our previous example in order to illustrate the definition above, we have the interval composition:

$$\alpha = \langle 11, 4, 10, 6, 2, 9, 8, 8, 7, 3, 8, 8 \rangle.$$

By reducing α to the interval composition $\mathcal{H}$ of the first hexachord of the row, we get:

$$\mathcal{H} = \langle 11, 4, 10, 6, 2, (9+8+8+7+3+8+8) \bmod 12 \rangle = \langle 11, 4, 10, 6, 2, 3 \rangle.$$

Now, we obtain the *T-matrix* ($\mathcal{M}_T$) and the *I-matrix* ($\mathcal{M}_I$) of $\mathcal{H}$:

$$\mathcal{M}_T = \begin{bmatrix} 0 & 11 & 3 & 1 & 7 & 9 \\ 1 & 0 & 4 & 2 & 8 & 10 \\ 9 & 8 & 0 & 10 & 4 & 6 \\ 11 & 10 & 2 & 0 & 6 & 8 \\ 5 & 4 & 8 & 6 & 0 & 2 \\ 3 & 2 & 6 & 4 & 10 & 0 \end{bmatrix} \quad \text{and} \quad \mathcal{M}_I = \begin{bmatrix} 0 & 11 & 3 & 1 & 7 & 9 \\ 11 & 10 & 2 & 0 & 6 & 8 \\ 3 & 2 & 6 & 4 & 10 & 0 \\ 1 & 0 & 4 & 2 & 8 & 10 \\ 7 & 6 & 10 & 8 & 2 & 4 \\ 9 & 8 & 0 & 10 & 4 & 6 \end{bmatrix}.$$

Then, by computing the number of occurrences of $x \in \mathbb{Z}_{12}$ successively in the respective *T* and *I-matrices*, we get the *T* and *I-vectors* of $\mathcal{H}$:

$$\mathcal{V}_T = [6,2,4,2,4,1,4,1,4,2,4,2] \quad \text{and} \quad \mathcal{V}_I = [5,2,4,2,4,0,4,2,4,2,5,2].$$

---

[20] For each i that satisfies $\mathcal{V}_{T_i} = 6$ or $\mathcal{V}_{I_i} = 6$, we compute it (respectively) as one *degree of* (*transpositional* or *inversional*) *symmetry* of $\mathcal{H}$.

At last, we notice here that $\mathcal{V}_{T_1} = 6$ and $\mathcal{V}_{I_6} = 0$, therefore α has *R-combinatoriality*[21] and *I-combinatoriality*. And since $\nexists\, i \in \{1,2,\dots,12\}$ such that $\mathcal{V}_{T_i} = 0$ or $\mathcal{V}_{I_i} = 6$, then α does not have *O-Combinatoriality*, *RI-Combinatoriality*, neither *ALL-Combinatoriality*.

Although the method presented here is useful enough, there is still a more practical way to verify the combinatoriality of a tone row through its interval composition only, without the tiresome work of obtaining the matrices and vectors.[22] However, its mathematical formalization involves a few more difficulties to be faced in another opportunity.

Conclusion

The representation of tone rows through their interval compositions, in addition to being something more suited to musicians, has the advantage of making their interval content explicit, bringing to light many of their properties such as symmetry and combinatoriality.

The interval approach proposed here echoes group theory, albeit by other mathematical means, and dialogues with important works in the specific area, which address group theory and twelve-tone music.[23] The segmentation of tone rows and row classes by dimension provides a new perspective of understanding those musical objects, opening the way for a broader and exhaustive taxonomy,[24] which will be done in the future.

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[21] This corresponds to the trivial transposition invariance, i.e., when the row is transposed by 0 semitones (T_0) resulting in itself.

[22] For an introductory guideline in this regard, see CHRISMAN (1977).

[23] See FRIPERTINGER and LACKNER (2015), HUNTER and HIPPEL (2003), and REINER (1985).

[24] For preliminary results in this direction and new data obtained through the *SerialGen* software, see again FEITOSA *et al.* (2022).

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6

The Principles of *Grundgestalt* and Developing Variation in a Bio-Mathematical Model[1]

Carlos Almada

Schoenberg's principles of Grundgestalt and developing variations represent perhaps the most far-reaching of his innumerous contributions to music theory. Essentially, a Grundgestalt of a musical work contains in a latent stage the basic material from which it will – ideally, at least – be constructed. Developing variation corresponds to the compositional techniques employed by a composer to explore the implications present in the Grundgestalt, which is based on intense and recursive derivative processes. The binomial Grundgestalt/developing variation occupies a central position in the Model of Derivative Analysis (MDA), a theoretical-methodological apparatus proposed by the present author, destined to the systematic analysis of music variation. This presentation aims to describe concisely MDA's main elements, exploring especially the close relations (not only in metaphoric terms) that associate developing-variation processes with biological-evolutionary variation.

[1] The content of this chapter is based on some parts of Almada (2023).

This paper is related to a research project that I have been pursued since 2011. Essentially, it corresponds to a systematic study of musical variation. In this work I would like to focus on how Schoenbergian principles of *Grundgestalt* and *developing variation* are incorporated to an analytical model, which involves an exam of the strong connections that exist between musical and biological variation.

Let me introduce the main elements of the theory adopted in the research. It is called the *Model of Derivative Analysis*, here in its most recent version.[2]

Some basic definitions

Firstly, let us see variation as an *isolated* phenomenon, out of time. Consider this scheme (Figure 1) as the basic model for variation, in which a given referential musical unit P, the *parent* Is transformed by an action V, a generic *function* into a related unit C, the *child*, in such a way that there is some amount of *similarity* between P and C An alternative representation depicts variation as a combination of similarity and divergence.

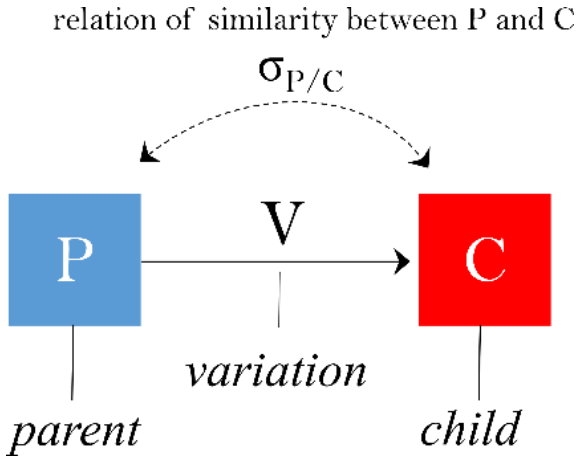

Figure 1: Representation of variation.

The model considers two types of variation (Figure 2): *Holistic*, when the transformation affects the unit as a whole in some way; or *decomposable* variation. In this case, there is a preliminary stage named abstraction, in which musical components, like pitches and rhythms, are isolated and then considered as referential for eventual independent variation.

[2] There is a considerable bibliographic production related to the model since the beginning of the research, in 2011. For more recent articles, see ALMADA (2019; 2020; and 2021).

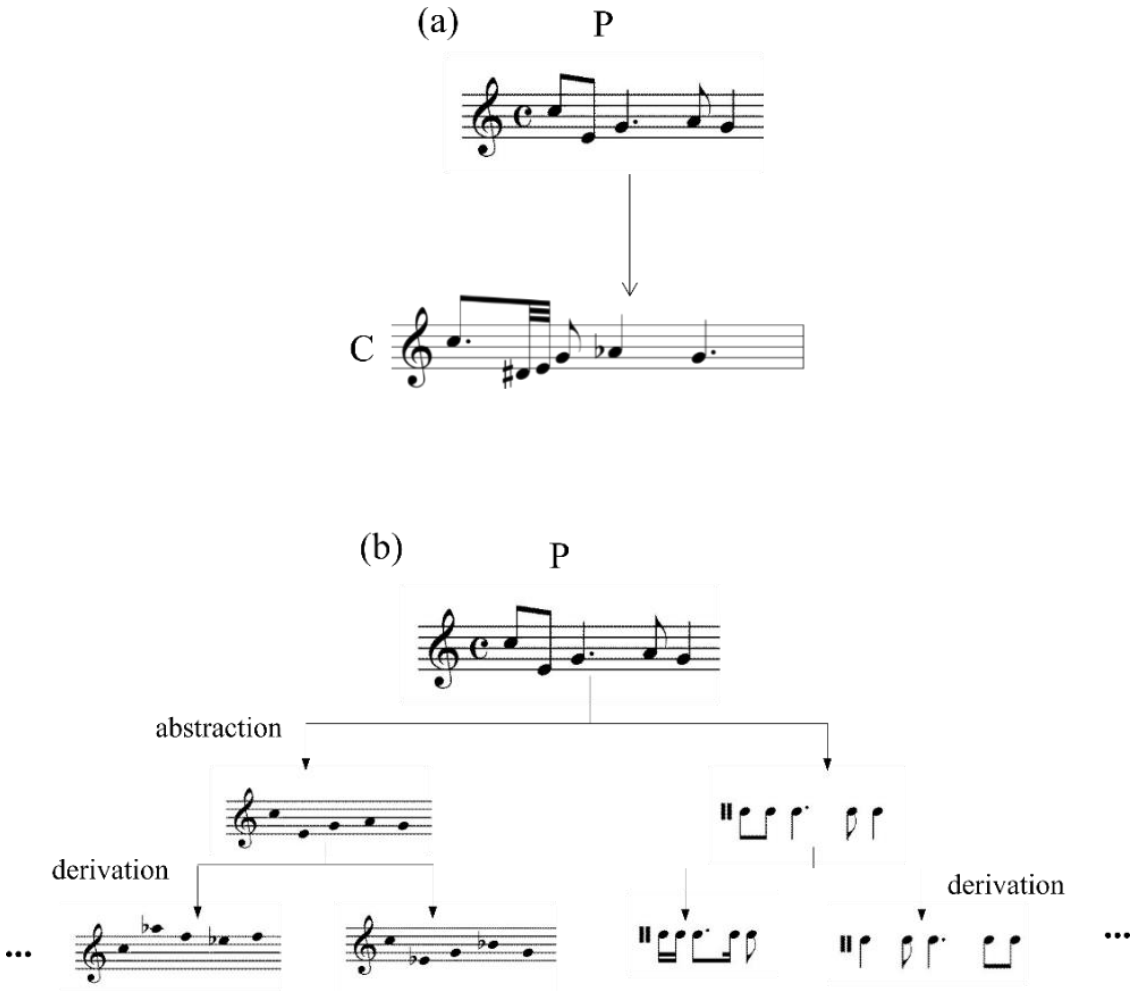

Figure 2: Examples of holistic (a) and decomposable variation.

Decomposable variation is associated with the notion of *domains*. There are two types: Primary and secondary. In the current version of the model, pitch and time are considered primary domains, and harmony a secondary one. However, it is possible to add other parameters to this category, like texture, dynamics, etc. Domains contain *attributes*. These can be seen as structural descriptors of specific aspects of a domain. The pitch attributes are (Figure 3):

- Pitch sequence (p1), represented with midi pitches (considering middle C as 60);
- Pitch-class sequence (p2);
- Intervallic sequence (p3);
- Melodic contour, an abstracted representation of a melody, considering just the relative positions of the pitches (p4);
- And the ambit, the interval between the first and last pitches (p5).

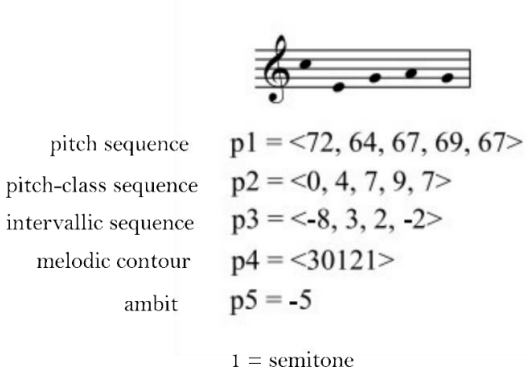

Figure 3: Example of pitch attributes.

The attributes of time are (Figure 4):

- The sequence of durations (t1);
- The sequence of inter-onset intervals (t2);
- The metric contour, analogous to the melodic contour, informing the relative metric **weight** of the events (t3);
- The durational span (t4).

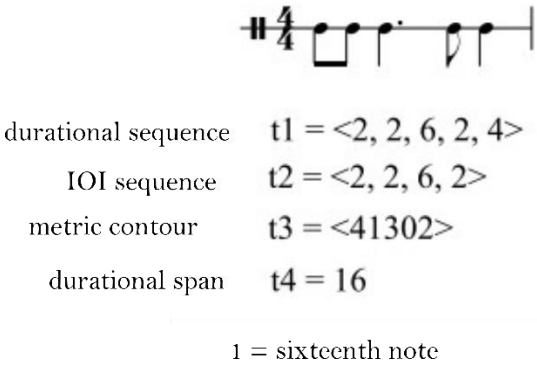

Figure 4: Example of temporal attributes.

Finally, the attributes of the harmonic domain, depicted in a hierarchical order.

- Key (h1), represented by a pitch class;
- Mode (h2), considering the major mode = 0, and the minor mode = 1;
- Chord quality (h3);[3]
- Root (h4), represented by a pitch class;
- Bass (h5), also represented by a pitch class.

With this information we can built a *matrix of attributes*, with 12 rows and a number of columns corresponding to the number of events (Figure 5). The first four rows refer to the pitch attributes. The next three to the temporal attributes.

[3] The qualities are arbitrarily encoded as integers (0 = major triad; 1 = minor triad; …; 4 = dominant seventh; etc.)

Harmonic information occupies the last five rows, distributed according to the events.

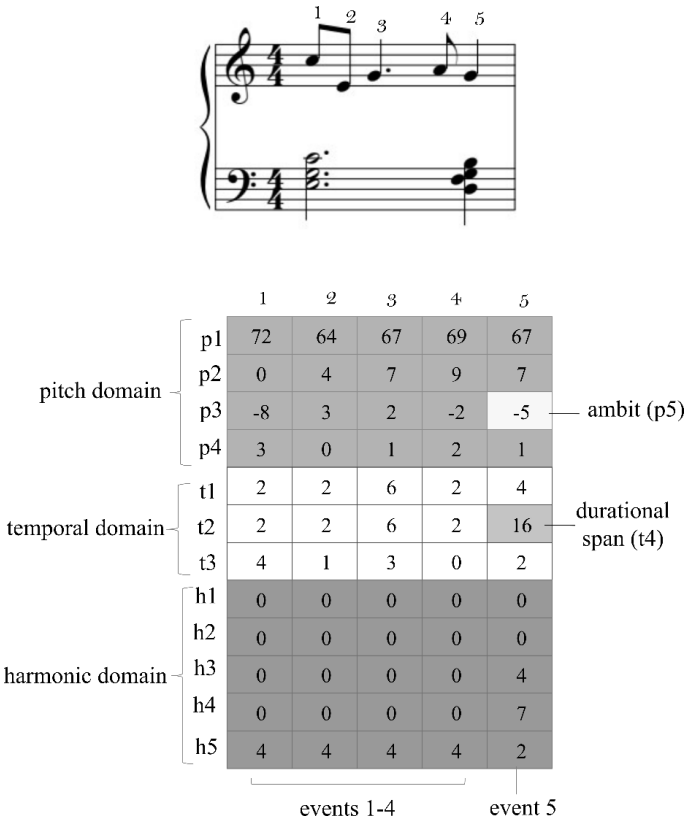

Figure 5: Matrix of attributes, considering the examples of Figures 3-4.

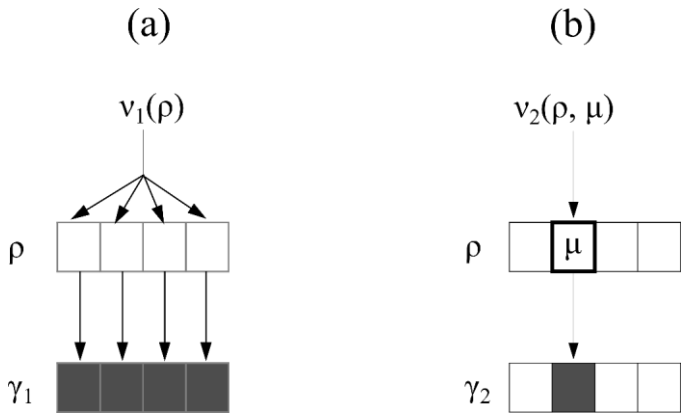

Figure 6: Representation of normal (a) and mutational variation (b) applied to a parent-attribute (ϱ) or part of it (μ), producing a variant (γ).

Operations

Another important element of the model is the group of operations. Operations are applied to a parent-attribute, producing a *variant* of it. Other operations can produce different variants of a same parent-attribute. Operations can basically be of two modes: *Normal*, when it affects all the components of the

parent-attribute, or *mutational*, when it affects just one component (or a selected portion) of the parent-attribute (Figure 6). Figure 7 exemplifies the application of the two modes, considering the operation addition (ADD) in two distinct attributes, intervallic sequence (p3) e IOI sequence (t2).[4] There are currently 26 operations formally defined in MDA (Figure 8).

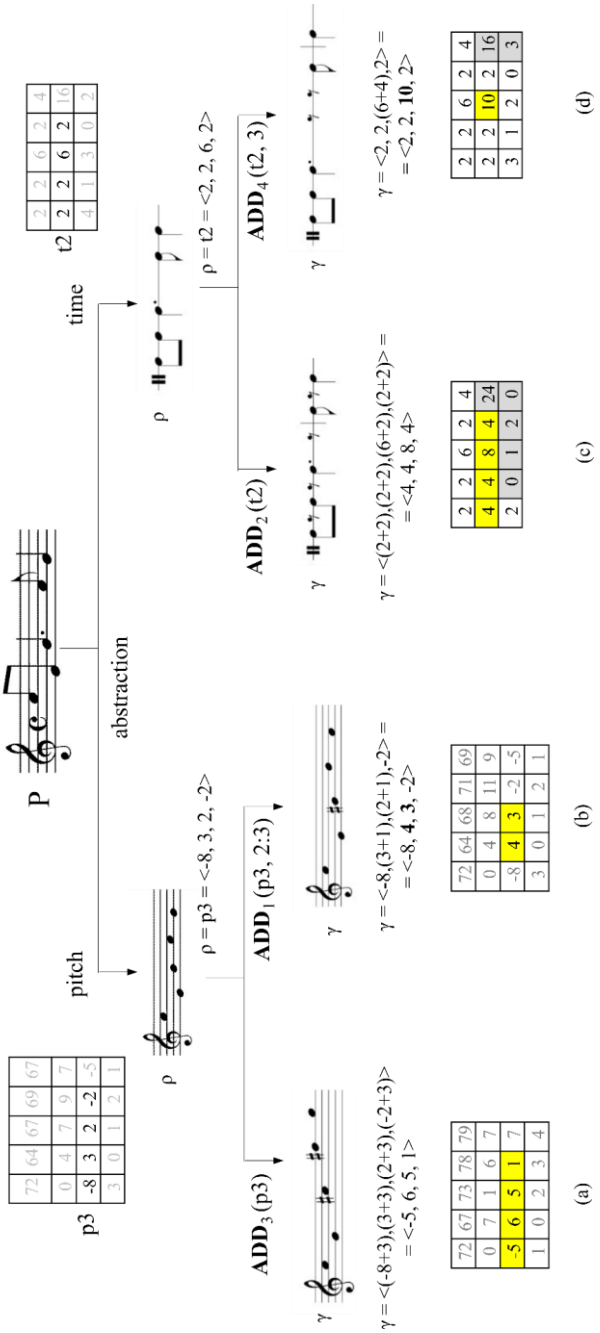

Figure 7: Example of application of operation ADD: (a) mode normal, pitch sequence; (b) mutational, pitch sequence; (c) normal, IOI sequence; (d) mutational, IOI sequence.

[4] ADD expands pitch and rhythmic intervals by adding n semitones (or 16th notes) to the selected argument.

operation		type		domain			scope		
name	symbol	can.	n-can.	p	t	h	nor.	mut.	dual
addition	**ADD**		x	x	x				x
augmentation	**A**	x			x				x
change of mode	**MOD**		x	x		x	x		
change of register	**OCT**		x	x				x	
chordal inversion	**CHI**		x			x		x	
chromatic alteration	**ALT**		x	x				x	
chromatic inversion	**I**	x		x					x
chromatic transposition	**T**	x		x					x
deletion	**DEL**		x	x	x			x	
diatonic inversion	**i**	x		x					x
diatonic transposition	**t**	x		x					x
diminution	**D**	x			x				x
extension	**EXT**		x	x	x		x		
interpolation	**INT**		x	x	x			x	
merging of durations	**MRG**		x		x			x	
metric displacement	**MTD**		x		x		x		
modulation	**MDL**		x			x	x		
permutation	**PER**		x	x	x			x	
re-harmonization	**RHA**		x			x		x	
re-partition	**RPA**		x		x			x	
rest-substitution	**RST**		x		x			x	
replication	**RPL**		x	x	x			x	
retrogradation	**R**	x		x	x		x		
rotation	**ROT**		x	x	x		x		
splitting of duration	**SPL**		x		x			x	
subtraction	**SUB**		x	x	x				x
suppression	**SUP**		x	x	x			x	

Figure 8: Chart of the 26 operations currently registered in MDA.

Contextualized variation

Let now us consider variation in function of time or, in other words, contextualized variation. This approach involves the principles of *Grundgestalt* and developing variation, both elaborated by Arnold Schoenberg. Essentially, a *Grundgestalt* refers to a musical unit that, ideally, contains the basic material of a whole piece. In a quite common metaphor, the *Grundgestalt* can be seen as a seed of a tree. As defined by Schoenberg himself, "everything within a closed composition can be accounted for as originating, derived and developed from a basic motive, or at least, a *Grundgestalt*" (SCHOENBERG, 2006, p. 27). This is another definition, proposed by Richard Taruskin:

> A *Grundgestalt* is a motivic complex that could serve as a source or quarry for everything that happened in the composition [...] All melodic shapes, all harmonies, all contrapuntal textures were to be derived from it by the composer – and therefore, at least theoretically, deducible through analysis, which thus constitutes a test of the composer's success (TARUSKIN, 2010, p. 325).

The second concept, *developing variation* refers essentially to the techniques that can be used for fulfilling the implications present in the *Grundgestalt*. Walter Frisch here defines the principle:

> By 'developing variation', Schoenberg means the construction of a theme (usually of eight bars) by the continuous modification of the intervallic and/or rhythmic components of an initial idea […] Schoenberg values developing variation as a compositional principle because it can prevent obvious, hence monotonous, repetition (FRISCH, 1984, p.8).

Ethan Haimo adds another interpretation for the concept:

> Developing variation is a special category of variation technique, one that implies a teleological process. As a result, later events – even markedly contrasting ones – can be understood as originate from, or grow out of, changes that were made in the repetitions of early musical unities (…). Developing variation offers the possibility of forwards motion, permitting the creation of new or contrasting (but still related) ideas, while local variation affects only the passage in question (HAIMO, 1997, p.351).

Let us now see how both principles were adapted in the analytical model. Firstly, the idea that the *Grundgestalt* can present *segments* containing musical structures that can be seen as potential *agents* for derivation. These elements are associated with *variables*,[5] which are used for labeling them (in algebraic sense). Figure 9 depicts a representation of these elements.

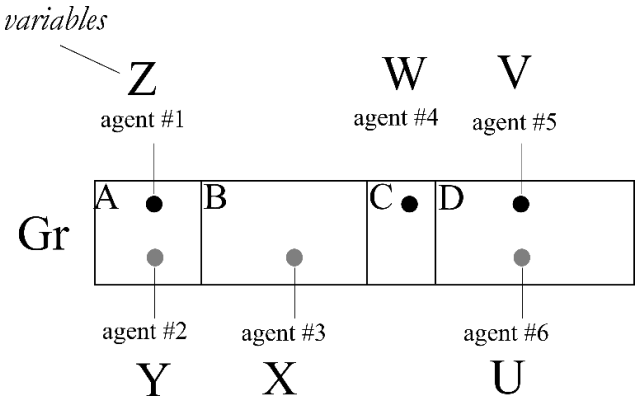

Figure 9: Representation of the segments of a Grundgestalt (Gr.) into agents, assigned to variables.

Using the assumptions adopted in MDA, a basic model for developing variation can be constructed in three stages (Figure 10):

(1) Firstly, the representation of an ordinary variation, with P0 being transformed into C0;

[5] Variables use the final letters of the alphabet in reverse order.

(2) Consider then C0 *becoming* a parent, P1;
(3) And having its *own child*, C1, by action of operation W.

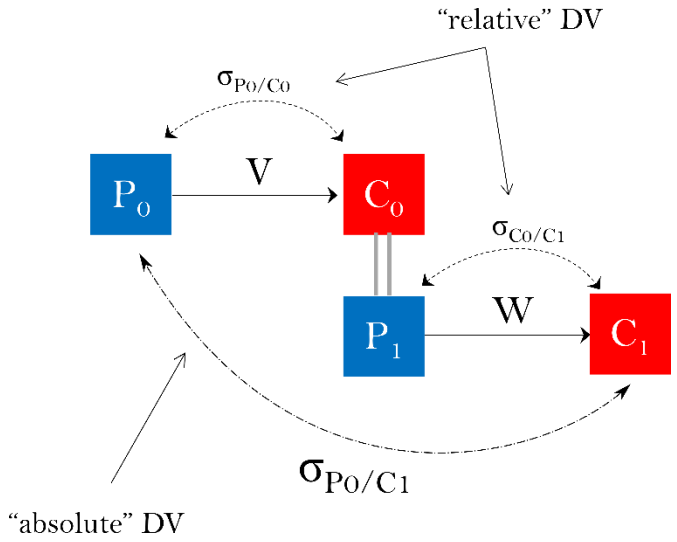

Figure 10: Basic model of developing variation (compare with Figure 1).

Observe that in this system we can consider the relation between C1 and its grand-mother P0. Let us name this as an absolute DV, and the contiguous relations as relative DVs. The generic model of developing variation can be extended, forming large *chains* of transformation, as shown in Figure 11.

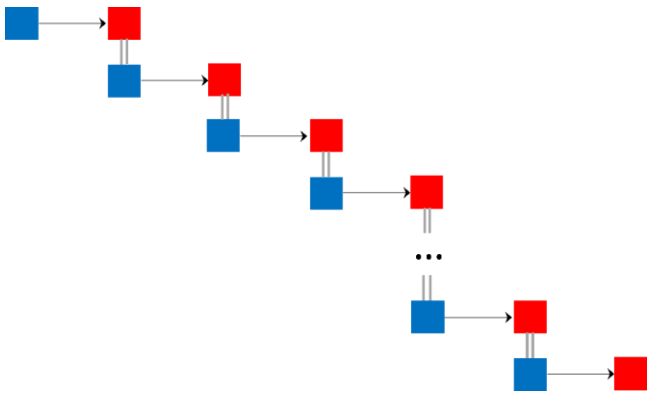

Figure 11: Representation of a generic DV chain, by extending the model of Figure 10.

The scheme of Figure 12 models abstractly the *organic growth* of a musical piece by the application of developing variation processes to a basic unit, the *Grundgestalt*.

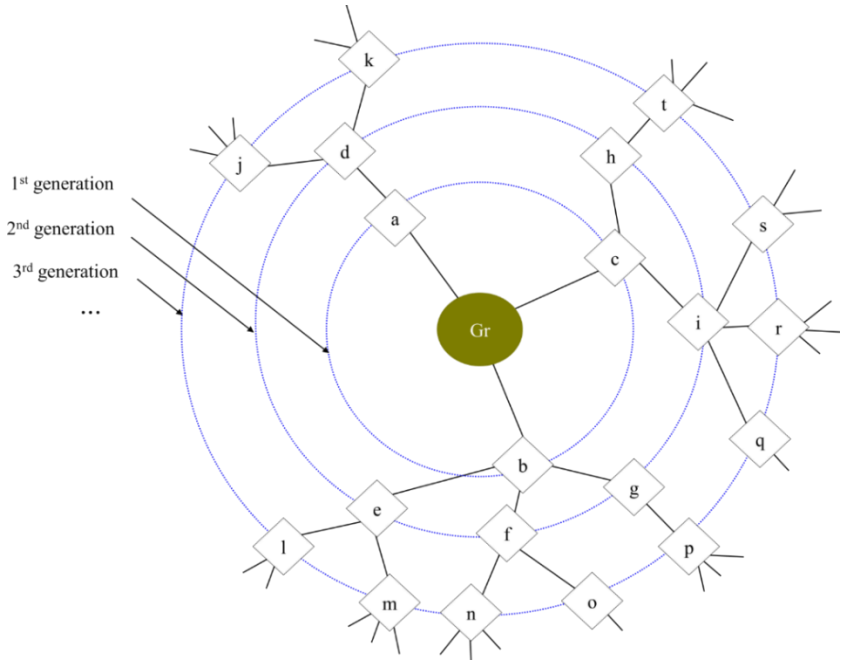

Figure 12: Representation of organic growth of developing variation from a *Grundgestalt*.

A biological-like analytical approach

In MDA there are some important concepts and methodological strategies of, so-to-speak, *bio-inspiration*. For time and space limitations, I will briefly describe only some of them in this section.

Firstly, the *genealogical notation* intends to precisely identify variants in developing-variation situation, which can be very complicated in some cases. Let us consider a given referential unit labeled as *a*, which gives birth to, for example, four variants, a_1, a_2, a_3, and a_4. Take now one of these, say, a1, which produces two children, $a_{1.1}$ and $a_{1.2}$, and so on. Dots separate generations and numbers indicate sequential order (Figure 13).

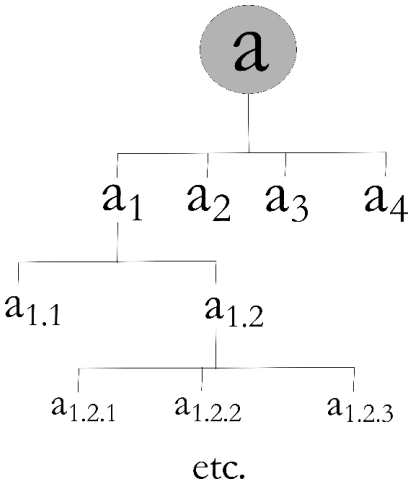

Figure 13: Representation of a hypothetical derivative lineage depicted in genealogical notation.

Consider now the *reverse* process. Suppose we have a genealogical label, say $a_{3.2.1.2.1}$, and would like to *reconstruct* its lineage. This can be easily obtained like this (Figure 14):

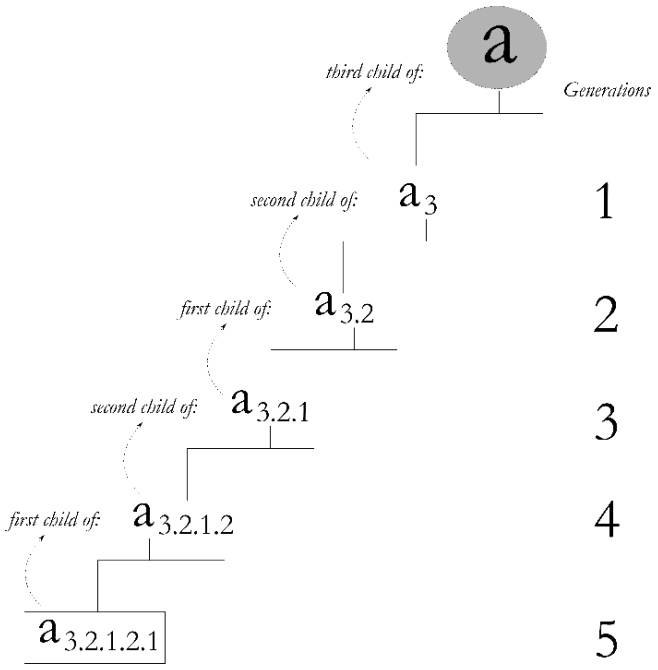

Figure 14: Representation of a hypothetical derivative lineage depicted in genealogical notation.

Moreover, a genealogical label can be mapped to a unique *code*, an especially useful information in a computer-assisted analysis. The order numbers become *exponents* of a product of sequential prime numbers (an algorithm known as Gödel Numbering, due to its creator, Austrian mathematician Kurt Gödel. Plugging the genealogical numbers of the label of the exemplified variant as exponents of the product of the five first prime numbers (considering that there are five generations in this case), we have:

$$G = 2^3 \times 3^2 \times 5^1 \times 7^2 \times 11^1 = 194{,}040.$$

The resulting integer (G) is called the variant's *Gödel address*, a code that ultimately and precisely represents its lineage.[6]

Another bio-musical concept is related to the notion of *holistic* or high-level D, being associated with the common idea of variation applied of concrete musical units (like motives). Under our biological lens we can compare it to a *phenetic* derivation, as considered in variation of *populations*. Figure 15 presents an example extracted from a real derivative analysis considering two related

[6] This labelling strategy is originally presented in ALMADA (2017). For other musical use of Gödel numbering, see MATHIAS; ALMADA (2021).

units, including measurement of similarity. The value of *k* indicates the amount of divergence between them (*k* varies from 0 to 1, which means that the two units are similar in a rate of 91%).

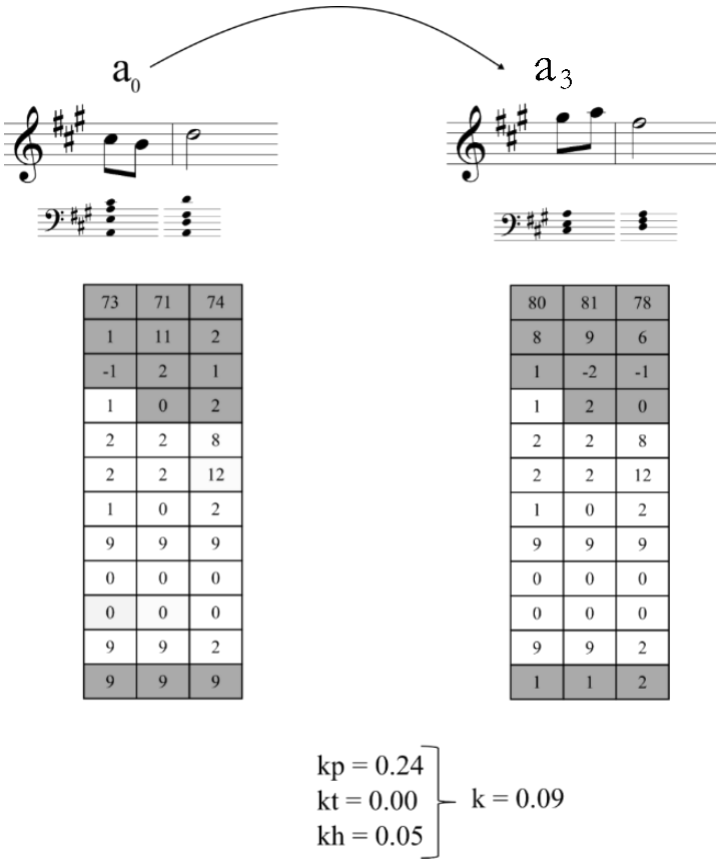

Figure 15: Derivation of unit a_3 from the referential a_0, evidenced by the differences between their respective matrices of attributes (in gray). Bottom: values for divergence referred to pitch (kp), time (kt), and harmonic context (kh). A definitive dissimilarity index is expressed by k.

A complementary concept is *decomposable* or low-level DV. In this case, derivation of variables is on the focus. It can be compared with variation on the level of *genes*. Figure 16 presents two examples of low-level derivative analysis, considering pitch and temporal attributes.

Phylogenetic trees (in several formats) are used in MDA in order to provide *global views* of the derivative processes, considering both levels, high and low. This abstract graph is the complete tree concerning the innumerous transformations of variable Z (related to intervallic sequences) in an analysis of Brahms's Piano Intermezzo op. 118/2 (Figure 17).

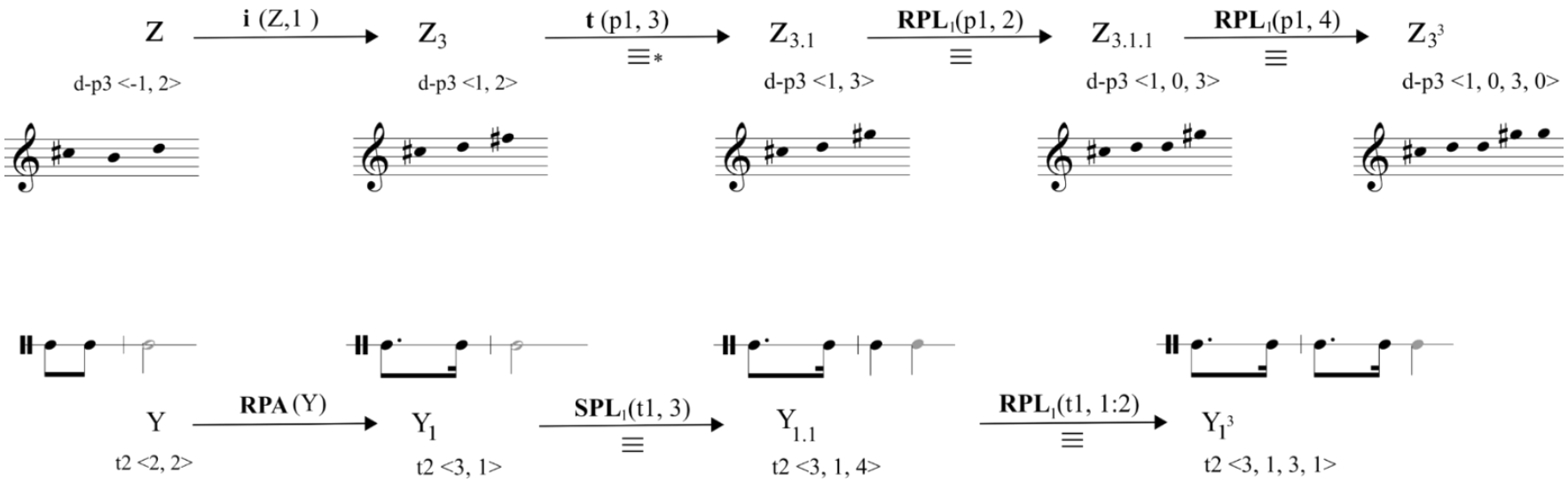

Figure 16: Examples of low-level DV, considering variable Z, related to the pitch domain (above), and variable Y, related to the temporal domain (bellow). Musical notation is added as an auxiliary representation of the derivations.

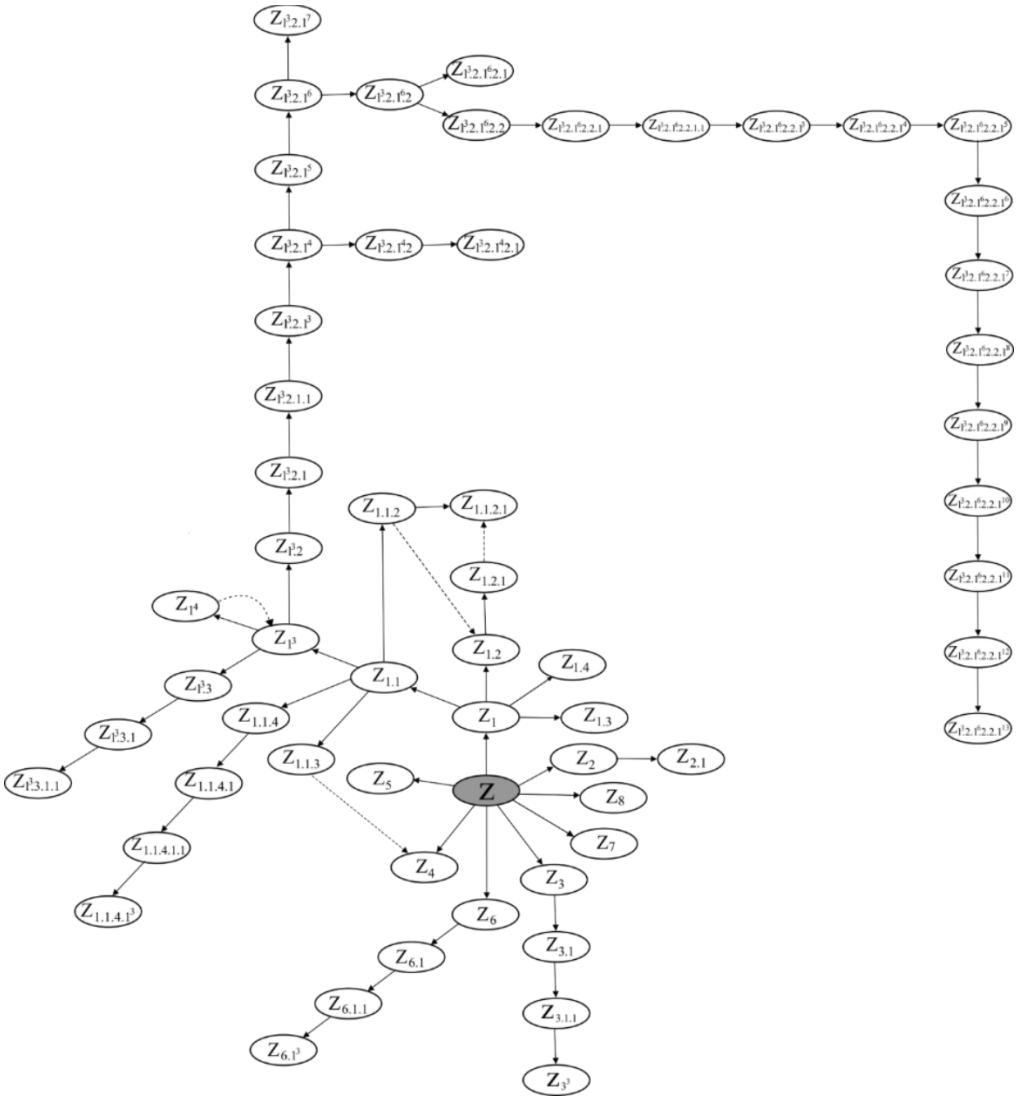

Figure 17: Phylogenetic tree representing the low-level derivative relations of variable Z in Brahms's Piano Intermezzo Op. 118/2. Since only the connections matter, the distances and directions are arbitrary.

Finally, Figure 18 depicts a phylogenetic tree of the derivation of *musical units* (the "population") in the same Brahms's piece. As one can observe, there are several derivative paths, which reveals the *multidimensional* nature of variation in a musical work. Observe the numbers on the arrows: they indicate similarity coefficients between the nodes. From this, we can consider many possible *lineages*, as that one highlighted in the figure, maybe the *longest*, linking the *Grundgestalt* a_0 to a very remote descendant.

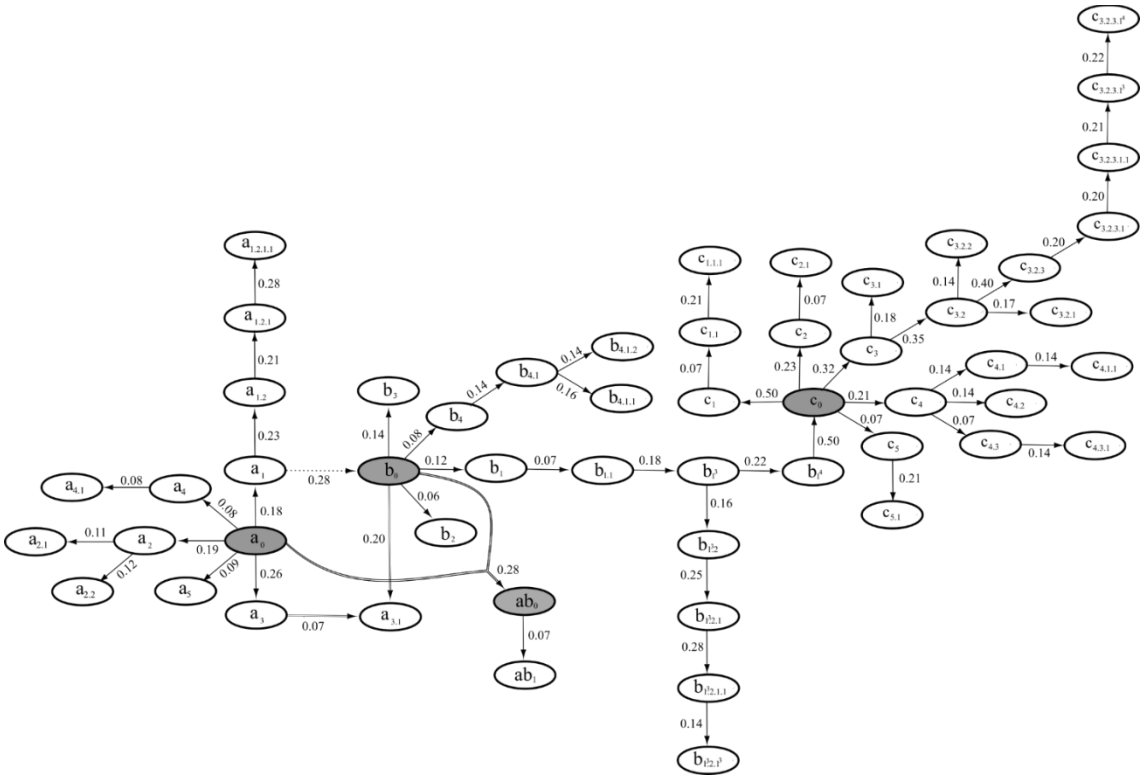

Figure 18: Phylogenetic tree representing the high-level derivative relations the musical units in Brahms's Piano Intermezzo Op. 118/2. Since only the connections matter, the distances and directions are arbitrary. Numbers over arrows indicate the related coefficient of similarity. Gray ellipses refer to referential units that initiate lineages.

Concluding remarks

This paper introduced the main elements of the Model for Derivative Analysis MDA, focusing on some special aspects that connect the idea of variation in musical and biological domains. This association – which involves not only metaphorical relations, but also concepts and original strategies – aims to systematize the analytical process. In this regard, Schoenberg's thoughts about variation – materialized in the principles of *Grundgestalt* and developing variation – play a very central role.

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7

The Painter Arnold Schoenberg: Transfigured Artist[1]

Guilherme Bueno

The last three decades have seen an increasing interest in Arnold Schoenberg's work as a visual artist. Overshadowed by his seminal role in the history of music, his paintings reveal more than a gifted artist: attentive to different experiments explored by European *avant garde* form late nineteenth century to German *Neue Sachlichkeit* during the Interwar years, Schoenberg maintained a profuse correspondence with Wassily Kandinsky, who by his turn, essayed in his books a whole theory on painting inspired by its parallels with music. A recurring subject among many other artists in the first half of the twentieth century, it was at the core of the quest for a "non-objective art", ranging from Piet Mondrian to Hans Richter and filmmaker Walther Ruttmann. This lecture will present to the audience a brief survey of such scene, focusing both the immediate context related with Schoenberg paintings, and some examples created by the numerous artistic movements active in the Old World during this period.

[1] This chapter is a transcription from Guilherme Bueno's lecture given at the 6th International Conference MusMat.

I am very honored to be with you today and share some impressions on Arthur Schoenberg's paintings. My main concern is to introduce you a survey of those works and the interpretation some authors essayed about them, references to which this lecture is much indebted. I warmly thank the kind invitation from Carlos de Lemos Almada and his research team MusMat, who gave me the opportunity to develop a couple of ideas about this rich part of Schoenberg's work still unknown to many people, discussing its place in the profuse context in which it took part, that encompass from the turn of the nineteenth to twentieth century up to the rise of avantgarde art during the Interwar period.

Our scope today is twofold, the reason why I will divide my lecture in two parts: in the first one we will draw the context of European visual arts during the period when Schoenberg was an active painter. For Peter Vergo (2010), since the last half of nineteenth-century, music offered to visual arts (painting, sculpture, even architecture) a variegated range of possibilities which enfranchised a detour from their conventional foundations, based on mimesis, the "unquestioned" emulation of a traditional repertoire of forms, and the assumption that visual arts always must depart from the interpretation of an outer motive.

The second part of our lecture will be devoted to Schoenberg, the painter. Although dealing with a context where the interplay between arts were at one of its apex points, one of its more emblematic and prolific moment, I will – as a layman on matters of music – avoid proposing objective parallels between Schoenberg's paintings and musical scores.

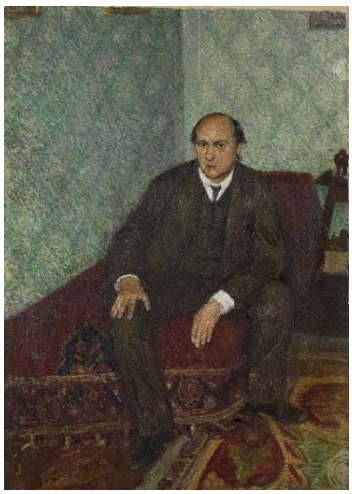

Figure 1: Richard Gerstl. Portrait of Arnold Schoenberg, 1906. Oil on canvas, 182 x 130 cm. Reproduced by kind permission of the Wienmuseum, Vienna.

Many have seen Schoenberg's paintings as an anecdotal episode of his biography, the testimony of the misfortunes of a short-lived and ill-fated friendship with Richard Gerstl (Figure 1), a young artist who gave him some lessons on painting and encouraged him to try his hand with the brushes,

committing suicide after an extra-marital affair with the composer's wife. The opinion of some of his contemporaries – even the modernist ones – ranges from the unfavorable criticism (August Macke, a member of the expressionist group Der Blaue Reiter, with whom Schoenberg exhibited his pictures, scornfully described Schoenberg's self-portraits as "green-eyed waterlogged breakfast rolls with an astral gaze". *Apud*. SIMPSON, 2004, p. 92) to an oscillating allegiance (Kandinsky). Although his works were along the last four decades the object of some in-depth studies (Eberhard Freitag pioneered this research with his writings published in 1973), such unfavored view remained to the point that in the late 1980's, the art-historian Kirk Varnedoe, who curated the exhibition Vienna 1900 – Art, Architecture and Design at the Museum of Modern Art in New York dismissed them as:

> […] even less easily categorized, as it wanders between caricature, naïve realism (primarily in a series of bluntly confrontational self-portrait heads), and more visionary, quasi-abstract imagery known as "gazes". […] Though they were an independent form of expression, it is doubtful these works would have found such notice had their author not been the musician he was. They have a faux-naïf feeling that, however fascinating for students of the composer, places them among the marginal curiosities of early modern art – in the company of the "proto-abstractions" of numerous naïve theosophist artists of this period […]. (VARNEDOE, 1986, p. 165)

I think it is part of our duty to object such a harsh judgment. Despite the limits I defined, focusing my analysis on the field of visual arts, it is important to remind the broader conception of modernism in which those works existed, when whatever reciprocal exchanges between arts (either correspondences like those between Joan Miró's works and Duke Ellington's improvisations in "Blues for Joan Miró", San Paul de Vince, 1966 – or the idea that painting could assume the inner structure of music, or that music – sound – could be produced by sculptures) were part of a debate that posed pivotal questions on artistic practices.

A Concise Survey on the Relationship between Visual Arts and Music under Modernism

Music as a metaphor for visual or spatial composition is a subject as ancient as art. It is not my intention to trace back all episodes of this long history. Suffice for the moment to indicate the existence of a topos often envisioned to subsume a theory of art whether allusive or partially independent from the burden of the Horatian *motto Ut pictura poesis* and the commanding idea of mimesis, comparing the degree with which every art tried to mime reality and consequently which one of them did it better.

In the late eighteenth century, for instance, the analogy music / arts was called for by French architect Nicolas Le Camus de Mézières, who, inspired by René Ouvrard's (a former Master of Music at the Saint-Chapelle) *treatise Architecture Harmonique ou Application de la Doctrine de la Musique à l'Architecture* (published in 1679), claimed in his book *Le Génie de l'Architecture, ou l'analogie de cet art avec nos sensantions* (The Genius of Architecture, or analogy of this art with our sensations, 1780) that the nature of Architecture was much closer to Music, as –

> At the sight of a fine building, the eyes enjoy a pleasure as sweet as any that the ears can receive from the sublime art of sounds. Music, the divine Art that enchants us, bears the closest relation to Architecture. The consonances and the proportions are the same. The City of Thebes, or so legend has it, was built to the strains of Amphion's lyre: a fiction that teaches us, at least, that the Ancients felt how intimately Architecture was allied to harmony, which is none other than the combination of different parts to form a concordant whole. Architecture is truly harmonic […]. (LE CAMUS DE MÉZIÈRES, 1780, pp. 11-12; 1992, p. 73)

– a definition congenial to Nicolas Poussin's (the great master of French classicism) reading of a painting, who in a letter dated 1647 to his patron Paul Fréart de Chantelou, teaches him to look at it acquainted with the principles of composition and tonality based on an intermingling of painting, poetry and music, those principles being inspired by the modes of Greek poetry:

> We must not judge by our senses alone but by reason. This is why I want to tell you something of great importance which will make you see what has to be observed in representing the subjects of painting. Those fine old Greeks, who invented everything that is beautiful, found several Modes by means of which they produced marvelous effects. This word Mode means, properly, the ratio or the measure and the form that we employ to do anything, which compels us not to go beyond it, making us work in all things with a certain middle course of moderation. […] As the Modes of the ancients were composed of several things put together, the variety produced certain differences of Mode whereby one could understand that each of them retained in itself a subtle distinction, particularly when all the things that pertained to the composition were put together in proportions that had the power to arouse the soul of the spectator to diverse emotions. Observing these effects, the wise ancients attributed to each [Mode] a special character […]. (Poussin *apud* HARRISON, 2000, p. 68)

Figure 2: Henri Fantin-Latour. Götterdammerung. Siegfried and the Rheinmaidens. 1884. Transfer litography, 46,5 x 37 cm. National Gallery of Canada

Being excused my abrupt chronological jump, I will devote my attention to more recent times. Such very summarized sketch of a much more complex theme just reminds us the fact that the debate on the exchanges between visual arts and music (with their nuanced conceptions), occurred along the second half of the 19th century and the first decades of the 20th century, derives from a long and polemical tradition (even when modernism claimed for such parallels to avoid tradition) which found at the turn of the century a promising sort of new dialogues. Observed retrospectively, one can perceive this debate along the last hundred fifty years reaching much more its turning point than its rising. It embraces the multitude of "wagnerians" painters (a cohort composed by post-impressionist, late romantics, symbolists and so on, each one of them interpreting in his own way the "lessons" of the romantic composer: it could be either the imagery of Wagner's operas (Figure 2) or some hidden compositional structures, or the ambition of an expanded and plenty aesthetic experience – all of this based on a noteworthy passage of Wagner's essay on Beethoven where the German composer says that music comes "from within" and the others arts "from without"[VERGO, 2010, p. 128][2]). It also included the mutual respect and "partnership" between Brahms and Max Klinger (who in 1894 printed an album of etchings – Brahms-Phantasie on behalf of the composer – Figure 3) and the Beethoven's shrine (Figures 4 and 5) endorsed by Viennese Secessionists, a sanctuary for the cult of modern art at the heart of the Austrian capital.

[2] Wagner supported Schopenhauer's point of view regarding the inner essence of music, who according to the composer "recognizes and defines the position of music compared with the other fine arts, as he ascribes to it a completely different nature from that of poetry and visual arts. […] From music a language immediately understood by everyone is spoken, as it does not depend on mediation by concepts, in which it differs absolutely from poetry, whose only material feasible to be used as an illustration of the idea is the concept. […] The ideas of the world and their essential appearance […] are namely the main object of the Fine Arts. […] [B]ut in Music the Idea of the World itself could be recognized […]. WAGNER, Richard (1870). *Beethoven*. Leipzig: Verlag von E.W. Frißsch, p. 5-6. (Author's translation)

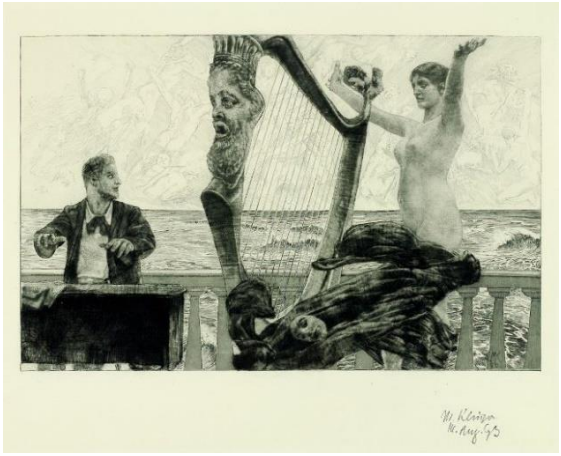

Figure 3: Max Klinger. Evocation from Brahms-Phantasie, 1894. Engarving, etching, mezzotint and lithograph, 29,2 x 35,7 cm

Figure 4: Max Klinger. Beethoven, 1901-1902. Pirinaean marble, marble from Scyrae (Greece), alabaster, ivory and bronze. Leipzig Museum of Fine Arts.

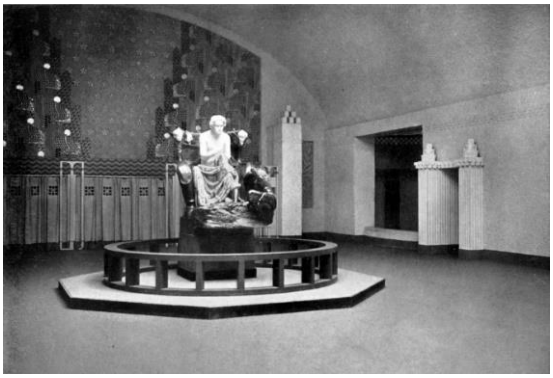

Figure 5: View of the original display of the monument at the Secession Building, Vienna, 1902.

Figure 6: Gustav Klimt. Beethoven Frieze, 1901. Fresco, Vienna.

At the moment of its unveiling (by this time, Schoenberg has already moved to Germany) the Faustian Beethoven monument sculpted with lavish and rare stones by the same Max Klinger (firstly exhibited in his studio in Leipzig and thereafter warmly acclaimed in Vienna), was displayed in the noblest hall of the Secession pavilion, built some years earlier after a project by the architect Joseph Maria Olbrich. The ceremony was celebrated with a performance in situ of an arrangement of a motif from Beethoven's 9th Symphony adapted by Mahler. For the occasion, architect Josef Hoffmann designed a special display for the sculpture, "by turning the central space into a kind of inner sacrum" and following Vergo's description:

> Determined that spectator's "should not be led in front of the main work unprepared", he had gone […] to devise a prescribed route around the building, in the course of which the Beethoven monument could be glimpsed at intervals through apertures which cunningly pierced the various internal walls […] On entering the vestibule, one was obliged to turn left, into the side aisle, where the first thing that greeted the eye was [Gustav] Klimt's monumental fresco-painting [fig.6], his Beethoven-frieze. […] Only after a lengthy detour, and having negotiated a short flight of steps, were members of the public finally allowed to enter the central space, passing in front of Klinger's monument as if before an altar. […] By the time they left [the building], [the spectators] must have felt that they had been drawn wilily-nilly into something that resembled a theatrical or even religious procession […] . (VERGO, 2010, pp. 125-126)

In the same token, but with a renewed perspective (sometimes motivated by the assimilation of new technologies and a new political agenda attentive to the crowds), when fine arts seek for a correspondence other than to figure out " descriptive scenes" for music, the Interwar generation enlarged – and reformulated in its own way – the emulation of a "direct experience" offered by music in conjunction with visual art: it could be a reciprocity between visual and aural patterns (Kandinsky); an interchanging compositional structure (Mondrian and the boogie-woogie, as Harry Cooper argues: "The essence of piano boogie is the vigorously competing rhythms of two hands, and sometimes two or even three pianos as well. It is a competition of likeness: the left and right hands nearly abandon their traditional roles of harmony and melody, instead sharing one repeated rhythmic motif and offsetting it between them to create a virtuoso polyrhythmic texture. The sound of good boogie-woogie, as early critics recognized, is a single mesh whose elements cannot easily be teased apart into "right" and "left" […] [COOPER, 1998, p. 135]); or a direct translation of time-sound structures into animated "pure" visual forms, explored by Hans Richter (Figure 7), Viking Eggeling (Figure 8), Walther Ruttmann (Figure 9), among others, in their movies, created as dynamic analogous of those pictures painted by their colleagues.

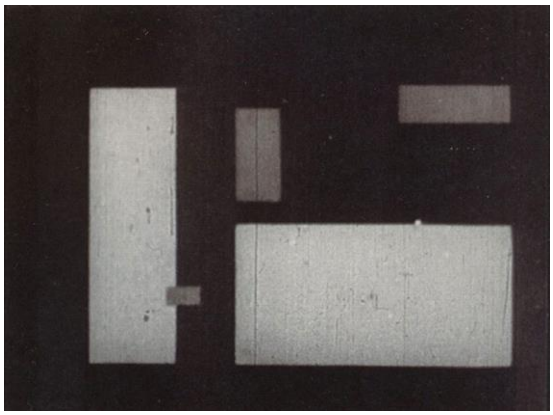

Figure 7: Hans Richter. Still from Rhythm 21, 1921. B&W movie.

Figure 8: Vikking Egeling. Still from Symphonie Diagonale, 1924. B&W movie.

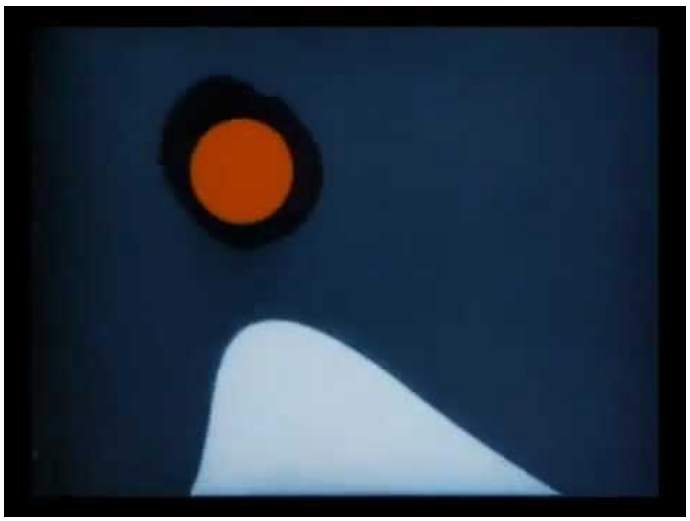

Figure 9: Walther Ruttmann. Still from Lichtspiel op.1, 1921. Film (color).

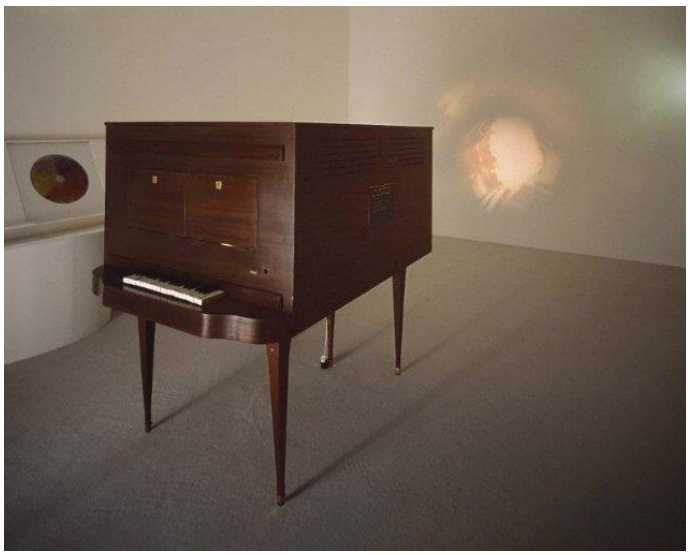

Figure 10: Wladimir Baranoff-Rossiné. Optophonic Piano (replica), 1912.

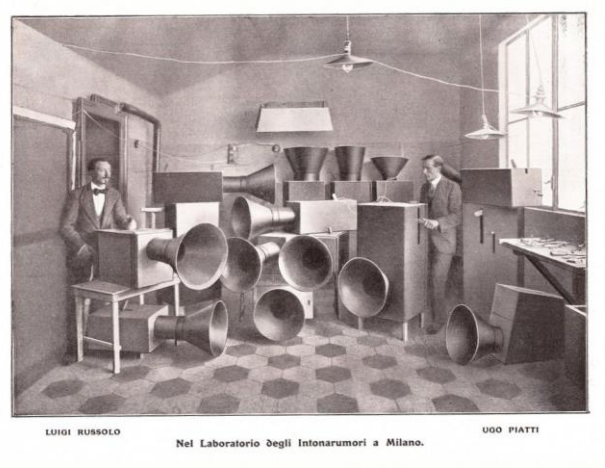

Figure 11: Luigi Russolo. Intonarumori, 1913.

Ironically, some experiences that seemed indeed groundbreaking – like Baranoff-Rossiné's Optophonic Piano (a device which assembled a kind of kaleidoscope of patterned-colors disks and a keyboard [Figure 10]) is – although its originality – rooted in a long-sought ambition to produce an enrapturing blend between sound and image that could be traced back up to Mézières' times, who mentioned the invention by the Jesuit Father Castel of a "merveilleux clavecin à couleurs […] un instrument que donnoit un concert de couleurs, en même temps qu'il en formoit un par le sons" ("the wonderful Harpsichord of colors. By a well-conceived and ingenious device, he constructed an instrument that gave a concert of colors, while at the same time producing another in sound" [MÉZIÈRES, 1780, p. 10; 1992, p. 72]). Some years earlier, in Italy, futurist Luigi Russolo tried with his Intonarumori (Figure 11) to produce a sort of mechanical sculptures that mimes the sound of motors, klaxons and so on, in order to invent a new "art of noises" able to rival music. Such quest also resulted in pioneering works of kinetic art, like that of Czech artist Zdeněk Pešánek's mixed-media sculpture Spectrophone (Figure 12).

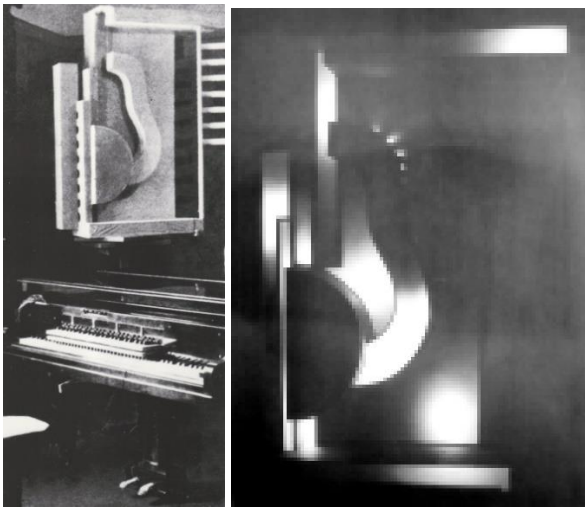

Figure 12: Zdeněk Pešánek's Spectrophone, 1926.Mixed-media sculpture.

On this respect, Vergo once again offers us a broad panorama:

> Today we may find it astonishing just how widespread was the interest shown by early twentieth-century artists and composers in devising instruments for creating color-music. These ranged from the "light keyboard" that was intended to play an integral part in Skryabin's symphonic poem Prometheus – the poem of fire (1911) to Morgan Russell's obsession […] with building a "kinetic light machine". During the 1920s and 1930s the numbers of such instruments and the scale on which they were conceived increased dramatically. At times it seemed that each new addition to the range was deliberately designed to be bigger, more expensive or more technologically advanced than its predecessors, while increasingly bizarre and colorful names soon became de rigueur. Largest and most spectacular of all was the giant "Chromatophone" invented by Baron Anatol Vietinghoff-Scheel: a monster of a machine that incorporated twenty-eight pedals and seventy-two spotlights, as well as apparatus for projecting short films. (VERGO, 2010, p. 284)

Such multitude of essays were nourished by many artists to whom painting was no more able to wholly fulfill the duty of producing the "work of art of the future". They were stimulated by the unprecedent possibilities opened by new technologies, which seemed to unfold to artists utopian new "poetical continents" of invention. It is something felt in the question made by one of those many pioneers, the Bauhaus' student Ludwig Hirschfeld-Mack, who asks if painting would still be capable of unite and express the ethos of a community. Hans Richter was for the same opinion, defending film (and therefore his experimental abstract movies) as the "art of the twentieth century" (VERGO, 2010, p. 287).

Figure 13: Piet Mondrian. Composition, 1929. Oil on canvas, 45,1 x 45,3 cm. Guggenheim Museum, New York.

Music, notwithstanding, serves also painting, what make us come back to Wassily Kandinsky and Piet Mondrian. Mondrian (Figure 13), as mentioned, believed that jazz and boogie-woogie due to their technique of improvisation and their treatment of harmony and melody, uncovered a compositional structure which permitted him to find a dynamical alternation of position between planes, – a same plane sometimes receding in the composition, sometimes jumping to the front, these relationships being activated according with the focal point the viewer looks at the picture. It implied a levelling of planes similar to boogie-woogie's piano technique and a "temporal" dimension in visual perception disruptive of the static condition of the image, their hierarchical disposition in a picture, and the segmentation of its parts. The resulting effect he envisioned, both in Neoplasticism and in Jazz music was the "destruction of the form", i.e., its abolition as the leading element of an descriptive image, as it was understood as a privileged and isolated figure whose protagonist condition reinstates a naturalistic conception of art. He explained it in his short (and ambitious) essay De Jazz en de Neo-Plastiek (Jazz and Neo-Plastic), from 1927:

> [Jazz and Neo-Plasticism] appear simultaneously with movements in various spheres that are trying to break with individual form and subjective emotion: they appear no longer as "beauty", but as "life" realized through pure rhythm, which expresses unity because it is not closed […] The more an art is limited by form, the less easily can it attain universal plastic expression, the new order. […] Jazz – being free of musical conventions – now realizes an almost pure rhythm, thanks to its greater intensity of sound and to its oppositions. Its rhythm already gibes the illusion of being "open", unhampered by form. But on the other hand Neo-Plasticism actually shows rhythm free of form: as universal rhythm […] Jazz and Neo-Plasticism are revolutionary phenomena in the extreme: they are destructive-constructive. They do not destroy the actual content of form: they only deepen form and annihilate it in favor of a new order. Breaking the limitations of "form as particularity", they make universal unity possible (MONDRIAN, 1927, pp. 421-423, HOLTZMAN, 1993, pp. 217-219).

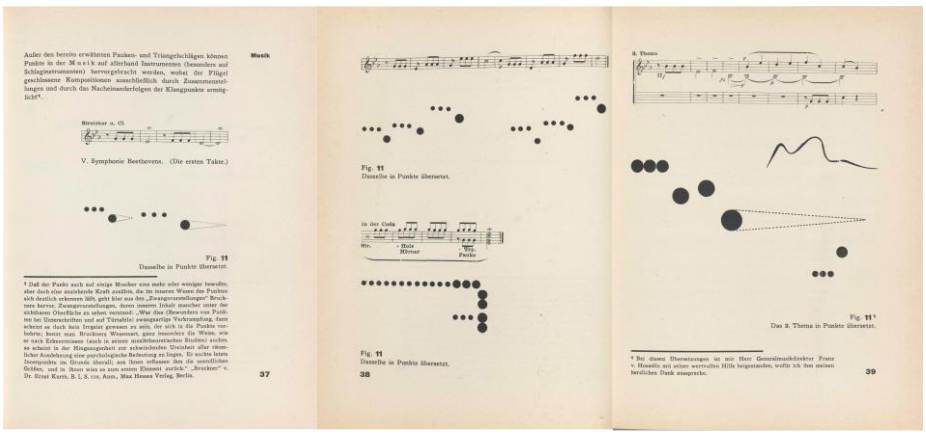

Figure 14: Wassily Kandinsky. Pages from Point and Line on the Plane, 1926. Bauhaus Archiv, Berlin.

Kandinsky (Figure 14), if not more explicit, was at least equally engaged to propose and "practically" demonstrate his theory on the hidden accordance of painting and music. Like a musician who says that a sound has a color, he refers the "sound" and "music" produced by the point and its combinations in and with lines. To support his view, he draws a sort of "visual transcription" of the opening measures of Beethoven's 5th Symphony, illustrated in his book Punkt und Linie zu Fläche (Point and Line on the Plane), published in 1926. He employed this analogy at least twice in the text, as follows:

> Music: What a musical line is, is well known [here Kandinsky evokes again his diagram after Beethoven's Symphony]. Most musical instruments are of a linear character ["linear" here brought a double sense, as this word was used a decade later by Swiss art historian Heinrich Wölfflin to describe the well-defined aspect of forms from styles like Renaissance which produced clear visual effects, distinct from the diffuseness of Baroque]. The pitch of the various instruments corresponds to the width of the line: a very fine line represents the sound produced by the violin, flute, piccolo; a somewhat thicker line represents the tone of the viola, clarinet; and the lines become broader via the deep-toned instruments, finally culminating in the broadest line representing the deepest tones produced by the bass-viol or the tuba.
>
> Aside from its width, the line is produced in its color variations by the diversified chromatic character of different instruments.
>
> The organ is quite as typical a "linear" instrument as the piano is a "point" instrument.
> It can be asserted that in music the line supplies the greatest means of expression. It manifests itself here in time and space just as it does in painting. How time and space are related to each other in the two forms of art is a question by itself which, with its distinctions, has led to an exaggerated scrupulousness and, thereby, the concepts of time-space or spacetime have been differentiated far too much.
> The degrees of intensity from pianissimo to fortissimo can be expressed in an increasing or decreasing sharpness of the line, that is, in its degree of brilliance. The pressure of the hand on the bow corresponds exactly to the pressure of the hand on the pencil (KANDINSKY, 1926, pp. 92-92; 1947, pp. 98-99).

Absorbed in his musical creation, Schoenberg's fine arts works (just paintings) could be felt by younger generations as something old-fashioned, as these paintings never insinuated any intention to bring together painting and music as one and only language, something often ambitioned by the attempts of the more radical avantgarde to go through (although he was in his way a worshiper of a totality). Discarded the eventuality of a possible generational non-alignment (a judgment not at all mistaken, by the way), he notwithstanding envisioned in some occasions a suggestive fluidity between arts akin to produce an enrapturing of the spectator through the meeting of different arts, as he did in

his instructions for the lighting design of some of his performances and his stage designs (we will discuss them later). Nevertheless, painting was just one of many ways to be modern at disposal during those years, and I guess Schoenberg, preferred to be modern in each one of them, responding to their respective demands. If, in a sense, it could be sound that Schoenberg – the painter – became a "conservative" artist (although he tried his hand sporadically as an amateur designer also, as we deduce from the furniture projected by him and other gadgets he invented, for example, a set of chess pieces and a musical typewriter), our point is not to discuss if he was a front-line revolutionary painter, but how a good, remarkable and modern one. It is this assumption that enfranchise us the way to think of his paintings and his personal approach in their own right.

On Schoenberg's Paintings

"Artists are different. They have no customers. Artists are the ones who place the orders with themselves". With such words, Adolf Loos, one of the leading modern architects in the early 20th century defined Schoenberg's artistic status. What at a first glance should seem a statement on the modernist artist professional condition as a Rimbaudian maudit, it leads us to a promising framework to discuss Schoenberg's paintings. There is some evidence that his works targeted two different publics: some of them sought for potential customers, with a hope for a financial relieve during hard times, while the other half of his production, if we keep in mind Kandinsky's endorsement of his Visions (a very ambiguous endorsement, as Kandinsky said about the more "personal" paintings by Schoenberg that they were painted "just as infrequently as the first [a group of landscapes and posed portraits], in order to allow those stirrings of the soul, which cannot find any musical form, to come to expression")… this other half of his oeuvre did not exist but by itself i.e., as an ultimate artistic credo beyond the appraisal of whoever, and a "complimentary" confirmation that his aesthetics choices as artist – either as painter and as musician – were the right ones. To some extent, Schoenberg would be a true expressionist, to whom the world is nothing but the object (and a subject submitted) to his will (I will resume it later). Once Kandinsky released Schoenberg from the exigences of skilled "professionalism" ("we see immediately that Schoenberg paints not in order to paint something "beautiful" or "engaging," but that he paints without even thinking about the picture itself. Renouncing the objective result, he seeks to affix only his subjective "feelings," and uses for that purpose only the means which seem to him indispensable at that moment", according with the Russian artist), he opened to him the doors towards expressionism and its goal to "give an outward expression to an inner impression". Nonetheless Kandinsky was enough aware on the difficulty to relate Schoenberg's paintings to a specific art movement of the early 20th century (commenting Schoenberg's "conventional" and the bold "inner" images, he concludes that "we see immediately that we are dealing here with painting,

whether or not this painting may lie 'apart' from the great 'movements of today'", as they went apart self-referential avantgarde agenda and were concerned strictly with a "vital" call). They "lie apart" due to their unusual coherence, not based on a recurrent but "evolutive" set of forms (such discernible "signature" that makes a Picasso a Picasso, a Matisse a Matisse etc. that crosses their many "phases") but from the "spiritual drive" they were made of. Indeed, both exhibited some of their works together with Franz Marc and other members of the *Blaue Reiter* (the exhibition Schoenberg was hesitant to accept the invitation, as we've seen above), a group pertaining to the second generation of Expressionism in Germany. But though Schoenberg's works reveals a decisive expressionistic attitude (curiously mixed with symbolist reminiscences) one can only partially identify some of the movement's main traits (above all in the "second group" of his works).

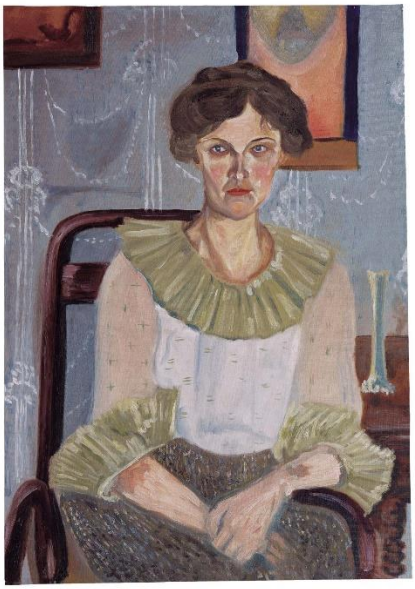

Figure 15: Arnold Schoenberg. Portrait of Helene Nahowski, 1910. Oil on canvas, 100 x 74 cm. Reproduced by kind permission of the Wienmuseum, Vienna.

The attribution of Schoenberg's paintings to Expressionism was in large part due to his friendship with Kandinsky, the first one to "theorize" his friend's works. Kandinsky's endeavor to find in music a pretext for his theory on painting, motivated him to contact Schoenberg, after he and Franz Marc attended a concert of the composer in Munich at the beginning of January 1911. His acquaintance with the composer's treatise on harmony worked as the point of departure to stimulate an exchange of converging ideas (VERGO, 2010, pp. 181-187). Schoenberg admitted to the painter in a letter dated March 8, 1912, referring to the latter's book *Über das geistige in der Kunst* (Concerning the Spiritual in Art) that "I must already write to you now that I like it extraordinarily. You are certainly right about so many things, particularly what you say about color in comparison to musical timbre. That is in accord with my own perceptions (AUNER, 2003 p. 102)." This is one of the reasons why (supported by facts and

characteristics of the remaining paintings, drawings and stage design created by the composer) some authors, like (once again) Peter Vergo and Courtney S. Adams generally assume such association with the Expressionists, although some portraits made by Schoenberg, even austere in their color scale, regulated by an intermediate tonality which loosely controlled the hue of each color and the luminosity of the whole picture (e.g. Helene Nahowski, 1910, Figure 15) did not resemble neither the bright and fulgurant colors of a turn-of-the-century painting produced in Vienna by artists like Gustave Klimt (Figure 16), Egon Schiele, and Oskar Kokoschka, nor the late post-impressionist and early Fauvism current in Paris (championed by Matisse – The Red Studio. Issy-les-Moulineaux, 1911. Oil on canvas, 181 x 219,1 cm, The Museum of Modern Art, New York) or even the intense and rough chromatism of early German Expressionism from the group Die Brücke (Figure 17) (in Schoenberg we hardly find even the sharp contrast of black and white explored by Expressionism's woodcuts).

Figure 16: Gustav Klimt. Birch Forest, 1903. Oil on canvas, 110 x 110 cm, Belvedere, Vienna.

Figure 17: Ernst Ludwig Kirchner. Portrait of Gerda, 1914. Oil on canvas, 70 x 57 cm, Von der Heydt-Museum, Wuppertal.

On the other hand, concerning the late self-portraits and the Fantasies (Figures 18 and 19), the panorama is much more complex, for one can then see sometimes a bolder use of colors and "empty" or "atmospheric" spaces alternating between a vast asymmetry and a blatant and imposing central axiality much like that employed by many expressionist artists.

Figure 18: Arnold Schoenberg. Blue Gaze, 1910. Oil on cardboard, 20 x 23 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 19: Arnold Schoenberg. Flesh, 1909. Oil on cardboard, 22,6 x 29,1 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Notwithstanding, we can also find pictures where the presence of formal expressionistic elements (mainly in similar impastos – thick layers of paint preserving evident traces of intense brushstrokes, translating the immediacy of an convoluted emotional state) were combined with a palette in which there

would be no "color" (Figures 20 and 21), but a dirty tonality (i.e. when mixed colors tended to gray or brown, losing their hue, such greyish tonalities working as an intermediary point in a chiaroscuro scale) only faintly linked with Die Brücke's approach and ironically (and probably involuntary) much more akin to the colors employed by Picasso and Braque in their first attempts on analytic cubism (as, e.g. in Georges Braque's Homage to J.S. Bach, 1911-12. Oil on canvas, 54 x 73 cm. The Museum of Modern Art, New York), as a strategy to avoid painting from the "sensuality" of colors (Kandinsky somehow recognized some "cubism" in Schoenberg, although he employed this term referring to a kind of structural order in his music). Regarding its color structure and visual motives, we can sometimes even find some actual resonances with alternatives currents of Expressionism, like the Dresdner Secession, especially in a Lasar Segall's Self-portrait from 1919 (oil on canvas, 58,50 x 68 cm. Lasar Segall Museum, São Paulo) – then active in Germany, before moving to Brazil –, where a similarity between the treatment of the "cavernous" eyes and the evolution of the brownish gradation of tones is recognizable.

Figure 20: Arnold Schoenberg. Portrait of Gustav Mahler, 1910. Oil on board, 45,6 x 44,5 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 21: Arnold Schoenberg. Gaze, 1910. Oil on cardboard, 32,2 x 24,6 cm. Used by permission of Belmont Music Publishers, Los Angeles.

So, what would be the expressionistic "trait" of Schoenberg? There is a deeper conveying sense: a search for extremes, and a comparison with his more "moderate" pictures give us some clues about this.

As aforementioned, although Schoenberg was not at all a "full" formally trained nor a full-time "professional" painter (but the only fact that he choose to work often with oil painting indicates how seriously he was taking painting, if we consider the weight of tradition and the technical challenges, exigences [and possibilities] related to this medium, signaling his concern with Painting), I think the intentional roughness assumed by him, his bare, crude technique mirrors his wish at once to defy tradition and to preserve (at the expense of abandoning painting's methods) a kind of primal energy and drama who motivated him to assume ugliness as a value – an ugliness that reflects the general, exasperating malaise felt during the later years of Austro-Hungarian Empire, resulting not in melancholy but bitter despair, cynicism and the drifting behavior typical of a decadent epoch: "the strategic decision to present oneself as loathed, diseased, and ugly began to dominate Viennese visual arts in the early twentieth century", as notes Kathryn Simpson in the introduction of her research about ugliness, hatred and self-representation in the city (SIMPSON, 2016, p. 4). In a word, he was not expurgating through another language what haunted him in music but seeking to deal with an ambience saturated with restlessness and immobility, resorting to whatever ways to not succumb to it. I do not hesitate to say that painting permitted Schoenberg to establish himself a place where he would feel like a very primitive: an alternative occasion when he manages his anxiety relieved from the burden and urging pressure of Technique (did he felt "unguilty" to deliver some "bad" paintings?) and "tradition". From a moment, to be an outsider seems to be a provisional advantage and a valve of escape. This is neither – remind us – a question of mere "translation" (i.e. music or painting solving each other dilemmas) nor a matter of equivalences (how a solution equals another), not even a compensation, but to put it simply – I hope so – how those elements must be fought whether in one or other language as their basic problems. I insist that such "bare" treatment of painterly elements – or the "extremes" above mentioned – could not be reduced to a lack of training (as the minutely cared treatment was easily related to academicism) precisely when one sees that in such "moderate" paintings, Schoenberg clearly organized the recession of planes, and a restrained brushstroke (i.e. he was fully aware of how a "conventional" painter establishes mediations from a plane to another one, using gradual transitions to order the composition). Both his Visions, Fantasies and the self-portraits (Figure 31) stresses how he decidedly opted for one initial abrupt contrast between a front plane and an empty background, resulting in a kind of shock created by floating or unfinished heads. Other works used a purportedly unbalanced distribution of elements, as e.g., the half silhouette occupying just a narrow part on the left side of Blue Gaze (Figure 18).

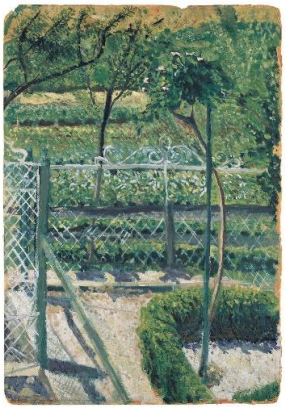

Figure 22: Arnold Schoenberg. Garden in Mödling, 1906-07. Oil on board, 71,6 x 49,8 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 23: Arnold Schoenberg. Portrait of Hugo Botsiber, 1910. Oil on cardboard, 73 x 50 cm. Private Collection. Reproduced by kind permission of the Arnold Schoenberg Center, Vienna.

The thick layers of paint sometimes employed in those more ambitious works reinforced the contrast with the "loose" and "smooth" treatment applied in Helene Nahowski (Figure 15), Hugo Botsiber (Figure 23) and in Garden in Mödling, 1906-7 (Figure 22). Could we assume Courtney Adam's conclusion that, quoting her, "the self-portraits [and some of the visions and gazes] reflect the inner world and its isolation" and "the emphasis on the eyes […] as a mirror of the soul"? I tend to agree with her, mostly when one parallel the famous statement of Schoenberg ("Art belongs to the unconscious! One must express oneself! Express oneself directly! Not one's taste or one's upbringing or intelligence, one's knowledge or skill") with the definition of Expressionism and the realm of inner reality presented by the poet Theodor Däubler in in his book Der Neue Standpunkt, published some years later (1919). Däubler says:

> Things we create must depart out of the self [ichbegabt sein]. Not from our vantage point, displayed in perspective, but outwardly crystalized by themselves. The middle-point of the World is in each self; in fact, in a work justified from the self [ichberechtigten Werk] (DÄUBLER, 1919, p. 180).

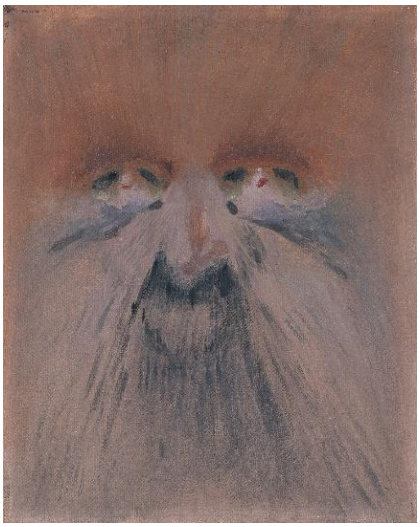

Figure 24: Arnold Schoenberg. Tears, 1910. Oil on canvas, 29,3 x 23,4 cm. Used by permission of Belmont Music Publishers, Los Angeles.

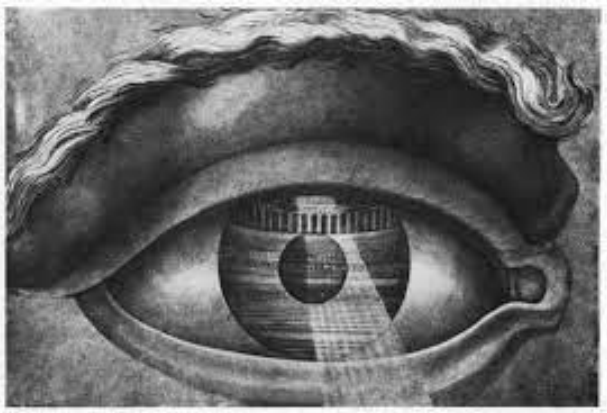

Figure 25: Claude Nicolas Ledoux. Theater of Besançon, circa 1800. Litography.

Figure 26: Odilon Redon. Eye-Balloon, 1878. Charcoal and chalk on colored paper, 42,2 x 33,3 cm. The Museum of Modern Art, New York.

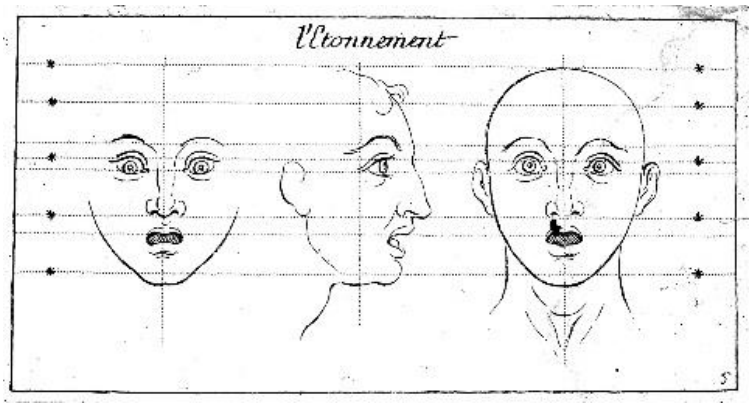

Figure 27: Charles de Brun. L'étonnement (Astonishment). From Caractères de Passions, 1695-1720.

An approach on the protagonist motive of the eyes identified by Adams and Däubler's inversion of the vantage point reveals its eloquence once we evaluate it from an art historical point of view. Perspective system was a rational definition of space dominated by the eye, but a visuality that, even dependent on the subject, only became possible through the transference from the subject to the world and conversely (an image is projected by someone placed in a specific place, but the space only could be realized once we "assimilate" this referential). This monumental conception preserved its driving force still during the late eighteenth / early nineteenth centuries, noticeable in a rendering made by the French architect Claude-Nicolas Ledoux for his project for a Theater in Besançon (Figure 25). Startled eyes, by their turn, were since the seventeenth-century a recurrent topic on the representation of human passion, exemplified in Charles Lebrun's treatise on humans's passions (Figure 27). If among modern artists the expressiveness of the gaze remained a valuable element to unfold the psychology of the portraited, the "rhetoric" of the perspectival eye, on the contrary, continually lost its favor: its cyclopean omnipotence was parodied by French symbolist painter Odilon Redon (a devoted Wagnerian) in a drawing depicting a ballooned-eye (Figure 26) and in René Magritte's The False Mirror, from 1929 (Oil on canvas, 54 x 80,9 cm. The Museum of Modern Art, New York). In "Expressionistic perspective", (in Däubler's opinion a step forward even compared to cubism, once everything could only exist if in accordance with what is imposed by the self over the world), the artist's eye becomes a egocentric, unidirectional and God-like ruler entitled to submit everything to its passionate reactions: the world just means something and only could be represented after the artist's will – an outward projection of his/her inner world fighting his/her torments and demons; there is no vision if not that born out of the artist's most transfigured attitude towards things, blurring the conventional borderline which formerly separated an "objective" world from imagination. Schoenberg's Gazes (Figure 24), especially those depicting crying eyes evokes, in a sense, a double meaning: if the "inner" tears protrude from the canvas towards us, they at once

project toward our world the artist's self, centralizing the eyes as the focal point. Thereafter, inverting the perspective scheme, i.e., the Renaissance model through which the outer world is reached, Schoenberg's eyes, conversely, engulfs our gaze through his abyss-like eyes.

We deal with what the artist's vision *envisions* – a kind of actuality only he or she access and intercedes for us. Artist sees what no ordinary people can see (or do not want to see) by themselves (although they live it). This is the core principle of Expressionism: to convince its audience that bare reality is not enough, but just a smooth appearance that one must pierce to feel its deeper sense, in order to overcome its malaise and reach a frail hope of redemption (if there is a redemptive "double" here, it faces psychoanalysis, and we can find it in Pabst's movie Secrets of a Soul). It is not hard to see how Schoenberg's paintings held this credo, notwithstanding his frank individualism. I would just like to add, respecting this very personal way to (re)present the world – as a confirmation of Däubler's description – the testimony of one of Schoenberg's disciples, the composer Karl Linke, in a short text he wrote in praise of his master, published in the Viennese journal Der Merker in June 1911. Linke's observations on Schoenberg's more ambitious goals strongly echoed Däubler's view registered some years later. Linke says:

> The eye, as a sensory organ is just the entrance channel of an impression came from outside. It is the medium [Mittel] to see and not the goal of Vision […] It must be possible the imitation not from what is exterior, but from the inner Nature. But this one not as an affection of things saw externally, but from well-known and incorporeal things floating in space. To make the iridescent point of a vision neatly visible and the other in the fog – as if we could physically see just one point […] In this consists Schoenberg's pictures of an inner face, that we can't come closer with words. So, they live in another world as the word and knowledge cannot find in words the expression at the end point of their way. (LINKE, 1911, pp. 710-711)

Once those images unfold a "different" reality, as they "imitate not from the outer, but the inner Nature", considering such points, one can initially suppose that Schoenberg's approach on color would seem a minor question, so distant it is from "naturalism" (we will try soon to show that this is a more complex issue than the mere adoption of stark colors or the use of tones). The key point begins with this – literally speaking – reversal of perspective that evades painting to be imitative, or, if it is permitted to say, to create "dissonances" in the image, considering the regular (or expected) development of its form. The spotted colors he added at specific points reveals at once a true perception of the way the leading painters of the period create interruptions on the "natural flow" of an image, as they were intended to break the expected continuity and soft, modelled, transition of forms determined by sequenced passages of light and shadow.

After those general considerations it is time to introduce the last part of our lecture, centered in the discussion of specific works and the series they are part.

Self-Portraits

> I am small; I have short legs; I am bald having a central Glatze and a small (crown?) of dark hair around it. My nose is big and hooked, I have dark big eyes, big eyebrows, my mouth is perhaps the best of me; I have usually my hands verschränkt [folded] on the back; my shoulders are round. (SCHOENBERG, circa 1937)

It is taken for granted that a self-portrait is always an act of self-examination, a personal statement fusing the image of the artist with the language he or she decided to present himself/herself (in literature, we have the publication of Memories, diaries, and similar kind of writings). Self-portraits literally embodies the motto "the style is the Man" to the point that a self-portrait that does not reflect someone's personal artistic "style" – i.e, presented himself/herself otherwise – seems a nonsense. Perhaps for visual arts and literature – precisely due their burden to be doomed to the dependence of the "inspiration after an outer motive", being rooted in an "exterior reality" – such task became more easier, or at last more promptly recognizable to the lay public than in music. We just see the artist's face and "here's the man", while in music it should require some subtlety to "decipher" how sounds "describes" someone's features, or to convey such "objective" features with his/her inner soul expressed through sounds.

These brief considerations bring us back to two delicate (but exciting) questions about the purportedly relationship between Schoenberg visual and musical works and, following the exhaustively mentioned link with Expressionism – I beg the pardon of the audience to be fastidious with this topic – about what reality is depicted by an artist, considering that the Fantasies and Gazes also "portraits" the artist's mind. I guess things will become more palpable if we concentrate in some formal aspects of this series.

Our first step is thus to describe which would be the parallels and antagonisms between both fronts in his production, according with some scholars on the theme. I will quote here in a very schematic way a passage where Courtney S. Adams synthetizes her hypothesis concerning potential points of comparison. According to her:

> The same elements of self-focus and alienation are prominent at this time Schoenberg's musical works with text. The most obvious example *is Die glückliche Hand*. […] The drama portrays the suffering of a great artist whose contributions were unappreciated by the world at large.

Erwartung concerns infidelity, loss, and death, which also give it autobiographical overtones. […] For *Pierrot Lunaire* Schoenberg selected 21 poems from a total of 50 written by Albert Giraud; several of these can be seen as self-referential, especially those that reflect martyrdom, persecution, or horror […].

The term "nightmarish" is frequently used to describe the impact of *Erwartung*, *Die glückliche Hand*, and *Pierrot Lunaire*. […] The description is *appropriate* to a number of the paintings as well. In [Gaze, 1910 and Vision, undated], for example, he achieves a nightmare quality through unsettling distortions […]. […] [T]here is the concept of design or balance as arts. Musically, passages of exact repetition that provide important guideposts in determining the organization of a work are rare in the atonal period. When Schoenberg wishes to provide a sense of recall, he brings back some elements, but alter others by modifying the material melodically, adjusting it rhythmically, or adding musical lines to the texture […] These sections of varied repetition and shortened recall appear in places that give a slight imbalance to the overall design. In other words, the repetition is not exact, nor does its placement exactly balance the original. Thus, we can speak of a near symmetry of design, as opposed to exact symmetry or asymmetry. […] His art reflects a similar approach. A casual glance through the figures […] shows a firm grasp of balance without the precision symmetry or the serious imbalance of asymmetry. […]

[…] A second element that functions in both arts is that of rhythm. In general Schoenberg's atonal music does not reflect a strong or regular sense of pulse. […] In fact, he had a systematic means of deemphasizing the first beats of measure by using a rest, tying a note over the measure, or writing phrasing that extended over the barline. In addition, several of his atonal works introduce frequently changing meters or tempos which reduce even further any effect of rhythmic regularity. […] The static ostinato holds back movement and often appears at cadential points in the atonal music. It is this latter kind of ostinato that predominates in the works between 1908 and 1912.

[…] One of the most direct ways to express rhythm in painting is through the repetition of lines, colors, or forms. A sense of motion in painting can also come from the subject matter itself, or it may result from techniques such as the use of individualized brush strokes, heavy repeated impasto […], or a linear directionality, such as strong diagonal lines. […] Another intriguing parallelism lies in the use of pictorial imagery in his music. […] [and] Another potential point of comparison between Schoenberg's music and art relates to athematicism in music. […]

In two areas Schoenberg's usage in the two arts differs dramatically - the first concerns complexity. Although the musical works in this period are short relative to composed earlier and later, the textures within them can be highly complex […] In contrast, complexity and attention to detail do not characterize the paintings. Usually, there is a single dominating focus that immediately strikes the viewer, and the work does not require lengthy study to unravel it. […]

The second element that differs in Schoenberg's practice of the two arts involves the use of color. […]

> That tone color in music had considerable significance for Schoenberg is clear from his development of *Klangfarbenmelodie*. […]
> His approach to painting is exactly the opposite, for he avoids sharp contrast. (ADAMS, 1995, pp. 9-20)

Although I do not agree fully with her – I mean, on which regards strictly painting – I think it is important to read her indications, as they suggest how one could try to discern not elements, but convergent concepts and anxieties at work, respecting their respective media. For my part, as I announced since the beginning of this lecture, I will center my analysis on the field of Art History and it would be up to the audience to evaluate to which extent those points concede a parallelism with Schoenberg's process of musical composition.

Figure 28: Schonberg, around 1926. Reproduced by kind permission of the Arnold Schoenberg Center, Vienna.

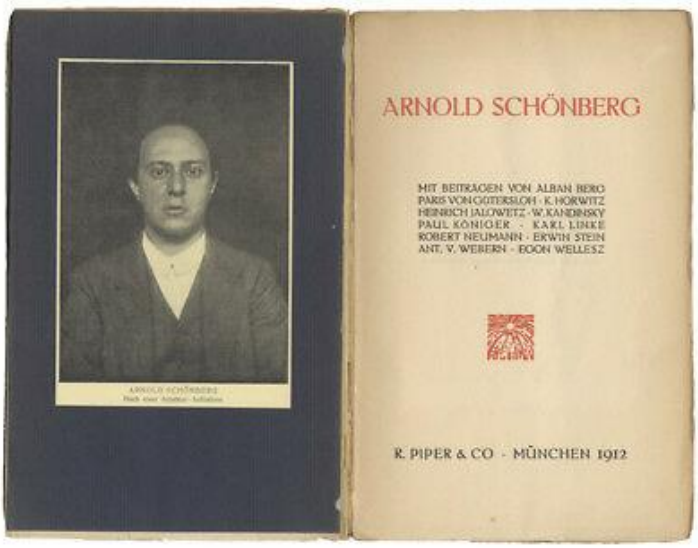

Figure 29: Photo of Schoenberg printed in Arnold Schoenberg, 1912. Reproduced by kind permission of the Arnold Schoenberg Center, Vienna.

We do not find always an absolute symmetry in Schonberg's self-portraits. I guess asymmetry was a value Schoenberg sometimes cared for when constructing his public personal image, as if it permitted to reveal his multiple personality through this, contrary to the more "atmospheric", "ethereal", affected and even base ways artists many times were represented. In a photography from 1926 (Figure 28), he poses frontally but a slightly smiling contorted mouth breaks symmetry of his face. Conversely in other photographs, like that appearing in the Festschrift dedicated to him in 1912 (Figure 29) (we will see that this image enfranchises compositional structures compatibles with those arranged in paintings), the symmetry which aligned the vertical axis reinforces the fixed gaze which dominates the upper half of the image, producing a picture intentionally static and almost ecstatic at once (just remember the "eloquence of the eye and the gaze we discussed on the theories of Expressionism). Symmetry here reinforces a "psychology" of the portrait and the compelling power of the visionary eyes of the portraited.

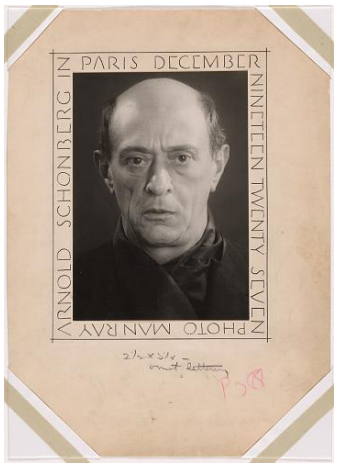

Figure 30: Man Ray. Arnold Schoenberg, 1927. Gelatin silver print, 29,8 x 23,8 cm. The J. Paul Getty Museum, Los Angeles.

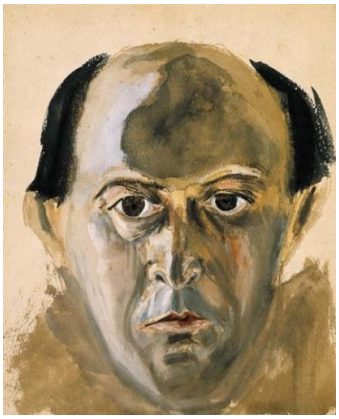

Figure 31: Arnold Schoenberg. Self-portrait, 1910. Gouache on paper, 23,2 x 18,6. Used by permission of Belmont Music Publishers, Los Angeles.

In a shot made by Man Ray – one of the main members of surrealism – taken in 1927 (Figure 30), the close stresses the centrality of the head in the picture and several secondary details nuanced this sharp contrast of his head against a neutral rear plane: the wrinkle marking the right side of his face, the zigzagging diagonals of the foulard around his neck. In short, those photographs – or better, the way Schoenberg poses for them – confirmed a strategy of self-representation we notice since his self-portraits: it goes beyond the only problem of symmetry x asymmetry, each one of them being adopted according with the way they intend to disclose the artist's "mind". Put Man Ray's photo side-by-side with a gouache dated 1910 (Figure 31): the similarity between the light spot on his bald, though the inversed general lighting of the whole scene, the contrast between the "floating" head against the unfinished and empty rear plane – what in photography is suggestively named "infinity wall", a backdrop that cancels the deepening effect of things behind the portraited – emulates a structure present in a vast amount of self-portraits: a monochromatic (sometimes neutral) background around the face, isolating it. This is a repeated motif also employed in other works, e.g. Portrait of Gustav Mahler (Figure 20), Red Gaze (Figure 21) and Christ (Figure 32), with specific variations if the forms partially dissolves (Christ) or stresses the opposition, demonstrating (above all when compared with his more conventional paintings) that such decision to restraint the structure, the design of the composition, to the depiction solely of the face was not at all an amateurish inaccuracy.

Figure 32: Arnold Schoenberg. Christ, 1910. Oil on board, 50 x 37 cm. Used by permission of Belmont Music Publishers, Los Angeles.

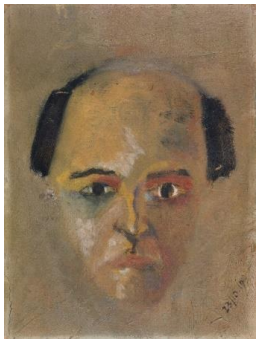

Figure 33 - Arnold Schoenberg. Green self-portrait, 1910. Oil on board, 33,2 x 24,7 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Anyways, Schoenberg did not reduce this striking duality to the absence of more subtle relationships inside them. There are some ordinary tricks well-known by painters to punctuate larger areas (the general scheme of the picture) with details which retain our attention and animates the image, enriching it: so, for example, the "notations" of colors that surround the main motif of the gaze in Green Self-Portrait (Figure 33), direct us to the focal theme of the eyes, from which resonates an opposite (and synthetic) balancing element, the gradually repeated, contrasted and rhythmically organized alternance between black and white, the extreme values in light and color scales (pupils, black predominating in the upper half of the head – hair stretching the face; white bellow; adjacent black and white in the right eyebrow; A darker spot in the left part of the bald diagonally opposed to a white spot, the last one echoing along the inferior part of the face). A trained painter also knows that the transparency of colors permits that the light and the color of an inferior layer of paint changes the luminosity and the "warmth" of the last layer. So, if you depict initially some areas with red, yellow and green, for instance, and then covers all of them with a same color (let's suppose with a lighter green), you will notice that this same light green will transit from a hotter to a colder temperature, according with the "subterranean" color with which it is contrasting). That is a very basic principle to work with contrast and harmony to avoid monotony or pedestrian simplicity in an image. Once the painter intentionally (as Schoenberg did) did not filled the main plane completely (as in *Blaues selbstportrait* / Self-portrait in Blue, 1910, Figure 34), revealing its process and subverting the traditional illusionism dependent on the wholly modelled surface covered in order to activate the effect of a window, the pink layer behind the face, "transpires", goes through the blank – i.e, incomplete – parts of Schoenberg's face up to the front plane.

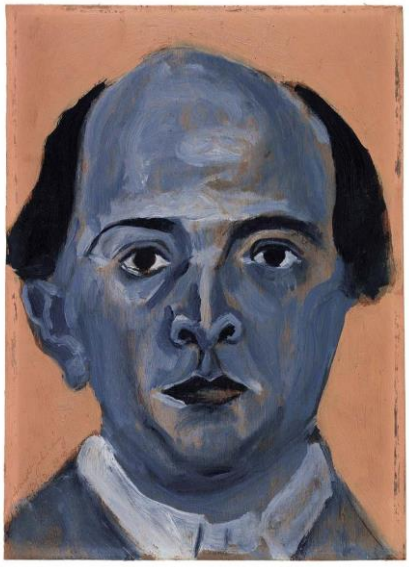

Figure 34: Arnold Schoenberg. Blue Self-portrait, 1910. Oil on three-ply panel, 31,1 x 22,9 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Considered from this point of view, contrary to Adam's suggestion, there is both complexity and contrast in his paintings. Contrast is obtained in two ways: by the (indeed) sharp contrast in design between the simplified plane against the face depicted (the "main" theme), which inside it registers some minor contrasts, like e.g., the many layers of planes which crisscrossed themselves. A rear plane can go forward to the front plane due to the interstices between brushstrokes and conversely, some other in the front can "retreat", resulting in a complex inner structure of the image as those details move to and fro instead of standing "static". In short, those categories are not restricted to color treatment (and Adams is right in which concern the controlled balance of colors if not in all, at least in the majority of his work) although it played an important role in its non-mimetic use, deepening its free imaginative use. In Blaues Selbstportrait, the blue-skin face has a double sense: interpreting it according with Expressionism's terms, it differs radically for an apparently analogous treatment seen in the green face Toulouse-Lautrec depicted at At the Moulin Rouge (Figure 35).

Figure 35: Henri de Toulouse-Lautrec. At the Moulin Rouge, 1892-1895. Oil on canvas, 123 x 140 cm. The Art Institute of Chicago.

In Toulouse-Lautrec's work, the woman's face probably reflects the light from an unseen lamp, a kind of effect enthusiastically explored by impressionists to discard the distinction between light and color (drawing and color) imposed since Renaissance. It was, in a way, a realistic effect, something you actually saw. Schoenberg's color in this self-portrait relates to personal mood or, better, the choice of the color intends to cast in the face something that could not be grasped through the observation of the "objective" world, it depends of the inner perspective claimed by Däubler. But this contrast of colors did not avoid the support of a middle point, a scale of balance: the highest contrasting color of blue is orange (and not pink), according to the theory of complementary colors, as orange is obtained with the fusion of the two other remaining primary colors (red and yellow); for the same reason, the opposite of the neutral pink – likewise that

used by Schoenberg here – would be a neutral green. Pink (a red lowered with white) and the blue (here also mixed with white) have not only a greyish aspect which controls the intensity of their hue, but produces at once a contrasting opposition between a warm and cold colors and a harmony, as the white works as the regulating term that unites them, working as the scaffolding of the color composition, that, finally organizes the transitions from the lighter colors (white, pink) to the deepest and darkest ones (blue, black). So, with a restricted palette (red, blue, white, black), he explored multiple variations in a simple scale. In some areas (lips, nose, pupils, eyebrows), blue is lowered, prevailing the dominance of white, punctuating contrasts between light and dark or between textures – a more solid or liquified (and translucent) paint. At the moment pictorial planes loosened an obligatory sequential ordering, we enter into modernism's concept of autonomy: lost its former rigidity of division, space becomes "undefined", but actual – likewise he did in other works, where some parts of the canvas were left unfinished, creating a fragmentary appearance of the whole and producing an abrupt "break". The visual discontinuity reminds us to be in front of the image's materiality, not eluded by an illusionistic double of the nature. These alternations of planes stress the fact that in the pictorial plane there must be solved simultaneously conceptual problems of its nature (formal language) and a condition of projection, where once again those movements to and fro mirrors the metaphor of the artist facing at once us and his/her phantasms during this moment of self-examination.

But in which conditions both projection and phantasm are faced? It leads us to a second question on self-portraits which deals with fragmentation, depiction of self-consciousness and modernist imagery of subjectivity. I listed here, in a very nonchalant way, a sequence of challenging concepts and my aim now is just to essay their correlation.

Figure 36: Arnold Schoenberg. Alliance, 1910. Oil on board, 39,3 x 63,7 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 37: Auguste Rodin. The Cathedral, 1908. Stone, 64 x 29,5 x 31,8. Musée Rodin, Paris.

About fragmentation, I suppose it is self-evident that the focus on the face (and likewise in one of his Fantasies, titled Alliance (1910, Figure 36), the touching hands evolves a theme once explored by Rodin (Figure 37), a representation of the deconstructed condition of the body) diverges from many usual self-portraits with their want to rhetorically enlarge the scenery of self-representation, including complementary elements which reinforces the spectacle of the self. But modernism's visual culture, after the advent of photography, made clear that representation means fragmentation: in other words, photo, working with instant, just shot. It delivered images that captures ephemeral angles, cropped images and so on. What about Schoenberg's self-portraits? Schoenberg is very direct at this point: just face – as a metonym of the self. Just depicting a face, eyes, two hands, subsume the whole body and we cannot help but to think on Freud dealing with the dismantling / rearranging of body and self in modern culture during these same years, as modern times did not only fragment its operative capacity (the assembly lines of industry) but also the stability of his subjective awareness. Anyway, even fragmented, there is still the gesture and the pose as a reactive symptom: one chooses how to be portraited, a personal statement of personality.

Figure 38: Max Beckmann. Self-portrait in Tuxedo, 1927. Oil on canvas, 139,5 x 95,5 cm. Busch-Reisinger Museum, Harvard University Art Museum, Cambridge, Massachusetts.

Figure 39: Arnold Schoenberg. Walking self-portrait, 1911. Oil on board, 49 x 44,9 cm. Used by permission of Belmont Music Publishers, Los Angeles.

From the early 1900's to his last self-portraits, Schoenberg worked with different poses that ranges from the austere self-portrait of 1910 (Figure 31) (austere too in its reduced tonal scale similar – and at once antithetical – to that used a decade later by Max Beckmann in his Self-portrait in Tuxedo (Figure 38, 1927), as if a moral code of artistic probity – just the essential – was announced) to the ironical anti-self-portrait – a scene where the portraited is absent, absorbed in his thinking while strolling in the streets, turning his back to the viewer, denying to us his face (*Gehendes selbstportrait*, 1911, Figure 39) – and the variations that goes from defiant eyes toward us (Self-portrait, 1918, Figure 40) to later works in which the hachured lines emulates wrinkles as the testimony of a personal history (Self-portrait, 1920, Figure 41; compare it with the deep trace we observed in Man Ray's photography), the ageing of wrinkles as indexes of a lived experience, to the crude and objective portrait of 1925 – Figure 42 – , an unpassionate look to himself akin to the frankness adopted by Neue Sachlichkeit (New Objectivity) artists active in Weimar Republic in those years. The later self-portraits seem oscillate between post-cubist (Figure 43), surrealist (Figure 44) and realistic overtones (respectively 1944, 1935 and 1936), the melancholic gaze of the 1936's version (Figure 45) assuming a somewhat retrospective and auto-biographical allure, if contrasted with the confident expression of the earlier years.

Figure 40: Arnold Schoenberg's Self-portraits: 1918. Used by permission of Belmont Music Publishers, Los Angeles.

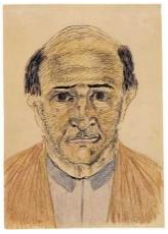

Figure 41: Arnold Schoenberg's Self-portraits: 1920. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 42: Arnold Schoenberg's Self-portraits: 1925. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 43: Arnold Schoenberg's Self-portraits: 1944. Used by permission of Belmont Music Publishers, Los Angeles.

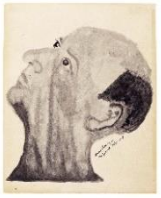

Figure 44: Arnold Schoenberg's Self-portraits: 1935. Used by permission of Belmont Music Publishers, Los Angeles.

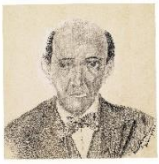

Figure 45: Arnold Schoenberg's Self-portraits: 1936. Used by permission of Belmont Music Publishers, Los Angeles.

Gazes, Tears, Eyes

"I've painted Gazes […] It is something that only I could have done, as it comes out of my Nature and is completely contrary to the nature of a consummate painter. I never saw Faces but only what I saw in people's eye, only their gaze. Then, it comes also that I can imitate someone's gaze. Notwithstanding, a painter seizes with one gaze a person in its entirety; I, just his/her soul" (BUDDE, 2004). Gazes, Tears among other works (Hatred, Christ, Flesh, Alliance, Thinking) consist of the part of Schoenberg's visual works he praised as his best, his authentic paintings.

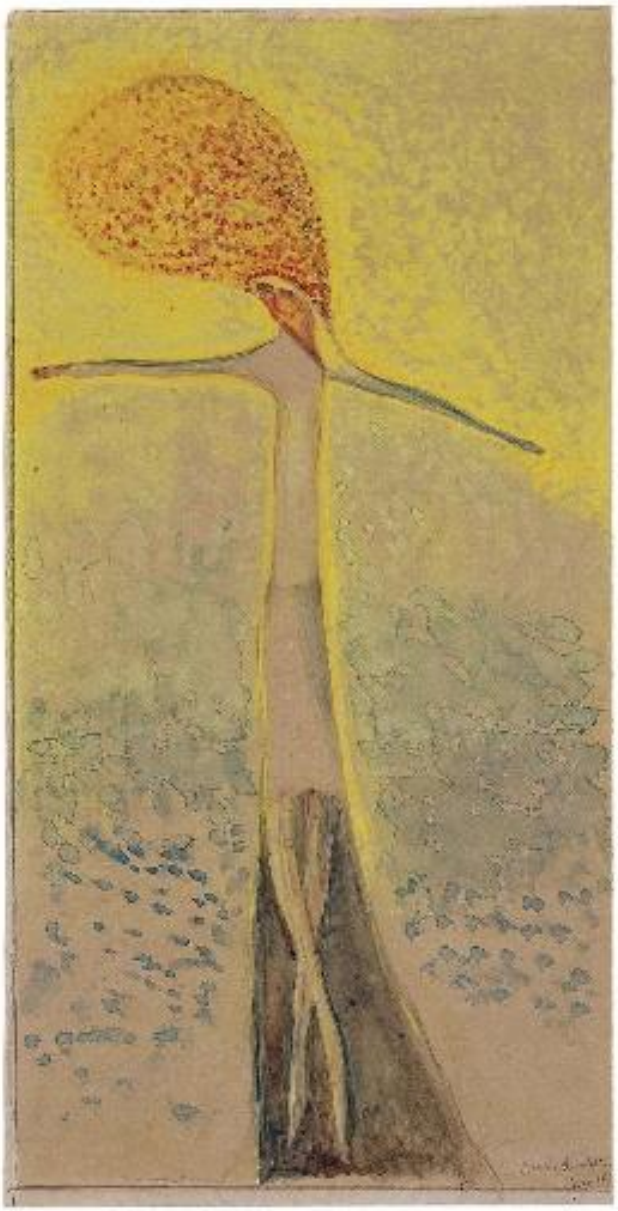

Figure 46: Arnold Schoenberg. Vision of Christ, 1919. Watercolor on paper, 20,7 x 10,6 cm. Used by permission of Belmont Music Publishers, Los Angeles.

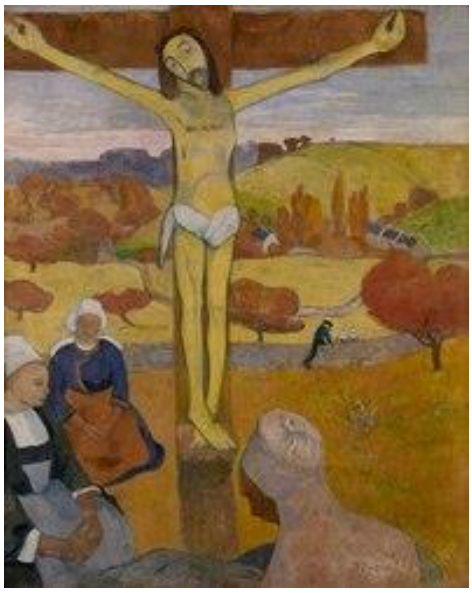

Figure 47: Paul Gauguin. The Yellow Christ, 1889. Oil on canvas, 92,1 x 73 cm. Albright-Knox Art Gallery, Buffalo.

Grouped under the titles Fantasies and Impressions, differently from the portraits, which he did continue to create along his life, those series dates from a restrict period, corresponding largely to the late 1900's to mid-1910's. Viewed as a whole, there are some eclectic elements in them (do not mistake it with eclecticism): in Christ (Figure 46), for example, one can retrace the dotted paintbrushes from post-impressionist painters (Henri Matisse. Luxe, Calme et Volupté, 1904. Oil on canvas, 98 x 118,5 cm. Musée d'Orsay, Paris) and the symbolist imagery of the early Gauguin (Yellow Christ, Figure 47); Thinking (Figure 48), otherwise, although not an abstract painting per se, have also a symbolic tone (Adams suggests that the arc represents Schoenberg's head) (ADAMS, 1995, p. 18).

Figure 48: Arnold Schoenberg. Thinking, 1910. Oil on board, 22,3 x 25,1 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Whether his head, a generic landscape or just a symbolic form which embodies an abstract concept, one recognizes in the whole series the same tendence identified in the self-portraits (there is a close resemblance between Brown self-portrait (1910, Figure 49) and Gaze (1910, Figure 21) to opt for the depiction of fragments of the body and sometimes dissolved forms, as if through this they crystalize a temporality.

Figure 49: Arnold Schoenberg. Brown Self-Portrait, 1910. Oil on canvas, 32 x 20 cm. Library of the Congress, Washington D.C.

In Flesh (Figure 19) two raising arms and a lying body dominates the inferior diagonal of the canvas: the breast and the legs almost metamorphose in two mountains against a stormy sky. The dominant diagonal of this body/mountain silhouette is balanced by the color contrast from the ultramarine blue at the upper left corner to the dark red carmine at the right inferior corner. In the juncture between body and sky, and area of light green and orange brought a yellowish stripe which organizes the transition from hot to cold colors. To avoid the risk that both opposite parts remain few connected, Schoenberg created a rhythmical structure with the arms echoing the main diagonal of the body and punctuating the blue area with dots of red in the nails. The juxtaposition of the nail and the green hands (using complimentary colors) distends the light in the whitish brushstrokes in the forearm, creating a secondary area of light.

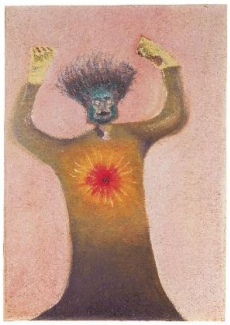

Figure 50: Arnold Schoenberg. Hatred, 1910. Oil on three-ply panel, 43 x 30,5. Reproduced by kind permission of the Arnold Schoenberg Center, Vienna.

Allegorical licenses can also be assumed in those works. Allegory is a special kind of symbolic representation of an idea, as it personifies it. So, concepts like “Flesh”, “Thinking” and “Hatred” (concerning the last one I can’t help but to see Mahler's face as model for his friend, Figure 50) find in those bodies its incarnation. Even “Tears” assume the same speculative aspect. There is an adherence – obvious in Tears – between the idea and the part of the body which it is related: the luxury of Flesh is projected onto the torso and the legs and the inviting hand; the feeling of hatred contraposing heart and head as focal points and pressing fists.

Figure 51: Arnold Schoenberg. Red Gaze, 1910. Oil on cardboard, 28 x 22 cm. Used by permission of Belmont Music Publishers, Los Angeles.

In Red Gaze (Figure 51) and Blue Gaze (Figure 18) something different happens, nonetheless. In both, the human figure works as an index to direct our gaze towards the vast areas of a misty colored plane. The slightly indication of a head in Red Gaze, with its very dissolved contour line; the incomplete silhouette at the corner of Blue Gaze (with beams of brushstrokes coming out of the mouth) stand as points from where the fantastical irradiates, and I believe they worked as a kind of hint to suggest us to grasp what the inner vision of the artist would have seen, imagined or envisioned – “things [that] we create [which] must depart out of the self. Not from our vantage point, displayed in perspective, but outwardly crystalized by themselves”, to remind Däubler's sentence.

Figure 52: Arnold Schoenberg. Expectation, op. 17, scene 2, circa 1911. Watercolor and white bodycolor, pen and Chinese ink on paper, 10,2 x 17,2 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 53: Arnold Schoenberg. The Lucky Hand, op. 18, scene 2, 1910. Oil on cardboard, 22 x 30 cm. Used by permission of Belmont Music Publishers, Los Angeles.

Figure 54: Arnold Schoenberg. The Lucky Hand, op. 18, scene 1, 1910. Oil on cardboard, 21,8 x 30,2 cm. Used by permission of Belmont Music Publishers, Los Angeles.

To substantiate such interpretation, the stage works designed by Schoenberg offer a valuable clue. Although arguable, I will recur to the ordinary perception of the stage as an illusionistic space, where the "double" of reality is staged. Such vision serves us to guess how he figure out a scenery for his inner world and fantasies and it does not seem a coincidence that he composed many of this set with a disposition of elements similar to this noticed in Blue Gaze (Figure 18) and its displacement of the figure to a small corner (compare with the sketches for *Erwartung* [Expectation] (Figure 52), and *Die glückliche Hand* [The Lucky Hand] (Figures 53 and 54). The asymmetry caused by the large empty space (occupied by our double – the singer) produces a scenic mirroring between stage (inner vision) and audience, resulting in the same visual effect supposed by me concerning Red Gaze and Blue Gaze. This "timing" of the image, as the "atmosphere" plausibly insinuated by visual elements was tested by Schoenberg in the special effects of lighting he planned for *Die glückliche Hand* (Figure 55),

marking a "color-crescendo" that precisely accompanies the bars (VERGO, 2010, p. 146).

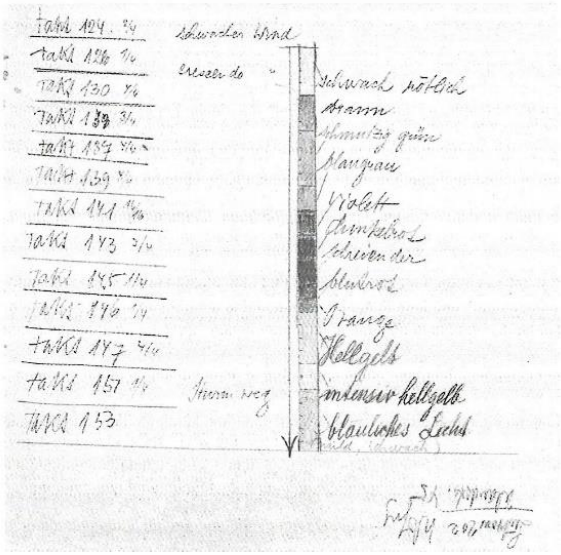

Figure 55: Arnold Schoenberg. Sketch for "color crescendo" in *Die glückliche Hand*, colored pencils on paper, 23 x 22,5 cm. Reproduced by kind permission of the Arnold Schoenberg Center, Vienna.

On the other hand, a direct association between the emptiness of space and solitude of characters arise. Empty space, due to the absence of elements to animate it, visually evokes contradictory forces: pause, silence, uncanny, unrest. "Was it a vision, or a waking dream? / Fled is that music – Do I wake or sleep?". With these nocturne verses by English romantic poet John Keats, I conclude my lecture, thanking the public for its attention.

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8

Schoenberg's "Pedagogical Project" – A Dialogue with Robert Maynard Hutchins and Mortimer Adler

Ernesto Hartmann

The last three decades have seen an increasing interest in Arnold Schoenberg's theories and writings. In this paper I will compare Schoenberg's ideas of musical education with Robert Maynard Hutchins and Mortimer Alder's thoughts about regular education. Hutchins and Adler are exponents of the so called Perennalist approach to education, also known as traditional school. Having as departure the main premises of this approach and Schoenberg's texts – mostly present in *Style and Idea* – I will promote a dialogue between the authors highlighting their many similarities. As a conclusion, I'll point out that what I here call the Schoenbergian Pedagogical Project, which the composer elaborated for UCLA in the end of the 1930's, has much in common with the later influential Perennalist perspective, although there are, indeed, some discordances.

Much has been discussed about Arnold Schoenberg's compositional techniques and style and his role as a composition instructor. The essential literature on this composer deals quite well with both dimensions of the artist's activity. From Leibowitz's *Schoenberg* to Walter Frisch's *Schoenberg and his World*, and with relevant articles about his life such as Dorothy Crawford's *Arnold Schoenberg in Los Angeles*, just to mention a very few, we are indeed well provided with information about these topics. In addition to that, there are also many other sources such as testimony from his students in America, such as Oscar Levant, Erwin Stein and John Cage and, of course, his own writings about composition and teaching composition.

Although this sources are very rich and its content have been systematically discussed by scholars all over the world, not much has been said about any kind of relation or possible dialogue on Schoenberg's ideas for musical education – mainly the ones he had in mind for UCLA – and the Perennalists, a group headed by Robert Maynard Hutchins (1899-1977) and Mortimer Adler (1902-2001) that had great influence in America's educational policies in the first half of the 20th Century.

Even though the Perennalists more significant works were published while Schoenberg was in the end of his life, and even after his death, there are too many common points between both perspectives and approaches, as I'll discuss further on. Since either Schoenberg and Hutchins played relevant roles in their Universities (UCLA and Chicago University) and Schoenberg and Adler – the later invited by Hutchins to the Department of Philosophy of Law at Chicago University – were both prominent jews, nonetheless there's no known register of their personal meeting, it's quite reasonable to assume that both men were aware of the other's work.

In this paper I'll present the context of what I call Schoenberg's pedagogical project, a brief exposition of the Perennalists ideas, and a review of Schoenberg's concepts on what should be the role of the student, and the teacher, the Schoenbergian Project and last, compare the main similarities and differences between the two perspectives. In this paper I'll use the following texts from the composer:

- *Style and Idea* - PART VIII – TEACHING and its eight texts about this subject:
    - Problems in Teaching Arts – 1911
    - Music – From Guidelines for a Ministry of Art – 1919
    - On the Question of Modern Composition teaching – 1929
    - Teaching and Modern Trends in Music – 1938
    - Ear Training through Composing – 1939
    - Against the Specialist – 1940
    - The Blessing of the Dressing – 1948
    - The Task of the Teacher – 1950

Also, I will consider:

- Models for Beginners in Composition – 1943
- Fundamentals of Musical Composition – 1967 (organized and edited by Gerald Strang and Leonard Stein).

Traditional School or the Perennialists

Essentially, the basic principles of Perennialism can be described in six categories (KNELLER, 1971):

1) Despite different environments, human nature remains the same everywhere; therefore, education must be the same for all. According to Hutchins it may vary from society to society... But the function of a man is the same in all ages and in all societies, since it follows from his nature as a man. The purpose of an educational system is the same in all ages and in all societies where such a system may exist – to improve man as man.
2) Since rationality is man's highest attribute, he must use it to direct his instinctual nature in accordance with deliberately chosen ends. Men are free, but they must learn to cultivate reason and to control their appetites, when a child cannot learn, teachers must not be quick to assign responsibility to an unhappy environment or an unfortunate sequence of psychological events. On the contrary, the teacher's task is to overcome these disadvantages through an essentially intellectual approach to learning that is the same for all their students. Teachers should not become condescending either, on the basis that this is the only way a child can relieve their tensions or express their true self. Nor should the child be allowed to determine his own educational experience, for what he wants may not be what he should have.
3) It is the task of education to introduce the knowledge of eternal truth. As Hutchins announces, 'Education implies teaching. Teaching implies knowledge. Knowledge is truth. The truth is the same everywhere. Therefore, education must be the same everywhere.
4) Education is not an imitation of life, but preparation for life. School can never be a 'real life situation', nor should it be, for it remains for the child an artificial arrangement in which he becomes acquainted with the highest achievements of his cultural heritage. Your task is to understand the values of that heritage and, where possible, add to the accomplishments through your own efforts.
5) The student must be taught certain basic subjects that familiarize him with the permanencies of the world. The child should not be forced into studies that seem important at a given moment. Nor should he be allowed to learn what appeals to him at a certain age. He must study his language - grammar and literature -, some foreign languages, History, Mathematics, Natural Sciences, Philosophy and Fine Arts. As

Mortimer Adler points out, 'the basic education of a rational animal ... is the discipline of his rational powers and the cultivation of his intellect. This discipline is obtained through the liberal arts, the arts of reading and listening, of writing and speaking, and, necessarily, of thinking, since man is both a social and a rational animal and his intellectual life is lived in a community that can only exist through communication between men. The three R's, which have always stood for formal disciplines, constitute the essence of liberal or general education (Reading, Writing, and Arithmetic).

6) Young people should study the great works of literature, philosophy, history, science, through which men have revealed, over time, their greatest aspirations, and achievements. The message of the past is never out of date. By examining it, the student learns truths more important than anything he can discover by exploring his personal interests or immersing himself in the contemporary scene. Mortimer Adler admirably sums up this point of view 'if there is philosophical wisdom as well as scientific knowledge; if the former consists of profound insights and ideas that change little over time, and is now as current as the past, the latter has many enduring concepts and a relatively constant method; If the great works of literature and philosophy address the permanent moral problems of humanity and express the universal convictions of men, involved in conflicts of a moral order – if all this is so, then the great books of antiquity and medieval times, as much as the moderns are a repository of knowledge and wisdom, a tradition of culture into which each new generation must be initiated. The reading of these books is not for the purpose of cold and heavy scholarship; the interest is neither archaeological nor philosophical... on the contrary, these books should be read because they are as current today as they were at the time they were written, and because the problems and the ideas presented in them are not subject to the law of perpetual and unending progress.

In the context of American education, the Perennalists (HUTCHINS 1936, 1953; ALDER, 1939) believed that the minds of the students were never submitted to appropriated stimulae nor exercised in intellectual matters. According to their view, a self-realized life required a great deal of self-discipline, which would only be possible through external means. This was one of the main tasks of the teacher and the school. Even though much of their work concerned undergraduate studies, it is evident that they may also be applied to higher studies. This ideal of self-realization is consonant with the concept of a man sovereign in some intellectual realm, a goal to be achieved to transcend an intellectual mediocrity. The mediocre, in their view, did nor could not enjoy a life of true freedom.

This conception was highly criticized, but doubtless it played an extensive role in the following educational policies in the USA, mainly in the mid-20th Century. Schoenberg's ideas may in some respects not be fully related to the later Perennalists, as for example his belief that not everyone was apt to perform the same functions in society. Also, Perennalists were accused of fostering an elitism founded in the intellect and restricting the whole curriculum to the classical Greek tradition, which, of course, it is not true.

As stated by Kneller,

> To subject students to the same kind of rigorous academic training that is administered to college-caliber students is to ignore this difference and to harm, perhaps, their personal growth...intellect is just one aspect of a man's personality. And although rational behavior is indispensable to human progress, the affective and unmistakably personal side cannot easily be subordinated to it' (KNELLER, 1971, p.59).

Nevertheless, as we shall discuss, there are many tangencies in the Schoenbergian pedagogical project, as we may call it, with the fundamental premises later stated by Hutchins and Adler, offering an interesting opportunity to compare both and see that they much more resemble than differ.

The teacher

In addition to his activities as a private teacher for a plethora of great composers and instrumentalists who decisively impacted the musical production of the 20th century, Schoenberg, upon emigrating to the United States in 1933 after a brief stay in France – having obtained American citizenship in 1941 – developed teaching activities at the Malkin Conservatory in Boston/M.A. and, in 1935, moved to the west coast of the United States, assuming the position of professor at the University of Southern California and visiting professor at the University of California Los Angeles – UCLA.

Perhaps, his initial experience in the new world was not quite as he expected as quoted by Eichler,

> But the excitement quickly faded. Soon after he arrived, Schoenberg learned that the school had no orchestra and only half a dozen classrooms. The students were few and underskilled, the tuition fees prohibitive, the soundproofing unacceptable. Within weeks, Schoenberg was teaching from his own apartment, where he lived with his second wife, Gertrud, and his young daughter Nuria (EICHLER, 2009, p.2).

Despite this odd challenge Schoenberg maintained a high opinion of his competence as a teacher. It was precisely this perception that made it possible for him to accept teaching positions at USC and UCLA after his short stay in Boston. As he himself stated, teaching composition was then his main source of income.

> I believe art is born of 'I must', not of 'I can'. A craftsman 'can': whatever he was born with, he has developed. And so long as he wants to do something, he is able to. What he wants to do he can do – good and bad, shallow or profound, new-fangled and old-fashioned – he can! … perhaps he was born with something; then he acquires it – manual dexterity, command of form, virtuosity. Not other people's, though, his own … so the genius learns only from himself, the man of talent mainly from others the genius learns from nature – his own nature – the man of talent from art. And this is the weightiest problem in teaching art. (SCHOENBERG, 2010, p.365).

> In my fifty years of teaching [1950] I have taught certainly more than a thousand pupils. Though I had to do it in order to make a living, I must confess that I was a passionate teacher, and the satisfaction of giving to beginners as much as possible of my knowledge was probably a greater reward than the actual fee I received. This was also the reason why I accepted so many pupils who could not pay, even if they had not the background to study with me, thus I said in such a case once: 'after all, if they cannot digest what I gave them, it will damage them less to study with me than with a poorer teacher' (SCHOENBERG, 2010, p.388).

In both universities he faced a rather precarious structure due to the recent Great Depression, besides, UCLA was in a transitional phase, from being a public-school teacher training school to becoming a major university. In this project, Schoenberg played an important role, as did Bertrand Russell, both were personae of relevance and international projection, as well as controversial, taking the institution to a new and higher level.

Nevertheless, the summer classes at USC were quite troubled since Schoenberg had to deal with an overcrowded class of about forty public school and college teachers, mostly middled aged that didn't show much interest beyond acquiring some credits to improve their incomes. Surely that wasn't what the composer expected neither the best environment to put forth his plans.

His original project was very similar to that he tried to deploy at UCLA, and he planned to begin it in 1937. It consisted in creating a Department of Music Theory and Composition to which he should be appointed as head. Even though that plan failed, it became clear its premises and adherence to some pedagogical principles guided by an almost religious devotion to art, in addition to a respect for the laws of morality, these, of course, more rigorous than those of everyday life. Indeed, for Schoenberg, artists should be models of behavior and morality for society in general and this connection were quite explicit in the objectives of this ideal school as to offer a panorama on other fields of knowledge, mainly those related to science and to develop the character of its students (CRAWFORD, 2002, p.23).

Quickly after understanding the situation of the two institutions to which he was linked, Schoenberg admitted many private students, focusing on

songwriters in the recording industry, which, financially, was one of the few profitable activities in the recovering US economy. Some of the main figures in the musical scenario had contact with Schoenberg, leaving us interesting testimonies, such as that of Oscar Levant,

> To my mind, Schoenberg is the greatest teacher in the world. The very contact with such a person either brings out something that is in you or lets you see that there is nothing to be brought out. Either way, it is helpful to know where you stand. Schoenberg not only permits each of his pupils to be completely himself, he insists on it. Father of the atonal system, he is passionate in his reverence for the classics and classic form. From him I learned that modernism is not merely a matter of hitting the keys with your elbow and seeing what happens; it is logical, and formed with an utterly logical if unconventional development. No one has to like modem music, but every serious musician owes it to himself to keep his ears open and listen to what is going on. (LEVANT in CRAWFORD, 2002, p.16).

It is quite interesting to compare Levant's testimony with the reflection present in Schoenberg's 1939 text *Ear Training through Composition*, since Levant's opinion echoes an important aspect of the Schoenbergian working method: a solid foundation and knowledge of the masterworks of the past. In this text, Schoenberg states that,

> Often, a young man who wants to study with me expects to be taught in musical modernism. But he experiences a disappointment … 'if you want to build an aeroplan [*sic*], would you venture to invent and construct by yourself every detail of which it is composed, or would you not better at first try to acknowledge what all the men did who designed aeroplanes [*sic*] before you? Don't you think that the same idea is correct in music? (SCHOENBERG, 2010, p.377).

However, another distinct aspect of Schoenberg's ideas may seem rather antagonistic to the testimonies. Schoenberg did believe in natural talent, one that may not be taught, and was native, as we displayed in the first citation, and again, well pointed out by Crawford,

> Many comments from Schoenberg's American students echo Oscar Levant's praise for Schoenberg as "the greatest teacher in the world," one who invested extraordinary care and energy in his work and whose methods were unique. But because he recognized that the ability to compose is inborn and cannot be taught, Schoenberg's attitude was paradoxical. He frequently made this comment about his own teaching: I always called it one of my greatest merits to have discouraged the greatest majority of my pupils from composing. There remain, from the many hundreds of pupils, only 6-8 who compose. I find such who need encouragement must be discouraged, because only such should

> compose to whom creation is a "must," a necessity, a passion, such as would not stop composing if they were discouraged a thousand times. (CRAWFORD, 2002, p.20).

Obviously, the idea of innateness would guide Schoenberg's pedagogical project. If a teacher cannot teach the student to have expressiveness and depth, to compose different themes, it remains for him to train his taste, give him a sense of form, balance, and present him with the criteria for evaluating the potential of an idea. Just as character must be developed in students, the teacher must serve as a model for them. In addition to moral issues, Schoenberg focuses on the ability of this teacher, clarifying that it is his duty, therefore, he must be able to not only steer the student verbally, but above all be able to fulfill all the demands that he makes to his students in loco, preferably improvising solutions to compositional problems that may arise. This logic is applied both in the *Models for Beginners in Composition* and in the posthumous *Counterpoint* and *Fundamentals of Musical Composition*, mostly in the multiplicity of examples offered, illustrating several solutions to a single problem in a very generous way.

Still, regarding the developing of the musician – as for that, it can be understood the one who will play the role of teacher – Schoenberg demands a much broader domain of the field of professional performance, something in a way, strange to the American logic, already strongly impregnated by Fordism, therefore, highly specialized. The following text quoted from 1940 'Against the Specialist' illustrates this situation well,

> I am opposed to the specialist. I admit that a great specific technique cannot be acquired without intense practice, even at the expense of being skillful in other or even in related, matters. But just as a doctor must know the whole body, so should a musician not only know harmony, or how to play the piano or the flute, or how to conduct, and should not be called only a harmony teacher, or a pianist, or a flutist, or a conductor. In order to deserve the name of musician, he should not only possess a specific knowledge in one field, but has to have an all-round knowledge of all the fields of his art (SCHOENBERG, 2010, p.387).

This concept of a fully developed musician, not only well trained in the traditional aspects of Music Theory and Performance, but also with considerable knowledge in the field of Arts, Humanity and Science clearly shows Schoenberg's pedagogical tribute to the past, even if this past is almost completely related to the German culture, for at least, as he exemplifies in his pedagogical works.

The student

> Nobody can give voice to an idea unless he could also think of it, and the true art of composition (like true science) will always remain a secret science. It already counted as such at the time of the

> Netherlanders, for all the doubting scorn of graceless historians. It has to be so, not just because the initiated are forbidden to make it known, but, particularly, because the others are unable to grasp it (SCHOENBERG, 2010, p.375).

As it can be understood as a profile of the graduate, that is, what Schoenberg expected to be the main goal of teaching composition in university, there is a paradox. If, on the one hand, Schoenberg claimed that the true nature of teaching composition could not be transmitted, on the other hand, he developed a large arsenal of didactic material specifically aimed at the acquisition of a technique. It is remarkable that the pedagogical proposal found in this material rarely alludes to real artistic issues. Probably, in *Models for Beginners in Composition*, where the author clearly indicates that the indications deal with exercises, because only later, the student 'try to create real melodies, instinctively and spontaneously' (SCHOENBERG, 1943, p.5), because 'the examples in this booklet are occasionally unbalanced and even un-melodious. They were not composed for beauty, but for his purpose and exclusively to illustrate the application of a technique '(SCHOENBERG, 1943, p.5).

> It is said of many an author he may have technique, but no invention. That is wrong; he has no technique either, or he has too. You don't have technique when you can neatly imitate something; technique has you. Other's people technique. If one is in a position to look closely, one must realize how fraudulent technique of that sort is. Nothing really works, everything is just neatly glossed over. Everything is approximate, none of the joints fit properly, nothing holds together; but from a distance it looks almost genuine. Technique never exists devoid of invention; what does exist is invention which has still to create technique. But the art's teacher's aim is to give his pupil this technique (SCHOENBERG, 2010, p.382).

Schoenberg reiterates that the composer's craft is not the simple and exclusive result of a systematic learning process, reinforcing the idea of innateness. However, it is the role of the art teacher to teach the technique on which the composition is based. Indeed, this is contradictory to his own affirmation of the indissolubility between technique and creation/invention. But even been contradictory, it becomes clear that he related this process with the scientific method, as he often mentions the similarities either directly or with analogies such as the construction of an airplane.

In this case the equivalent to the literature review could be understood as a thorough knowledge of the masterworks of the past. That correlates his project with the Perennalists ideas that were to gain force shortly after his death.

> It is obvious that not even a small percentage of music students will become composers. They cannot and they should not. It is also evident

> that many would be composers and musicians who, through some study, have acquired a superficial knowledge of music, may presume to judge the activities of good artists and real creators. This is where a correct attitude on the part of the teacher becomes important. He must convince his students that the study of composition will not make them experts or acknowledged judges, that its only purpose is to help them understand music better, to obtain that pleasure which is inherent in the art. (SCHOENBERG, 2010, p.382).

Therefore, the graduate of the Schoenbergian pedagogical project is not necessarily a composer, but, in a way, a dilettante, a remnant of the amateur culture typical of the 19th century.

A pedagogical project

> So even a new way of teaching composition, to be of use to many people, would simply teach again, as I have already said, how it is done, not what is! And so, I believe there are problems for the modern composition teacher, but none demanding a new way of teaching composition; rather the kind that demand an old one (SCHOENBERG, 2010, p.376).

> Schoenberg never drew on his own music. "One had to master the past, and the forms out of which the present came.... What was terrifying was his acute analysis of a student's weakness. He would correct, immediately, in whatever personal style the student was using ... Schoenberg believed that modernism, like composing, "cannot and ought not to be taught. But it might come in a natural way, by itself, to him who proceeds gradually by absorbing the cultural achievements of his predecessor (CRAWFORD, 2002, p.24).

For Schoenberg, the best thing to do would be to reincorporate music into the elementary school curriculum. Recognizing the logistical and financial difficulty of this proposal, he advises the following three actions:

a) The division of schools into three levels – elementary, secondary and higher, differing both in the level and objective to be achieved and in the time allotted for it;
b) Qualify talent as the sole criterion for admission to the highest level;
c) Allow access to the poorest by charging fees proportional to the family's assets, a system that should be extended to the entire education network, not just music education (SCHOENBERG, 2010, p.370 and 373).

It is important to emphasize the difficulties that Schoenberg encountered in arriving on the west coast of the USA. Its classes were huge with 25 students in music analysis; 25 in composition; and an impressive 60 in counterpoint (EISCHLER, 2009). Naturally this forced him to request several assistants, in addition to building a library.

From a structural point of view, Schoenberg's idea was very similar to the creation of a traditional conservatory, only the methodology employed was quite different. There would be divisions into Departments, an Orchestra, and courses in Orchestral and Choir conducting. The proposal also contemplated the creation of a school for copyists and a time reserved for listening to music with the accompaniment of the score, in addition to a strong incentive for students to actively participate in the city's concert schedule.

According to his premise on the innateness, there would be different curriculae for composers in order to separate the most talented from those who were only interested in a diploma or certificate.

> It seems to me that a great number of talented students to be found in this community could be stimulated to take a more serious attitude towards art if they were forced to," he wrote. His suggested plan for this process involved progression through six undergraduate courses in harmony, counterpoint, and analysis, to the seventh, "Composition for Composers" (with the consent of the instructor). Graduate classes for composers would be in different aspects of composition. Following the precepts of the great Viennese musicologist Guido Adler, Schoenberg wanted to require of graduate students a thorough knowledge of specific works (CRAWFORD, 2002, p.24).

This 'serious attitude' surely articulates with the repertoire, which naturally should be mainly, if not all, of 'serious music' just as he stated in his text Ear Training through Composition. Due to this levelling up of the quality of music, there would be a whole new generation able to understand and capable of judging music, drawing a clear division between high and low cultures.

> Of course the best way to train a musical ear is to expose it to as much serious music as possible. Musical culture would spread faster if people would read music, play music or even listen to music much more than they do today. Extensive familiarity with serious music is the foremost requirement of musical culture. But even this is not enough without a thorough ear-training … A trained ear is valuable, but not especially, so if the ear is the gateway to the auditory sense rather than the musical mind. Like harmony, counterpoint and other theoretical studies, ear-training is not an end in itself, but only a step towards musicianship (SCHOENBERG, 2010, p.379).

Regarding the evaluation activities, there would be oral exams judged by a panel formed by members of other Departments of Art, Philosophy and even Physics. Those interested in compositional theory could defend a thesis. The entire program could last from 5 to 8 years depending on each student's entry level and individual development.

Schoenberg's most radical proposal for UCLA foresaw a creation of an interdisciplinary forum for Art and Aesthetics, seeking to dialogue with various

areas of the humanities. This would be the solution to some problems that the composer foresaw as, 'a possible 'end of art', caused by the erosion of the social class structure and the consequent inability of artists to resist the temptation to satisfy the demands of the masses.

Leonard Stein realized that while many of the musical examples and exercises used by Schoenberg were similar to those he employed in Europe, his didactics and texts written in the USA, while maintaining their basic principles, were adapted to the needs of 'beginners'. Schoenberg took a certain pride in this ability to teach non-composers to the point where he once claimed that he could 'teach composition to chairs and tables' (STEIN, 1984). Nevertheless, as we clearly see in *Fundamentals of Musical Composition* these examples almost completely rely on the works of German composers.

One of the biggest paradoxes, once the Schoenbergian ideas and project for music teaching are understood in general terms, is the fact that, for many times, he has directed and built material for students who would certainly not reach a high level of development. When developing the program *Eartraining through Composition* in 1939, Schoenberg was addressing himself directly to beginners, in the same perspective that he understood that anyone could be trained to draw, paint, write, etc. of the mediocre, use your learning in order to enhance your sensitivity. In any case, he believed that every good musician should go through this process.

A comparison on Schoenberg's Pedagogical Project and the main concepts of the Perennalists

Some of Schoenberg's main concepts as a teacher can be summarized by the following statements.

a) Devotion completes art as the holder of moral principles and character formation of the individual, - specifically of the artist;
b) Systematic and organized learning, based on the clear division between high and low culture, privileging the study of the classics.
c) Use of exercises of inferior artistic quality, contradictorily to his speech, detaching the technique of creativity;
d) Innate premise – a gift already originating in the composer, cannot be taught;
e) An attitude of the teacher not of stimulation, but of guidance through the presentation of a large repertoire of solutions. Technically capable and skillful teacher, but never a specialist;
f) The idea of the artist as a chosen one, a leader of society anointed by metaphysical forces.

Of course, in this context, the association with the Perennalist movement is inevitable. As stated, this movement had a strong influence on the US university environment, particularly in the 1930s and 1940s, precisely the period

when Schoenberg emigrated to that country. Even though there are no specific records of contact of the composer and its two main theorists, Robert Maynard Hutchins (1899-1977) and Mortimer Adler (1902-2001), the ideas of the second, much younger than Schoenberg, can be taken as an example about the main principles of this theory as described by Kneller:

> Young people should study the great works of literature, philosophy. History, science through which men have revealed, over time, their greatest aspirations and achievements. The message of the past is never out of date. By examining it, the student learns truths more important than anything else he can discover by exploring his personal interests or immersing himself in the contemporary scene (KNELLER, 1971, p. 58).

When we parallel Schoenberg's ideas with those of Hutchins and Adler we see many points of accordance. I shall highlight some.

For Adler, the purpose of an educational system is the same in all ages and in all societies where such a system may exist – to improve man as man – as to Schoenberg there are no new problems in teaching composition, but just the old ones. What has changed must be understood from the past, as a natural outcome – as he spoke about modernism – of what has preceded it.

Rationality as a necessity to control the instincts. Even though it seems paradoxical or maybe incoherent, Schoenberg believed that composition could and ought not be taught as a scientific method. But, his pedagogical works, illustrates, nevertheless for the sake of comprehension, how to acquire a technique. This seems rather strange for the one that said that there is no technique if there is no invention. However, as he states that what should be taught is how to do and not what to do, this warring enigma may have some kind of awkward coherence in itself.

Again, for Adler, education is not an imitation of life, but preparation for life – and indeed, Schoenberg's pedagogical works attests that the lack of artistic purposes in some of its exercises are not at all trying to aim at a work of art, but rather they target the building up of a technique that should support the idea of how to do it.

Also, as stated by Adler, the student must be taught certain basic subjects that familiarize him with the permanencies of the world, so the three R's, which have always stood for formal disciplines, constitute the essence of liberal or general education – in fact Schoenberg always made clear that the equivalent to those three R's in music are Harmony, Counterpoint and Composition. His opinions about the specialist and its inadequacy as a music teacher are very elucidative since he believes in a solid foundation on a kind of art that can be separated into higher and low culture. His used of the adjective "serious" to music is a great proof of that. Also, this is related to another premise of the Perennalists, that young people should study the great works of literature,

philosophy, history, science, through which men have revealed, over time, their greatest aspirations and achievements.

And the last one I wish to highlight deals with the almost monastic ideal of art as a superior achievement of the intellect, which would imply in a moral oriented behavior from the artist as a reflection of this Olympian level. For Schoenberg as for Hutchins and Adler, only the contact with one's culture – here understood as the high culture could either free him or heighten humanity and, in consequence by the artists agencies, humanity.

Conclusions

Although Schoenberg's ideas are rooted in his own experiences as a teacher that lasted for 50 years, and only in the last decade of his life Perennalists began to publish and to influence US educational policies, there are many parallels, between both.

The Schoenberg Pedagogical Project for UCLA, despite having some innovations, still is a representant of some very solid values of the 19th century, such as great respect for the past and the ideal of art as morally superior – particularly if this art is music.

By comparing Schonberg's text and his project with the main premises of the Perennialists (Hutchins and Adler) essentially the only difference that can be highlighted is the belief of the composer's innateness. Otherwise, his project is an excellent dialogue with this approach that was gaining forces in the United States at the end of his life.

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9

Werker, Bach, Schoenberg, and Symmetry

Severine Neff

For Schoenberg, symmetries found in his Serenade, Op. 24, were analogous to those that the early twentieth-century German theorist Wilhelm Werker discovered in Bach's Well-Tempered Klavier. The third movement of the Serenade, entitled "Variations," uses a 14-tone set doubling three pitches, leaving out one, and omitting the intervals of the perfect fifth and fourth. This essay will demonstrate that the pitches and intervals absent from the set of the "Variations," determine its so-called composition "problem" and create symmetries akin to those in Bach's fugue.

Arnold Schoenberg rarely read music-theoretical texts in detail, and when he did, it was with little enthusiasm. A stunning exception is the formidably entitled *Studies of Symmetry in the Structure of Fugues and in the Motivic Coherence of the Preludes and Fugues in the Well—Tempered Klavier by Johann Sebastian Bach*, authored by the German composer, choral director, and Bach scholar Wilhelm Werker (WERKER,1922/1969; GURLITT, 1961, pp. 329–30, HINRICHSEN, 2001, pp. 50–53). Schoenberg received Werker's book as a 1924 Christmas gift from his student, Josef Polnauer, the future teacher of the eminent North American music theorist, David Lewin (SCHOENBERG, 1928, p. 1; HARTEN, 2005, online; GEWERTZ, 2003, online). Several months later, Schoenberg wrote to the composer–theorist Josef Hauer saying that "this book interests me very much. It treats a subject to which I in part feel very close . . . I have observed numerical symmetries in my own works" (SCHOENBERG, 1925, p. 1; see also GUSTAFSON, 1979, p. 21).

Ironically, Werker had little respect for contemporary composition and held that improvements could only be possible if composers implemented structural and quantitative analysis of Bach. In his words: "Moderns must pass via Bach toward a high, mathematical, well–founded, and imaginative art of the future" (WERKER, p. 1).

Schoenberg would seem to have taken Werker's advice before he had seen the book. In a 1924 lesson with the Catalan composer Roberto Gerhard and in a 1928 manuscript he wrote on Werker, Schoenberg revealed that the temporal symmetries and mirror designs which Werker found in Bach, reminded him of his recently premiered *Serenade*. In his words:

> I went through some of it with my pupil Robert Gerhard (Castrells)[1] back then in Mödling (!) and, even at that early date, I pointed out to him that I was not unaware of some of the things Werker writes about. Especially, I drew attention to the third movement (variations) of my *Serenade*. (SCHOENBERG, 1928).[2]

This essay will first summarize Werker's analysis of symmetries in Fugue 1, Book I, of the *Well–Tempered Klavier*, thus functioning as a case study of his interests and methods. Secondly, I will comment on analogous symmetrical designs in Schoenberg's "Variations" movement, focusing on the structural role of pitches and an interval omitted from the work's 14-tone set.

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[1] In Vienna (1923–25), Schoenberg's Catalan student, Roberto Gerhard, who was aware of the Viennese penchant for titles, assumed the pseudonym "Dr. Castells, Professor of the Spanish Language" when he gave lessons to support himself (GRADENWITZ, 1998, p. 56). Gerhard confirmed his study of Werker's book with Schoenberg in his writings (MEIRION, 2000, p. 48).

[2] Translation by Grant Chorley.

Werker on Bach

In his analysis of Fugue 1, Book I, Werker understood the strong arrival on a tonic A minor triad in m. 14 as symmetrically dividing the work into two large sections of 13.5 measures each (see Figure 1). He then demonstrated that the constant repetition of ascending and descending fourths—the main intervallic characteristic of the work's subject—appeared in 96 statements, 48 before the cadence to A minor, 48 after it, thus supporting his position that the fugue presented a two-part symmetrical form (see Example 1; compare WERKER, 1922/1969, pp. 10–13). He further understood the 96 fourths in the piece, a multiple of 24, symbolic of the number of keys in the preludes and fugues of the *Well-Tempered Klavier*.

Werker was intrigued by the fugue's subject. Traditionally subjects end on a pitch in the tonic triad arriving on a strong beat, usually at or slightly before the entrance of the answer (see Example 2). By this definition, the subject of the C–major fugue would conclude on the sixteenth–note E within the third beat of m. 2. In this fugue the traditional end of the subject is part of the ongoing line of sixteenth notes, concluding with the quarter-note G on beat 2 of m. 3. Werker focused on this entire line indicating that it contained 24 pitch attacks rhythmically parsed into units of 4, 6, and 14 beats, respectively (see Example 2; compare WERKER, 5). These subdivisions become part of his analytic reasoning as clarified in his diagram entitled "Structural Plan of the Fugue" presented in its original form and in transcription and English translation in Figure 2 (compare WERKER, p. 14). Here he proposes that the first half of the work should be divide into a "First Development" (mm. 1–7) and a "Second Development" (mm. 7–14); a "Third Development" constitutes the entire 2nd half of the piece (mm. 14–27). "First Development" has 4 entries of the subject and answer, the "Second Development," 6 entries, and "Third Development" 14 entries. Together they mirror the subdivisions of the "extended" subject (compare Example 2).

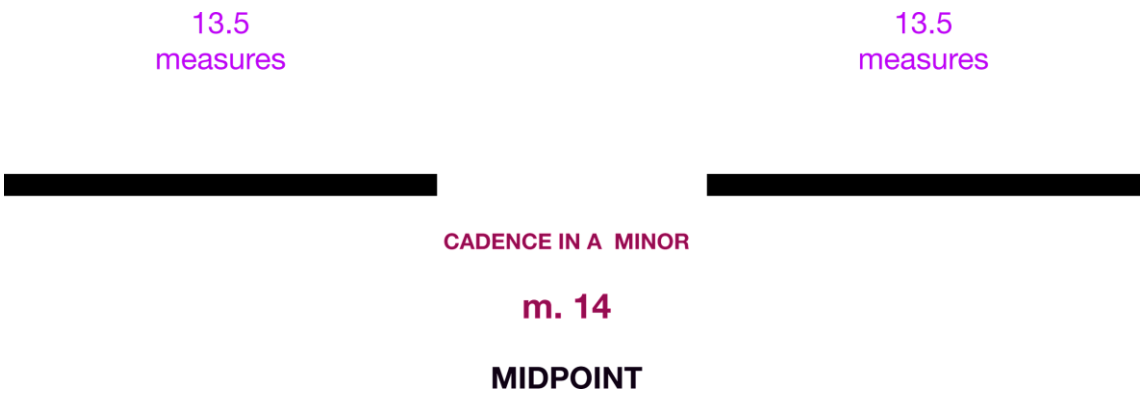
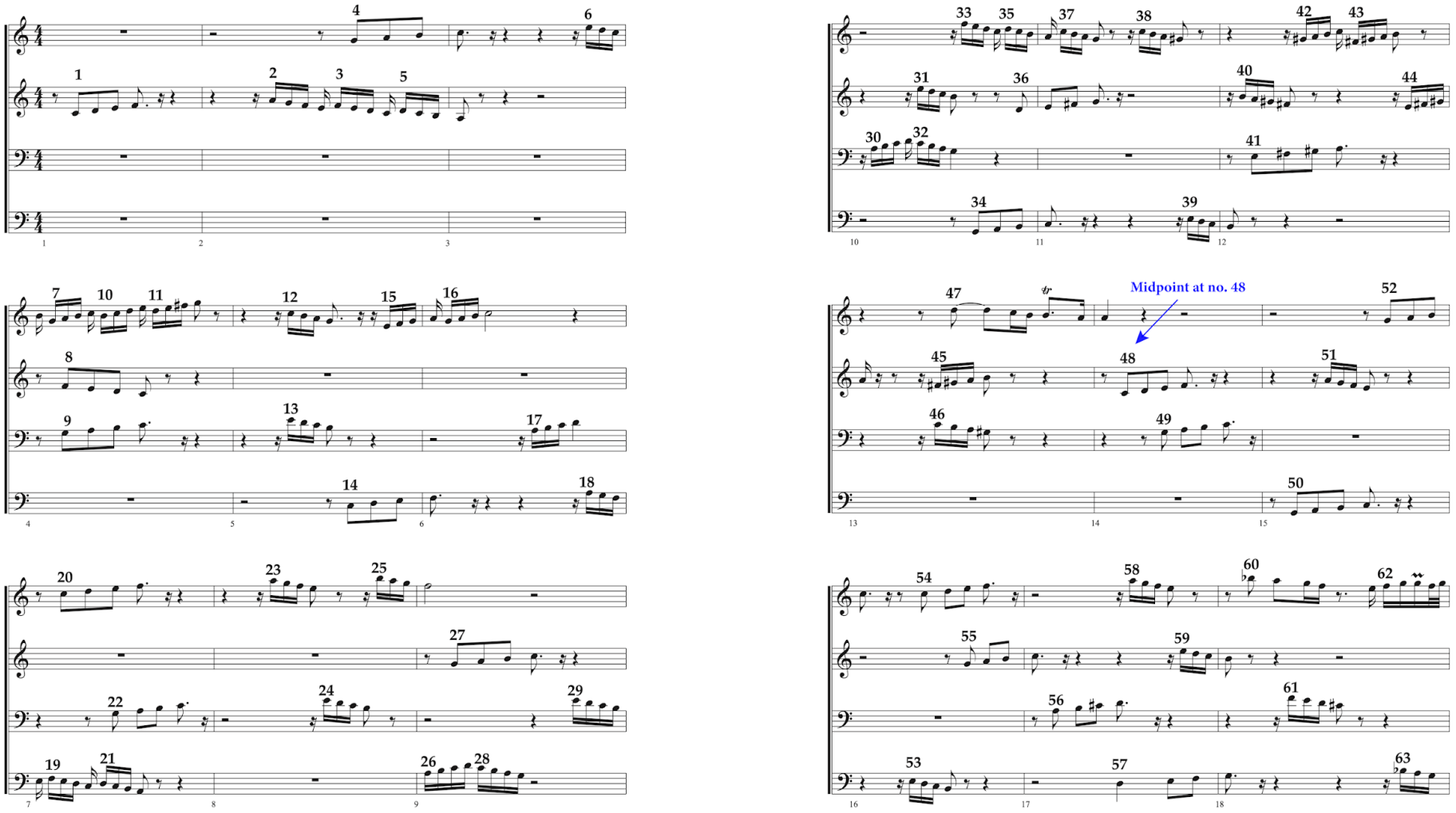
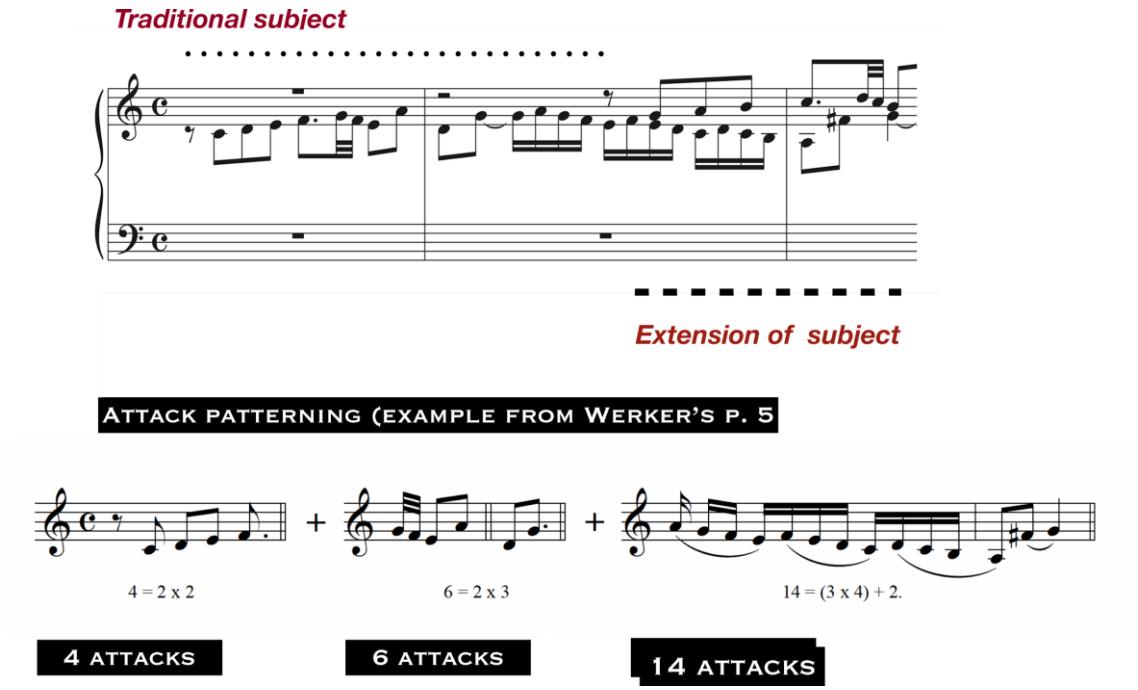


Figure 1: The bifurcated form of Fugue I, Book I, *Well-Tempered Klavier*.



Example 1: Illustration of ascending and descending fourths articulating the fugue's bifurcated form: mm. 1–14/14–27.



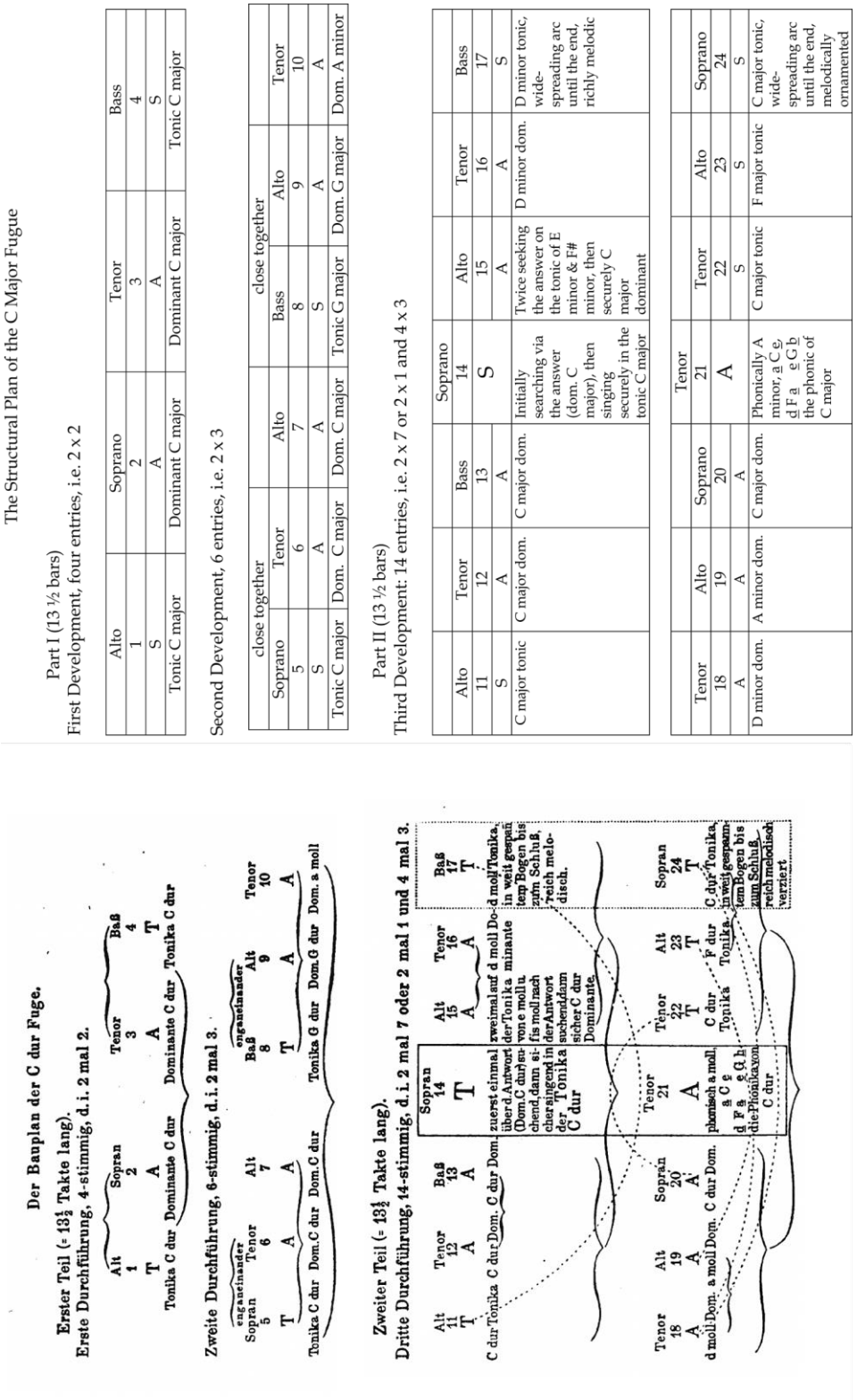
Example 2: 4, 6, and 14 pitch–attack patterns in the subject and its extension.

In the "First Development" in measures 1–7, beat 1 (see Example 3), the four entrances of subject material have an unusual symmetrical design of Subject-Answer/Answer-Subject (The usual design would have been Subject-Answer, Subject-Answer, with the subject beginning on the tonic and the answer on the dominant.) My example specifically shows that the Subject-Answer and Answer-Subject are presented symmetrically, in timespans of 13 beats each, balancing around the dominant harmony in m. 4.

The "Second Development" in mm. 7–14 (beat 1) contains six statements, symmetrically divided between the first three entrances and the second three (see Example 4). Both sections contain 15 beats, overlapping at the midpoint on a G-major triad on the third beat of m. 10. The next entrance following this triad is in the bass and beginning on G (m. 10), followed by the Answer on D (m. 10), the first entrance on a pitch other than C or G and reinforcing the strength of G major. The tonal turning point is the subsequent entrance on E (see m. 12, beat 1), leading to the A-minor cadence dividing the entire piece in two (m. 14).

Whereas Werker's analyses of the "First" and "Second" Developments are convincing, his understanding of the "Third Development" is somewhat questionable. He has valuable comments only on the balanced positioning of subject statements around incomplete entrances: (compare WERKER, 1922/1969, pp. 7–9). Specifically, the initial three entrances in mm. 14–17 (see S11, A12, A13 in Example 5) are in C major, the last entries in mm. 16–19 (A15, A16 and S17) move to D minor, and the incomplete entrance morphing into the complete entry on G (S inc.14, m. 15, S14, m.16) acts as a bridge between the first three entries (S11, A12, A13) and the last three entries (A15, A16 and S17). In the second half of the "Third Development" (mm. 19–27), the first three entrances (A18, A19, and

S21 in mm. 20–21) move from D minor/major toward G major and the last three (answers S22, S23, and S24) ultimately proceed to and ultimately define the tonic C major. The incomplete tenor answer on E (see m. 20) acts as the bridge between the foregoing statements (see Example 5).



The Structural Plan of the C Major Fugue

Part I (13 ½ bars)
First Development, four entries, i.e. 2 x 2

Alto	Soprano	Tenor	Bass
1	2	3	4
S	A	A	S
Tonic C major	Dominant C major	Dominant C major	Tonic C major

Second Development, 6 entries, i.e. 2 x 3

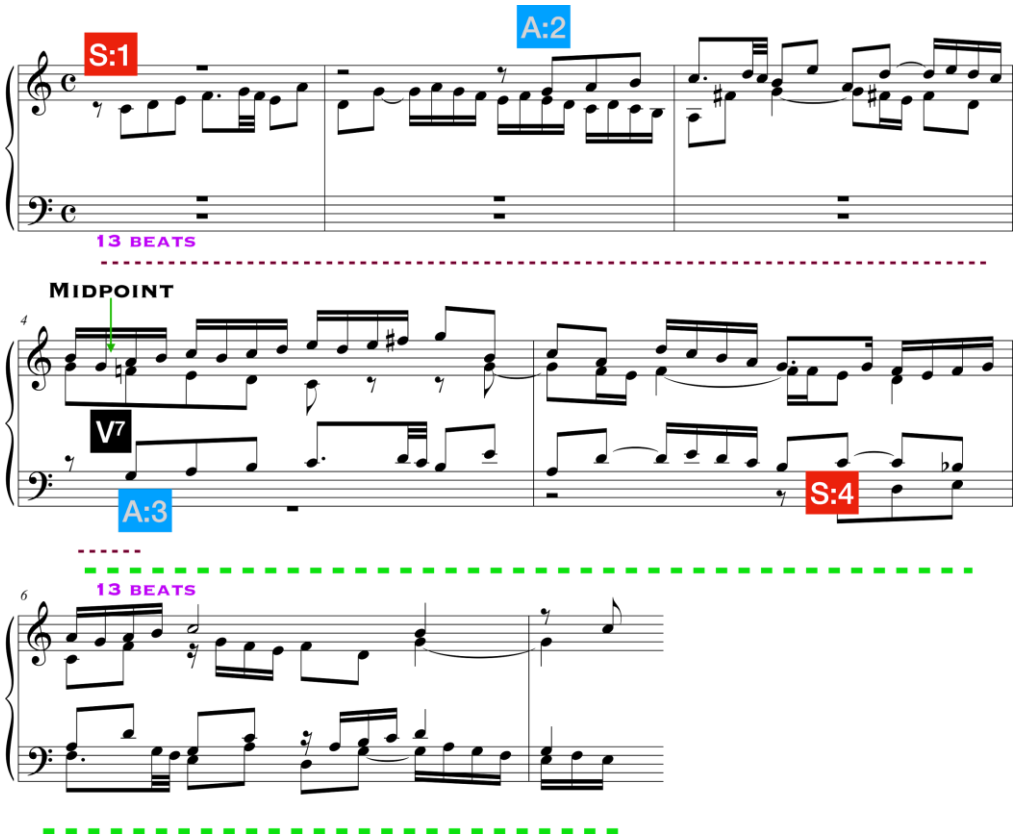
close together			close together		
Soprano	Tenor	Alto	Bass	Alto	Tenor
5	6	7	8	9	10
S	A	A	S	A	A
Tonic C major	Dom. C major	Dom. C major	Tonic G major	Dom. G major	Dom. A minor

Part II (13 ½ bars)
Third Development: 14 entries, i.e. 2 x 7 or 2 x 1 and 4 x 3

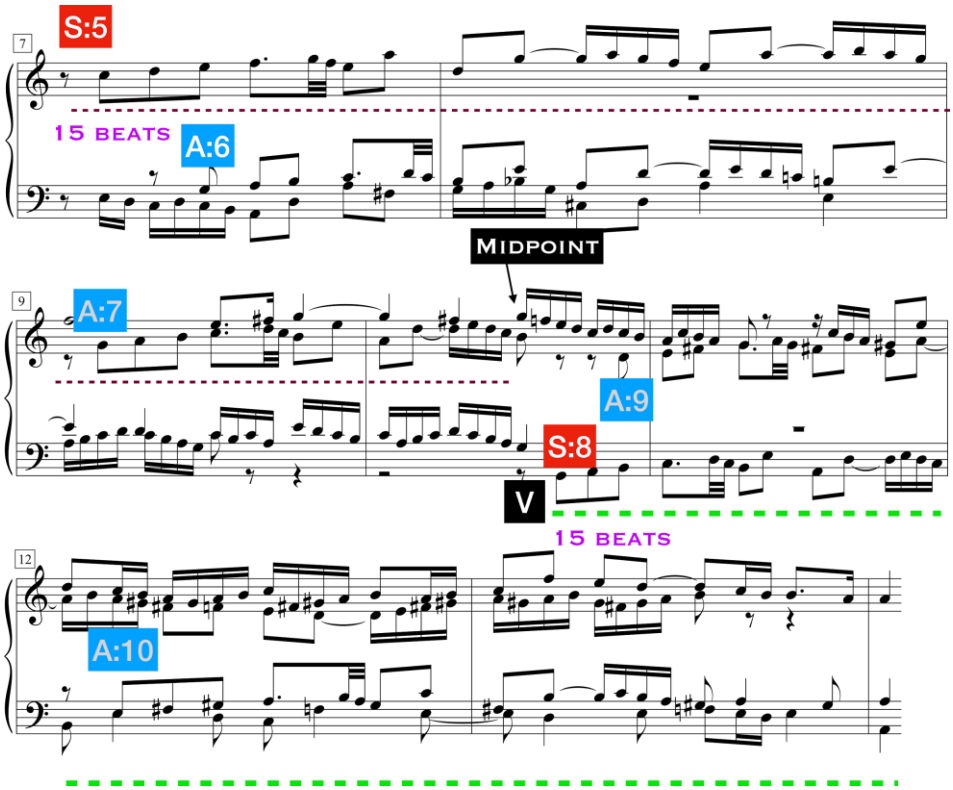
			Soprano				
Alto	Tenor	Bass	14	Alto	Tenor	Bass	
11	12	13	S	15	16	17	
S	A	A		A	A	S	
C major tonic	C major dom.	C major dom.	Initially searching via the answer (dom. C major), then singing securely in the tonic C major	Twice seeking the answer on the tonic of E minor & F# minor, then securely C major dominant	D minor dom.	D minor tonic, wide-spreading arc until the end, richly melodic	

Tenor	Alto	Soprano	Tenor	Alto	Soprano	
18	19	20	A	22	23	24
A	A	A		S	S	S
D minor dom.	A minor dom.	C major dom.	Phonically A minor, a C e, d F a e G b the phonic of C major	C major tonic	F major tonic	C major tonic, wide-spreading arc until the end, melodically ornamented

Figure 2: Werker's original form chart and its transcription/translation.



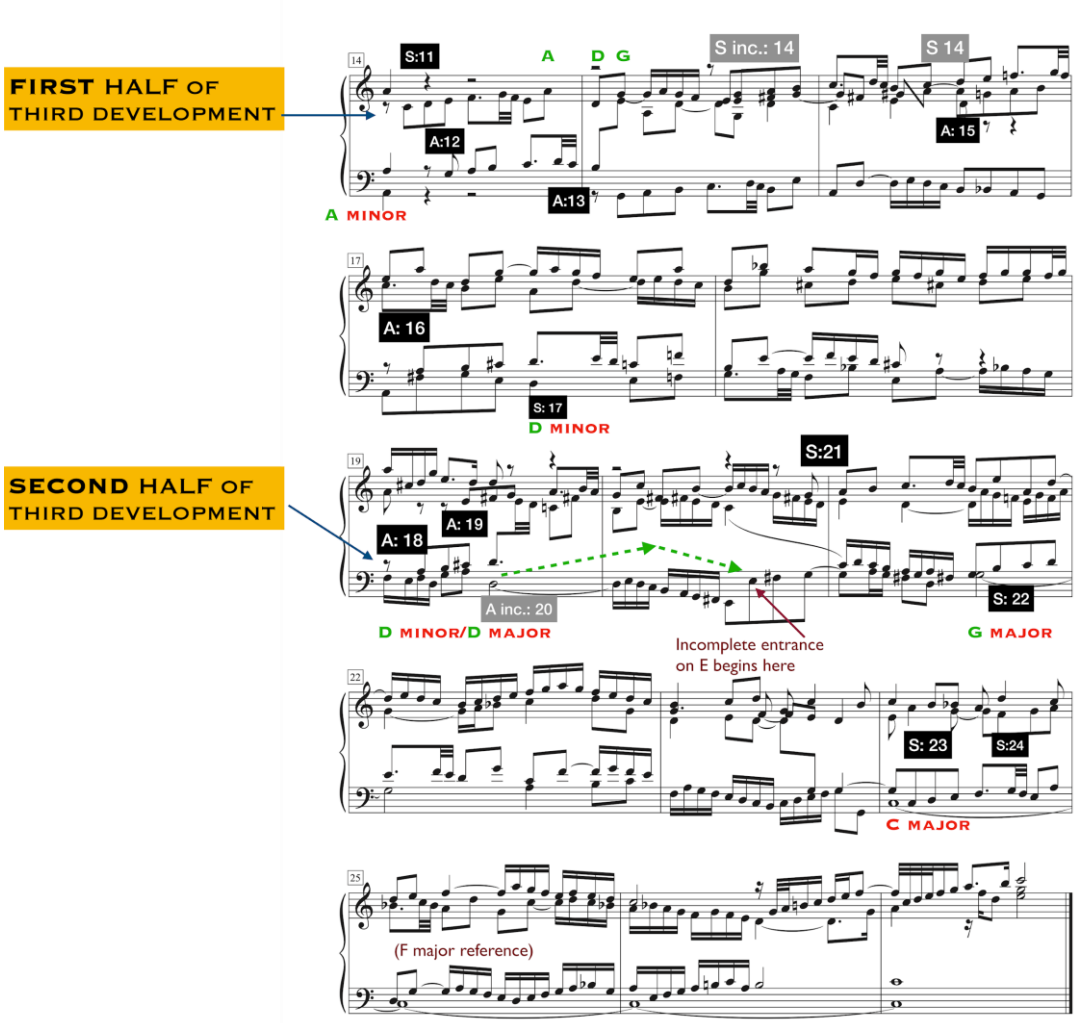
Example 3: "First Development": equal number of beats for S–A/A–S around the dominant harmony.



Example 4: "Second Development": equal number of beats (i.e., 15) for S–A–A/S–A-A around the dominant harmony.

It is odd that Werker never discussed the powerful *symmetrical* circle of fifths motion stretched across the fugue's second half which moves from A minor in m. 14 to D minor (m. 17), D major (m. 19), and G major (m. 21) before its final motion to C major (m. 24) (see Example 5). These key references constitute an emphatic augmentation of the central section of the subject, which presents the same relations in simple eighth notes (compare A on the fourth beat, m. 14, moving to D–G on beat 1, m. 15, highlighted by the letters in Example 5).

The strong cadence in A minor in m. 14 marks not only the end of the work's first half but also the beginning of the forthcoming fifth cycle as well. Motions to its remaining keys in the cycle are signaled by increasingly long pedal tones in the bass, pedals which are often on entries of the subject in complete, incomplete, or sometimes *ornamented* forms. This process starts on the third beat of m. 17 with a bass entrance in D minor in which the first pitch is lengthened from an eighth note to a quarter. Two bars later, on the third beat of m. 19 the same D appears in the bass, two quarter notes in length and tied across the bar. This pitch D is simultaneous with F♯ in the alto; within the next beat, C and F♯ clarify that D represents a strong V/V, leading two bars later to the arrival of a held bass G, now sustained for four quarter notes.



Example 5: The symmetrical cycle of fifths in "Development Three".

The motion between the D in the bass of m. 19 and the bass G in m. 21 constitutes a unique, highly ornamented presentation of the subject on D (beat 3, m. 19). I conceive the linear consequent of the held D in m. 19 as E in m. 20—a stepwise motion interrupted by sixteenth-note ornamentation (see green arrows on Example 5). The line D–E is succeeded by F♯–G–A–G–F♯–D–G, and together they form an incomplete subject on D, literally connecting the held D of m. 19 to the held G of m. 20, which in turn ascends to A–B–C, creating an augmentation of the subject on G. This rendition of the subject also remains incomplete; but since the initial, ascending linear fourth of the subject is ubiquitously heard throughout the fugue, its meaning is clear, and it leads directly into the long C pedal beginning at m. 24 (see Example 5).

Symmetrical designs in the Variations

Like a Bach fugue, the Variations movement of Schoenberg's Serenade, begins with a single voice—an eleven-measure unaccompanied clarinet solo divided symmetrically into two phrases of five and a half bars each (see Example 6). This same design of 11 bars divided in two is replicated by all five Variations and the Coda (see Table 1).

Table 1: The bi-furcated, 11-measure Theme, Variations, and Coda.

Theme || bars	Var. I	Var. II	Var. III	Var. IV	Var. V	Coda
mm.:	mm.:	mm.:	mm.:	mm.:	mm.:	mm.:
1) 1-5.5/	1) 11-15.5/	1) 23-28/	1) 34-39/	1) 45-49/	1) 56-61/	1) 67-72/
2) 5.5-11	2) 15.5-22	2) 29-34	2) 40-45	2) 49-55	2) 62-66	2) 72-77

The Variations is a transitional piece, written in August 1920 and March 1923, when Schoenberg's twelve-tone method was emerging as his main compositional focus. In this work we find his only use of a 14-tone set. Table 2 shows the set-forms Schoenberg uses in the work: the Prime, Inversion, Retrograde, and Retrograde Inversion lie only at one pitch level (there is no transposition of them). Thus, the Prime and Inversion begin on B♭ and the Retrograde ends on it, resulting in crucial consequences throughout the piece. Turning again to the two phrases in the Theme (see Example 6), we find that the first phrase presents the Prime form beginning on B♭, the second half presents its retrograde ending on B♭. B♭ hence frames the Theme and will frame the entire movement (cf. m. 1 and m. 77); see Table 3. B♭ also becomes ubiquitous in numerous, imitative entrances of the Prime and Inversion, especially in Variation II, which includes the greatest number of set-statements (see arrow on Table 3).

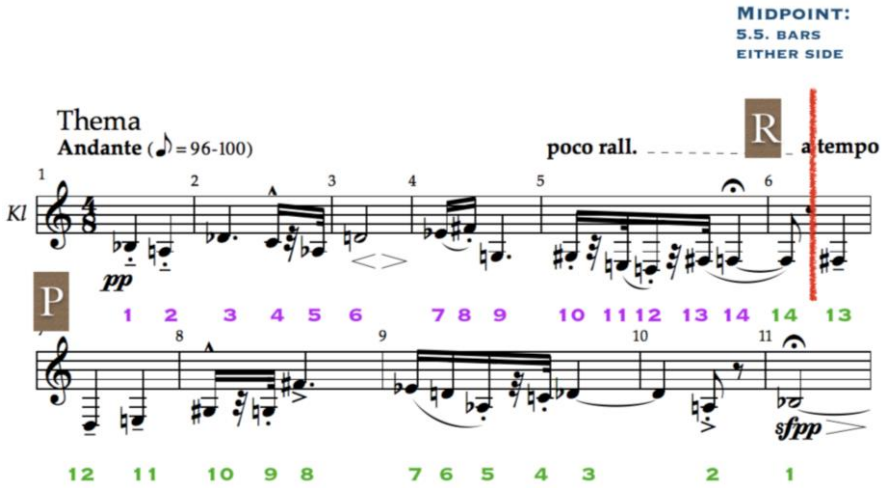
Thus, B♭ seems to form a reliable and thus stable component within the work's pitch materials.

Table 2: The only forms of the set in the piece (NO transposition).

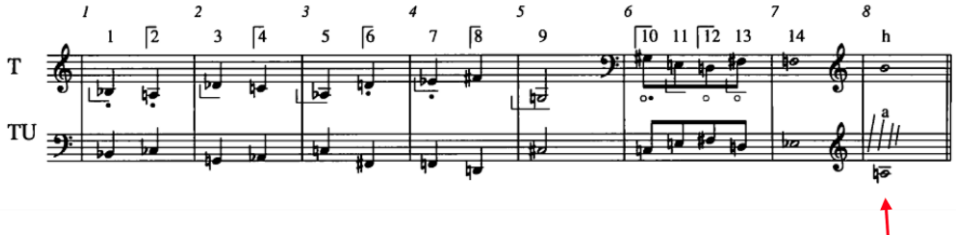
P	B♭	A	D♭	C	A♭	D	E♭	F♯	G	G♯	E	D	F♯	F
R	F	F♯	D	E	G♯	G	F♯	E♭	D	A♭	C	D♭	A	B♭
I	B♭	C♭	G	A♭	C	F♯	F	D	C♯	C	E	F♯	D	E♭
RI	E♭	D	F♯	E	C	C♯	D	F	F♯	C	A♭	G	C♭	B♭

Table 3: The bi-furcated, 11-measure Theme, Variations, and Coda and their succession of sets.

Theme || bars	Var. I	Var. II	Var. III	Var. IV	Var. V	Coda
mm.: 1) 1-5.5/ 2) 5.5-11	mm.: 1) 11-15.5/ 2) 15.5-22	mm.: 1) 23-28/ 2) 29-34	mm.: 1) 34-39/ 2) 40-45	mm.: 1) 45-49/ 2) 49-55	mm.: 1) 56-61/ 2) 62-66	mm.: 1) 67-72/ 2) 72-77
mm.: 1) 1-5.5: P	mm.: 1) 11-15.5: I, I, RI	mm.: 1) 23-28: R, P, I, RI, P, I, RI	mm.: 1) 34-39: P, I	Sets are reordered in odd and even pitches	mm.: 1) 56-61: R, P [sets incomplete or reordered]	mm.: 1) 67-72: I, P, I
2) 5.5-11: R	2) 15.5-22: P, P, R	2) 29-34: R, P, I, R, P, I, RI, P, I, P, I	2) 40-45: I, P, R, I		2) 62-66: R, I, P R, I	2) 72-77: P, I, P, P, I R, RI, R, I [all but last two sets incomplete]



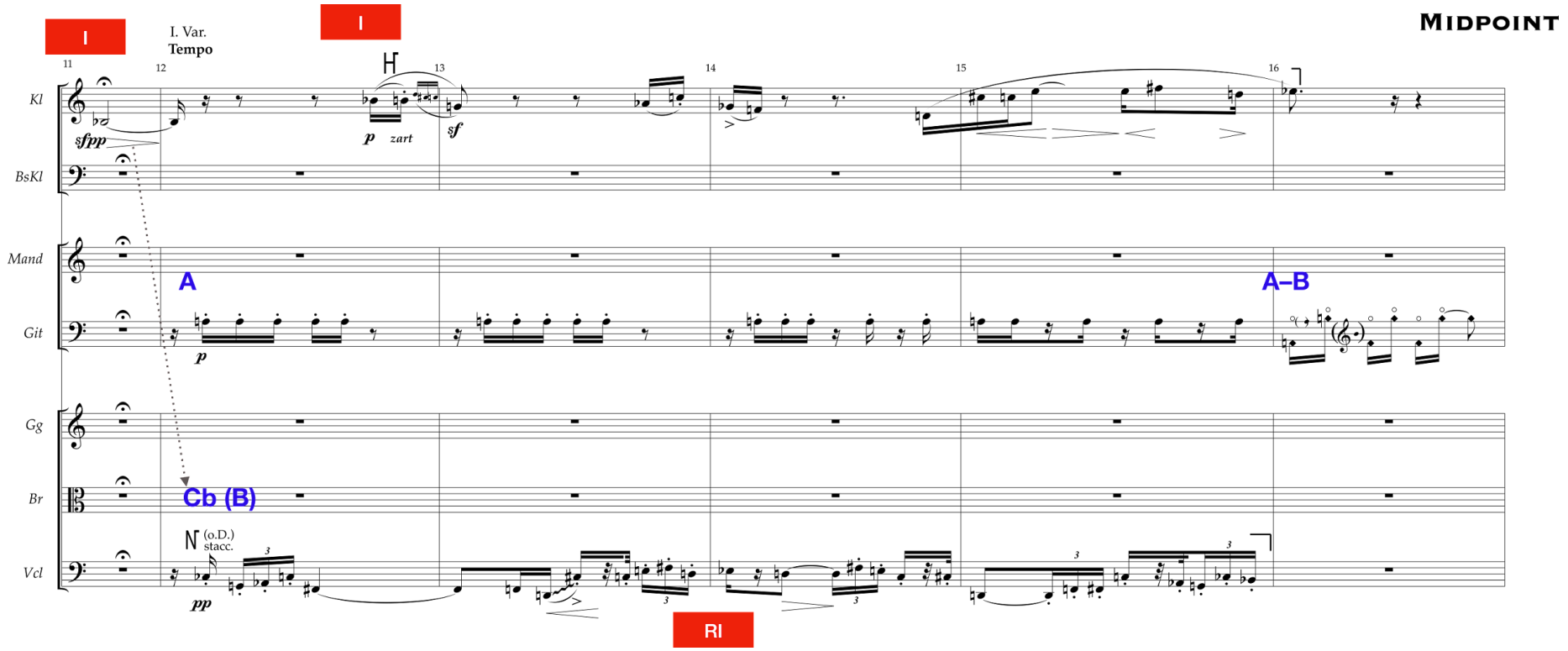
Example 6: The Theme of the Variations.



Example 7: Schoenberg's sketch showing the missing pitches B from P and A from I, singled out from the set [T = Theme]; TU = *Thema Umkehrung* (Theme inversion); [h = B].

Schoenberg's early sketch of the Prime and Inversion (see Example 7) immediately reveals that the set includes only 11 of the possible 12 pitch-classes; thus, one pitch-class is left out of every set-form. The Prime and Retrograde omit B or C♭; in turn, the Inversion and Retrograde Inversion exclude A-natural. (Note their separate positioning in the right of the sketch marked by the arrow.) A and B will constitute a crucial contrast in the work's pitch structure challenging the supremacy of the B♭. I understand the A and B, foreign to the set, as the work's compositional "problem," but not in the exact sense of the "problem" in a mature 12-tone work (Boss, 2014, p. 33). For here the pitches A and B cannot return to sets of which they were never members.

For example, in Variation I, (see Example 8), after the Theme ends with a fermata on B♭, the first Variation begins with A and C♭ appearing simultaneously between the cello and the guitar (m. 12)—note their symmetrical relation to B♭ is weakened by their timbre and registral position. The guitar A is then extended into a four-bar pedal point (mm. 12–15), and after the Variation's midpoint, the mandolin begins a five-bar, tremolo pedal on B. Thus, their resonant repetitions divide the Variation into equal timespans. Moreover, A and B alternate in striking guitar harmonics before the midpoint. After the midpoint, the cello ends with a mercurial fragment on B♭. The violin and bass clarinet imitating in renewed mercurial fragments add two more B♭s. The stable B♭ and the challenging A and B above are thus juxtaposed.



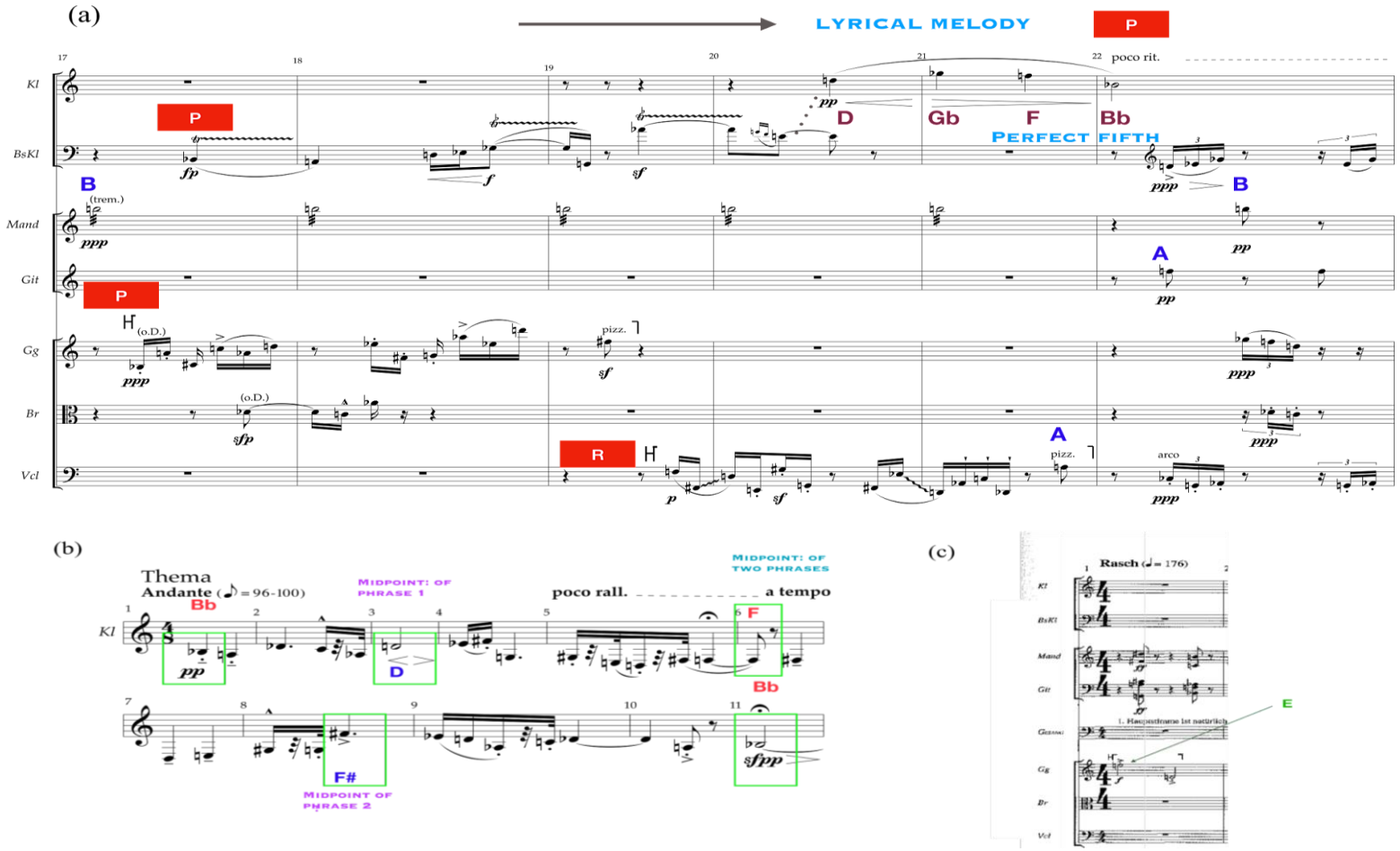
Example 8: Variation I: The entrance of the missing pitches A and B and their timbral distinction.

The lyrical melody with a perfect fifth

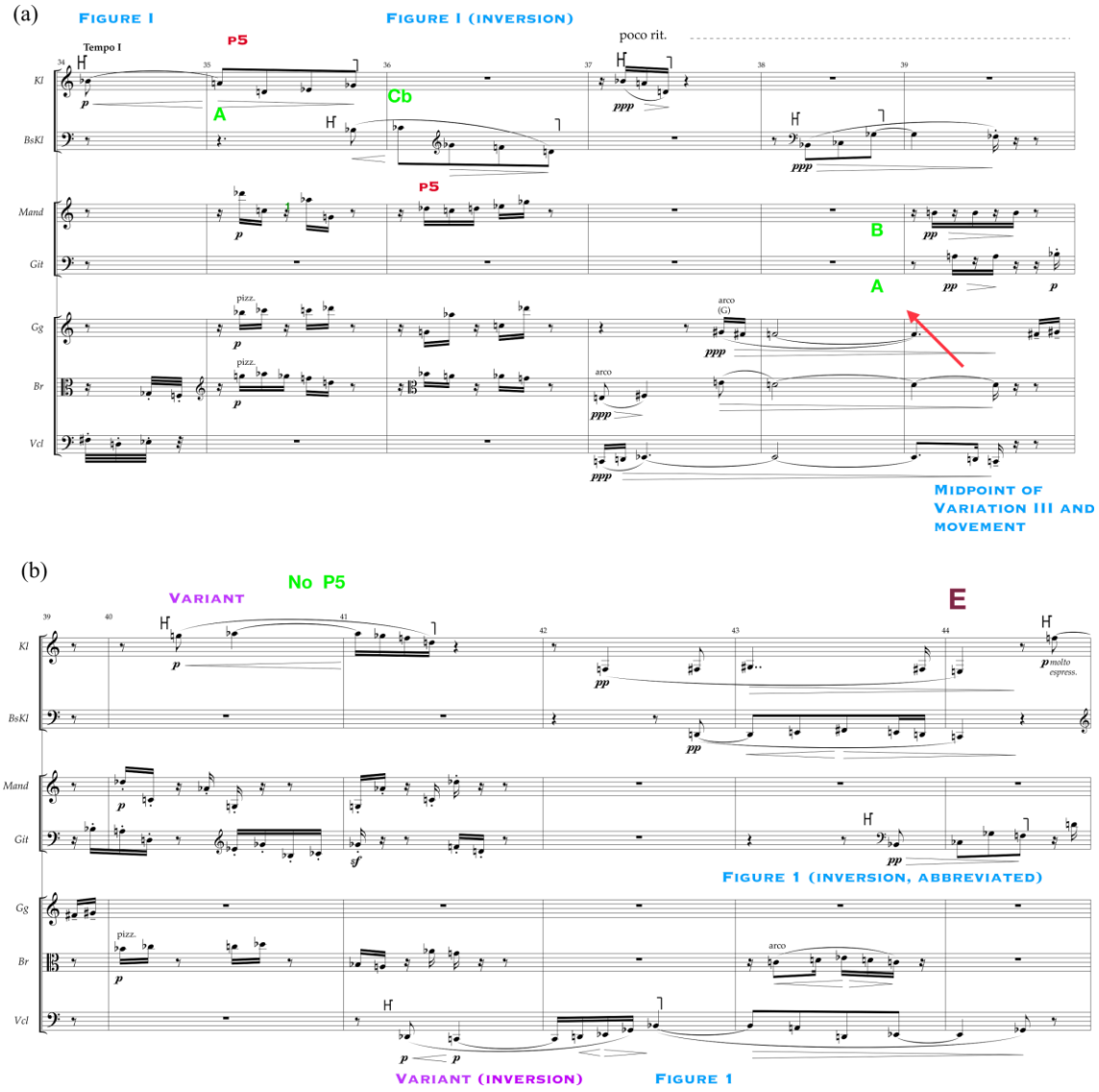
Toward the end of the Variation I, the texture changes with the entrance of a striking lyrical line in the clarinet, recalling the melodic character of the opening Theme, creating a slow, steady expressive movement quite distinct from the rest of the texture in timbre and rhythm (see Example 9a). The melody is derived from the framing or symmetrically placed held notes of the Theme: that is, the held B♭ framing the Theme; the held F is at its temporal midpoint; the held D (m. 3) is the temporal midpoint of the first phrase; and held F♯, the midpoint of the second phrase (see Example 9b). The lyrical melody concludes in m. 22 with a descending perfect fifth from F to B♭. Although the B♭ begins and ends the Theme and contrasts with F at its midpoint, a literal statement of a perfect fifth is never used as a pitch adjacency in the set or Theme. Thus, this lyrical melody introduces an *intervallic* element omitted from the set.

The lyrical melody seeks its own, later development, postponed to Variation III (see Example 10a). The last three pitches G♭–F–B♭ of the lyrical melody in m. 22 are here transposed to B♭–A–D in the clarinet, followed by E♭ and G♭. I term this melody Figure 1. Its mirror Inversion follows in the bass clarinet, (mm. 35–36), again on B♭: B♭–C♭–G♭–F–D. Notice that Figure 1 and its Inversion are built from the elements both absent in the set: A and C♭ and the perfect fifth. Here A and C♭ appear *downbeats*, while B♭s on upbeats; A and C♭ are thus strengthened compared to B♭. At the *midpoint* of this variation and of the entire movement and at the end of a long ritard (m. 39), A and B again assert their importance and independence, alternating between the guitar and mandolin, as they had at the midpoint of Variation I.

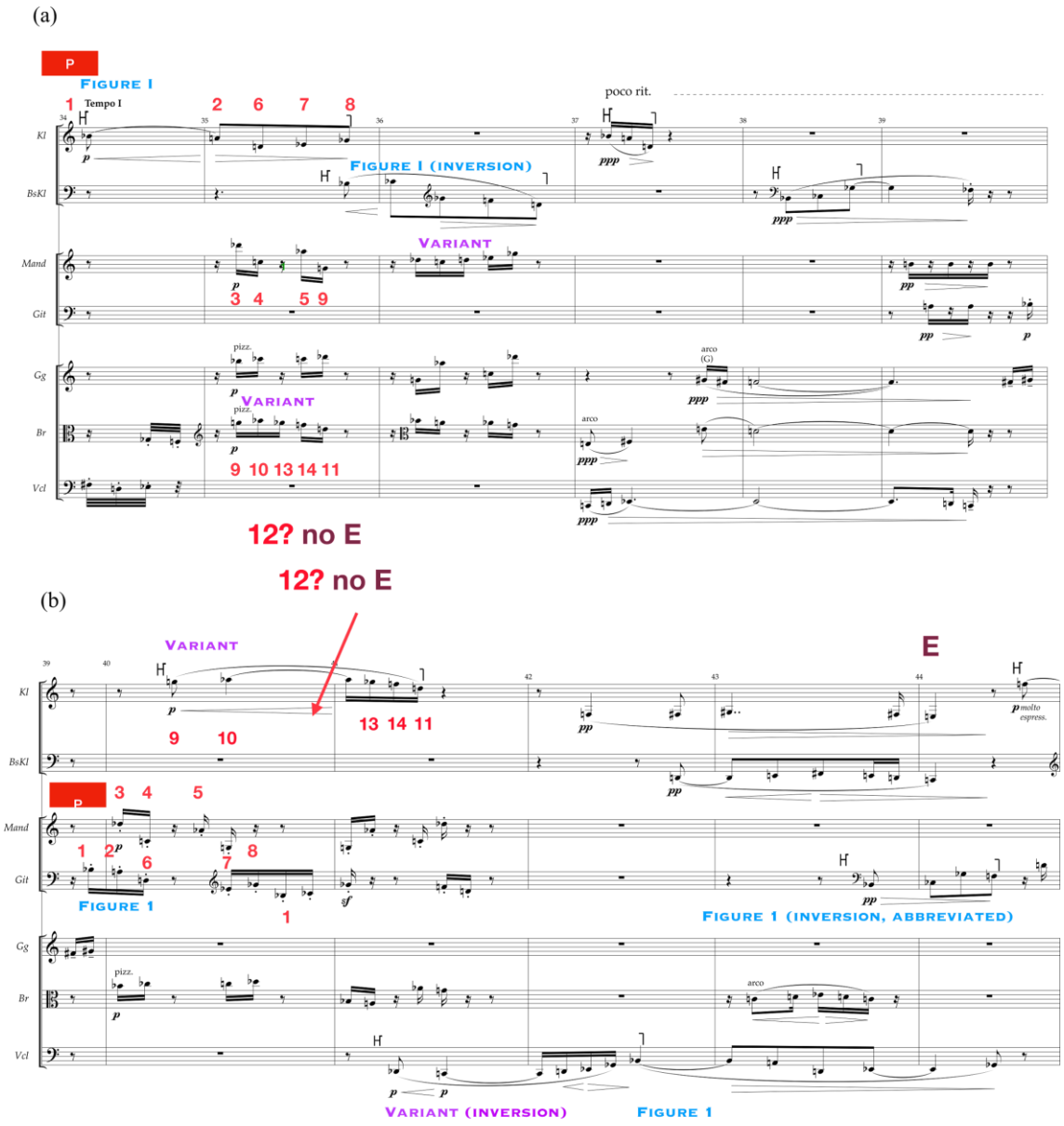
A variant of Figure 1 enters at the return of the original tempo, (see Example 10b), beginning with a semitone and ending with three pitches, but omitting Figure 1's signature perfect fifth altogether, followed by its inversion in the cello (mm. 41–42), continuing directly into a return presentation of Figure 1 and its inverted form. The entry of the variant of Figure 1 at the "a tempo" in m. 34 is totally seamless. It had already appeared within the mercurial, unobtrusive accompaniment to Figure 1, first in the viola, (m. 35, Example 11a), then inverted in the mandolin. All the set's pitches missing from Figure 1 are present in this accompaniment except E, thus carrying on the process of omitting single pitches from a set–form. In the second half of Variation III, the roles of Figure 1 and its variant are reversed (see Example 11b): Figure 1 becomes part of the mercurial accompaniment to the *variant*, filling in all possible pitches of the set, except for the pitch, E. In fact, there are only five appearances of E in all of Variation III (mm. 34, 37 (twice), 43 and 44) and only the last one, in the clarinet in the last bar of the Variation is particularly prominent on the work's surface.



Example 9: Sustained pitches at midpoints with and between phrases of the Theme; compare lyrical melody, mm. 21-22; the set contains no perfect fifth as a pitch adjacency.



Example 10: a) Figure 1, its inversion and A, B(C♭) and b) Variant of Figure 1.



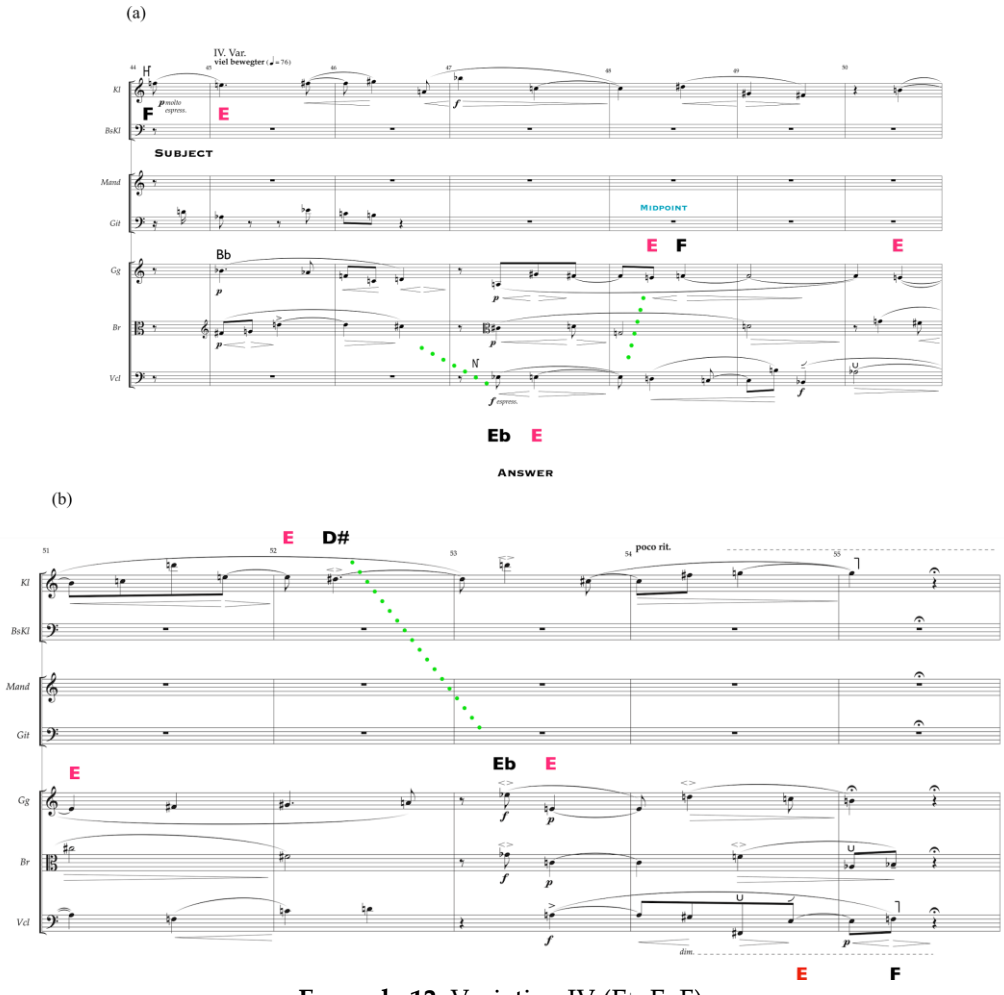
Example 11: a) Variant as accompaniment to Figure omits E and b) Figure 1 as accompaniment to variant omits E.

E and the perfect 5th are emphasized

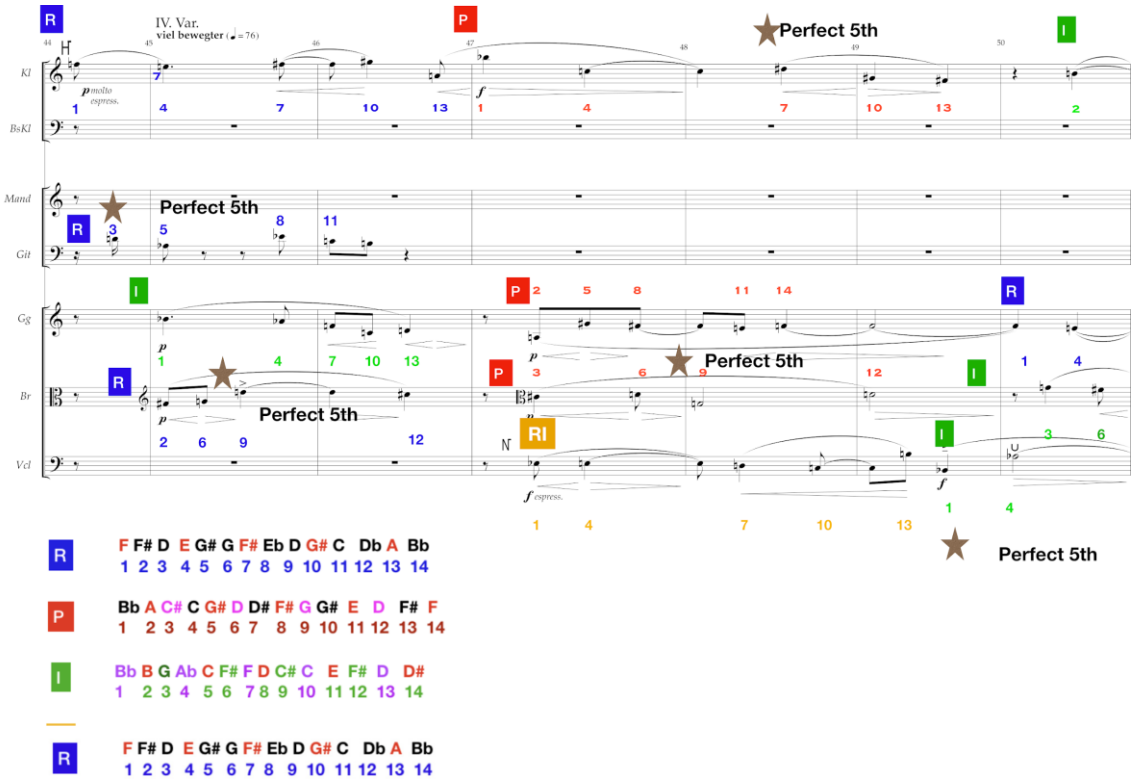
Variation IV begins with E (mm. 44–45) two octaves higher, introduced by F (see Examples 12a). F–E in the clarinet is answered by an inversion in the cello, naturally beginning with E♭–E. Now F and E♭ gradually assume the role previously played by A and B surrounding B♭ they encircle the pitch center E, which becomes ubiquitous throughout Variation IV (see Examples 12b). In addition to the initial clarinet and cello entrances, E♭–E and F–E later appear between the cello and violin (mm. 47–48), the clarinet (m. 52) and violin (m. 53), and violin and the cello (mm. 51–55). Thus, the B♭–A/B♭–B relations recently omnipresent in Figure 1 and its Inversion have now been temporarily replaced by their transposition at the tritone, E♭–E/F–E♭; at the same time, the guitar,

mandolin, and bass clarinet drop out for the first time in the movement leaving only four melodic voices in a stunning contrapuntal framework. The transition from this variation to the next is also unique: the only one marked by silence—a quarter rest with a fermata.

Completed on 8 August 1920, Variation IV has one of the most sophisticated deep-structure designs in Schoenberg's early path to twelve-tone music. In 1988 Dr. Fusako Hamao demonstrated from her study of the brackets on Schoenberg's early sketch that all the lines in this passage are re-orderings of the set derived from every third pitch (HAMAO, 1988, pp. 180–87). The opening of Example 13 shows that main clarinet line of the Variation is formed from set row re-orderings of R, P, I, and RI. However, sub-phrases in the guitar present an anomaly. In a consistent patterning, D in the guitar would move to G not A♭, proceeding to E♭ (see first star in Example 13). Like the guitar line, the viola's melody also highlights the perfect fifth. Interestingly, its F♯-G–D is related by transposition to the inversion of Figure 1 in Variation III (see second star in Example 13). And indeed, the viola line remains continuously filled with fifths throughout the Variation (see all of the starred fifths in the viola in Example 13), thus highlighting the interval omitted from the main set.



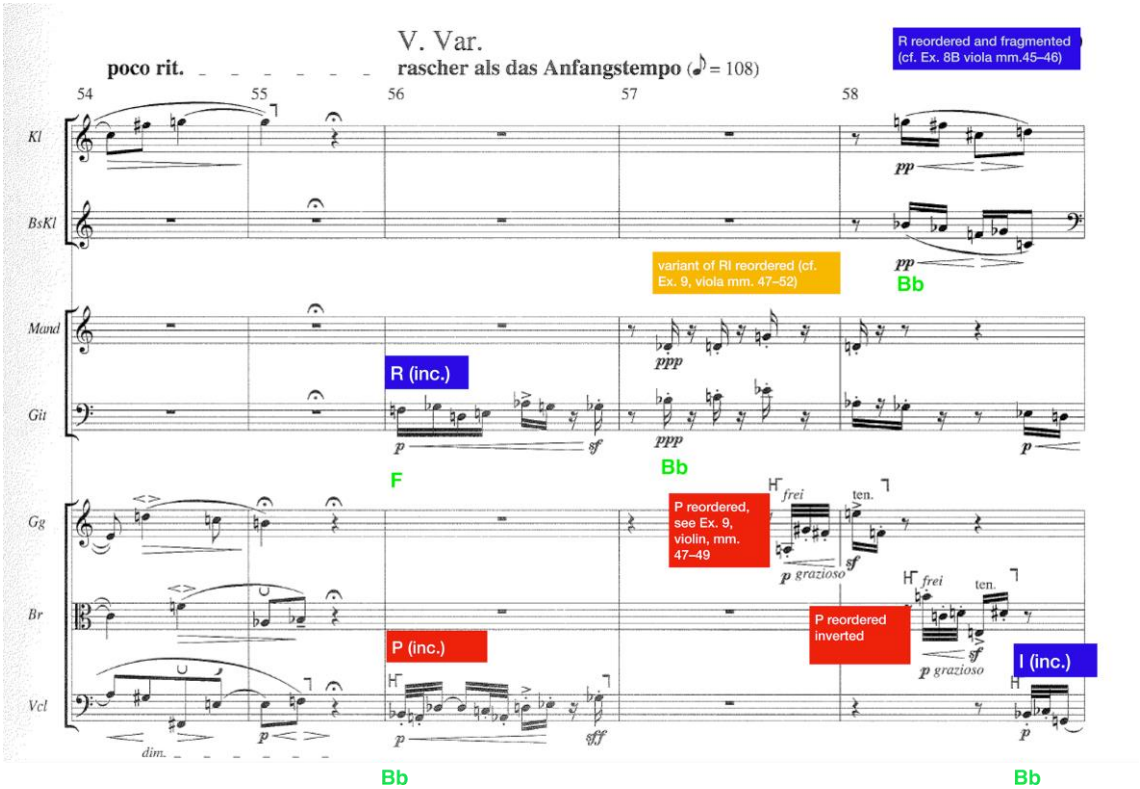
Example 12: Variation IV (E♭–E–F).



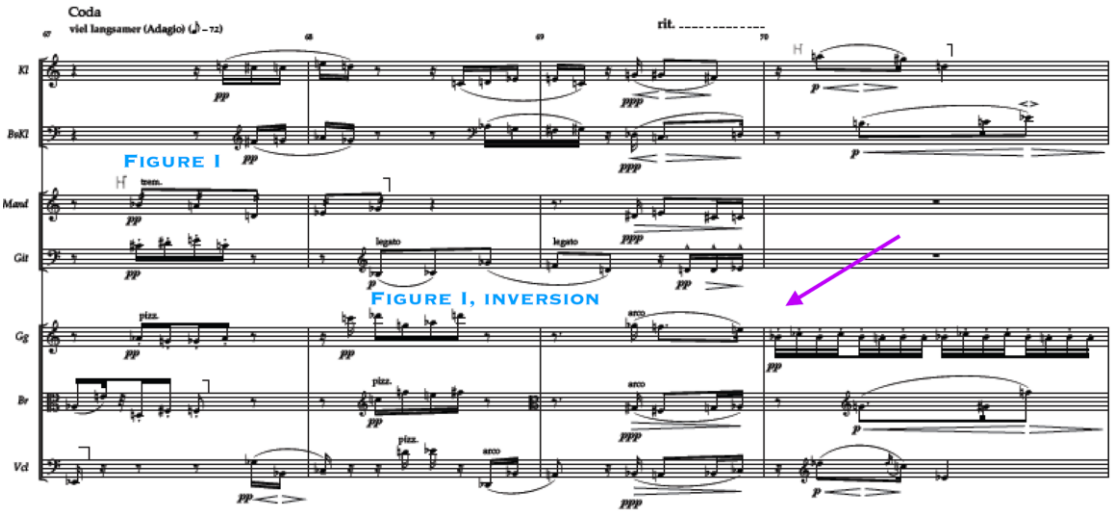
Example 13: Reordered sets in Variation IV and P5.

The return to B♭ and E at and after the final cadence

Variation V begins by reasserting the centrality of B♭, which appears in the cello (m. 56) acoustically supported by a fifth above in the guitar: the power of E as a central pitch now has ended (see Example 14). Mercurial fragments of partial sets and re-orderings from Variation IV follow in mm. 57–58, four of them beginning with B♭. This texture continues through the Variation V's rhapsodic conclusion leading into the Coda, where full linear statements of the set in mm. 65–67 re-introduce Figure 1 and its inversion (see Example 15). Whereas Figure 1 and its Inversion previously led to the development of "problematic" pitch relations of E and the interval of perfect fifth, they now lead to continuous pedal on the pitches B♭–A—B, starting in m. 70 and continuing to the end of the movement.



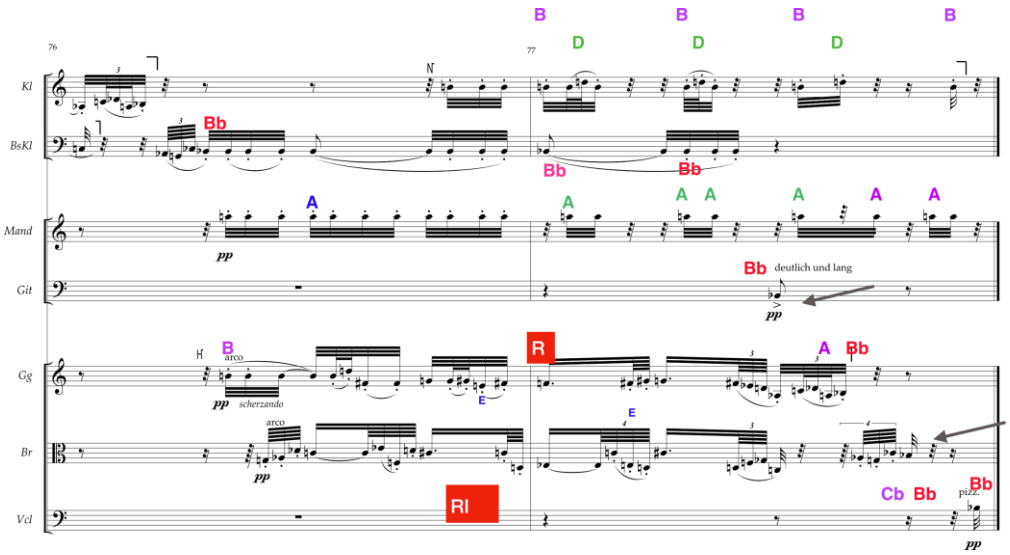
Example 14: B♭ restored as stable pitch in incomplete and reordered sets.



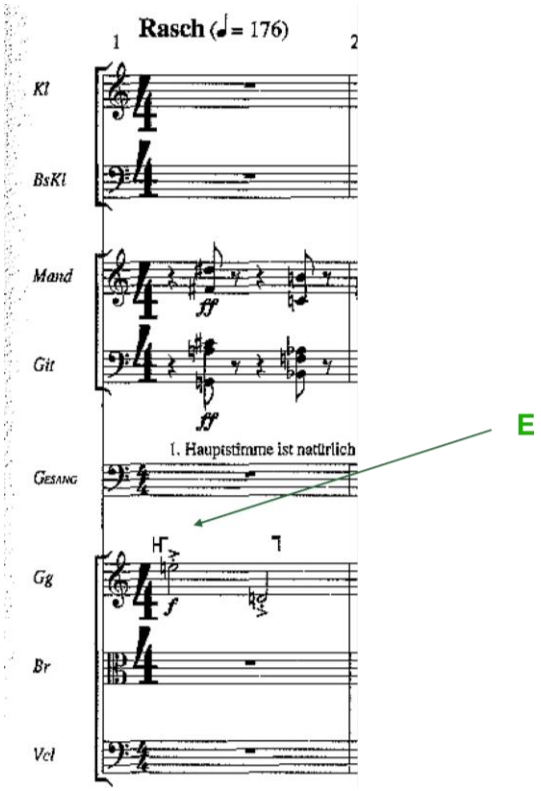
Example 15: B♭ accentuated in B♭–C♭/B♭–A.

In the final measures A and B♭ separate into their own repeated tones and join with a repeated D—altogether they create the sound of the opening the pitches of Figure 1, B♭–A–D in Variation III (see Example 16); moreover, the pedals A and another on B alternate in the final bar as they had in Variations I and III; here the mercurial sets R and RI sound against them in the strings. This summary of the crucial "problematic" pitch relations, A and B and the perfect fifth, A–D, is contrasted by the almost complete omission of the pitch E—it appears only twice in a 32nd note ornamentation. At the movement's very end,

32nd-note reiterations of the pedals A and B lead to the subtle pluck of a pizzicato B♭ in the cello on last beat of movement: a questioning gesture to be answered by an emphatic violin E, at *forte* opening of movement four, a setting of Petrarch's Sonnet 217 (see Example 17). Thus, the problematic pitches have made their final questioning statements, and the missing, E, signals something new as it had done in Variation IV.



Example 16: B♭–A–B and B♭–A–D at the final cadence.



Example 17: The cadence/beginning the ensuing movement called "Sonnet".

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10

Grace and Clarity: Schoenberg's Other School

Jeffrey Perry

Arnold Schoenberg's relocation to Los Angeles in the 1930s gave rise to a much different compositional milieu than the famed Second Viennese School of which he had been the center earlier in the century. Despite Schoenberg's attempt to provide them with the fundamentals of a middle European musical education, the composers (mostly American) of the Los Angeles Schoenberg "school" had a much different relationship to Schoenberg than did his earlier European acolytes. The numerous pedagogical works he wrote during his years teaching privately and on the faculty of USC and UCLA testify to his attempt to bridge the enormous gap between his cultural set and setting and that of his American pupils. The most well-known and problematic student/teacher relationship of Schoenberg's American years was undoubtedly the one he shared with John Cage. Although the specifics of their interaction have proven difficult to document, there are resonances of Schoenberg's compositional practice in much of Cage's music and musical thought. This paper examines details of Schoenberg's String Quartet No. 3 Op. 30—the only one of his own works that he lectured on to Cage and his classmates—and explores links between Schoenberg's mature twelve-tone craft and Cage's developing aesthetic in the 1930s and '40s, e.g. in essays such as "Grace and Clarity" (1944) and "Lecture on Satie" (1948). Paradoxically one may draw a more direct line from Schoenberg's rhythmic syntax in Op. 30 to Cage's works of the 1940s than from the Satie works that Cage claims as models for those works. Cage's need to both claim Schoenberg as his teacher and distance himself from him suggests a pattern that was to repeat throughout the former's creative life in works like *Cheap Imitation*, *Song Books*, *Mureau*, and the *Writings Through Finnegans Wake*.

This essay explores how John Cage misreads Arnold Schoenberg, one of his most notable American students. A "misreading" takes a source work and distorts it in some way—by exaggerating some aspects of it and minimizing others, for example, or by carrying some feature of the model work to an extreme that the original author did not intend. Such a process of creative appropriation and revision summarizes how Cage received and used what Schoenberg taught him; it also provides a lens through which to explore the ambivalence that Cage felt about his teacher in later years.[1] The exploration of aesthetic topics of interest to them both reveals further connections between the two composers.

Schoenberg's influence on American composers spread far beyond his immediate circle of students; in part this is due to the vigorous intellectual atmosphere of 1930s and '40s Los Angeles, which included two émigré communities—one of intellectuals, authors and artists, and one of working musicians—who had fled the Nazification of central Europe, and the subsequent world war it caused. In addition, Los Angeles was home to a community of (mostly American) writers, actors, and other creatives who had come to work in Hollywood—a metonym, then as now, for the film industry, which was at the time far more localized in the greater Los Angeles area than it is in the 21st century. Schoenberg's ideas found their way into these intersecting groups through vectors such as Thomas Mann and Theodor Adorno (among the émigré intellectuals), Otto Klemperer and Ernst Toch (among the émigré musicians), George Gershwin and Harpo Marx (among the Hollywood folk), and Peter Yates (a promoter of new music who was to figure prominently in Cage's career as well).[2]

The American composer Mel Powell (1923-1998) did not study with Schoenberg; he did, however, work as a staff performer and arranger in the MGM studio in Hollywood in the immediate postwar years, working with some of the musicians who performed in the Evenings on the Roof series that formed a nucleus for contemporary music in Los Angeles at the time.[3] Originally a jazz pianist, later a student of Ernst Toch and Paul Hindemith, Powell became more and more indebted to Schoenberg's music and began to compose serial music in the 1960s while serving on the faculty of the California Institute of the Arts (of which he was founding dean) with Schoenberg's student and assistant Leonard

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[1] The theory of creative misreading originates with Harold Bloom. See Bloom (1973) and Bloom (1975). Bloom states that a misreader is a "revisionist, who wishes to find his own original relation to truth" in the texts he misreads (BLOOM, 1975, p. 3).

[2] Kael (1971), although controversial, provides a glimpse into the world of Hollywood writers in the 1930s-40s. Ross (2020) discusses the community of émigré writers and intellectuals; Crawford (2002) provides an account of the intersections of Schoenberg's circle and that of the émigrés in general.

[3] Crawford (1989) provides a history of the Evenings on the Roof concerts and their role in the cultural life of southern California.

Stein. In 1986 Powell critiqued the state of contemporary composition, saying that he found a great scarcity of real, heard multiplicity in the music of the day. The source of this scarcity, he felt, was, quote,

> the fact that over the years, the art of counterpoint has been badly conceived and badly taught…Whether we teach sixteenth- or eighteenth- or twentieth-century counterpoint, we are constantly laden with pitch-intervallic relations and that, I am more and more persuaded, is not at all what actual contrapuntal multiplicity is about… we have not studied what its true essence is, which I'd call the temporal structure of individual, linear components…an understanding of conflicting periodicities, and conflicting in the deep sense, with respect to the incipients, the termination, the agogic accent, those are the keys to real multiplicity. (ROBINS, 1986, pp. 482-483)

How Schoenberg taught counterpoint as a means of creating multiplicity, and how Cage generalized the concepts of counterpoint and multiplicity, is of relevance to my topic.

There are two things to note about how misreading applies to the relationship between Cage and Schoenberg. First, as his student, Cage misreads both Schoenberg's pedagogy and his music: the lessons that the teacher may have wished to impart are not necessarily those that the student carried away from their encounter—Schoenberg's music (to his ongoing frustration) was received differently in Los Angeles in the 1930s than it had been in the Vienna of the 1910s and '20s, and Cage (or one of his Los Angeles classmates) would not take away from his classes with Schoenberg the same lessons that Webern and Berg had. Second, misreading honors the composer whose teachings are misread, but if the misreading is a fruitful one it represents a kind of triumph of the misreader over the misread. Bloom makes this into a masculinist, Oedipal affair that does not necessarily help us understand either Schoenberg or Cage, so I will discard this part of Bloom's model, drawing instead on Joseph Straus's adaptation of it as a tool for musical analysis in his 1990 book *Remaking the Past*.

David Bernstein, Dorothy Lamb Crawford, Severine Neff and others who have investigated Arnold Schoenberg's teaching in America (privately, at the University of Southern California, and at the University of California Los Angeles) have shown that he seems to have been less a composition teacher than a pre-composition teacher (NEFF, 2014; BERNSTEIN, 2002). For the most part he felt that his American students were horribly underprepared, and thus focused on teaching them the fundamentals he felt they needed before they could compose in earnest, if that was their goal. Crawford notes that "Many of his private students were film composers, some of whom must have been told… 'First you must learn something about music'" (CRAWFORD, 2002, pp. 15-16).

Like his earlier teaching in Vienna and Berlin, the curriculum in Schoenberg's private classes and at the two universities on whose faculty he served (the University of Southern California (USC) and the University of California Los Angeles (UCLA)) leaned heavily on his own reinterpretation of the conservatory staples of harmony, formal analysis, and (above all) counterpoint (NEFF, 2014, p. 459). But Schoenberg refused for the most part to discuss his own works, or the twelve-tone method, with his students; moreover, for the most part he did not listen to or critique his students' original works. This American Schoenberg circle was therefore qualitatively different from his original group of pupils in Vienna, performances of whose works he championed.

Table 1: Schoenberg's American curriculum. The last date for each item is the date of its publication.

Title	Date
Harmony (abridged translation of *Harmonielehre*)	1911/48/78
Suite in G (Old Style) for String Orchestra	1934/35
Models for Beginners in Composition	1942/43/72
Preliminary Exercises in Counterpoint	1936-50/1963
Fundamentals of Musical Composition	1937-1948/1967
Structural Functions of Harmony	1948/54

Table 1 lists the textbooks and related materials that Schoenberg wrote (or began) during this period. There is one musical composition on the list, the Suite in G for String Orchestra of 1934. Drawing on the extensive notes of Cage's fellow student Gerard Strang, Severine Neff has provided a thorough account of the pedagogical function of the Suite, focusing on Schoenberg's pervasive use of canonic imitation in its first movement. What Schoenberg's writing such a work just as he arrived in the New World seems to imply is that in order for prepare America for his twelve-tone music, he first had to introduce them to tonality as he understood it, and to the contrapuntal technique that underlay his conception of organic development of the *musikalische Gedanke*. Although he doesn't discuss the suite itself, Straus (1986, 1990) provides a convincing account of Schoenberg's various *Rückblicke* at the tonal past, suggesting that his recompositions of works by Handel and Monn are necessary exercises in misreading; Straus sees them as providing a justification for Schoenberg's own assertion of the centrality of motivic relationships in tonal as well as in atonal music, and thus retroactively justifying his own leap into atonality and providing the logic of his progression into serialism.

Although his students knew (and were able to hear a performance of) this suite, only once, as far as we know, did Schoenberg relent and offer his students a lecture on his own *twelve-tone* music—specifically, on the String Quartet no. 3

Op. 30 of 1927. In April 1935, after hearing Schoenberg give this lecture, his pupil John Cage wrote that "About a week ago…I heard the entire III. String Quartet in my sleep" (NEFF, 2014, p. 452). This work seems to have haunted his waking hours to some extent as well. Although Cage never elaborated on what in the work specifically influenced his compositional thought, I believe I have an idea what it was that led to its intruding on his dream life.

It was not the sonata-form analysis that Erwin Stein provided for the Universal Edition score of the work, with the authorization of Schoenberg (Table 2); this analysis, however, is not entirely irrelevant to the issue.

Table 2. Erwin Stein's analysis of Schoenberg, String Quartet no. 3, I.

mm. 1-91:	*Quasi exposition*	
	mm. 5-32:	*Theme I*
	mm. 62-94:	*Theme II*
mm. 95-173:	*Quasi development*	
mm. 174-277:	*Quasi recapitulation*	
	mm. 174-208:	*Theme II*
	mm. 239-263:	*Theme I*
mm. 278-341:	*Quasi coda*	

As Straus points out,

> for Schoenberg, the sonata was primarily a convenient way of articulating the proportions of a work through thematic contrast and associated changes of texture, articulation, and instrumentation (…) Schoenberg's reinterpretation of sonata form in this string quartet depends above all on his concept of inversional symmetry and balance…the form depends (…) on the symmetrical balance of exposition and recapitulation around a central development section. To heighten this sense of formal balance, the order of the first and second themes is reversed in the recapitulation (…) This reinterpretation of the sonata form as an expression of inversional balance rather than dramatic polarity is Schoenberg's boldest stroke. (STRAUS, 1990, p. 121)

Although it was probably the Quartet's surface continuity, not its resemblance to tonal formal models, that held the most interest for Cage, Straus shows that the work's architecture and its surface continuities are clearly linked by a conception of form rooted in Schoenberg's sense of proportion. What is

more, he shows that the concept of inversion animates the work both in terms of the intervallic structure of its row and its temporal ordering.[4] I return to this idea and how it is relevant to Cage below (Figure 1).

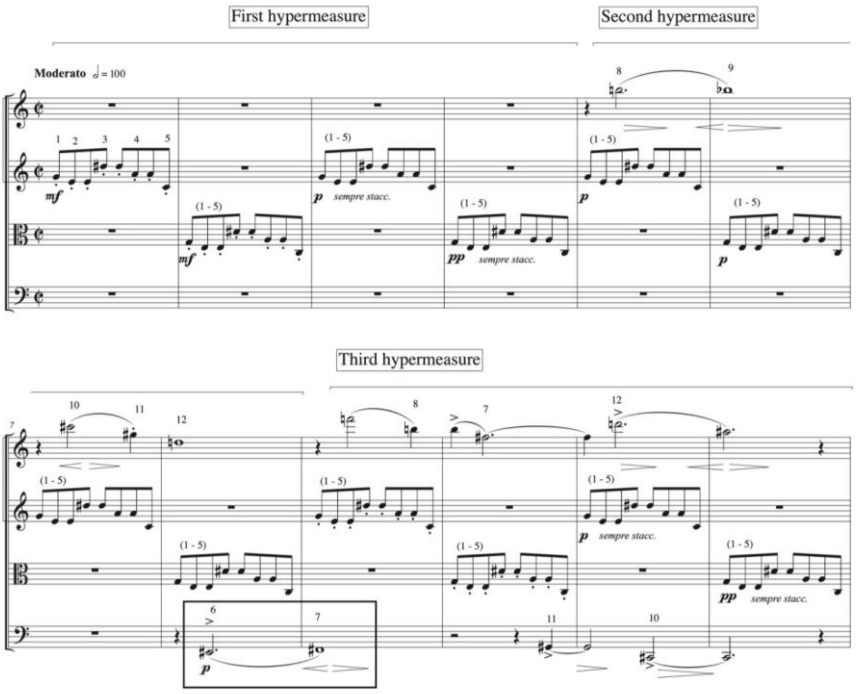


Figure 1: Schoenberg, String Quartet no. 3, I, mm. 1-13.

The Quartet's opening ostinato propels the entire movement— Stein characterizes it as "a continuous eighth note agitation." The ostinato alternates between two different octaves and two different instruments, suggesting two-measure groupings; the first violin's entrance in m. 5, however, invites us to hear the movement, at this point, in terms of four-bar hypermeasures. Figure 1 places row order numbers above each note in the passage. The ostinato figure articulates the first five notes of the row throughout; the first violin enters in m. 5 with row members 8 through 12, divided into a dyad plus a trichord. If there is a still deeper organizing element at work in the quartet, it is found in the distinction between figure and ground, the ostinato serving as the latter, the thematic utterances and interjections as the former.[5] The fact that this distinction persists even in the absence of the syntactical formations of tonality is another feature of Schoenberg's mature music that I will return to below.

[4] Boss (2014) extensively explores the role of the twelve-tone series in establishing dynamic formal symmetries. He avers that "a twelve-tone piece can give rise to problems regarding the relationship of its opening gestures to a symmetrical ideal." (BOSS, 2014, p. 33).

[5] The figure/ground duality is derived from Gestalt psychology. It has primarily been used as an analytical tool in visual art, as in KENNEDY (1985), but an application to music analysis (by way of Deleuze) is found in FITCH (2017). I explore this in greater detail below.

Shaftel (2009) writes perceptively about the work's tonal problem, or *Grundgestalt*, which he identifies with the semitone B/B-flat; violin I (mm. 5-6) provides the first intimation of it. Note how the two "missing notes" of the row, order numbers 6 and 7, E-sharp and F-sharp, appear in the cello's lowest register just as the first violin concludes its first five-note segment, providing the inverted shadow of that instrument's descending semitone. The position of this cello dyad, boxed in the figure, might have interested Cage even more than this artful first unfolding of the row form. This dyad creates overlap between the second and third hypermeasures of the passage. The exposition of phrase structure and row structure are thus somewhat in conflict—or at any rate in counterpoint. Such dovetailings are not uncommon in Schoenberg's serial works, but the relatively foursquare rhythm here makes this one especially prominent.

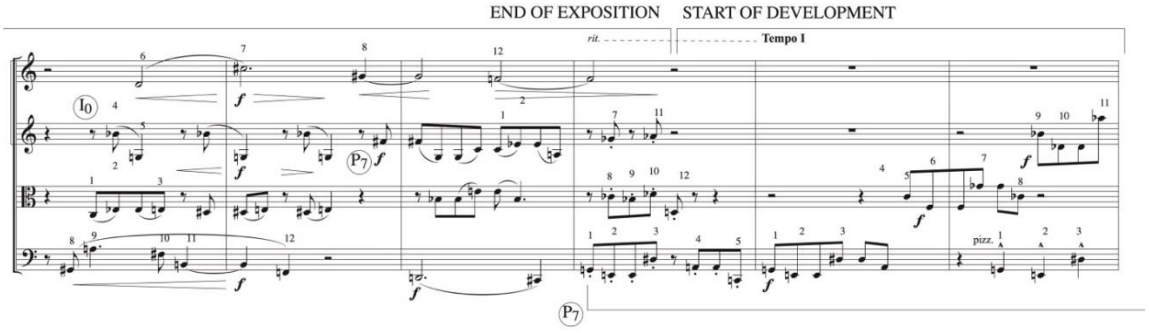


Figure 2: Schoenberg, String Quartet no. 3, I, mm. 91-96.

A slightly more subtle bit of dovetailing of row structure and phrase structure appears later on in the movement, where the exposition's second theme ends and the development section begins (Figure 2). A persistent feature of the movement's second theme is the displacement by one quarter note of the accompanimental eighth-note ostinato that it shares with the first theme, causing it to unfold across the barline; this play against the movement's notated time helps individuate the theme, thus helping to articulate the sonata-derived form of the movement, as does the difference in cardinality that marks its two themes (the first theme unfolds mainly in pairs of pentachordal figures punctuated by the dyad that fills out the row; the second theme's melodic utterances are hexachordal). Note Schoenberg's quite conventional use of tempo inflections in places like this to signal a structural point of demarcation.

When the recapitulation begins in m. 174 Schoenberg indulges in an homage to the reversed recapitulations occasionally found in sonata-form works of eighteen-century Classicism (e.g. Mozart's Piano Sonata in D K. 284) and nineteenth-century Romanticism (e.g. the Fourth Symphonies of Brahms or Tchaikovsky); as we have seen, Straus is able to make a larger point concerning Schoenberg's sense of sonata form in terms of symmetry and proportion. (It is also true that nineteen years earlier Schoenberg had employed a reversed

recapitulation in his Second String Quartet, so it also seems to be the case that he simply enjoyed the play against formal norms that such a reversal entails.)

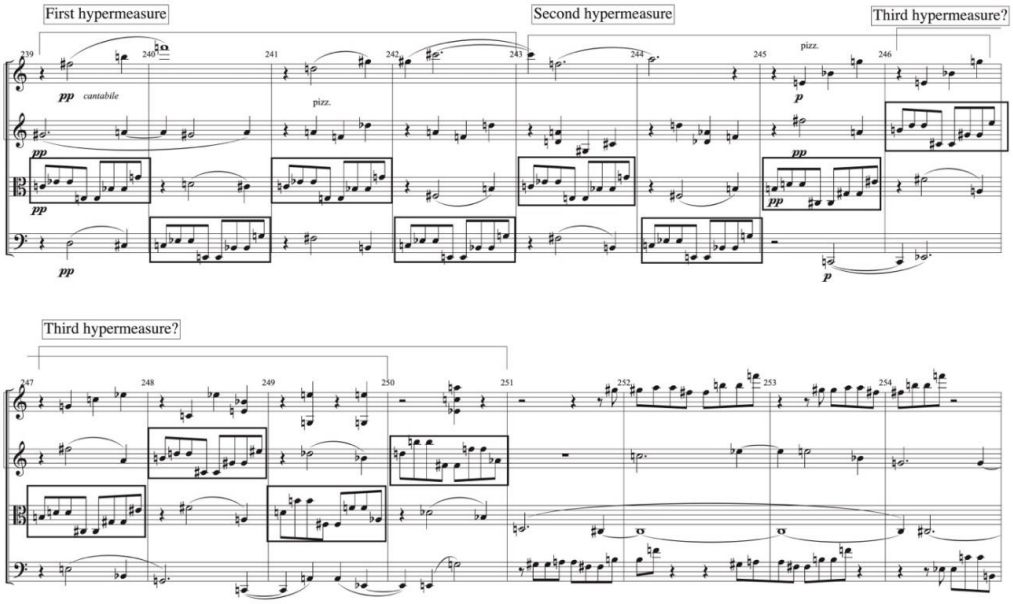


Figure 3: Schoenberg, String Quartet no. 3, I, mm. 239 ff, showing recapitulation of Theme I.

When the opening theme does return (see Figure 3), the ostinato accompaniment and alternation between viola and cello (boxed in the figure) reasserts the four-measure hypermeter of the movement's opening. Note that the last measure of the second hypermeasure alters this pattern, the ostinato moving to the second violin; this is either an extra measure that elongates this hypermetrical unit, or an instance of hypermetric overlap. This occurs as Schoenberg liquidates the ostinato figure—loosens its intervallic and contour profile—prior to replacing it with a more irregular eighth-note melody. There is subtlety to Schoenberg's manipulation of the passage's phrase rhythm that I believe Cage would have taken to heart.

In January of 1937 Cage withdrew from Schoenberg's courses, feeling stifled by his teacher's rigidity and lack of encouragement.[6] For the time being, however, he remained nearby, working as an accompanist for dance classes at UCLA. His work for the next several years would be with percussion music and, increasingly, music for the dance, neither of which interested his former teacher (Schoenberg famously informed Cage that he would not be free at any time to attend one of his concerts; see HICKS, 1990, pp. 132-133). In a process familiar to any former student of a strong-minded teacher, Cage took many years to come to grips with his relationship with Schoenberg. Distancing himself from his teacher was a process that took place in stages, over a number of years. In 1937

[6] The chronology of Cage's studies with Schoenberg has proven controversial. Neff (2014) provides the best evidence for this date as the endpoint of their work together.

Cage wrote the first version of his manifesto "The Future of Music: Credo" in which, speaking of future developments in contemporary music, he prophesied, "New methods will be discovered, bearing a definite relation to Schoenberg's twelve-tone system" (Cage, 1961, p. 5). Somewhat later, in his 1944 essay "Grace and Clarity" Cage framed music in terms of the duality of rhythmic structure, which he simply called structure, and musical content, which he called form. In this essay he takes as his model the best of modern dance as he then knew it, drawing extensively on the work of dancer, teacher, and choreographer Bonnie Bird, who hired Cage to teach at the Cornish School in 1938; Syvilla Fort, the dancer for whom he wrote the first work for prepared piano in 1940; and Merce Cunningham, with whom Cage's creative and life partnership was just beginning. "With clarity of rhythmic structure, *grace* forms a duality," he begins.

> Together they have a relation like that of body and soul. Clarity is cold, mathematical, inhuman, but basic and earthy. Grace is warm, incalculable, human, opposed to clarity, and like the air…The two are always present together in the best works of the time arts, endlessly, and life-givingly, opposed to one another (CAGE, 1961, pp. 91-92).

As shown above, the play with and against rhythmic structure is present in Schoenberg's music, and particularly in the Third Quartet, even if it isn't a feature that the composer himself foregrounds in his own theoretical, analytical, and polemical writings. On the other hand, such "play with and against" is at the center of counterpoint in its most essential sense, and counterpoint is the one aspect of Schoenberg's own pedagogy that both Cage and his other American students emphasize in their recollections of his teaching.

Comments that Stein makes in his analysis of the Third Quartet, and an all-caps dictum that Schoenberg includes in his *Fundamentals of Musical Composition* are suggestive in this regard:

> The style of the work, however, as well as its form, deviates far from its classical models. First, the themes hardly ever recur in the Recapitulation in their original form. To be sure, the original rhythm is frequently retained upon restatement; the melodic line, however, is generally altered—often inverted or otherwise changed (Stein in Schoenberg, 1927, p. iv).

> THE PRESERVATION OF THE RHYTHM ALLLOWS EXTENSIVE CHANGES IN THE MELODIC CONTOUR (Schoenberg, 1967, 30).

There is a strong suggestion here that rhythm, in Schoenberg's mature serial works, functions as a grounding element against which melodic figuration can be varied and developed.

(a) (b)

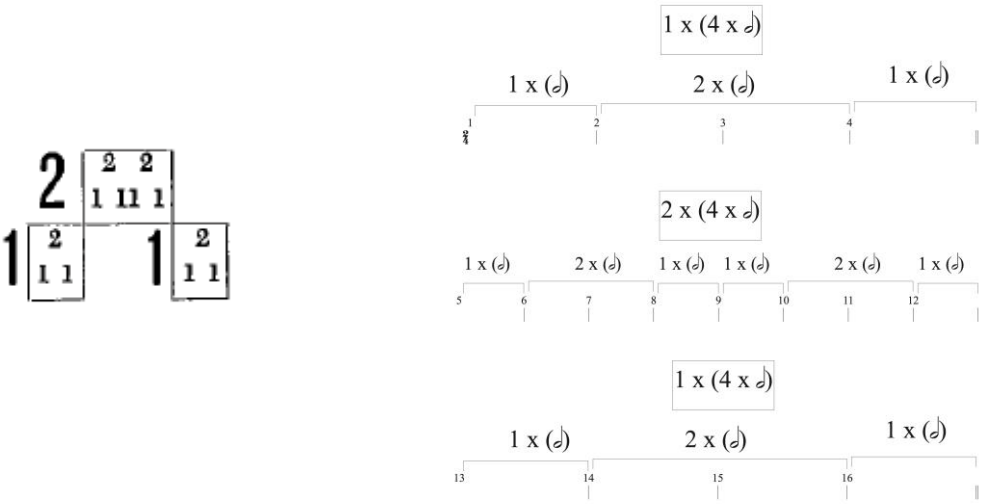


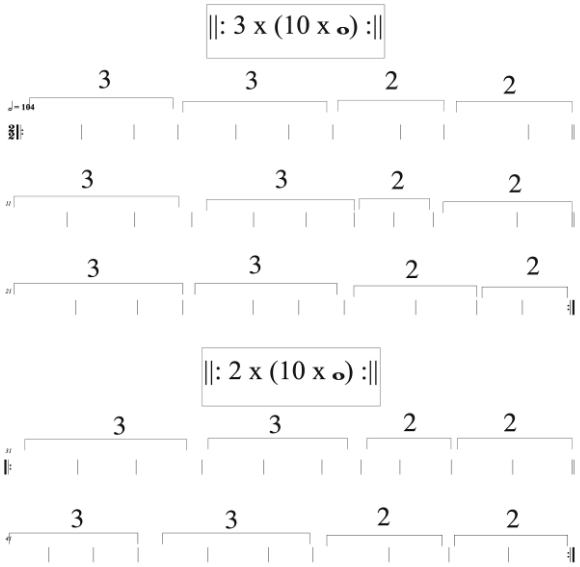
Figure 4: (a) John Cage, example of "square root" (macro-microcosmic) structure from 1958 essay "Erik Satie." Reproduced in Cage (1961, p. 81) (b) Realization of 1:2:1 structure in 2/4, as in Satie (see Figure 7).

The connection between Cage's work with Schoenberg in 1935-37 and his subsequent formulation of the duality of structure and form is important to note, although Cage himself attributes his discovery not to his teacher Schoenberg but rather to Erik Satie (1866-1925), whose music he probably had already discovered while a traveling college dropout in Paris earlier in the 1930s. In a 1958 essay he traces the origin of his "square root" or "macro-microcosmic" structure to Satie, without specifying *where* in Satie he finds it (Cage, 1961, pp. 80-81). This technique begins by defining a time span in terms of a given number of large sections that have a fixed durational relationship to each other. These large sections are defined in terms of a common rhythmic value—a number of beats or measures—and each is divided into smaller sections that have the same proportional relationship to one another as the large sections have to each other.[7] This is illustrated in Figure 4. The fourth Sonata from Cage's *Sonatas and Interludes*, composed in 1946, exemplifies this (Figure 5): the piece consists of two repeated reprises. The meter is 2/2; the first half is thirty measures long, the

[7] There are cogent explications of how Cage's "square root" structure works in PRITCHETT (1993, pp. 13-22), JENKINS (2002), and BERNSTEIN (2002).

second half twenty measures long. Taking the repeats into account this yields a macro-structure of 30+30+20+20: three ten-measure units (repeated) followed by two ten-measure units (also repeated). Each ten-measure unit is divided similarly: three plus three plus two plus two measures.

(a)



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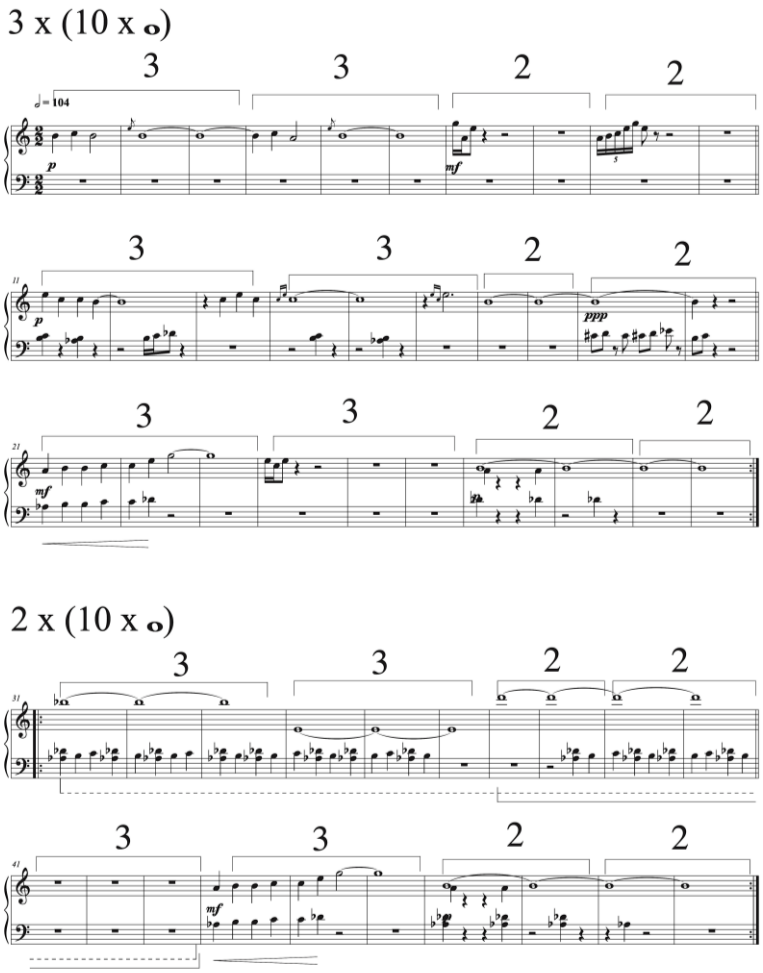
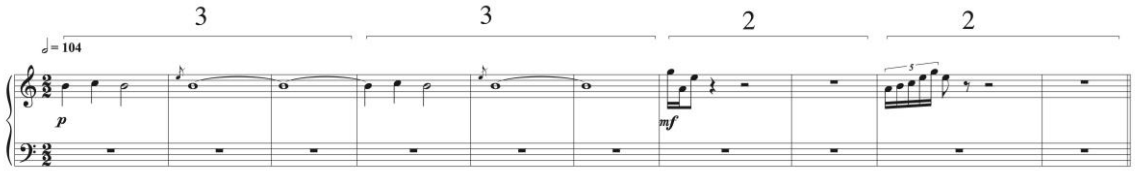
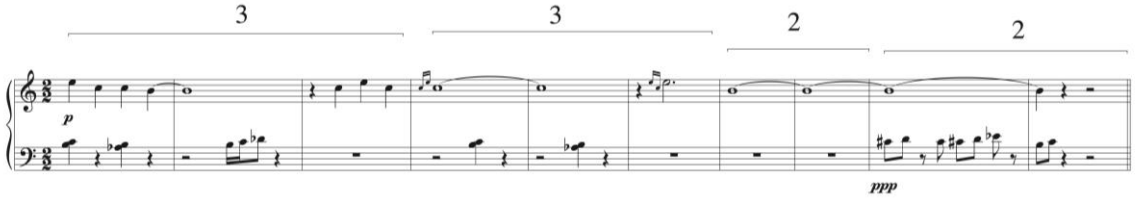


Figure 5: Cage, Sonata IV from Sonatas and Interludes for Prepared Piano (1946-48). (a) Macro-microcosmic structure of the sonata; (b) annotated score of the sonata.

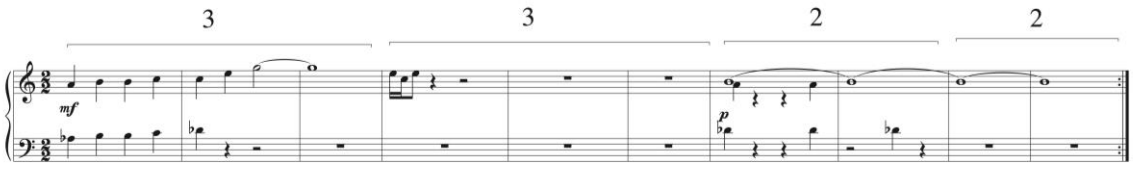
(a) mm. 1-10. First macrounit of the first reprise



(b) mm. 11-20. Second macrounit of the first reprise



(c) mm. 21-30. Third macrounit of the first reprise



(d) mm. 31-50. Second reprise (both macrounits)

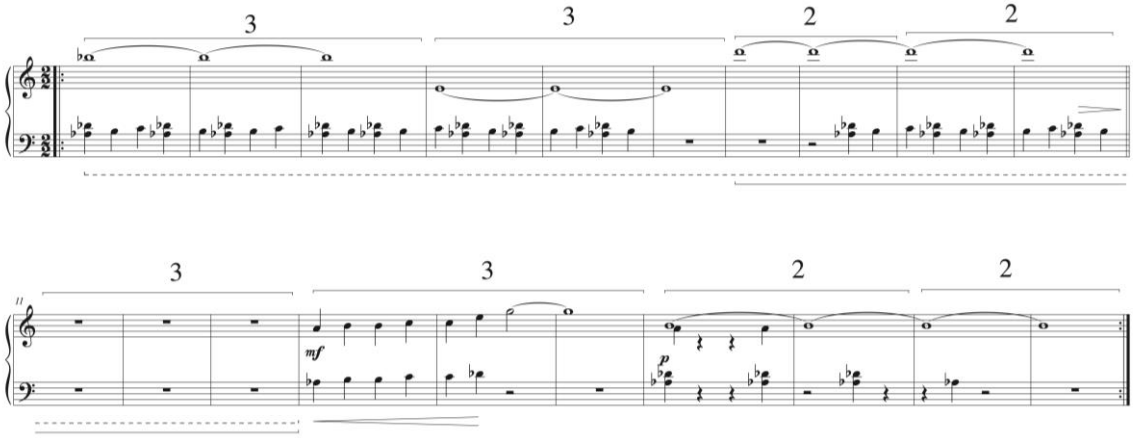


Figure 6: Cage, Sonata IV from Sonatas and Interludes for Prepared Piano (1946-48).

Cage fills this rhythmic framework with sounds—elsewhere he compared selecting the sounds he'd use to collecting shells on the beach. For the most part his placement of sounds invites us to hear the sonata in terms of its rhythmic structure—note how each of the first four microunits (Figure 6(a), mm. 1-10) begins with a new melodic figure, a slight inflection being provided by the tie across the barline in mm. 3-4. There is a kind of enjambment between the first and second microunit of the next macrounit (Figure 6(b), mm. 11-16), but the start of the last microunit in this section is articulated by the left-hand figure that starts in m. 19. A new, striking melodic idea heralds the start of the third macrounit in m. 21 (Figure 6(c)), while the last two microunits in this passage are run together due to the absence of melodic activity in m. 29.

It is in the second reprise of the sonata (Figure 6(d)) that the interplay of grace and clarity becomes a major factor. Here, although single held notes (and, in one case, the absence of any notes whatsoever) mark the start of most microunits, a syncopated five-quarter note figure dissonates rhythmically against the structure of the section.

There is a duality in Schoenberg's Third Quartet between (on the one hand) row deployment and (on the other) surface figure and ground events that is also evoked by Cage's very different, "prepared" musical surface and its interplay with his two-tiered rhythmic structure. By the time he wrote the *Sonatas and Interludes* in the mid-to-late 1940s, however, Cage had decided (as mentioned above) that he owed this technique not to his former teacher, but rather to Erik

Satie.[8] In letters he wrote at this time, however, he is more explicit about where he claims to have found inspiration for this technique. Here are excerpts from two 1948 letters to the new music advocate Peter Yates:

> You…in no sense give [Satie] the importance due him, which is, I believe, to have consistently structured his music on lengths of time rather than harmonic relations. I'm sure he was aware of doing this but I doubt whether he knew its real importance, which is *real*: liberation from the Beethoven yoke, far more real than granted by S[choenberg] with the 12-tone row…
> With Webern he is, from my point of view, the 20th century. (CAGE, 2016, p. 81)

> I do not know whether I am being rabid about Satie or not. However I give him first place with Webern and I fight for them both…the fourth [of the *Cinq Grimaces,* 1915] [is] very important from my own point of view because it is written in the same rhythmic structure that I have employed in all my work since 1938. (CAGE, 2016, p. 82)[9]

In the first letter, note that a decade after his studies with Schoenberg Cage still refers to him as granting a measure of "liberation" from Beethoven and all that Beethoven symbolized for him—indebtedness to tradition, being yoked to tonal harmony—but has already shifted his allegiance to Erik Satie as a musical liberator. The second letter provides the exact source: an extremely minor work of Satie's composed in 1915 for a planned circus-like performance of Shakespeare's *Midsummer Night's Dream* organized by Edgard Varèse and directed by Jean Cocteau. The 1:2:1 proportions that Cage provides in his 1958 article as an example of how macro-microcosmic structure operates in his own music come from this tiny sixteen-measure piece by Satie (Figure 7).

[8] Satie's influence on Cage is discussed in greater detail in PERRY (2014).

[9] The context for these letters may have been a performance of Satie's *Messe des pauvres* that Yates added to an otherwise all-Schoenberg Evenings on the Roof program in May 1948. (CRAWFORD, 1989, p. 194).

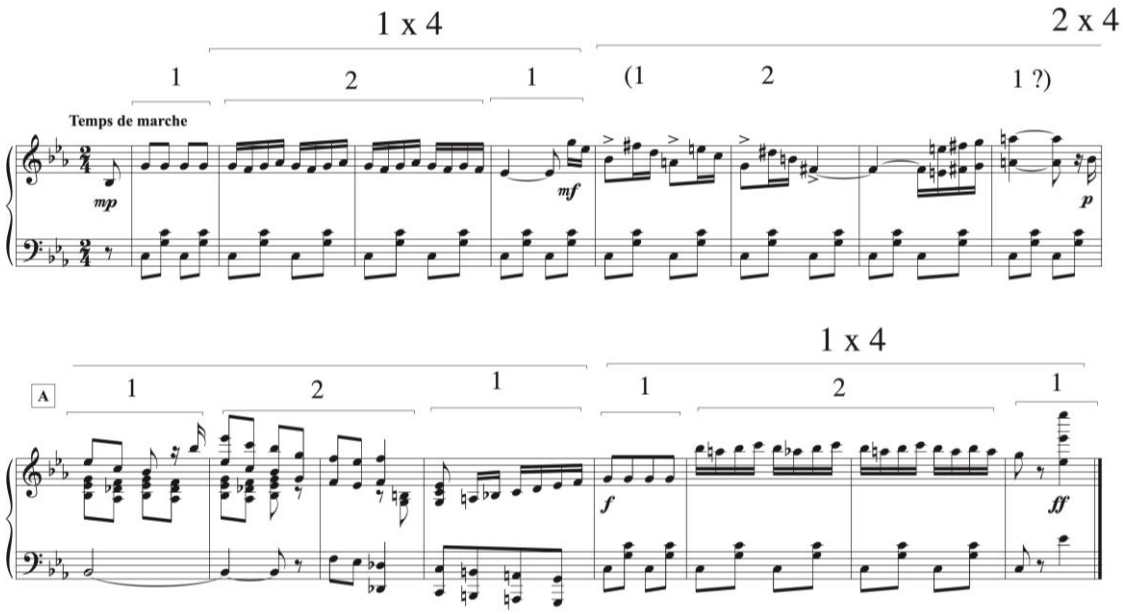


Figure 7: Satie, "Fanfaronnade" (Cinq Grimaces pour "Le songe d'une nuit d'été," iv, 1915) This is the source of the 1:2:1 "macro-microcosmic" structure shown in Figure 6(a). From Perry (2014).

Cage's imputation of such temporal planning to Satie appears somewhat specious; it's unclear whether or not Cage would have detected a 1:2:1 rhythmic composition of the piece unless he were already looking for it. Cage seems to have known only the piano version of the *Cinq Grimaces*, but Satie also produced an orchestral version, possibly with the assistance of Darius Milhaud (ORLEDGE, 1990, pp. 309-310); Figure 8 shows that the added percussion parts contradict the rhythmic proportions that Cage detects in the piano version.

The speciousness of Cage's claims on behalf of Satie actually strengthens the importance of Schoenberg's teaching and example as a source of Cage's insights about rhythmic structure. Schoenberg's emphasis on counterpoint and the examples he provided in his own music of the interplay of clear structural boundaries with more fluid content provide a clear model for Cage's opposition of grace and clarity, structure and form.

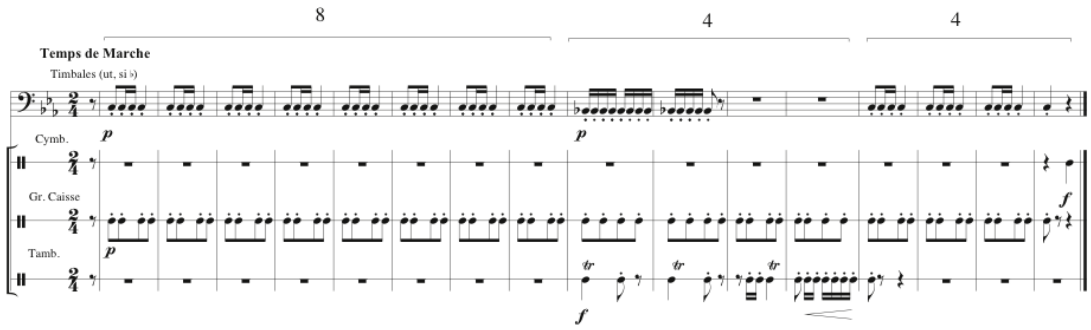


Figure 8: Satie, "Fanfaronnade" (Cinq Grimaces pour "Le songe d'une nuit d'été," IV, 1915), percussion parts. From Perry (2014).

Schoenberg, further (unlike Heinrich Schenker, who often writes about rhythm as something that merely distracts the listener from perceiving the underlying unity of pitch structures), considers the rhythm and phrase structure of a theme to be an integral part of that theme's identity (SCHOENBERG, 1975, pp. 225-226). Criticizing his contemporaries in 1923, he claims that "my themes (with few exceptions) are such that they *could not* be harmonized with the old harmony, that in the manner in which their phrases are combined they diverge altogether from the conventional, that the differences among the lengths of their phrases, as well as the changing location of their points of stress, inevitably bring about the creation of new forms" (SCHOENBERG, 1975a, p. 167). This statement affirms an agential role for rhythm, proportion, and accent in Schoenberg's music.

In 1948 Cage gave a lecture at Black Mountain College in which he asserted that at some point Western music had taken a wrong turn—that wrong turn epitomized by the music of Beethoven. Satie was the remedy to this wrong turn. Returning to his grace/clarity duality and elaborating on it somewhat, Cage opined that

> Music…must have a structure; that is, it must have parts that are clearly separate but that interact in such a way as to make a whole…
> In the field of structure…there has been only one new idea since Beethoven. And that new idea can be perceived in the work of Anton Webern and Erik Satie. With Beethoven the parts of a composition were defined by means of harmony. With Satie and Webern they are defined by means of time lengths…Beethoven was in error, and his influence, which has been as extensive as it is lamentable, has been deadening to the art of music. (CAGE, 1970, pp. 78-79, 81)

Cage's reference to Webern is a bit anomalous. Although in his Satie marathon at Black Mountain College in 1948 he included only one non-Satie work, Webern's *Three Little Pieces* for violoncello and piano Op. 11 (1914), it is unclear what Cage heard in either Webern's music that might have given him the idea that the latter, of all people, was on his side of the rhythm versus harmony issue. Perhaps Webern represents a waystation in Cage's gradual distancing of himself from his teacher, Schoenberg. Like Boulez at approximately the same time, Cage may have felt that what Schoenberg got wrong, Webern had gotten right; in a later article Cage would, by implication, praise Webern's "concern for discontinuity" (NATTIEZ, 1990, p. 22). As a fellow pupil of Schoenberg's, Webern may have served for a few years as a kind of *Doppelgänger* for Cage; like Cage, Webern had once been the subject of Schoenberg's harsh, discouraging pedagogical methods, but ultimately, to some extent, broke free of his teacher's influence and forged his own path. In any event, Cage felt that he no longer needed Webern, or needed to champion his music, particularly once the Darmstadt composers began to take him up as their patron saint in the early '50s. Satie remained his role model of choice.

There are significant parallels between Cage's 1948 Black Mountain College lecture and certain passages in Schoenberg's *Fundamentals of Musical Composition*. For example,

> Without organization music would be an amorphous mass, as unintelligible as an essay without punctuation, or as disconnected as a conversation which leaps purposelessly from one subject to another.
>
> […] The presentation, development, and interconnexion of ideas must be based on relationship. Ideas must be differentiated according to their importance and function.
>
> […] Man's mental limitations prevent him from grasping anything which is too extended. Thus appropriate subdivision facilitates understanding and determines the form.
>
> […] No beginner is capable of envisaging a composition in its entirety; hence he must proceed gradually, from the simpler to the more complex...It will be useful to start by building musical blocks and connecting them intelligently.
>
> These musical blocks (phrases, motives, etc.) will provide the material for building larger units of various kinds, according to the requirements of the structure. Thus the demands of logic,

> coherence, and comprehensibility can be fulfilled, in relation to the need for contrast, variety and fluency of presentation. (SCHOENBERG, 1967, pp. 1-2).

The organization of Schoenberg's text, which is admittedly the result of posthumous editing by two of his American students, reflects its author's belief in a nesting taxonomy that rises from the level of the motive and phrase up, through the period or sentence, to the simple sectional forms and thence up to sonata form, which in turn is typically understood and taught mainly in terms of its constituent segments. Although his American pupil did not come to share Schoenberg's Germanic organicism, Cage's system of macro- and microcosmic rhythmic proportions, therefore, resembles Schoenberg's *Formenlehre* in its unfolding in terms of multiple, nested levels of structure.

To the end of his life Cage would advocate for Satie as the essential composer of the modern era and draw on his music as a source of inspiration and material for his own. Schoenberg is mentioned somewhat later in his Black Mountain essay as having "provided no structural means, only a method—the twelve-tone system" (CAGE, 1970, p. 82). Despite this negative acknowledgement of his old teacher's contributions this sentiment—that conscious use and manipulation of rhythmic structure is the one feature essential to the music that Cage found worth championing—has as much to do with what Cage learned from Schoenberg as it does with Satie or Webern (or with the classical music of India, another topic of the lecture).

The common thread connecting Cage's thought at this crucial juncture in his compositional development with that of Schoenberg lies in a shared Aristotelian view of musical form—a conception of form in terms of a whole that divides into large parts, each of which in turn divides into smaller parts—a nested structural scheme that Cage adopted for his own starting shortly after his studies with Schoenberg concluded. This is laid out in the textbook Schoenberg was writing for his students in California, and thus presumably in what he taught to Cage and his classmates. This connection is far clearer than the somewhat fanciful connection that Cage makes between his rhythmic strategies and anything in Satie.

In his exploration of the commonalities between language and music—and the limits thereof—David Lidov posits an opposition between inflection and articulation. The latter "implies a point of *division*," while "an inflection of sound is a shape of *continuous* change imposed on it" (LIDOV, 2005, p. 3) A conflict between means of continuous inflection and a means of form-delineating articulation is a feature of both Schoenberg's and Cage's work; such a thread may be traced through the mature work of the former and the early and middle work (at least) of the latter.

Considering Schoenberg's Third String Quartet (the work of his that Cage knew best) in terms of the interaction of its phrase rhythmic structure and its

twelve-tone content we might find the seeds of both Cage's notion of the interplay of grace and clarity, and of the latter's focus on rhythmic proportions as the essential element of the "new idea" that Cage posited for contemporary music. Although Schoenberg's use of tempo inflection as a means of delineating formal boundaries in the Third Quartet may not have appealed to Cage as a stylistic remnant of Romanticism, it might have interested him as a means of reconciling continuity and division, to refer back to Lidov.

Schoenberg's refusal to attend any of the percussion concerts that Cage organized in Los Angeles in the years following his studies with Schoenberg is one indication that Cage did not gain a satisfactory level of validation from his teacher, despite the well-known pronouncement by Schoenberg that Cage was the one interesting student he had taught in the 1930s, and an "inventor of genius." By the time that Cage wrote the "Grace and Clarity" essay and delivered the Black Mountain lecture he seems to have decided that Schoenberg could not possibly have been the source for his ideas about macro/microcosmic structure and the rhythmic bases of musical form. The substitution of Satie for Schoenberg in Cage's personal pantheon is perhaps more an example of the anxiety of influence than an accurate account of where, in fact, Cage got his ideas.

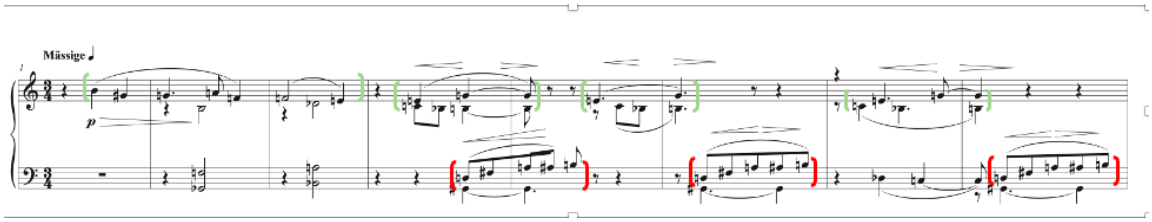


Figure 9: Schoenberg, Klavierstück Op. 11 no. 1, mm. 1-8, showing interplay of figures (green) and emergent figures (red). Accompanimental materials (ground) are unlabeled.

I have mentioned the persistence of the dichotomy between figure and ground in Schoenberg's post-tonal music. Clearly there are melodic lines and accompanimental gestures throughout his music, from the earliest phases of his abandonment of tonality through to the mature serial works. For example, Op. 11 no. 1 begins with a series of melodic figures undergirded by chordal gestures (Figure 9). A countermelody in eighth notes begins on the anacrusis to m. 5, establishing that the dichotomy is a dynamic one—there are emergent details that mediate between figure and ground.

In the Op. 19 pieces of a few years later (see Figure 10), although with respect to form Schoenberg has progressed far beyond the Wagner-tinged tone poems of Op. 11 he retains not only the interplay between figure and ground, but also the possibility of entities that mediate between them, entering as subsidiary details but acquiring foreground status along the way. The ability for material to

emerge from the background and assume foreground status persists as a dynamic element of Schoenberg's style.

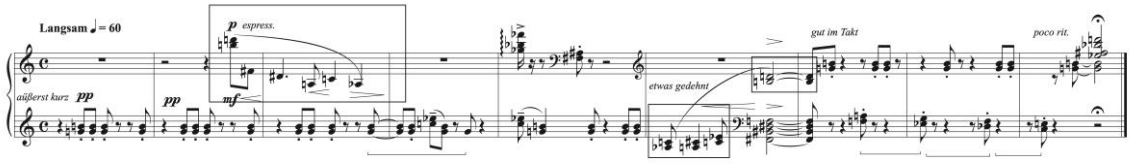


Figure 10: Schoenberg, Kleine Stück Op. 19 no. 2, showing interplay of figure (boxed) and ground. Brackets indicate emergent figures. Ground ostinato and accompaniment materials are unlabeled.

His occasionally obsessive notational precision (examine, for example, the score to the Fourth String Quartet Op. 37, where in certain passages almost every note bears an articulation marking) suggests a need to assign each aspect of his musical surface a specific role, to define a foreground, middleground and background which, while often clearly delineated, may merge into one another, creating a dynamic musical space. In much of his later music, and especially in both of the last two string quartets his *Hauptstimme* and *Nebenstimme* indications draw our attention to the dialogue between figure and ground, thus further differentiating figures that occur in the near and middle distance from unmarked details in the score, which often fulfill background functions. Such notational precision would, as he must have known, have been unnecessary if the musical gestures that populate his music's surface were presented in the context of conventional tonal harmonies.[10]

The way in which many of Schoenberg's paintings seem to struggle with the concept of forced perspective suggests an awareness by Schoenberg the artist that separating space into foreground and background was not an easy thing to do, given the constraints of his style, rooted in Expressionism with its intermingling of the psychological and the representational. Consider two of Schoenberg's paintings from 1911, the year of the Op. 19 *Klavierstücke: Burial of Gustav Mahler* and *Gehendes Selbstportrait.*[11] In both the background is atilt; there is no clear sense of up and down, and the figures in the foreground seem to be in some danger of being enveloped by, or falling off, the background. Schoenberg's representational canvases suggest that, in the absence of gravity, an unproblematic sense of up, down, far and near is by no means a given; the problematization of the viewer's sense of their orientation within the space evoked by the painting is a recurring topic of Schoenberg's art—and of his music.

[10] I thank Walter Frisch for adroitly exploring connections between Schoenberg's paintings and his compositional path in his keynote talk during MusMat 2021.

[11] These two oil paintings are listed by the Arnold Schoenberg Center, Vienna as having catalog raisonné numbers 153 and 18, respectively. Photographs of them may be viewed at https://www.schoenberg.at/index.php/en/component/joomgallery/selbstportraits/018-25 Accessed 15 March 2022.

One might suspect that Schoenberg's fondness for painting self-portraits in very tight closeup provided a way to avoid this issue, but one need only examine the self-portraits to see that he found other ways to explore questions of distance and depth. A sampling of self-portraits from a single year, 1910, suggests a gradual dissolution of the foreground figure into an unknown, nebulous background—again, a feature of artistic Expressionism.[12] As a painter he was freed from the demands he felt upon him as a composer to find means of order and form generation; in his music he was led back to some of the textures he had learned to manipulate from Haydn, Wagner, and the composers that came between them precisely because he needed textural differentiation (and the duality of foreground figure and background) to assist his project of creating coherent temporal forms in the absence of tonal architectonics.

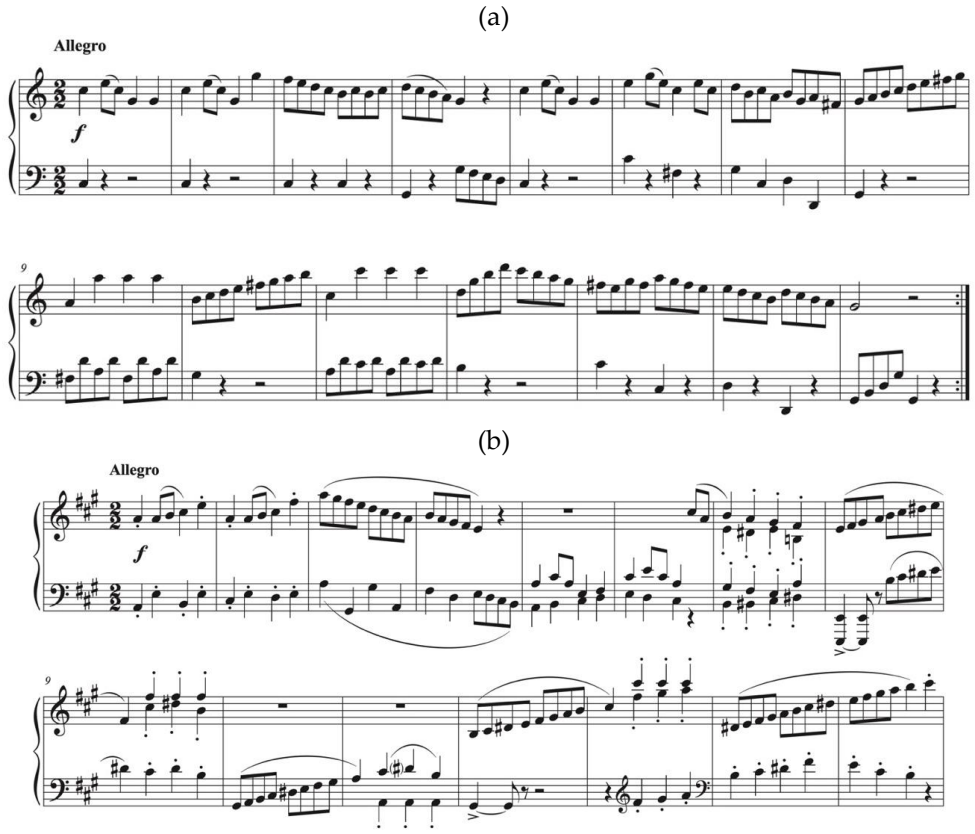


Figure 11: (a) Muzio Clementi, Sonatina Op. 36 no. 1, mm. 1-15; (b) Corresponding passage of Erik Satie, *Sonatine bureaucratique*.

[12] The self-portraits I am thinking of are the well-known blue head, the disembodied, realistic head on a gold-yellow background (currently in the Verne Knudsen Collection, Los Angeles), and the nightmarish "red gaze":
https://artsandculture.google.com/asset/blue-self-portrait-arnold-sch%C3%B6nberg/jQF4_n0_uz2ROA
https://montecristomagazine.com/magazine/spring-2009/revolution-9
https://www.schoenberg.at/index.php/en/gazing-into-the-soul-with-schoenberg?arnold-schoenberg-blick-karl-kraus-die-chinesische-mauer
Accessed 15 March 2022.

Schoenberg's divorce of rhythmic and textural markers of syntactic function from the tonal system that originally gave birth to them is one of the things that led to Boulez and the Darmstadt school's disdain for his music; his stylistic evolution might be summarized as a search for a new syntax to pair with the textures he had inherited from the eighteenth and nineteenth centuries, now that they were shorn of the form-delineating and form-generating possibilities that harmony afforded the composer of tonal music. To summarize the matter rather simplistically: Having freed his musical surface from its original syntactic underpinnings, Schoenberg sought a surrogate in the twelve-tone row; that surface carries forward textures familiar from tonal music, causing a sense of displacement or disorientation analogous to that felt when one looks at one of Schoenberg's paintings.

This act of syntactic emptying and replenishing might have interested Cage, since many years later he adopted something like it as one of his favored working methods. It also allowed Cage to connect his real teacher, Schoenberg, with his imagined mentor, Satie. In 1969, Cage was to arrange Satie's *Socrate,* a chamber drama for voices and orchestra, for piano solo, to accompany (and be accompanied by) a dance that Cunningham had created for his company. Alas, Satie's publisher insisted on an exorbitant royalty. The dance already created, Cage needed music with the same duration and temporal structure as *Socrate* but without the encumbrances of copyright.

Cage knew that Satie had accomplished a similar act of syntactical emptying in works such as the *Sonatine bureaucratique* (1917), which is at least in part a parody of Clementi's easy sonatina in C, Op. 36 no. 1 (Figure 11). As Robert Orledge points out, Satie was fond of taking melodies by composers like Clementi, Gounod, Mozart, Chabrier, and Chopin, retaining their rhythmic profile, and effacing their contour and intervallic properties in whole or in part. The effect is parody, or even ridicule; Cage, however, seems to have seen a deeper point in these exercises, taking Satie's own rhythmic structure from *Socrate* and using the *I Ching,* his favorite oracle, to add new pitches and intervals. His work *Cheap Imitation* accomplishes a selective erasure that posits the independent coherence of rhythmic structure, even torn from its original tonal context. Figure 12 provides an illustration of Cage's process.

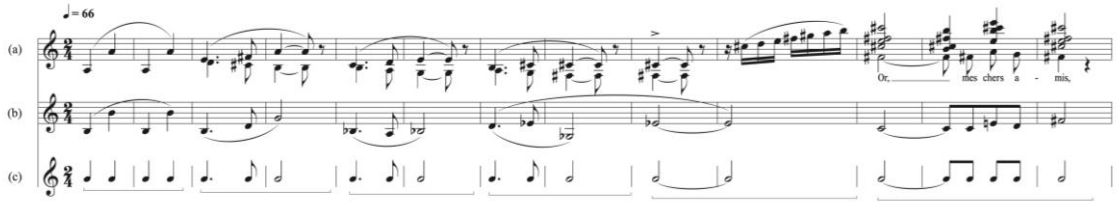


Figure 12: (a) Satie, Socrate, I, mm 1-12**.** (b) Cage, *Cheap Imitation*, I, mm. 1-12**.** (c) Rhythmic structure that Cage extracts from Satie. Brackets show microcosmic groupings.

Like Schoenberg, Cage seeks to preserve the surface rhythmic life of his model while substituting a new pitch vocabulary for the old one; where Schoenberg sought a new syntactic system to replace that of tonality and felt that he had found one that spoke through the twelve-tone row, Cage begins to move beyond syntax entirely. By the 1960s, in his music Schoenberg's lessons in counterpoint has become generalized into a disciplined interplay between system and freedom, between grace and clarity.[13]

Coherence remains in Schoenberg despite the absence of tonality. Even granting that the preservation of coherence is largely due to the relationships made available by the twelve-tone system, the ordering of the musical surface afforded by the persistence of the figure/ground relationship serves at least an ancillary role. What Cage may have taken from his work with Schoenberg (rightly or wrongly) was a sense that musical coherence may be obtainable when all that remains at the composer's disposal is this relationship, this way of ordering a musical texture. Even despite the coherence that serial methodology provided him, Schoenberg had, it seemed, loosened the relationship between the way that thematic and subordinate shapes occupy the musical surface, and the way that pitch relations shape that musical surface. Once Schoenberg had loosened this relationship Cage was free to remove it altogether. In Cage's hands the compositional multiplicity that Powell found in Schoenberg's music largely became a matter of counterpoint between syntactic and asyntactic methods of composition (CAGE, 1973, p. x).

In a 1973 letter to the composer Dieter Schnebel Cage reflected at length about his relationship with Schoenberg. He repeated what by then had become some well-known statements about himself and Schoenberg (the "inventor of genius" quote, for example) but also spoke frankly about his frustrations:

> How did I get along with him? How can I say? He could do no wrong, while I could do no right...Let us say that he was devoted to discipline and that he was gifted to transmit to others that devotion. My understanding of discipline is that it frees us from the tyranny of likes and dislikes that arise within us. Could Schoenberg have seen my composition using chance operations as following essentially his teachings? I doubt it. He tended, I believe, to look backward (to Bach, Beethoven, Brahms, Schoenberg) more than to look forward, though he did look forward (Webern). [...]
> I was deeply struck when he said, before a large class at USC: My purpose in teaching you is to make it impossible for you to write music. I determined then and there to devote my life to nothing else. (CAGE, 2016, p. 440)

It is significant that Cage's recollections of Schoenberg are negational; Cage sees Schoenberg's gift of discipline in terms not of what it affords the

[13] I explore this topic extensively in PERRY (2021).

composer but rather in what it takes away ("the tyranny of likes and dislikes that arise within us"). Here Cage seems to admit that his misreading of Schoenberg, even more than his beloved Satie, was the source of his subsequent musical innovations and aesthetic orientation. By 1950-51, in composing his Concerto for Prepared Piano and Orchestra, Cage sought to move beyond personal choice, preferring rather to find ways of "throwing sound into silence," as he said in a 1950 letter to Boulez (CAGE, 2016, p. 141; see also PRITCHETT, 1993, pp. 70-73). He thereupon adopted the I Ching, conceiving of the oracle as a means of asking questions rather than imposing system. In the following year he tells Boulez that adopting chance procedures "I freed myself from what I had thought to be freedom, but which was only the accretion of habits and tastes" (CAGE, 2016, p. 150). In this negational manner the discipline that he learned from Arnold Schoenberg indeed made it possible, not impossible, for Cage to move forward and write music—a negation of Schoenberg's negation!

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11

Arnold Schoenberg and the Composition Movement in Bahia: The Interplay of Organicity and Inclusivity

Paulo Costa Lima

This chapter was built from the rejection of the idea that influence is a unidirectional phenomenon, and from the conviction that it is necessary to understand the Schoenbergian heritage in Brazil in cultural terms. And, in this sense, it sought to imagine a possible dialogue between this heritage and the context of the Composition Movement in Bahia, taking the composer Ernst Widmer (1927-1990) as a legitimate representative. After pointing out several paths taken from the identification of serial strategies present in the work of Widmer and his first students — Lindembergue Cardoso, Fernando Cerqueira, and Jamary Oliveira —, it devoted attention to the process of formulating laws of composition and its teaching, Widmer (1988). By identifying the cultural dimension of the laws of organicity and relativization (or inclusivity), it revealed an articulation between the Germanic organicist tradition and the maximalization of simultaneities typical of Bahian cultures, discussing possible implications of this articulation for our multicultural times, and even for the friction between modernism and postmodernism. From the imagination of this dialogue, it was possible to ask about the relationship between Schoenbergian thought and the function of inclusivity, and to gather elements to suggest that the universe of rhythmic constructions in Schoenberg can become a promising field for a future reflection on this kind of dance between organicity and relativization (or inclusivity) in Schoenberg himself.

I wept as I listened to the Fourth Quartet. Now I know for certain that you are the last Classical composer: your cradle was Beethoven's Grosse Fugue, where there is none of that Russian, French, or English folklore, and the barbarism of presenting a symbol instead of a direct experience (...) Bach, Beethoven, and Schoenberg are the last composers capable of erecting a musical structure that must be regarded as an organic world.
Oskar Kokoschka to Arnold Schoenberg, 19 August, 1949

Organic change was a central aspect of his creative approach (…) Schoenberg's absolute commitment to rigorous discipline and structural unification became in a sense a condition of inner survival
Alexander Ringer

What is most important to me, and I did that a lot in Brazil, is inclusivity, or, for example, addressing the audience differently[1].
Ernst Widmer

I write this chapter from the perspective that has been built up over the years by our research on the Composition Movement in Bahia. We started this investigation in the 1990s, from the interest in describing and analyzing the pedagogy of Composition set in motion by the Swiss-born Brazilian composer — Ernst Widmer (1927-1990) —, who began his teaching activity in Composition among us in 1963, at the Music Seminars of the Federal University of Bahia, and this teaching (by the way, no longer tutorial but in group) was the catalyst for the creation of the *Grupo de Compositores da Bahia* (Group of Composers from Bahia) — GCB in 1966, through the launch of a quite original manifesto, presenting an entire program in just one line: "In principle, we are against any and all declared principles" — Lima (1999).

So, Ernst Widmer had arrived in Bahia in 1956, at the invitation of the German composer Hans Joachim Koellreutter (1915-2005), creator of the Music Seminars, widely recognized as the introducer of the ideas of Schoenberg's dodecaphonism in Brazil. This scenario is important to emphasize that the environment where this Composition Movement was born was not far from the impact of Schoenberg's thought, quite the contrary, since it was idealized by the composer who for the first time spread these ideas in Brazil.

So, in the midst of the interest in the pedagogy developed by Ernst Widmer, the awareness of the importance of the relationship with culture, leading to the identification, in several works of this author, of representations of a crossing, the voyage that crosses the Atlantic and connects the background of Germanic culture with Bahian cultural polysemy[2]. This perception ended up

[1] *Was mir am meisten am Herzen liegt, und das habe ich in Brasilien sehr viel gemacht, ist Inklusivität, oder eben z.B. das Publikum anders ansprechen*. Ernst Widmer, 1988, an interview given in Aarau, Switzerland.

[2] In general, we have used the work *De canto em canto: uma possível resposta* op. 169 as a reference in this direction, especially due to the intriguing encounter between the Symphonic Orchestra of

having a significant impact on the most recent configuration of the research, which then began to focus on the cultural perspectives associated with composing, or rather, the cultural perspectives inherent to composing.

The Composition Movement in Bahia has been an observatory, a laboratory of musical experiences that offers itself to our questions as an exemplary case for the formulation of paths and possible solutions, which, despite being formulated at the local level, can have a much broader relevance. Such was the context, for example, of the creation of the notion of "resignifying distance" — Lima (2018) and (2020) — an analytical tool that allowed dealing simultaneously with the construction of structures and narratives in works by composers of this Movement, thus offering a possible way to the broader question of the intertwining of Composition and Culture.

We thus envision the environment in which our research questions are being constructed, and it is from this perspective that we affirm that Schoenberg's enormous contribution to Brazilian music — whether as a paradigm of compositional invention, or as a theorist, teacher and leader of a movement — also needs to be understood in cultural terms, that is, abandoning the model of a single way of agency, thinking, therefore, of the performance of local agents who receive and work with the impact of the heritage of Schoenbergian thought, and thus, getting involved with shared values and senses of belonging, in other words, facing the challenge of intertwining musical creation and culture. It is not a simple thing. It seems appropriate to recall an essential reflection by Bohlman (2003, p. 46):

> Above all, why does the historiography of Western art music, which includes historical musicology, music theory, ethnomusicology, and popular-music studies, cling to the counterintuitive assumption that music and culture are separate? (...) I wish to suggest that one reason there is resistance to accepting the relatedness of music and culture results from the paradoxical unwillingness to admit to the full range of cultural work that music accomplishes.

We are here before one of the most radical criticisms of recent times of the body of discourse on music, and we are witnessing the subtle appearance of the notion of "cultural work", a dimension that is associated with compositional decisions or attitudes - whether it be the invention of a method for composing with twelve tones that are only related to each other, whether it is the appreciation of the notions of *Grundgestalt* and Developing Variation, or the

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Bahia and the *Afoxé Filhos de Gandhy*; but the figure of the crossing is also present, for example in the *Sonata Monte Pascoal* op. 122 for solo piano, among others.

"maximization of simultaneities"[3] that marks Bahian culture, from the Baroque to Tropicalism and the *avant-garde*.

Thus, the way this text has been conceived may be outlined: to place our research context — the Composition Movement in Bahia, which has been a precious theoretical-analytical observatory of cultural perspectives inherent to composing — as a fundamental part of the reflection on the cultural dimension of Schoenberg's heritage and contribution in Brazil. We will work, therefore, reverberating the format of the case study, that is, seeking to imagine this experience in Bahia as an exemplary case for the broader theme. From the point of view of our research, we will be asking how to understand the Schoenbergian contribution in the context of the Composition Movement in Bahia, with emphasis on the work of Ernst Widmer (1927-1990), hoping that the possible answers also bring elements about the cultural dimension of the Schoenbergian thought and its reverberation in Brazil.

Well, this perspective of the issue needs to go hand in hand with the awareness that, in the case of the Schoenbergian heritage — as in many others — it is not simply a matter of expanding a set of ideas and visions, or even of translating a movement from a society to another. It is about seeking to understand the dynamics of the interaction between the contribution of new ideas and proposals and the context in which they are received. The value of heritage refers to interpretive work in relation to the legacy, and not necessarily to an unrestricted adherence to its paths.

How could we expect to deal only with influences, if the object of composing (ideas, problems, theories) does not remain unchanged as it moves from one society to another? And, in a special way in Brazil, given that Brazilian society is so distinct? European historicity does not overlap with the complex historicity of this complex tangle of people and cultures that flourished here. To ask about Schoenberg's influence in Brazil is, in fact, to ask about this dialogue of historicities — or else, to face the enormous risk of dealing merely with an exercise in the celebration of hegemony and dependence. A society marked by this somewhat dizzying confluence of people and cultural contributions will demand a certainly differentiated relationship with the construction of the imaginary; therefore, however necessary and comprehensive Schoenberg's formidable theoretical-musical construction may be, the Brazilian cultural trajectory will bring into play additional perspectives.

The opening to discuss the dynamics of the reverberation of Schoenberg's contribution in Brazil will require talking about local agents, how they interpreted this contribution and interacted with it, and thus, the cultural work involved therein. This is, of course, a political dimension of the issue, a vision that bets on decolonization. We are thus in the field of reasons and justifications

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[3] This expression was coined by José Miguel Wisnik when dealing with the *Tropicália* movement, but it fits as a broad reference to Bahian cultural contexts.

for this text to have taken this possible exemplary case as an object of attention, reconstructing links between the thinking elaborated by a Brazilian leadership as part of an active context (in this case, Ernst Widmer and the Grupo de Compositores da Bahia) and the Schoenbergian heritage. But why can we consider the connection with Ernst Widmer and the Composition Movement in Bahia as a kind of exemplary case?

Well, firstly, because they are two leaders of the Composition area, widely recognized, and both immersed in the construction of a movement — despite the difference between the contexts. In addition, both were involved with teaching composition, a dimension that is added to the movement, and besides that, with the creation of composition theory. When talking about each of them, we are also talking about marks that were built in each of the disciples, and with each of them, giving rise to different reverberations. In the case of Widmer and the creation of the Grupo de Compositores da Bahia (1966), formed basically by his students, a radical horizontality is identified in the teaching relationship, and a decisive investment in the uniqueness of each student. We are dealing with an important movement in Brazil, I would even say that it is unique in terms of maintaining the same dynamics for sixty years of existence4, without characterizing at any stage the creation of a school of composition, in the sense of stylistic uniformity or vision, forming several generations of composers — from the first ones which included Lindembergue Cardoso (1939-1989), Fernando Cerqueira (1941-), Jamary Oliveira (1944-2020), passing by Agnaldo Ribeiro (1943-), Ilza Nogueira (1948), Paulo Costa Lima (1954) -), Wellington Gomes (1960), to the most recent ones, designed from the 1990s onwards, with Antonio Fernando Burgos Lima (1948-2008), Pedro Augusto Dias (1966-), Alexandre Mascarenhas Espinheira (1972-), Alex Diniz Pochat (1974-), Pedro Ribeiro Kröger (1974-), Marcos da Silva Sampaio (1977-), Guilherme Bertissolo (1984-), Paulo Rios Filho (1985-), Eric Barreto (1985-), Vinicius Borges Amaro (1988-), among several others.

Despite all this, we understand that it is necessary to start with palpable elements about dealing with Schoenbergian ideas in Bahia in the 60s. During the first stage of the research mentioned above, in the interviews carried out with the composers Fernando Cerqueira (1941-) and Jamary Oliveira (1944-2020), members of the *Grupo de Compositores da Bahia* and first generation of students, we deal with the teaching of serial techniques by Ernst Widmer. It is worth noting some of the answers — Lima (1999, p. 156):

> PCL – And serialism, dodecaphonism, did that also come in early, from the first year? [with 1963 being that first year]:
> FC - Yeah, it came in...

[4] Taking the year 1963 as a starting point, the first year in which the subject Composition was taught in a group format by Ernst Widmer, at the School of Music at Federal University of Bahia (UFBA).

> PCL – But I speak about the teaching of techniques in the classroom. Did you do traditional exercises?
> FC – He tried to apply the same rigor that he used in the search for a cantus firmus that would be good for you to make a good counterpoint; he tried to apply it in the series, in a series that was good in terms of relationships and that did not have harmonic fields that made it easily fall into a tonal relationship, so there was this rigor of asking us to do it like this, and then let each one do it as well understood, whether or not using these relationships; when it was an exercise, he applied it, demanded it rigorously, laid down the rules, but when it came to composing, he didn't.
>
> PCL – Now, serialism, did you work with twelve-tone matrix?
> JO – Ah, yes, but European-type, not Babbitt...
> PCL – And did he comment on his own relationship with serialism? If he used series...?
> JO – You find little serial stuff, there's a tribute to Dylan Thomas.
> PCL – Ceremony after a fire raid?
> JO – That one... it was exactly in the period of Dylan Thomas' death, and in the period when Stravinsky was starting to write serial. (...) The other piece of his that is serial is also the Ave Maria...
> PCL – But she uses a set of nine notes...
> JO – One thing that Widmer never adopted was Viennese serialism, for him serialism was closer to the meaning we use today for serialism, it was never twelve-tone...
> PCL - ... it was all about small sets.
> JO – Yes, more about small sets.

Three things deserve emphasis: the conviction that the subject "serialism" should be introduced as early as possible to students of composition; a significant openness towards composing, allowing new ideas to emerge from this path; the differentiated interest in the serialism of sets smaller than the aggregate.

Thus, as it was possible to demonstrate throughout the research, the Schoenbergian heritage reached the composer Widmer directly in his creative plot. When dealing with serial strategies in Widmer's work, I observed the following — Lima (1999, p. 313):

> Taking these processes of approximation of the aggregate as an object of attention, we can register in Widmer's work, the segmentation made by trichords (Ignis, op 102), segmentation by tetrachords (Ceremony after a fire raid op. 28, Partita II – Louré op. 23, Block I op. 27, Prismas op. 70), symmetry as a criterion for segmentation (Quintet II op. 63), cycle of continuous transformations between segments (Trégua op. 93-b), strict

> twelve-tone serial canon (Block I op. 27), approach to the aggregate as an initial totality from which some specific contours are outlined (Synopsis op. 64), approach to the aggregate through the interaction of octatonic formats (Sonata Monte Pascoal op. 122), aggregate reached by a succession of thirds (Vértice op. 112), melodic presentation of the aggregate in two halves (Cosmophonia II op. 162), and, lastly, but quite frequently, what we could call quasi-aggregate situations — arriving at complex sets of 9, 10 and 11 elements, hit at from basic cells (Suite op. 6, Bahia Concerto op. 17)

Regarding the identification of the series used, it is worth illustrating some examples:

a) *Ceremony after a Fire Raid* op. 28-1962,
   [(11, 1, 2, 0), (7, 3, 6, 10), (5, 4, 8, 9)];
b) Wind Quintet op. 63 - 1969/1975
   [(8, 11, 0, 7), (9, 4, 10, 3), (6, 1, 2, 5)];
c) *Prismas* op. 70, Pn and Orchestra - 1971
   [(11, 1, 2, 0), (10, 6, 7, 3), (8, 5, 4, 9)];
d) *Ígnis* op. 102, Mixed Ensemble – 1977
   [(4, 7, 9), (11, 6, 8), (3, 1, 10), (0, 5, 2)].

The work *Ígnis* op. 102 should deserve special emphasis. Like the others mentioned, it conceives the series in terms of subsets, but uses only subsets of type [025], a sonority closely linked to the melodies produced by different Brazilian cultural traditions. There would be space here to deal with this subject as a kind of hybridization between the procedure inspired by Schoenberg and the clearly native oriented sound. It doesn't hurt to remember that his work *Coco op. 22* - 1961, the first officially linked to Brazilian cultural materials, abundantly designed sets of type [025]. Below, the serial scheme used in *Ígnis* op. 102 and presented right at the beginning of the work. Also noteworthy is the mirror game that guides the presentation of the series (Table 1).

Table 1: Serial scheme used by *Ernst Widmer in Ignis op*. 102.

4	7	9	11	6	8	8	6	11	9	7	4
3	1	10	0	5	2	2	5	0	10	1	3
E	G	A	B	F♯	G♯						
E♭	D♭	B♭	C	F	D						

To conclude this brief mention of the serial strategies used by Widmer, it is worth noting the theme in Figure 1, which appears at a certain point in *Ópera da Liberdade* (a work left incomplete), in fact, derived from the motet *Crux*, by Father Nunes Garcia.

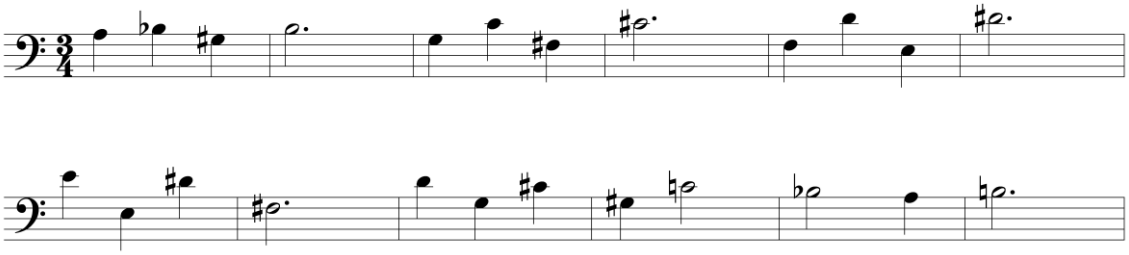

Figure 1: Ernst Widmer – *Ópera da Liberdade* op. 172 (1989) – *O povo e Tiradentes* (mm. 185-196).

This construction repeats aspects of previous experiences — a structure segmented into tetrads, in which the first and third are the set [0123] and the middle one the set [0156] — but there is a novelty, it plays with the series of all intervals, which unfolds like a cycle, from semitone to octave, and back to the point of origin.

This game of identification of marks of Schoenberg's thought could be extended to several members of the GCB, and also to all other generations of composers trained in the scope of the Bahia Movement up to the present day. Some examples: Silva (2002) identified compositional procedures in works by Lindembergue Cardoso, involving dialogues between serial practices, tonal references, and rhythmic planning. In a special way, the tonal implications present in the series used in *Voo do Colibri* for Harpsichord and String Orchestra. Castro (2007, p. 54) records this statement by Fernando Cerqueira about his relationship with serialism:

> I use serialism in the critical way I mentioned above, I use complex and multi-serial serial processes, starting from original elements, no matter if tonal, modal or atonal, to generate sound material that allows continuous and contrasting reordering of fragments to compose the structures and textures of the works, always with the intention of building an expressive and clear poetic-musical sense.

Still in Fernando Cerqueira, Castro (2007, p. 106) comments in detail on the series used and the transformation processes in the 1967 Trio *Metamorfose* (Figure 2).

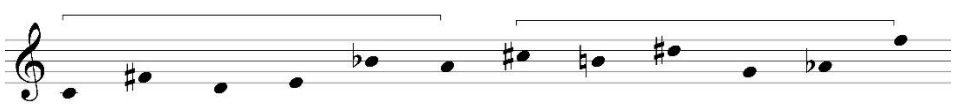

Figure 2: Fernando Cerqueira, Série do *Trio Metamorfose* (1967).

As for Jamary Oliveira, we have already seen his own statement above about the compositional interest in small-set serialism. The work itself, music for solo piano, is a kind of culmination of this line of development, a virtuous example of the use of this perspective of construction of musical discourse, at the same time extremely rigorous in its procedures of concatenation of materials and extremely open in terms of references and stylistic accommodations, being able to activate an arc of affects that goes from minimalism to jazz and *candomblé* music, being still a "true musical experiment", as the composer Herbert Brün exclaimed when listening to the premiere of the piece in Salvador, in 1991, with the interpretation of José Eduardo Martins. In addition, Jamary Oliveira was the professor of "serialism" for a whole generation of composers in the 70s and 80s in Bahia and took this theme to the field of computer music, having developed the computer program PCN (Pitch-Class Processor).

In addition to the use of serial strategies such as those mentioned above, there is, in the whole of Widmer's work, an undeniable orientation towards the concatenation of pitches from logics that involve smaller sets that operate as motive forces, and that often address the aggregate, even when they do not fully achieve it. Lima (1999) identifies and comments on these logics, which favor sets of the type [014] and [025], with plenty of details in more than 30 works of all their phases of creation, between 1952 and 1989.

It is even understandable that when dealing with Widmer's music, attention falls more easily on the attraction of interactions between different sound universes, involving the play with timbres, dynamics, rhythms and certainly many musical references, that which is generally called eclecticism. — an inappropriate term, insofar as it uses external references to define the 'internal' work of composing. We cannot fail to observe that this entire edifice is supported by motivic-serial procedures, movements of complexification from the small motif-set to the aggregate, allowing the notions of motif and set to be shaped by the same processes.

It is impossible not to mention that this interest in the tissue of relationships created by small sets refers to an approximation between serial and motivic thinking. Now, from the point of view of Schoenberg's work, we can find in the analytical literature an interpretative perspective connecting the notion of *Grundgestalt* to the impulse of creating a method of composing with twelve tones. This is a very important connection to highlight the proximity between the two composers that we engage in dialogue here. According to Epstein (1979, p. 17), in a book prefaced by Babbitt:

> Schoenberg formulated the concept of the Grundgestalt in the early years of the period in which he was developing twelve-tone

> theory (...) He found in the Grundgestalt concept an underlying link between musical tradition, especially that of the classic-romantic Viennese school, and his own compositional theories, which he saw as growing out of this tradition.

All this heritage that manifests itself in compositional acts extends to the production of theory — an environment that will allow us to move forward to address the cultural nature of these constructions — and we can see this in the formulation, late in its trajectory, of a binomial of laws of composing. and his teaching, which represents a kind of synthesis of his thinking and practices — organicity versus relativization (or inclusivity), Widmer (1988).

> The first law has to do with the creative act, which consists of the following phases: conceiving, giving birth, letting sprout, avenging, flourishing and maturing - therefore, a rigorously organic process from which the form results, and which also implies pruning, and criticizing uninterruptedly (...) The second law is based on the relativity of things, of points of view (...) We must admit that it is no longer a question of dualisms such as 'either this or that' (...) but rather of the paradoxical reality of 'this and that'. Inclusivity instead of exclusivity.

In fact, from Lima (1999), it is possible to demonstrate how these two laws (seen then as interpretive categories) transit between the three fields of data taken as references by the initial phase of the research — Widmer's discourses (the discursive mode), teaching strategies, and compositional choices revealed by analysis of his works; in all of them the unquestionable presence of attitudes of organicity and relativization (or inclusivity). This means that these values are scattered throughout Widmerian production – what he writes, how he teaches, how he analyzes and how he composes – and span several decades.

More recently, we advanced the following reflection on the two laws, Lima (2020):

> Organicity involves a logical field around operations that stem from the connective "if this, then…" — in other words, derivational work is involved in connecting antecedents and consequences, it is an investment in the construction of the perception of coherence and unity. In addition, another important criterion is also required, the concatenation of sonic ideas has to be perceived as fluid, gradual, and, at times, organic. Widmer created a metaphorical description of the creative act, as having phases such as: conception, bringing it to life, letting it germinate, becoming ripe, blooming and maturing.

> On the other hand, the major aspect of relativization seems to be that of an all-encompassing idea, the possibility that distinct logical fields give birth to a whole, an unexpected one, not foreseen from the standpoint of each of the logical systems; and, in this regard, it responds to the connective "and if…". Relativization (or inclusivity) becomes associated to "turning tables" and a moment of inflection, but this is just the point at which the two fields come together.

It is impossible not to mention that organicity has a strong relationship with all this care of concatenation, made through motivic and serial processes, and that refers to the Germanic tradition.

When searching in Widmer's trajectory for the possible origins of this construction of laws, we find a very significant moment in 1962, recorded by a text that accompanies the work *Bloco I op. 27* for mixed group — *apud* Lima (1999, p. 239):

> Just as in a block of granite, quartz, feldspar, and mica are fused together; or just like a carnival group, in which people and masks, indeed very distinct, dance among themselves; so, one imagines, here, distinct musical events that happen in parallel and that interpenetrate each other. A real development no longer exists: things are fixed. It happens that they are illuminated from different angles, and each one vivificates the other. And so, we have 'music strata' (structures), in which one no longer searches for the solution of the conflict, but rather for contiguity and the interpenetration of contraries that transform themselves into elements which define form.

Here we already have, even if in an embryonic form, the presence of the two laws. There is a way to compose with "real development", read "organicity"; and a new attitude that no longer seeks a solution to the conflict, but rather the "contiguity and interpenetration of opposites", read "inclusivity" and "relativization. This meeting with a carnival group ended up materializing in the composition of one of the most representative works of Widmerian thought, *De Canto em Canto: Uma Possível Resposta, op. 169*, twenty-six years later, in 1988, where a meeting takes place between a symphony orchestra and an *afoxé* group from the Bahian carnival, prepared by many musical allusions to Charles Ives' unanswered question. The ironic key being that perhaps the *afoxé* can answer the metaphysical question taken up by Ives. When analyzing this work looking for evidence of the intertwining of composition and culture — Lima (2020) — I found subsidies to elaborate the concept and analytical tool of the "resignifying distance", something that arises from the veiled representation of the crossing

carried out by Widmer himself, between his background in Germanic culture and the dizzying diversity of Bahia.

> The narrative design of Widmer's op. 169, with its clash between symphonic thinking and carnival, made it possible to understand the cultural distance portrayed in this way as part of the interpretive effort of the piece, its critical dimension – in other words, as the play of organicity and relativization (or inclusivity), to use concepts proposed by the composer. This distance can be understood as a meaningful opposition, touching on the field of expectations, the values connected to the musical experience and the sense of belonging generated by its presence, with all of this connecting the dimensions of compositionality: the invention of worlds, criticality, reciprocity, a field of choices and the intertwining of theory and practice.

We are here emphasizing the reflection on the cultural implications of the theoretical formulations developed by Ernst Widmer within the scope of the Composition Movement in Bahia, and, in this way, how they can be understood as a dialogue with their own formative background, and certainly, a dialogue with the Schoenbergian contribution. In doing so, we are touching on something very relevant — the cultural dimension of theory construction — although such an undertaking brings with it the same difficulties pointed out above by Bohlman (2003, p. 46), even insofar as the production of theory values a certain "neutrality" in relation to the phenomena to which it is addressed. We point out that it is also from this 1962 vision that we can understand the impulse to establish the contrast and articulation between the laws of organicity and relativization (or inclusivity). It is, therefore, the experience of Bahian cultural diversity, which ends up being elaborated as the justification for this second law, which is dynamically (and even dialectically) related to the first, organicity.

In fact, this pair of laws of composing (and of its teaching) even seems to establish a close relationship between creation and the opening to the paradoxical — this being another formulation by Widmer himself —, an opening to cultural difference, for example, insofar as it is admitted that there is a clash of distinct, sometimes apparently incompatible, logics. It is as if Widmer was saying that a theory of composing needs to be shaped from the possibility of challenging encounters, such as the one between the Symphony Orchestra and the Carnival *Afoxé*. Would we be in full transit towards an ethno-theory of composition?[5]

[5] It is extremely interesting the strategy of presentation of the idea of Inclusivity used by Widmer in an interview given during his trip to Switzerland in 1988: "What is most important to me, and I did a lot of that in Brazil, is inclusivity, or, for example, addressing the audience differently. I have a feeling that not only are we a world of specialists, but also the people in the audience specialize in one way or narrow down in one way or different ways. We still live in a world that is so realistic, with so many possibilities, that people are terrified and limit themselves. In this

Another possible interpretative bias would be the relationship between these two laws, in fact, the relationship between the Composition Movement in Bahia itself and the articulation between modernism and postmodernism. As a cultural environment, Bahia has always been somewhat aloof from the horizons of modernism — which arrived here late, in fact, from the 1940s onwards. for the "maximization of simultaneities" that characterizes Bahia from the Baroque of a Gregório de Mattos to *Tropicália* and *avant-gardes*. In other words, historicity, and culture dialogue in this perspective.

It was very natural for Gilberto Mendes, when the topic of postmodernism emerged among us, to point to Bahia as a place that had produced antecedents in this direction. And, strictly speaking, the manifesto of a single line of the GCB can be seen from this perspective. When rethinking its meaning, in this last text where he presents the laws, Widmer (1988) states that it is simply a matter of "accepting suggestions". As if the way out of what he criticized as dualism — "this or that" — went this way. And this is a very important point, including for the fundamental distinction between this Bahian conception of multiple cultural representations and the tradition of nationalism, where there is always an idea of totalization, of celebration of an identity.

Here we come to a curious turning point in our journey. From the question about Schoenberg's contribution to Brazilian music, we arrive at the solution engendered by Ernst Widmer and Movement of Composition in Bahia around the dynamics that surround the two laws. Now, if these laws reflect a cultural construction, and if the place of organicity can be associated with the Germanic tradition, then we can elaborate a question directed to the Schoenbergian contribution from the Brazilian experience: what to say about a possible presence of relativization (or inclusiveness?) in Schoenbergian thought? Schoenberg's organicity is quite visible, but what about its relativization? If Brazilian developments led to a conception that relativizes organicity based on inclusivity, what can we say about this last function in Schoenberg?

A brief inspection of twenty concepts elaborated by Schoenberg (1995, p. 97-99) and used in the manuscript of The Musical Idea confirms this extraordinary emphasis on organicity. Starting with the concept of Idea (*Gedanke*) which would be the origin and even the justification for maintaining the coherence of the following choices, supported by relationships such as:

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regard, I would like to create catalysts that would somehow remove, break down, or even better: melt these limitations or barriers. Our barriers are mostly intellectual in nature. You set them up so you don't get lost. People then say they are concentrating on something. In the end, that is the most important thing, vital. But this concentration in no way entitles us to put on rigid blinkers. People still suspects certain things, has perhaps forgotten them, but can still sense them. It is very likely that we are in an age of transition. That's what's so fascinating about life: if everything were fixed, we could leave our lives to the computer. You had better be a little less responsible, a little more open, more direct. Mankind is terribly stubborn - stubborn is a good, bad word".

. Durchführung (Realization)
. Widerspruch (Contradiction)
. Konsequenzen (Consequences)
. Schiksale (Destinies)
.Ableitungen (Derivations)
.Umgestaltungen (Transformations)
Kombinationen (Combinations)
.Erreichung des Höhepunkts (Achieving the climax)
.Verlassen (Abandoning the climax)
. Inselbindung ("Island" formation)
. Abrundung (Rounding off)
.Mannigfaltigkeit (Variety, avoidance of monotony)
.Tiefe (Profundity)
.Fernliegendes (Remote matters, glimpses, perspectives)
.Entwicklung (Development)
.Haupt – und Nebensachen (Main and secondary matters)
.Fortschritt vom Einfachen zum Komplizierten (From the simple to the complex)
.Variation und Kontraste (Variation and Contrasts)
.Aufbau (Construction)
.Liquidierung (Liquidation)

The comprehensive investment in the direction of organicity is impressive. Two notions spark some hope of relativization and inclusivity: *Mannigfaltigkeit* (Variety) and *Liquidierung* (Liquidation). However, a closer look will show that in the case of Variety, it is really about "avoiding monotony", and this is a criterion of organicity. Also with Liquidation, we can see that it is described as the gradual dissolution of the characteristics of the original motif, that is, a transformation referenced by organicity.

So, what would this attitude, which could perhaps be described as a kind of refusal towards relativization and inclusivity, mean? Now, since the domain of heights represents a kind of climax for organicity, we should think about other dimensions. For example, some statements by Schoenberg himself about rhythmic constructions. We see in some fragments of *The Musical Idea*, Schoenberg himself (1995, p. 199) making an effort to bring the logic of motivic thought to the domain of rhythm, that is, of organicity:

> Rhythm (in the sense applicable to the musical work of art) is surely not just any succession of stressed and unstressed attacks; it is also necessary that this succession behave like a motive. In other words, it forms an enduring gestalt that can indeed be varied, can even be entirely transformed and dissolved, but which, like the motive, will be repeated again and again (varied or unvaried, developed or liquidated etc.) Just as in **primitive forms** the motive of tones will be repeated, unvaried or slightly varied, so too the rhythm. [June 9, 1934].

This logic of motivic thinking is undoubtedly a process of generating meaning, represented here by the notions of motif, basic form, repetition, variation, development, dissolution or liquidation — conceptual mesh that we are used to evoking through the synthesis "developing variation". In fact, it is worth remembering that at the 2019 Meeting (MusMat / TeMA), taking up an article published decades ago, I sought to demonstrate how *Klavierstücke* op. 11 no. 1 by Schoenberg illustrates the conception of a rhythmic Grundgestalt, showing how the rhythmic construction sheds light on the organization of pitches, thus exercising a role of unquestionable centrality.

Returning to our interest in rhythmic thinking in Schoenberg, we see that in this same passage he goes on to observe that:

> And just as in higher forms the motive is developed, so too a rhythm would have to be developed, even if it were not associated with tones but merely with sounds. Perhaps it then becomes evident that this stage of merely sounding rhythms was not attained, that **a purely rhythmic art was not capable of a higher development. [**July 4, 1934]. (SCHOENBERG, 1995, p. 199).

This last statement brings a series of elements to an in-depth discussion of the relationship with rhythmic creation — insofar as there is a multiplicity of cultural contexts spread around the world that cultivate music that could undoubtedly be described as "purely rhythmic", even knowing that this purity is already part of the problem, being quite relative; and even more, questioning the meaning of the expression "higher development", for example, based on criteria endogenous to each context (in other words, the autochthonous hope of building historicity). And, in this sense, we see that Widmer's formulation is positioned for a world where cultural encounters, the intertwining of different traditions, are becoming increasingly frequent and important. A world that needs to overcome dualisms, as Widmer said.

But there is another angle of interpretation, quite different, that needs to be mentioned. Schoenberg's entire creative effort is directed towards a relativization of the tonal tradition, and, in the best spirit defended by the composer, an expansion of tonal relationships (inclusiveness, therefore), made possible by the new method of composing. We are thus faced with the paradox that a high investment in organicity ends up resulting in relativization and inclusivity.

Therefore, there would not be a refusal on the part of Schoenberg's thought in relation to inclusivity, but the creation of a path, of (organic) criteria to advance in this direction. According to Ringer (1997, p. 23):

> Organic change was a central aspect of his creative approach, sustained as much by the precipitous events of the disturbing

> times in which he lived as by that intangible inner necessity he felt compelled to obey. By the same token, however, that very quality served to reinforce the overarching sense of lawful unity at the heart of all of his varied artistic responses to a world in turmoil (...) Schoenberg's absolute commitment to rigorous discipline and structural unification became in a sense a condition of inner survival.

By understanding the method for composing with twelve notes that relate only to each other as a way of expanding tonal possibilities, and not as the proposition of an atonal world, Schoenberg would be in full cultivation of a clash between organicity and relativization (or inclusiveness). But from a solution that allowed him to declare to Rufer that he had discovered a path that could guarantee the hegemony of German music for another hundred years... In other words, he himself recognized the cultural marks on the path he was taking. Furthermore, this observation about the relationship between Schoenberg's desire and the law, the unity of the law, as Ringer says, deserves attention.

So, would the clash between organicity and relativization (or inclusivity) as thought by Ernst Widmer be an encounter between distinct and sometimes even antagonistic organicities? But in that case, what would inclusiveness be? Now, it would be the possibility of establishing a musical coexistence between distinct organicities, so distinct that they can even be apparently incompatible — the *Afoxé* in the concert hall, playing with Ives' question... — without one interpreting the other as barbarism, or if that is the case, a necessary barbarism in the face of the heterodox criteria of inclusivity.

I cannot conclude this journey without asking about Schoenberg's swing, that is, the sense that is made fluid in music and that directly involves the body, with the possibilities of movement that the experience offers. We can very well feel the presence of something of this order in his identification of a tendency towards lower denominations. When only one appears, it increases the possibility of others appearing, until the entire texture is filled. Schoenberg highly values intensification, but he is also very concerned about its control. Now, this investment in sonic excitation via the rhythmic universe is undoubtedly one of the most significant factors in the Trio's experience for strings, for example. But care is taken to ensure a healthy irregularity of intensifications and quieting. Would this dimension of Schoenberg's musical experience be an important interface for the dialogue between organicity and relativization (or inclusiveness)? This is a question that deserves future contributions of reflection and research.

In recent years, several works on rhythmic constructions in Schoenberg have come to light. As Muniz (2021, p. 3) points out:

> Rhythm in Schoenberg's atonal music has attracted well-deserved scholarly attention in recent decades. Two factors have

> contributed to this renewed consideration: first, a resurgence of rhythmic theory in general, and second, research by Harald Krebs (1984) and Paul Johnson (1988) showing the importance of rhythm in Schoenberg's creative process.

These authors revealed how Schoenberg conceived some of his works in a primarily rhythmic way, as he sketched several versions using the same rhythm, but different pitches. Roeder (1994, pp. 232-233) develops an analytical model in this direction:

> Essentially the theory represents rhythmic polyphony as two or more concurrent 'pulse streams' created by regularly recurring accents. These pulse streams are considered to be distinct continuities, not 'levels' or groupings of each other, so this approach does not involve meter in the exclusive and hierarchical sense...

Now, this scenario of refusal of the hierarchy of metrics seems quite suitable for a discussion on the intertwining of organicity and relativization (inclusiveness). Another direction of research continuity.

This chapter was built from the rejection of the idea that influence is a unidirectional phenomenon, and from the conviction that it is necessary to understand the Schoenbergian heritage in Brazil in cultural terms. And, in this sense, it sought to imagine a possible dialogue between this heritage and the context of the Composition Movement in Bahia, taking the composer Ernst Widmer (1927-1990) as a legitimate representative. After pointing out several paths taken from the identification of serial strategies present in the work of Widmer and his first students — Lindembergue Cardoso, Fernando Cerqueira and Jamary Oliveira —, it devoted attention to the process of formulating laws of composition and its teaching, Widmer (1988). By identifying the cultural dimension of the laws of organicity and relativization (or inclusivity), it revealed an articulation between the Germanic organicist tradition and the maximalization of simultaneities typical of Bahian cultures, discussing possible implications of this articulation for our multicultural times, and even for the friction between modernism and postmodernism. From the imagination of this dialogue, it was possible to ask about the relationship between Schoenbergian thought and the function of inclusivity, and to gather elements to suggest that the universe of rhythmic constructions in Schoenberg can become a promising field for a future reflection on this kind of dance between organicity and relativization (or inclusivity) in Schoenberg himself.

The tension between organicity and relativization (or inclusivity) still remains today, in the Composition Movement in Bahia, as a backbone of what is being elaborated and developed. A very convincing example is the adoption of research designs in the Doctorate in Composition that admit a field phase (as in

Ethnomusicology), that is, that bring to the academic court of reason very different logics of understanding the world, and of composing, such is the case of the dissertations elaborated by Bertissolo (2013), Pochat (2017), and Amaro (2019), involving the world of capoeira, a popular fair in Salvador and a Candomblé community, respectively, in this game of relativization and inclusivity, which depends intrinsically on a base of organicity, and there remains Arnold Schoenberg's contribution as fundamental.

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12

Tracing the Dissemination and Selected Manifestations of Dodecaphony, circa 1921–2021

Sabine Feisst

Much has been written about the twelve-tone ideas and compositions of Arnold Schoenberg, his disciples and followers in Western Europe and the United States. Much less is known about Schoenberg's legacy and the reception of dodecaphony in the so-called Global South, including Latin America, Asia, Africa, and Oceania. The efforts of totalitarian countries to ban dodecaphony in the early twentieth century propelled its dissemination through Schoenberg's and his followers' forced migration across the globe before mass media facilitated its spread. Due to its pliability and great potential to organize sound and think about music in a fresh way, twelve-tone principles have been absorbed into many different musical cultures. This paper addresses the histories and geographies of twelve-tone music and dodecaphony's malleability in selected manifestations across musical genres and styles in the last 100 years.

Reception Histories and Geographies of Dodecaphony

When Schoenberg explored dodecaphony in the 1910s – evident in the sketches for his unfinished Scherzo and *Die Jakobsleiter* – and enthusiastically confided his discovery to his students in 1921, he sensed this technique's abundant structural potential in terms of symmetry. The prominence of the number twelve in this method, its divisibility as a composite and superior highly composite number, as well as its presence in Western timekeeping, mythology, and Judaism must have strongly resonated with him and his disciples. But Schoenberg could not foresee its impact on artists and scholars around the globe. Hoping it would help sustain the high reputation of German music, in little more than ten years, he had to flee Hitler's Germany and the Nazis banned his music to eliminate Jewish influence in their culture. Yet already in the 1920s, as Schoenberg was still refining the twelve-tone technique, he attracted students from all over Europe and the United States, who eagerly adopted it and/or disseminated it in their home countries. These included the Dane Paul von Klenau, the Greek Nikos Skalkottas, the Spaniard Roberto Gerhard, and the American Adolph Weiss, the first to publish and teach dodecaphony in the United States in the 1920s.

At that time, Schoenberg and his disciples drove the globalization of dodecaphony via guest performances, talks, score sharing, and article and book publications. Newly established Western-style musical infrastructures and new-music networks in countries around the world, including the Americas, provided fertile ground for twelve-tone-music and -theory. Although the globalization of Western music is controversial (particularly in postcolonial discourse) due to the marginalization and hybridization of non-Western musical practices, dodecaphony's abstract properties seemed to transcend national boundaries. In spite or because of the fact that it was developed by a Viennese-born Jewish composer, it was widely understood as an "international" and "progressive" concept in line with modern and open-minded societies, allowing for new departures and connections between music scenes across the world. Thus, twelve-tone music appeared to many avantgarde musicians in the mid-twentieth century as an antidote to nationalistic neoclassical styles aimed at merging European classical and distinct nationalistic elements. Yet as is well known, as a technique, dodecaphony could merge with other techniques and diverse styles, including neoclassical ones, and with genres outside of European classical music.

In the 1930s, when the Nazis expulsed Schoenberg and his followers and banned dodecaphony in Europe, it often became a symbol of creative freedom and resistance against totalitarian politics in other parts of the world. Schoenberg and other exiled artists who settled in the United Sates, including Paul Dessau, Hanns Eisler, Erich Itor Kahn, Ernst Krenek, Ursula Mamlok, and Igor Stravinsky, helped this country grow into an influential center of twelve-tone composition and research during World War II and the Cold War era. Similarly,

exiled Schoenberg aficionados who settled in Latin America, Asia, Africa, and Oceania became ambassadors of twelve-tone music (see Figure 1).

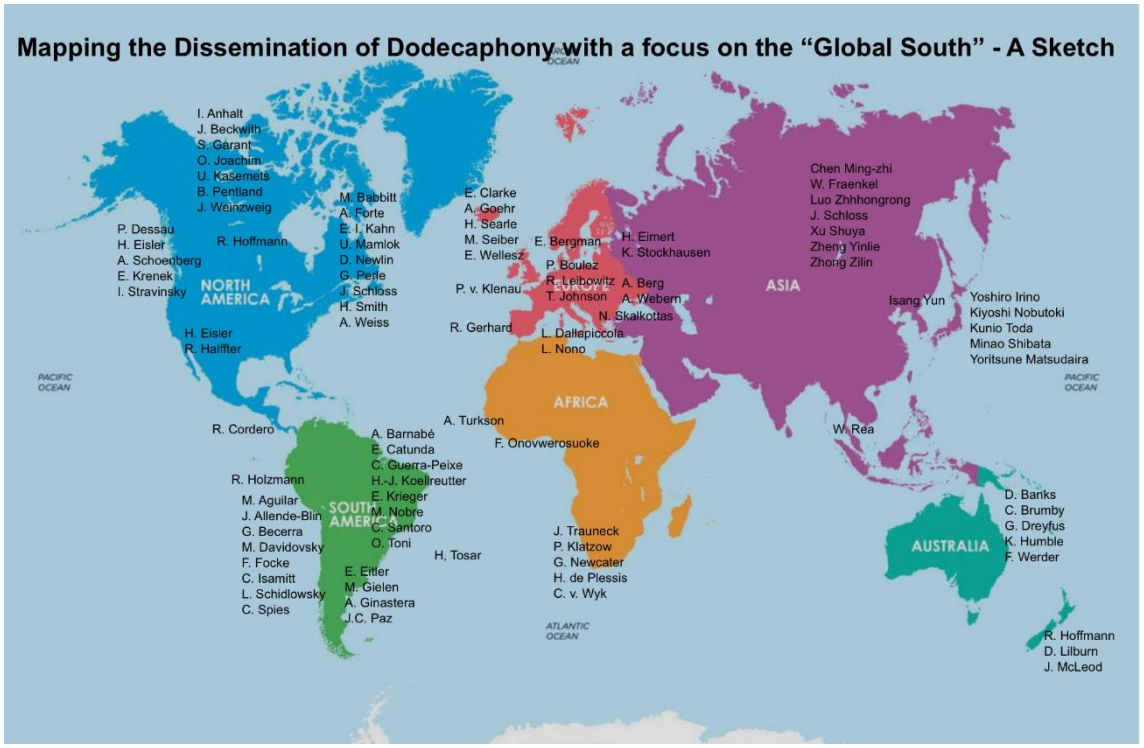

Figure 1: Mapping the dissemination of dodecaphony with a focus on the "Global South" – a sketch.

Through complex acculturation processes, they instigated a wide variety of responses to dodecaphony which challenged national boundaries, aesthetics, styles and genres, pointing to the "erosion of territories of knowledge marked by the nation-state" (FRASSINELLI *et al*., 2011, p. 2). Yet despite the advances of communication technologies, important twelve-tone composers, repertoire, and theorists have remained obscure as dodecaphony's entanglement in worldwide music networks have received little attention.

Latin America

Latin America, for instance, has fascinating histories of dodecaphonic music dating back to the early 1930s. As Argentina-born composer Graciela Paraskevaídis explained, its many representatives hoped to connect with scenes abroad to be on a par with musical modernists in Europe and the United States and exchange creative experiences (PARASKEVAÍDIS, 1984, pp. 135–36). Inspired by Egon Wellesz's writings on Schoenberg, Argentina-born Juan Carlos Paz (1897–1972) was among the first in Latin America to study dodecaphony. Discovering this technique around 1930, he used it in such works as the *Primera* through *Cuarta composición dodecafónica* (1934–38). He also introduced such composers as the Panama-born Roque Cordero (1917–2008) to twelve-tone music,

wrote about it and communicated with Schoenberg, informing him about the new music concerts he organized in Buenos Aires (see Figure 2).

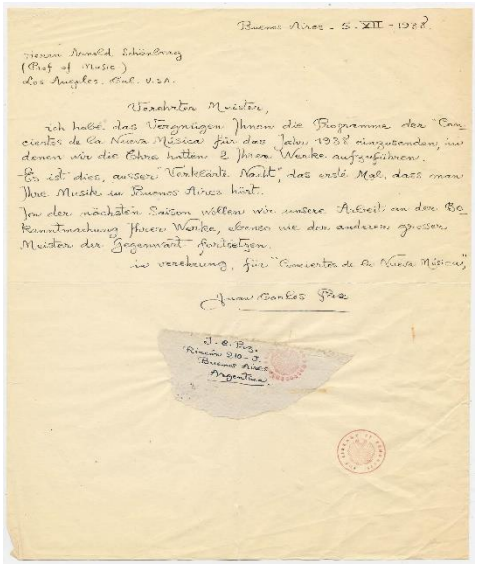

Figure 2: Juan Carlos Paz to Arnold Schoenberg, letter of 5 December 1938, Arnold Schoenberg Center.

Meanwhile in Chile, the painter and composer Carlos Isamitt Alarcón (1887–1974) began to explore twelve-tone music in 1938. At this time, the first musical modernists fleeing Nazi Europe had arrived in Latin American countries. Esteban (formerly Stefan) Eitler (1913–60), followed by Richard Engelbrecht (1907–2001) and Michael Gielen (1927–2019), settled in Argentina in 1936, Hans-Joachim Koellreutter (1915–2005) in Brazil in 1937, Rodolfo (formerly Rudolph) Holzmann (1910–92) in Peru in 1938, Hanns Eisler (1898–1962) in Mexico in 1939, and Fré Focke (1910–89) in Chile in 1946 to name a few. These artists helped grow existing or new contemporary music groups and stimulated lively debates on the relationship between music and cultural identity.

Brazil is an interesting case in point. A student of Hermann Scherchen and fervent Schoenbergian, Hans-Joachim Koellreutter fled Nazi Europe due to his political leanings, his Jewish fiancée, and collaboration with Jewish musicians. Active as a composer, conductor, flutist, and teacher in Rio de Janeiro and São Paulo, in 1938, he initiated the new music endeavor Música Viva, and introduced the twelve-tone technique to such composers as Eunice Catunda (1915–90), César Guerra-Peixe (1914–93), Edino Krieger (1928–2023), and Cláudio Santoro (1919–89) and used it in his own works since 1939 starting with Inventions for oboe, clarinet and bassoon (KATER, 2001). After that piece which shows an orthodox approach to dodecaphony, Koellreutter often used multiple rows per work in unorthodox ways as in *Música* 1941 which is based on two rows subjected to pitch alterations, repetitions and omissions, small changes in the ordering of the pitches, and permutations of row segments (GADO, 2005, pp. 164–173). In later

works, he wedded dodecaphony with experimental music elements and activist perspectives. As Marxists, he and the other members of Música Viva saw twelve-tone works as "free music in a free world" compatible with manifold styles and functions. Through his idiosyncratic use of dodecaphony Koellreutter wanted to challenge academic and government-favored populist styles. Hosting performances of dodecaphonic music, Música Viva generated heated debates around nationalistic neoclassical trends in vogue since the 1920s. In 1949 Música Viva congratulated Schoenberg on his 75th birthday (see Figure 3).

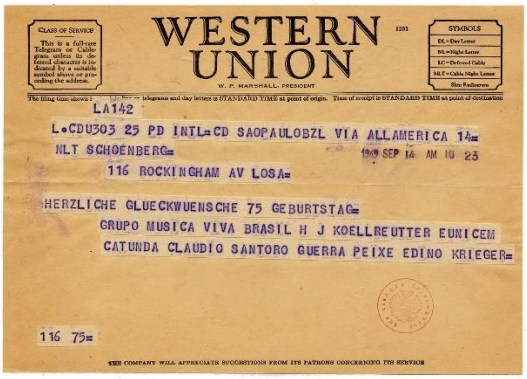
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Figure 3: Música Viva's Birthday Telegram to Arnold Schoenberg, 14 September 1949.

But a year later, the group disbanded as a result of nationalist composer Camargo Guarnieri's public critique through a *Carta Aberta* (published in *O Estado de São Paulo* on 17 December 1950), calling dodecaphony a "crime against the mother country" and perpetuating colonialist hegemony (ALMON 2019, p. 49). In a rebuttal, Koellreutter clarified that dodecaphony was "not a style," but a "compositional technique" and continued to use and teach it, but most Música Viva members abandoned twelve-tone composition – at least temporarily (PIMENTA, 2010, p. 332). Thereafter few Brazilian composers including Marlos Nobre (1939–), Santoro, and Oliver Toni used dodecaphony and serialism (as did such other Latin American composers as the Uruguayan Héctor Tosar and Chilean Gustavo Becerra). But they never received institutional support as did composers in Argentina through their Centro Latinamericano de Altos Estudios Musicales (1964–71) which facilitated Anton Webern's approaches to dodecaphony and European integral and experimental serialism.

But in the 1980s, dodecaphony made an appearance in Brazilian experimental popular music, in Tropicália-inspired artist Arrigo Barnabé's (b. 1951) notorious album *Clara Crocodilo* (1980).[1] Equally fascinated with the music of Luigi Nono, Karlheinz Stockhausen, Caetano Veloso, and Gilberto Gil, Barnabé fused socially critical lyrics about urban life in Brazil with popular music rhythms, modernist meter, as well as experimental and twelve-tone elements in the album's pieces *Acapulco Drive-in*, *Orgasmo total*, *Instante*, *Infortúnio*, and *Office*

[1] For a recorded performance of Barnabé's album *Clara Crocodilo* see https://www.youtube.com/watch?v=co17YM3fVVE.

boy. In *Infortúnio*, for instance, Barnabé employs the prime and retrograde versions of a twelve-tone row, in combination with tonal and freely atonal pitch patterns (CAVAZOTTI, 2000, p. 9). Barnabé may have used dodecaphony and serialism – which he had already explored in the early 1970s – symbolically to criticize Brazil's culture industry (DURÃO; FENERICK, 2021, p. 224).

The 1980s also saw the introduction of Babbitt's twelve-tone theories in Brazilian academia's newly established graduate music programs which has led to new scholarship on Brazilian twelve-tone music, rich contributions to the global history of dodecaphony (ALMADA *et al*., 2018, n.p.).

Asia

In the 1930s, Schoenberg's twelve-tone ideas also made it to Asia. In Japan, for instance, dodecaphony was first discussed in 1937 in a special issue of *Ongaku Kenkyu* [Music Research] dedicated to Schoenberg and by composer Kiyoshi Nobutoki (1887–1965), thirteen years before Yoshiro Irino (1921–80), Kunio Toda (1915–2003), Minao Shibata (1916–96), and Yoritsune Matsudaira (1907–2001) wrote their first twelve-tone works (FUKUNAKA, 2008, pp. 60–61). In the early 1950s Irino composed his dodecaphonic Concerto da Camera for seven instruments, penned two articles on Schoenberg's compositional techniques for the magazine *Ongaku Geijutsu* [The Musical Art], exchanged letters with Schoenberg on these activities (see Figure 4) and soon after made a Japanese translation of Josef Rufer's *Die Komposition mit zwölf Tönen* (1956; first edition in German: 1952).[2] In 1951 when after the Japanese occupation (1942–45) Singapore/Malaya was again under British control, Irish-born composer William Rea (1914–93) brought twelve-tone music to this island performing his own dodecaphonic Piano Sonata (1950) a year later.[3]

In China, dodecaphony entered the musical arenas through Wolfgang Fraenkel (1887–1983) and Julius Schloss (1902–72) who settled in Shanghai, a haven for Jewish refugees from Nazi Europe in the 1930s and 40s despite the fact that the Second Sino-Japanese War (1937–45) caused many difficulties.[4] A professionally trained violinist and music theorist from Berlin and immersed in

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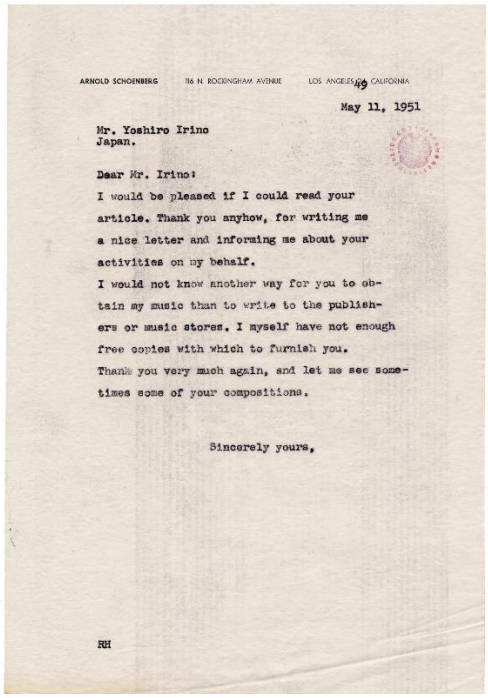
[2] Irino's two articles are titled "Schoenberg's Composing Technique" and "What is Twelve-Tone Music?" Incidentally, Irino's Japanese translation of Rufer's *Die Komposition mit zwölf Tönen* prompted Korean composer Isang Yun (1917–95) to explore dodecaphony and, in the late 1950s, he moved to Europe to study this technique in depth.

[3] In 1951 Rea won the Festival of Britain Competition's first prize for his dodecaphonic Piano Sonata (1950) which he himself performed on 9 April 1952 in Singapore, just a few months after his arrival. A few weeks later the twelve-tone technique was described in an article on Rea in *The Singapore Free Press* of 23 April 1952. For more information on Rea see David Byers website: https://www.byersmusic.com/william-rea---belfast-born-pianist-and-composer.php.

[4] Apparently composer Zhu Quing (Shang Guo Liao, 1893–1959), who had studied law in Berlin, published a positive article about twelve-tone music, "Reactionary Music?," five years before Fraenkel and Schloss's arrival (see LING, CHEONG WAI *et al*, 2020) https://aims.cuhk.edu.hk/converis/portal/detail/Publication/151811249?auxfun=&lang=en_GB.

twelve-tone composition since the early 1930s (although not part of Schoenberg's circle), Fraenkel fled to China after his release from the concentration camp Sachsenhausen in 1939 and was musically active there as a performer, teacher, writer, and composer until 1947 (UTZ, 2004, p. 122). Among the compositions Fraenkel completed in Shanghai are his Three Orchestral Songs (1941) and Three Two-Part Preludes for piano (1945), both of which are twelve-tone works. While the piano pieces were probably composed for pedagogical use, the Three Orchestral Songs show Fraenkel's acculturation to Chinese culture as they bring together texts about spring by Chinese poets from the Tang- and Song-Dynasties, Meng Haoran, Su Ding-Bo (Shu Shi), and Li Tai-Bo (Li Bai), translated into German by Vinzenz Hundhausen, and the twelve-tone technique. The three songs share the same twelve-tone row with its four basic forms (prime, retrograde, inversion, and retrograde inversion) and several transpositions and also feature such sonorities as trombone glissandos, string tremolos, and arpeggios that Fraenkel may have derived from Chinese and Japanese traditional music which he studied through transcriptions of recordings (UTZ 2004, pp. 138–139).[5] Besides such compositional projects, Fraenkel also taught the twelve-tone technique to several students, including Yang Yushi and Sang Tong who composed atonal works such as *Yejing* for violin and piano and *Zain a yaoyuan de fang* for piano (both 1947). Yet none of these students wrote dodecaphonic music in the 1940s perhaps due to the difficult political situation.



ARNOLD SCHOENBERG 116 N. ROCKINGHAM AVENUE LOS ANGELES 49, CALIFORNIA

May 11, 1951

Mr. Yoshiro Irino
Japan.

Dear Mr. Irino:

I would be pleased if I could read your article. Thank you anyhow, for writing me a nice letter and informing me about your activities on my behalf.

I would not know another way for you to obtain my music than to write to the publishers or music stores. I myself have not enough free copies with which to furnish you.

Thank you very much again, and let me see sometimes some of your compositions.

Sincerely yours,

RH

Figure 4: Arnold Schoenberg to Yoshuri Irino, letter of 11 May 1951, Arnold Schoenberg Center.

[5] A performance of Fraenkel's Three Orchestral Songs from 28 March 2019 in Beijing with mezzo-soprano Guanfu Chen and the Orchestra Academia China under the baton of En Shao can be found on YouTube: https://www.youtube.com/watch?v=sF627m7ZyCk.

In the 1920s Schloss had been a student and assistant of Alban Berg and a promising twelve-tone composer and, similarly to Fraenkel, emigrated to Shanghai upon his release from the concentration camp in Dachau in 1939 (FRICKE, 2000, pp. 171–72). Schloss joined the National (Shanghai) Conservatory as a composition professor in 1947 where he taught Sang Tong and Tchen Gi-vane (1928–).[6] At this point, after a hiatus of twelve years in his compositional career, he wrote two dodecaphonic *Chinese Rhapsodies* for violin and orchestra (1947–9) which stand as his most important works of this period. Schloss based these *Rhapsodies* on Chinese folksongs. The first *Rhapsody* involves two love songs from the Suiyuan province, *We Meet Every Day but I Cannot Marry You* and *Do not Take the Road in Front of the House*, which Schloss transcribed. The songs' most prominent intervals are reflected in the basic row of the first *Rhapsody* with the first five pitches forming a pentatonic scale (CAI, 2021, pp. 12–14). Schloss uses the basic row's four forms and transpositions thereof, and rather than presenting row forms in a linear thematic fashion, he partitions them to excavate melodic material for use in contrapuntal textures (CAI, 2021, pp. 14–21).[7] Perhaps, Schloss knew Schoenberg's choral piece *Der Wunsch des Liebhabers* (The Lover's Wish) from opus 27, a setting of a Chinese poem by Hung-so-Fan freely translated into German by Hans Bethge and published in *Die chinesische Flöte* (1907). Interestingly, this choral piece features a delicate instrumental accompaniment (mandolin, clarinet, violin, and cello) and a basic twelve-tone row that includes "orientalizing" pentatonic segments (HAIMO, 1990, pp. 142–144).

Due to the Chinese civil war, Fraenkel and Schloss among many other exiles moved to North America in the late 1940s and twelve-tone music was largely suppressed during the Cultural Revolution. A rare article on twelve-tone music in a 1956 issue of *Renmin Yinyue*, *Fandui shieryin tixi* (Against Twelve-Tone Music), translated by Huang Hui, did not change this situation as it was influenced by Soviet ideology and thus very negative (ZHOU 1993, p. 87).

In the late 1970s, interest in dodecaphony arose again in China's so-called "open-door" and reform era at the end of the Cultural Revolution, which prompted the *xinchao* ("New Wave") or the "Second Chinese Modernity." As a result Chinese composers discovered and freely explored dodecaphony, often combining it with elements of traditional and contemporary Chinese music: pentatonicism, folk music and instruments, and drama to reflect both their backgrounds and current interests (ZHOU, 1993, pp. 212–214; UTZ, 2004, p. 120). They thus seemed to build on Fraenkel and Schloss's teachings which stressed a thoughtful and critical adaptation of Western techniques. Luo Zhongrong's

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[6] Tong wrote about his experience of studying with both Fraenkel and Schloss (TONG, 1990, pp. 1–15). Tchen who moved to Paris in 1951 has engaged in piano performance, composition, visual art, architecture, fashion and politics.

[7] For a performance of Schloss's Second *Chinese Rhapsody* for Violin and Orchestras with Yoram Youngerman, violin solo and the China Youth Philharmonic Orchestra under Jin Ye, see https://www.youtube.com/watch?v=Od4OA9aqFxI.

Shejiang cai furong (Picking Lotus) for voice and piano (1979), arguably the first post-revolutionary twelve-tone work in China, merges a twelve-tone row and its basic transformations with pentatonicism. Born in 1924, during the Cultural Revolution, Luo learned about dodecaphony from Sang and occasionally from translated Soviet and Czech writings (RAO, 2002, pp. 228–229; SCHRÖDER, 2017, n.p.). A translator of books by Schoenberg, George Perle, and Allen Forte, he taught dodecaphony to others, including Chen Ming-zhi (1925–2009) who adopted the technique similarly to Luo in his emotionally charged Eight Character Pieces and Preludes and Fugues (No. 13).[8] Other composers like Xu Shuya (b. 1961) contrasted twelve-tone sections with non-twelve-tone sections and punctuated melodically laid out rows with non-dodecaphonic cluster chords in such works as his 1982 Violin Concerto (ZHOU, 1993, pp. 212–214).

In the 1980s, musicologist Zhong Zilin who taught courses on the history of Western Music at Central Conservatory of Music in Beijing was one of the first to give lectures dedicated solely to Schoenberg's music, dodecaphony and serialism. He specifically discussed works by Schoenberg, Pierre Boulez, and Milton Babbitt (ZHOU, 1993, pp. 119–121). In 1980, the British composer Alexander Goehr gave guest lectures at the Central Conservatory, discussing such music by such composers as Schoenberg, Webern, and Boulez, besides some of his own music. Zilin's and Goehr's lectures attracted hundreds of attendees (ZHOU, 1993, p. 123). Furthermore, in the early 1980s, Zheng Yinlie, a music theorist at the Wuhan Conservatory of Music offered the first course on twelve-tone music in China, prompting more such courses and twelve-tone compositions in China in the following years (ZHOU, 1993, p. 217). In the early 2000s, the Arnold Schoenberg Center's collaboration with Conservatories in Beijing, Chengdu, and Shanghai also furthered interest in twelve-tone composition and research in China.

Oceania

Dodecaphony began to take a hold in Australia in the 1930s and New Zealand in the 1950s. Interestingly, in 1933, Australia may have had the opportunity to welcome Schoenberg himself to its shores and learn about the twelve-tone technique first hand. That year he applied for the director's position of the New South Wales State Conservatorium (later called Sydney Conservatorium). But his application (along with that of Ravel) was rejected for "undisclosed – though easily imaginable – reasons" (PLUSH, 1998, p. 48). Performances and discussions of twelve-tone music were rare in Australia before 1933, but in 1938 composer-pianist Roy Agnew (1891–1944) began to play Schoenberg's and Webern's music in his weekly ABC radio program: "Modern and Contemporary Composers' Session" and thereafter twelve-tone music was mentioned – although not always

[8] Chen Mingzhi used the twelve-tone row of Luo's "Shejiang cai furong" in his Eight Small Piano Pieces, and Prelude and Fugue.

in positive terms – in such papers as *The Sydney Herald, The Age, The Australian*, and *Angry Penguins* (SHAW, 2003, p. 41). In 1949, on the occasion of Schoenberg's 75th birthday, music critic Wolfgang Wagner compiled a special Schoenberg issue of the Australian music journal *The Canon* with several contributions focusing on dodecaphonic works (SHAW, 2003, pp. 31–57).

Felix Werder (originally Bischofswerder, 1922–2012) was arguably the first composer in Australia who wrote dodecaphonic music and became one of the strongest promoters of the twelve-tone technique in this country. A mostly self-taught and prolific artist, Werder, who had known Schoenberg through his father Boas, a cantor, came from Berlin via England to Melbourne as an enemy alien in 1940 and used the twelve-tone technique in his Symphony No. 1 while he was at the internment camp in Tatura in 1943 (MARTIN, 2001, n.p.). Feeling a strong affinity with Schoenberg throughout his career, Werder used dodecaphony freely in the 1940s and 1950s in his often highly expressive and stream-of consciousness-style works and in the 1960s and 1970s explored experimental serialism.[9] Critical of his conservative cultural environments and struggling with acculturation to Australian society, Werder advanced dodecaphony through his compositions, teachings, music criticism for *The Age* (1963–77), and in his many radio appearances thanks to his charismatic, provocative and persistent personality.

Numerous Australian-born composers, including Don Banks (1923–80), Colin Brumbly (1933–2018), George Dreyfus (b. 1928), and Keith Humble (1927–95), were inspired by Werder and/or their studies abroad to work with twelve-tone and serial techniques. Equipped with a strong background in jazz, Banks who, like Werder, was based in Melbourne went abroad to study with twelve-tone composers Matyás Seiber, Milton Babbitt, and Luigi Dallapiccola. He began using dodecaphony in the 1950s – without hiding his experience in jazz performance. While Banks instilled works such as his *Three Studies for Cello* (1954) and *Episode for Small Orchestra* (1958) with subtle rhythmic jazz references, he later composed, similarly to Gunther Schuller, "third stream" pieces, including *Nexus* for jazz quintet and orchestra (1971) which fuses octatonic and twelve-tone sets, improvisation, and dodecaphonic row transformations. In the 1960s, inspired by Alexander Goehr, Brisbane-based composer Colin Brumby wrote primarily dodecaphonic works, including the *Fibonacci Variations* and *Antitheses* before turning to experimentalism in the mid-1970s. Originally from Wuppertal, Germany, Dreyfus explored serialism after studying Herbert Eimert's 1950 *Lehrbuch der Zwölftontechnik*. His *Music in the Air* (1961) and *From Within Looking Out* (1962), both written for small ensemble and dating from the 1960s, are primarily inspired by the techniques of Webern, Boulez, and Stockhausen. In the 1970s, dodecaphony began to fade into the background of Australian new music

[9] Werder, however, never reached out to Schoenberg during his Australian years.

scenes, but it is still having an impact on compositional thought as it is studied in academia to this day.

In New Zealand knowledge about dodecaphony was sparse before 1945 due to limited Western-style music infrastructure such as professional orchestras, conservatories, concert venues, and music journals. None of the musicians who arrived from Nazi Europe promoted Schoenberg's music. However, Schoenberg's distant relative, Richard Hoffmann (1925–2021) who fled with his family from Vienna to New Zealand in 1935 and graduated from the University of Auckland with a Bachelor in Music in 1945 would have been familiar with some aspects of twelve-tone music. In 1947 he left for Los Angeles to become a student and amanuensis of Schoenberg (1948–51) and thereafter dedicated his whole career to the advancement of dodecaphony as a composer and teacher in the United States. In the 1950s and 1960s, talented New Zealand composers such as Douglas Lilburn (1915–2001) and Jenny McLeod (b. 1941) who had formerly written tonal music with nationalistic features, began, after studies in Europe and the United States, to embrace the twelve-tone technique. Furthermore, Schoenberg's dodecaphonic music, such as his *Ode to Napoleon* made appearances on university concert programs in the 1950s (CROWE, 1955, n.p.). Lilburn turned to dodecaphony in 1955 after his sabbatical sojourn in the United States and Europe where he had encountered much avantgarde music and explored it through the early 1960s, producing such major twelve-tone works as his Third Symphony in one movement (1961).[10] In this composition, he merged a basic twelve-tone row and its classical transformations with non-dodecaphonic materials. Yet he presents the rows primarily melodically and does not always state them in their entirety as his focus is on variant and informal set segmentation (NORMAN, 1983, pp. 533–546). Similarly to Lilburn, Jenny McLeod explored the twelve-tone technique, after studying with Messiaen, Boulez and Stockhausen in the mid-1960s, in such works as *Piano Piece 1965* for piano solo (1965) and *For Seven* (1967) (MCLEOD, 1992, pp. 29–31). The piano piece is based on a single twelve-tone row, partitioned into tetrachords with distinct rhythmic and timbral features, but the ordering of the twelve notes is not always strictly observed.

Africa

Twelve-tone music has been composed, performed and/or listened to in a variety of African countries since the 1940s, including Ghana and Nigeria, but arguably most prominently in South Africa.[11] Olomouc-born conductor and

[10] A performance of this work by the New Zealand Symphony under John Hopkins can be heard at: https://www.youtube.com/watch?v=QCCQwB5Wj0s.

[11] Interestingly at around this time, South African-born music scholar Noel Heath Taylor who had come to the United States promoted Schoenberg's music and ideas in the form of articles ("The Schoenberg Concept," *Music & Letters* 20/2, 1939, pp. 183–88, and "Arnold Schoenberg: Music in Motion," *ETC. A Review of General Semantics*, 1944, pp. 1–9), and served as Schoenberg's

Schoenberg student Joseph Trauneck (originally Travniček; 1898–1975) gets credit for introducing Schoenberg's ideas to South Africa. A Jew and a political leftist, he was among more than a dozen Austrian musicians who fled Nazi-occupied Europe and settled in South Africa in the 1930s and 1940s and arrived in Cape Town in 1934, settling in Johannesburg. Trained in conducting under Alexander Zemlinsky, he directed and founded various orchestras and a festival, presenting programs that advanced musical modernism and catered to both black and white audiences. Although Trauneck wrote to Schoenberg on 8 April 1935: "I do everything I can to make you popular here – and you do need that," he was never able to conduct one of his twelve-tone works, as he had to walk a fine line in designing his concert programs (TRANCSIK, 2009, p. 134). While not interested in South Africa's abundant indigenous musical cultures, as a socialist, he fought against the rampant racism and segregation. Disillusioned about the National Party's apartheid and anti-Semitic politics, music infrastructure challenges, and musical conservatism, Trauneck left South Africa in 1955 to move back to Europe. But his endeavors as both conductor and teacher began to bear fruit as he was leaving.

Scottish-born composer Erik Chisholm (1904–65) who settled in Cape Town in 1946 to join the University of Cape Town shared Trauneck's political and aesthetic convictions and built on his work. Chisholm used twelve-tone elements only occasionally as part of his eclectic compositional toolbox in such works as *Dark Sonnet* (1952) and *Simoon* (1953). But as the founder of the South African ISCM chapter in 1948, he pioneered Schoenbergian ideas until his death in 1965 and may have inspired the then fledgling jazz pianist Chris McGregor (1936–90) who studied at the South African College of Music, led by Chisholm, to give the South African premiere of Schoenberg's dodecaphonic Piano Piece, op. 33a in the early 1950s (MARTIN, 2013, 219). Modernist compositional techniques also became attractive to such South-African-born musicians as Hubert du Plessis (1922–2011), Graham Newcater (b. 1941), and Carl van Wyk (b. 1942) who all used dodecaphony in their works. Arguably the first South African-born composer to adopt twelve-tone techniques in the early 1960s and "South Africa's most celebrated 12-tone composer," Newcater encountered dodecaphony when hearing Berg's Violin Concerto in his teen years (MULLER 2018, n.p.). He embraced the technique after studying in England with twelve-tone composer Humphrey Searle in the 1960s and has often used it classically and sometimes freely in his highly expressive works. Critical of integral serialism, Newcater is more inspired by Webern than by Schoenberg. Thus he often treats rows like Webern and also used some of Webern's rows in compositions such as his Violin and Trombone Concertos (1979) and Symphony No. 3 (1978). Many of his rows are marked by thirds and minor seconds. It is also

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assistant in the late 1930s, and as secretary of the Arnold Schoenberg 70th Birthday Committee in Los Angeles in 1944.

noteworthy that he based several of his works on the same tone row. For instance, *Variations de Timbres* for orchestra (1967) and *Palindromic Structures* for piano and orchestra (1977) are based on the same row and his Violin and Trombone Concertos, Trio for violin, horn and piano (1982), *Threnos* for orchestra (1983), and String Quartet (1984) also share the same basic tone row. He believes that the twelve-tone technique "is a way of thinking rather than a method of composing … [It] produces a music of the subconscious … What comes through in twelve-tone music is a direct product of the will – the will unhampered by traditional yardsticks and sterile academicism" (RÖRICH, 1987, p. 107). According to Newcater, his music is not influenced by Africa and could have been written anywhere in the world, although (or perhaps because) it was long funded by the pro-apartheid government through commissions and other subsidies and has catered to mostly white South African audiences. Newcater who has composed dodecaphonic music to this day has been able to make a living as a composer despite the fact that his compositions do not provide for easy listening (RÖRICH, 1987, p. 105).

Dodecaphony's Malleability

The gradual adoption of dodecaphony by composers around the world in the last hundred years – sometimes facilitated or hindered by severe political, social, and economic circumstances – has contributed to an ever-growing variety of twelve-tone works that reflects a wide range of personal motivations, identities, geographies, cultural environments, politics, technical understandings, and aesthetics different from the ways Schoenberg used it. As is well known and has been shown above, it has often been made compatible with existing tonal and modal structures. Recently British pianist and composer Stephen Hough (b. 1961) demonstrated with his Third Piano Sonata "Trinitas" (2015) how tonal a twelve-tone composition can sound (NEWMAN, 2016, n.p.).[12] In the 1950s and 1960s, American composers Ben Johnston and Lejaren Hiller integrated twelve-tone ideas with microtonality in some of their string quartets, while at the same time many others in North America and Europe – among them Babbitt, Mario Davidovsky, Stockhausen, and Luigi Nono – extended the pitch ordering principle to other parameters in both acoustic and electroacoustic music. Earle Brown, Lukas Foss, and Barbara Pentland, to name a few, made dodecaphony compatible with experimental music's performance freedoms in their open-form works. William Duckworth (1943–2012) approached dodecaphony conceptually in *Pitch City* (1969), providing a twelve-tone map which wind players individually navigate. Larry Austin, Banks, Gunther Schuller, and Hale Smith, for instance, blended twelve-tone principles with jazz and various types of improvisation, an approach for which Schuller coined the term "third stream."

[12] A live recording of Stephen Hough performing his Third Piano Sonata "Trinitas" can be accessed at: https://www.youtube.com/watch?v=2FmvkYURUKc.

In *Evocation* for piano (1966), Smith (1925–2009), who along with such other Black composers as Arthur Cunningham and George Walker, explored twelve-tone writing, merges classic dodecaphonic principles with African-American vernacular elements, such as blues and jazz gestures and a quasi-improvisatory quality (MAXILE, 2004, p. 123).[13] Furthermore, twelve-tone rows and serial processes have been used in popular culture: in sound tracks of feature and animated films scored by Hanns Eisler, Scott Bradley, Leonard Rosenman,[14] Benjamin Frankel, and Elisabeth Lutyens, in songs by Frank Zappa, Barnabé, and such metal bands as Peculate, Thinking Plague, and Blotted Science. [15]

Decried as creator of musical mathematics and celebrated as the Einstein in music (FEISST, 2011, p. 235) – regardless of his elementary Realschule training in mathematics –, Schoenberg inspired music theorists from Babbitt and Allen Forte to Robert Morris and Guerino Mazzola to use mathematical concepts, above all group theory, that initially facilitated the identification of different varieties of twelve-tone music and, later along with graph theory, helped discover commonalities between dodecaphonic, tonal, and modal pitch organizations. While never seeking to show that twelve-tone music is sounding mathematics, these theorists – along with critics of dodecaphony – may have prompted many contemporary composers to keep the details of their compositional techniques to themselves (FEISST, 2011, p. 247). But twelve-tone theorists have motivated Paris-based American composer Tom Johnson (b. 1939), trained under Forte at Yale and famous for his 1972 minimalist *4-Note Opera*, to compose a new kind of twelve-tone music that emphatically sonifies combinatorial mathematics. Long interested in basing his works on found mathematical objects such as Pascal's triangle and the Narayana series, Johnson, a critic of complex post-World War II serialism, now – one hundred years after Schoenberg's announcement of his discovery – writes twelve-tone music. His dodecaphonic music, however, is based on block design, blocks of numbers (12,4,3), not tone rows. This type of music observes the equality of all twelve notes more painstakingly and shuns octave equivalence. Johnson's piano works *Twelve* (2008) and *Twelve Years Later* (2020–2021), for instance, musically model (12,4,3) block designs of which there are more than 17 million possibilities. In the mathematical (12,4,3) block design, a set of twelve elements falls into blocks of four with each pair of elements joining together thrice in three different combinations (JOHNSON, 2008, p. 1). Twelve notes are assigned to numbers 1–12 in the form of specific scales which are divided up into 33 four-note subsets, whereby each pair of notes occurs thrice in one of the subsets. *Twelve* comprises twelve short pieces with each piece featuring

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[13] An outstanding performance of Hale Smith's *Evocation* by pianist Karen Walwyn can be found here: https://www.youtube.com/watch?v=JRcHyfipdZk.

[14] Listen to the opening of Rosenman's score for Vincente Minelli's film *The Cobweb* (1955) https://www.youtube.com/watch?v=K8Tge55VQag.

[15] For an explanation of the use of dodecaphonic processes in Blotted Science's 2007 song "Cretaceous Chasm" see https://www.youtube.com/watch?v=IVyUHFl0iB8.

33 four-note chords or eleven twelve-tone aggregates and reflecting processes specifically designed by mathematicians (see Figure 5).

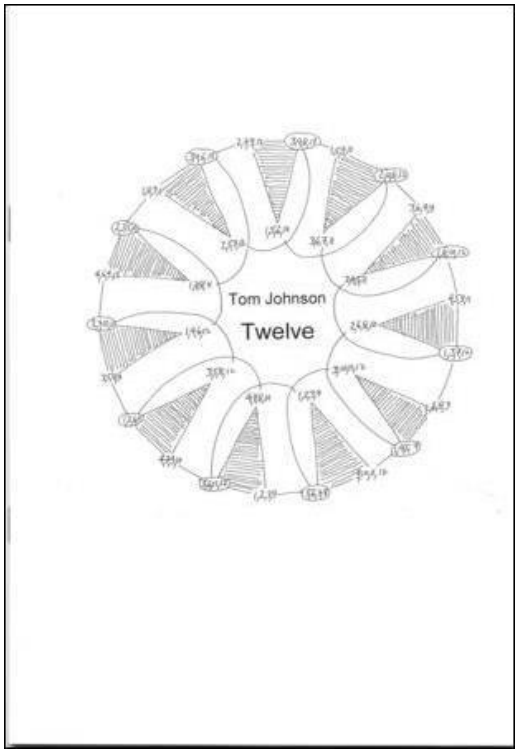

Figure 5: Tom Johnson, *Twelve* for piano (2008), cover image and sketch for No. XI, presenting eleven triangles each with three complementary four-note sets to create a twelve-tone unit.

In *Twelve*'s piece No. X, for instance, each measure includes all twelve tones divided into three four-note chords whereby each measure's last chord has two notes in common with the previous measure's last chord (see Figure 6) (JOHNSON, 2008, p. 14).

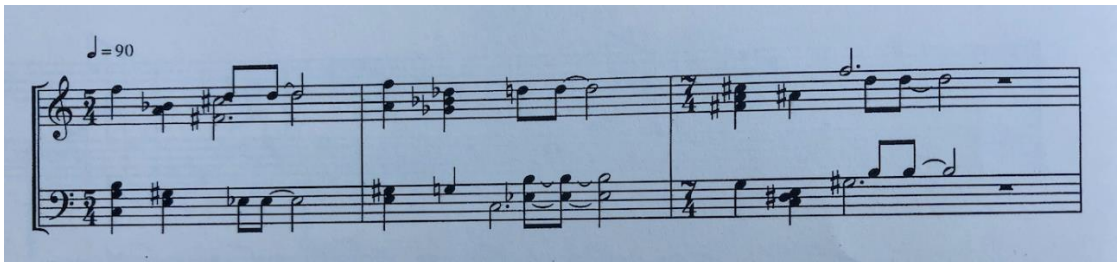

Figure 6: Tom Johnson, No. X from *Twelve* for piano (2008), mm. 1–3.

Written twelve years after *Twelve*, *Twelve Years Later* contains 22 pieces based on (12,4,3) block designs Johnson obtained from such mathematicians as Paul Sampson, Leonard Soicher, and Edward Spence. The collection offers a rich variety of structures, textures, and timbres. The nineteenth piece, "Rising with Spence", reflects a design developed by Edward Spence at the University of

Glasgow. Here Johnson added the numbers of the four notes of each chord to receive their sum and to order the chords beginning with the one with the smallest sum 10 (1,2,3,4) to that with the largest sum 33 (1,9,11,12) (JOHNSON, 2021, p. 37).[16] Johnson's recent approach to twelve-tone composition demonstrates not only the mathematical beauty of (12,4,3) block design, but also the inexhaustible structural potential of dodecaphony that will surely lead to more music building on Schoenberg's ideas in the coming years.

Conclusion

Instigated 100 years ago, dodecaphony has evolved far beyond Schoenberg's imagination and had a wider and deeper impact on music than is apparent in the current Euro- and U.S.-centric music history and theory literature. It has inspired musicians, music theorists, and artists from other disciplines across the gender and race spectrum in Europe, the Americas, Asia, Oceania, and Africa since the 1930s and generated vital reflection around artistic identity and modernity while serving larger political, social, and cultural purposes. Dismissed as "mathematical," Schoenberg's dodecaphonic music galvanized theorists spearheaded by Babbitt and Forte to shed new light on it through the lens of mathematics and, in turn, at the centennial of dodecaphony, inspired Johnson to compose mathematical twelve-tone music. Having traversed many national, genre, and style boundaries, this technique has broadened, complicated, and advanced our understanding of what music can be. However, much work remains to be done to document, analyze, interpret, and compare the many histories, geographies and transformations of dodecaphony – and to recognize the many marginalized artistic and scholarly voices – in the years to come.

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[16] A live recording of Johnson's *Twelve Years Later* by Samuel Boré with commentary by the composer is available at https://www.youtube.com/watch?v=nQLKPXLJ51k.

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In 2021, the MusMat Research group promoted the 6th International Conference on Music and Mathematics honoring Arnold Schoenberg (1874-1951). We had the presence of a significant number of world-renowned Brazilian and foreign researchers, invited to lecture on their most recent research associated with the theoretical and artistic work of this important composer. The conference was conducted entirely online, through video conferences, keynotes, virtual debates, and concerts.

This book is a result of this conference, and its release coincides with the 150th anniversary of Schoenberg's birth, joining a myriad of tribute initiatives honoring his legacy around the world. It presents a comprehensive prospection of Arnold Schoenberg's music, paintings, as well as his theoretical writings. Thus, each chapter provides a unique perspective on different aspects of his extraordinary multifaceted work presented by a team of esteemed scholars and experts who delve deep into the intricacies of his groundbreaking contributions (theoretical and artistic).

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