Music as a Carbon Language: A Mathematical Analogy and its Interpretation in Biomusicology

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Abstract: Carbon dynamics influence human physiology, culture and social patterns. Along centuries, linguists had been sufficiently discussed how breathing and cardiovascular performance set preconditions for word segmentation, phrasing, repetition, iteration, variation and expressiveness. Less attention had been paid to this influence as reflected in music, due to the belief that music can be "purely instrumental", and therefore far away from speech. However music, dance, respiration and verbal language share common evolutionary grounds, as well as important physiological features and constraints related to the organic properties of carbon and to its role in biological evolution. In this context, this contribution interprets chemical proportions in bioorganic compounds as analogies of their musical parallels, with consequences to music theory. Mathematical evidence is suggested for sketching a carbon hypothesis of music. From this perspective, music is more a feature and a consequence of chemical and biological constraints (not exclusive of humans), than a product "purely social" or "uniquely cultural".

Keywords: Carbon. 1/f Noise. Zipf. Music Language Self-similarity.

I. A FINDING OF QUANTITATIVE LINGUISTICS

Ittle known outside his technical domain, Luděk Hřebíček's (1934 -) research on speech self-similarity is a major contribution to quantitative linguistics including developments on semantic attractors, grammar structures as graphs and networks, and word-phrasing length variation as self-similar dynamics (particularly in [24, 25]). Making part of a new paradigm in language investigation, these theoretical devices fit strikingly well with their corresponding analogies in music, even when musical semantics and syntax preserve important differences in respect to verbal language.¹

Hřebíček published his article "Fractals in Language" (1994) as a first attempt to explain the Menzerath-Altmann law, with relevance to the theory of phrase extension, in pursuit of a theory of sentence aggregates in natural language. This law applies to the discrete probability distribution in the frequency of data which can be syllables, words or phrases in a text, and is closely related to Frumkina's Law, a probabilistic model for the occurrence of linguistic units in text passages [21].

Linguist Reveka M. Frumkina (1931-) systematically investigated the distribution of words in text blocks of fixed length. Later, also the recurrence of syntactic structures and functions was analysed with similar results in [27]. Applying this probabilistic law in random samples of

¹For a discussion on musical semantics and syntax, see the work of Raymond Monelle, David Lidov, Eero Tarasti, Kofi Agawu, and Lidia Goehr, among other semioticians involved with music theory. Their specialised developments are not referred to in this contribution.

texts and voice recordings, provides us with a useful tool for determining authenticity of styles in literature, but also, in very distinct contexts, for the identification of patterns in ecolects and idiolects (i.e. contextualised pragmatic variations of speech). An equivalent technique in music and sound analysis has had major applications in automata for the identification of musical style and authorship (e.g. [16]).

Music and speech diverge in a complexity of variables and outputs, however it suffices to compare intervals of duration, intonation, emphasis and repetition, in order to obtain useful information that ultimately can be interpreted in terms of rhythm, pitch, amplitude, timbre and texture. In fact, most samples of speech and music can be characterised as "melodies" containing these features; therein the interest that Hřebíček's approach may receive in musicology: whether in linguistics an "aggregate" denotes a group of sentences in a text containing a length-variating word or lexical unit (Hřebíček 1997:104), the analogous concept in music may reflect the structure of motive-phrase variation, and so forth the variation of melodic components (i.e. how a melody performance is enriched by second or third-level complexity, with variations of rhythm, pitch, amplitude, timbre and texture). Contrapuntal densities can also be conceived as relatively simultaneous melodies, because, as Julián Carrillo (1948/1967:152) thought: "counterpoint is the amalgam of melodies with same or different rhythms". In strict sense, musical counterpoint can be described by a set of rules acting under an *aggregates law*.

Complementarily, the Menzerath-Altmann law states that the longer an aggregate in a number of sentences, the shorter its sentences in their number of words; this implies a tendency to concentrate the periodic recurrences in the structure of a phrase, towards a compact group of structural units (something familiar to musicians thinking of motivic commonplaces). Although this concept is not verbatim mirrored in counterpoint theory, actually the variety of contrapuntal "species" portraits the historical intuition around this law (in its background, a *power law* in a thermodynamic context),² from first species to florid counterpoint. Furthermore, Schenkerian schematism also captures—in its fashion—the level of the most "compact group of structural units", over which successive structured layers arise. Then a musicological version of the Menzerath-Altmann law would state that, the longer a melody is obtained by a set of rules, there will also be a greater tendency to segmentation.

At this point, musicology and linguistics embodies the shape of a garden of connecting paths, since different theoretical approaches, proper of each discipline, start to unveil a single phenomenology. Thus, Hřebíček's theory of speech self-similarity may be traced by its connections with the Zipf's law of language *harmonicity* (in terms of physical, compositional trends), with the *law of least effort* (ethological empiricism that emphasizes constant power laws in biological behavioural economy), and with the balance between noise and syntax correctness in Markovian systems as probabilistic-statistic collections of codes under postulates of information theory.

Zipf's law is an empirical law employed in probability and statistics, which reflects an approach to a probabilistic distribution applicable to a variety of samples in many different fields including physics, biology and social sciences. The law, originally proposed by linguist George K. Zipf (1902-1950), states that in a generalized sample of verbal expressions, the most frequent word will occur approximately twice as often as the second most frequent word, which occurs twice as often as the fourth most frequent word, and so on (see [63, 64]). In the end only a few words are used very frequently, whilst most of the words are little used (an experience common to conscious processing of a new language acquisition, comparable to a musical repertoire learning process). This principle is summarized in the formula

 $P_i \sim P_1/i$

²See [50] for a general introduction to the concept of *power law*.

where P_i is the probability of selecting an object *i*, and P_1 is the probability which a first term of a series has to appear in a repertoire of objects. Given that the successive probabilities are positive and ordered ($p_1 \ge p_2 \ge ...$), and for all *i*, $p_i \le 1/i$, it is suggested that a second term occurs approximately 1/2 as often as the first term does, whilst the third term occurs approximately 1/3 as often as the first one, and so on. From this law it is also conjectured that the most common words tend to be shorter, and that when they tend to be too short, then they are replaced by longer words [52, p. 27].

[42, p. 238-249] description of Zipf's law, focused on the idea of efficiency in terms of an ability to "emphasize some choices at the expense of others", has a special significance for music, assuming that many musical strategies for consistency are based on a same kind of efficiency. As a matter of fact, the structuring form of Zipf's equation, as the series

$$1\sim 1, \ 2\sim 1/2, \ 3\sim 1/3, \ 4\sim 1/4, \ 5\sim 1/5...$$

constitutes a self-referential sequence, analogue to the aliquot division of an acoustic system, with a fundamental frequency and its natural harmonics. Such a self-referentiality can also be interpreted as a sequence with statistical self-similarity, i.e. not necessarily with "obvious" superficial self-similarity.

Acoustic (i.e. molecular) patterns in biology involve patterns of *activation-inhibition*, or *perturbation-absorption*, as happens in the mechanical vibrations of a string or a membrane. In a diversity of layers, these patterns tend to synchronization and consequent self-similarity, as it can be noticed in phenomena such as the intricate network of changing figures in a surface of water shaped by the wind; or in the Chladni figures of fine sand upon a plate periodically vibrating; but also in respiration and cardiac pulsation in individuals, and in a variety of practices coordinated among groups of individuals. The rhythms shaping these patterns cannot be explained uniquely as a linear physical interaction, because of the relative autonomy of physical sources participating within a same environment, as noted in [43]. Therefore the application of nonlinear methods – sometimes in cooperation with the linear ones - may be useful for systematic musicology, as explained below.

II. MUSIC AND SPEECH AS BIOACOUSTIC PATTERNS

From evolutionary physiological and structural views, respiration is of first importance for the foundations of music and speech. As a physical pattern, respiration has a constant, quasi-periodic relationship with the brain's oxygenation, and thus, with the rhythm of mental functions and mind-performative processing. Accordingly, it is not surprising that cardiorespiratory quasi-periodicity synchronises with brain performance and the nervous system's functions involved with music and speech outputs.

Respiration has at least three analogous – although importantly differentiated-scalar levels, closely associated among them: (1) cardiopulmonary carbon dioxide release and absorption of oxygen from external environment, (2) cellular respiration which, by degrading glucose with O₂ participation, allows organisms to obtain energy, and (3) carbon self-structuring at DNA-bases sequences' recursion. Conversely, the electric features of carbon and carbon oxidation have a capital role building protein blocks, making up the sequences of the genetic code and self-repair of tissues and limbs. This major role of carbon also affects the endorhythmic coordination of organs, including motor and cognitive functions.

At each of these levels of cycles of energetic-chemical interaction, these cycles match at least two different systems; for example, the cardiopulmonary cycle is coupled to the muscular/locomotor

cycle, but also to the cycle of involved brain/modular performance (in its turn, a system of electrochemical couplings). Abundant literature [7, 8, 9, 23, 43, 51] reports natural synchronization of physical cycles in many animal species and in quite varied biological processes. The mathematical modelling of these coupled cycles usually employs the *circle mapping* representing quasi-periodicity of the organic cycles. Quasi-periodicity ratio in coupled cycles fall into the so-called *mode locking patterns* in the circle, helpful for measuring synchronization [43] (a concept somehow familiar to musicians, but in contexts of tuning, harmonic structure and physical empathy).

Mode locking in human cardiorespiratory patterns crucially include ratios 1/1, 1/2, 2/3, or 1/4, mentioned in [9 and 23]. Then it is not just casual that ratios 1/1, 1/2, 2/3 (i.e. triplet implication), 4/4, 2/4, 3/4 and 6/8 are commonplaces in music; particularly in dance music and its derivations. Since human body is nearly symmetric, and dance is usually a group practice reflecting human symmetry in a given space, dance steps are mirrors of bodily ratios such as the mentioned ones. Ancient Pythagoreans did notice that these simple symmetries easily coincide with small ratios they identified as "harmony" in acoustic phenomena; thus they discovered a natural harmony represented by a succession of rational numbers, where the smallest ratios were considered as more pleasant or "more harmonic", and conversely higher ratios would correspond to less pleasant and "less harmonic" musical sounds. Now we may add that prime numbers progressively larger would also progressively produce a feeling of sound unprocessing, a concept that Barlow (2001, p. 6-8) labels as "indigestibility of primes". Table 1 suggests this ratio progression, from higher symmetry and easier predictability, to levels increasingly embedding sub-symmetries and bigger prime factors, which may be enriched by introducing probabilistic variation of inner accentuation (in metric patterns) and *intervallic chord composition* – both musical concepts in Riemannian sense [48].

Songs and other vocal repertoire may reflect an interplay between bodily symmetries, such as the mentioned dance metres, and Pythagorean ratios. Congruently, vocal repertoire intuitively identifies simple ratios with the Zipf's law series described above, as the vocal functions and recurrences also follow the *law of least effort*. In fact, the use of the so-called "harmonic series" 1/1, 1/2, 1/3, 1/4, 1/5, 1/6..., when applied to tuning voices and instrumental practice, cannot ignore the Weber-Fechner law on the relationship between the physical magnitude of a stimulus and its perceived value. The effect of this law makes that, in musical experience, the harmonic series cannot mean a simple succession of abstract ratios, but instead may involve a deployment of epistemic-cognitive deviation reflected in experiential varied interpretation. Table 1 illustrates this correspondence among the simplest tuning intervals and the most used musical metres, suggesting a Weber-Fechner complexity for the human interpretation of progressively smaller intervals, gradually increasing anti-intuition, i.e. gradually going beyond the arrow in the lower row of the table (a progression that Figure 1 suggests in more detail).

Moreover, if we interpret the harmonic series as a probabilistic arrangement, we may say that – using the simplest example in tonal classical harmony – there is a total probability of 1/1 for a *fundamental pitch*, to be the ubiquitous signature in a tonal piece. Then we would have a probability of 1/2 for the second tonal hierarchy (commonly the so-called interval of *perfect fifth*), and 1/3 for the probability of a third interval (the interval of *third*), and so forth. Although this may be interesting for some aspects of tonal theory, it is evident that musical practice does not consist uniquely on arranging ranks of pitch-span probability according to the hierarchies of a rigid structure. This is why Barlow (2001, p. 4) theorizes on probability as a function of *musical priority*; nevertheless, one may ask what "musical priority" is exactly, as it directly concerns to form and style from the very foundations of music. Barlow (*ibid.*, p. 2-3) proposes that – for the sake of simplification, let the metrical one be an illustrative example – musical priority is distinguishing a diversity of *probabilistic weights* in a given metre: "In the case of ametric music, all the pulses are

equally probable [...] But if you want to make the music more and more metric, you have to then decide how probable or how important the individual pulses ought to be. This assumes there might be a correlation between their importance and their probability" (*op. cit.*, p. 2).

For the Riemannian theory of musical metre and phraseology, the correlation between the diversity of musical weights and the extension of musical phrases and periods was already a big concern. In fact, Riemann (1903, p. 200-201) conceives musical metre as an *analogy* (i.e. proportionality) of musical harmony, since "it is clear that, as the proportions of a measurement grow, the answering member is increasingly likely to lead to an ever more noticeable resting point." Altogether with this notion, Riemann introduces probability ("likelihood") for estimating the weight and extension of musical notes, motifs, phrases and periods, in a fashion analogous to the linguistic concept of "aggregate" used by Hřebíček (1994, 1997). Thus tension and extension (the latter identified by [59, p. 337] as "complementary cadences" in a Neo-Riemannian context), as well as the function of what Riemann calls resting points, are aspects of a whole system common to speech and music practices, where a diversity of parameters are frequently correlated; therein the importance of measurement, contrast and punctuation, both in lyrical and musical traditions where balance between periodicity and aperiodicity has a capital structural and semiotic function. As a matter of fact, the Riemannian theory is an elaboration upon Koch's Versuch einer Anleitung zur Composition (1793), a work of enormous influence throughout 19th and 20th centuries, which emphasizes the analogies of *basic unit transformation*, *periodicity*, and *structure* in speech and music, as well as the intuition of symmetry, for verbal and musical composition and diversification.

In contrast to diversity, speech and music also necessarily seek for structural and functional economy. Since early times of systematic musicology (see [13, 44, 34, 35]), information theory receives particular attention for being helpful as a method estimating the balance between *noise* and *code*; between *randomness* and *meaning* within a musical system. Once again, we may invoke the relationship between *probability* and *priority*, as possible, often desirable, equilibration in tension and extension of periodic-aperiodic systems.

Whether the economy of the code and the gradual sophistication of the "message" are biological characteristics starting from chemical organic self-assembling, the *ordering function of the code* also gradually leads to structural coherence manifested as self-similarity within in a vast range of diversities. Self-similarity can be understood, then, as a mechanism preserving information at low cost (see [39, p. 209-210])). In music, a basic example of this is the geometric-arithmetic relationship $3/2 \leftrightarrow 12/8$ (see Table 1) which contains both the whole and the half-step of the diatonic system (i.e. its *diazeuctic feature*), and simultaneously allows the tonal system cycles, also represented in a two-dimensional periodic space by the *Tonnetz*, the Euler-Riemannian honeycomb lattice that characterizes tonal functions. The recurrence of this structural relationship, including all balances between periodic-aperiodic repetitions and extensions within a given grammar, guaranties the efficient economy of music as an information system, nonetheless *continuously productive* (i.e. *poietic*, both in biological and cultural senses). For organic chemistry this is not a new kind of systematic relationship; on the contrary, the cycles of periodic-aperiodic relationships lie at the bottom of chemical self-organisation, as noticed by crystallographers since the ending of 19th century.

III. TOWARDS A *carbon hypothesis* OF MUSIC AND SPEECH

The philosophical association between cardiorespiration and harmony delves into the darkness of antiquity. Plato's harmonic concept of cardiorespiration in his *Timaeus* (70 b-d) is just one example within an endless collection of historical sources. However this issue comes into clarity in relatively recent days. By mid-20th century, Carrillo wrote that "the vibrations our heart produces are of

Table 1: Columns from left to right: musical ratios ordered by numerator size, starting at 1/1; decimal expansion of the same ratios, and their conventional denominations in Western music; correspondence to the harmonic series (acoustics); and, in the rightmost column, common metrical signatures in music and dance, e.g. from the single-beat bar 1/1 to the 12/8, common in distinct cultures albeit with different inner accentuation. Notice the 12/8 ratio closes the first cycle of music self- structuring towards 3/2, connecting the diapente (perfect fifth) with the diapason (octave), allowing the circle of fifths and expressing the diazeuctic feature of the diatonic set that embeds the chromatic scale. Complex metrics and harmony arise progressively going downwards in this conceptually endless table (continuation is suggested by an arrow and ellipsis for each column), which mathematical ordering is suggested further in Figure 1.

ratio	decimal expansion	harmonic interval (tonal degree)	harmonic equivalence	music & dance metre signature
1/1	1	generator (fundamental or tonic)	1/1	1/1, 4/4, 2/2, 8/8
2/1	2	diapason (octave)	2/1	4/2, 8/4
3/2	1.5	diapason (perfect fifth, dominant)	1/2	2/4, 6/4, 4/8
4/3	1.333	diatessaron (perfect fourth, subdominant)	1/3	1/3:4/4 (triplet)
5/3	1.666	major sixth (submediant)	2/3	1/4:3/4 (quadruplet)
5/4	1.25	fifth harmonic (major third, mediant)	1/4	1/4, 2/8
6/5	1.2	minor third	1/5	1/5:4/4 (quintuplet)
7/4	1.75	seventh harmonic (subminor seventh)	(1/2 + 1/4)	3/4, 6/8
7/6	1.1666	septimal minor third	1/6	1/6:4/4 (sextuplet)
8/7	1.142857	major second (supertonic)	1/7	1/7 : 4/4 (septuplet)
9/8	1.125	major tone (epogdoon)	1/8	9/8
10/9	1.111	minor tone (lesser tone)	1/9	1/9 : 4/4 (nontuplet)
11/8	1.375	eleventh harmonic (tritone)	(1/4 + 1/8)	3/8
12/8	1.5	diazeuctic feature of the diatonic/chromatic proportion (12/8 : 3/2) and fifths cyclical.feature	1/2	12/8
13/8	1.625	thirteenth harmonic (tridecimal neutral sixth)	(1/2 + 1/8)	5/8
15/8	1.875	major seventh (subtonic or leading tone)	(1/2 + 1/4 + 1/8)	7/8
\downarrow				

musical nature as they fall within the human acoustic thresholds and within the *ratios* of musical sounds [...] and they are the cause of *empathy* or lack of it, between human beings and animals of all species" [12, p. 167-169, 409] (my translation and emphases). In a parallel investigation, quick developing cardiology soon discovered and registered the "harmonic" patterns of cardiac behaviour, as extremely useful signs for understanding the heart as a dynamical system (see [5, 6, 7, 9, 14, 20, 23, 28, 32, 45, 49]).

In human cardiorespiratory performance, according to [32, p. 1] "the heart can act as a pacemaker for respiration". This is a mechanism of synchronization that in physical terms signifies that heart and lungs, and the whole cardiovascular system tend to adjust pressure and electric potentials within a same harmonic system with constant variation and re-adjustment. [32, p. 5] proposes a diversity of *tunings* — although not exactly using this term — of cardiovascular human synchronization that behaves as a system of harmonic couplings (in its physical sense). Whether brain oxygenation strongly depends on this process of synchronization, [29] provides arguments to hypothesize that Hebbian synaptic plasticity (the adaptation of neurons in the brain during memorising, learning and comparing processes) shares the same kind of proportionality. In few words, music would be an expression of empathy and coordination of a complex selfness, a connection and articulation of endorhythms and exorhythms oriented by carbon signals at different levels: from organic chemical bonds, to cellular coordination, and then to cardiorespiration that provides rhythmic assortment of oxygen and hydrogen to the brain and the emerging mind.

Carbon has an exceptional role in biochemistry: leading and performing electrochemical bonds and structures with the nitrogenous bases in RNA and DNA; with its structural-energetic self-organisation constructing organs and organisms; and with its central participation in cardio-vascularity, respiration and brain-nervous system operation. At the genetic level, each nucleotide consists of three components: a five-carbon monosaccharide (*pentose*) called *ribose*, a phosphate group, and a nitrogenous base. At the muscular and locomotor systems, CO2 release comes from the breakdown of glucose — as said before. As well, besides carbon dioxide and water, aerobic respiration produces Adenosine Triphosphate (ATP), the *molecular unit of currency* in metabolism and the intracellular energy transfers, which has remarkable molecular plasticity thanks to its multi-faceted topological features including carbonic bonds (see [60]). Besides [14, p. 6] describes how *tyrosine-protein kinase* (an enzyme encoded by the Abelson-related human gene, localised in stress fibers and cardiocyte disks) does stimulate rhythmic pulsation of the cardiac system:

Receptor tyrosine kinase protein phosphorylation plays a crucial role in a wide variety of cellular processes that control signal transduction [in the cardiac system]. Protein phosphorylation is a rapidly reversible process that regulates the intracellular signaling in response to a specific stress [...] Signaling by activated tyrosine kinase receptor protein is initiated by the phosphorylation of cytoplasmic proteins, which in turn potentiate the intracellular signaling cascade.

A tyrosine kinase (TK, a subclass of protein *kinase*; from Greek *kinein*, "to move") is then an enzyme that can transfer a phosphate group from ATP, produced in respiration, to a protein in a cell. TK and ATP are closely related in cardiovascular and respiratory processes where the phosphorylation of pentose sugar molecules (carbon atom ribose) directly participates in DNA synthesis and cellular oxygenation [20]. In this context, TK operates as a chemical *on-off* switch of cellular functions related to the patterns of activation-inhibition involved in motor and cognitive human functions with quasi-periodic behaviour [20, 23, 32].

[56] believes that DNA frequencies can be traced as chemical noise, and mentions that "individual base positions in DNA sequences' [...] measurements demonstrate the ubiquity of low frequency $1/f^{\beta}$ and long-range fractal correlations as well as prominent short-range periodicities." [56, p. 7]. In this fashion, [56] associates 1/f noise (called "carbon noise" in electric circuits context), to "large averages over classifications in the Genetic Bank data bank [including] primate, invertebrate, plant [...] [with] systematic changes in spectral exponent β with evolutionary category." Sumarising, [55, 56] interpret the symbolic autocorrelation function for measuring DNA, in terms of low frequency 1/f noise. The musicological meaning of this "noise" is exhaustively studied in [39].³

i. Empirical evidence and theoretical expansion

Observing dynamics in carbon quasi-periodic cycles, analogous to physical dynamics modelling, we may assume that this model is useful to investigate a wide range of biological quasi-periodicity. From this analogous systematization, the circle mapping of a locally-constant rational rotation number that produces Arnold tongues (see Figure 1) emulates quasi-periodicity in physiological transduction (i.e. couplings of electric and mechanical systems) as happen in cardiorespiration and nervous dynamics.

Vaughn (1990) is probably the first author to report emergent physiological harmonic synchronization in a human being singing a melody. This and further research on the same topic employ time series analysis in order to estimate and describe emotional complexity in musical performance and self-perception. The obtained results illuminate the structure of physiological quasi-periodicity. [6, 29, 45] confirm the adequacy of this approach that connects cardiorespiration, and nervous-cerebral dynamics, with the analysis of speech and music employing the circle map and the Arnold tongues as a set of associated analogies.

The circle map exhibits certain regions of its parameters where it is locked to the driving frequency (*phase-locking* or *mode-locking* in the jargon of electronic circuits) in periodically forced nonlinear oscillators. The Arnold tongues is a resonance zone emanating out from rational numbers in a two-dimensional parameter space [46, p. 130-131, 217]. Within the Arnold tongues, the orbits of the circle map are periodic and they are called *mode* (or *frequency*)- *locked solutions* [47, p. 135], a feature useful for mapping rational periodic — and therefore musical — intervals.

Figure 1 (equation originally published in [3]) displays Arnold tongues obtained by iterating the function shown upper left in the diagram. Between the tongues asymptotically sprouting from K = 0, the dynamics are quasi-periodic, and the frequency ratio is irrational. As K increases, the Arnold tongues broadens, finally leading to an overlap between two tongues, and the system can display chaos ([2, p. 65], [51, p. 122]). However, as the tongues' broadness decreases in the lower part of the diagram, the rotational values of the function change until mapping the set of rational numbers (lower limit). Notice that a zoom-in between any of these intervals will display subsequent harmonic hierarchies nested among the infinite rational numbers as intervals of classical music from distinct harmonic regimes.⁴ The distribution of these musical intervals is neither successively continuous, nor perfectly symmetric, but harmonically segmented in hierarchies; thus, for example, the interval 3/5 (perfect fifth) has a higher structural hierarchy than 4/3 (major fourth), and the latter has a higher hierarchy than 5/4 (major third), and so on, in a sense plotting a *generalised musical harmony*.

The lower part of the diagram in Figure 1 suggests a quasi-musical harmonic proportionality for biochemistry, with the ratio hydrogen-carbon as "generator interval" or first-order harmony,

³The relationship of 1/f noise reported from DNA autocorrelation, is a sequel from [57, 58], after the primitive "carbon noise" originally reported by [10]. Actually, the circle map equations (such as in Figure 1) also models the phase-locked loop in electronics, as typically happen in carbon circuits [22, 37, 53]. 1/f noise wavelets and signals, in scalar invariance, are related by their generalized self-similarity.

⁴For a more in-detail explanation of the Arnold tongues in the context of musicology, see [39, p. 354-371].





and potassium as a following hierarchy, before phosphorus, sulfur, nitrogen, oxygen, and lower harmonic hierarchies laddered in smaller intervals. These hierarchies are "visible" across the comparison between the tongues' areas, i.e. the blank areas between the sigmoid lines in the diagram. Table 2 includes corresponding values for this comparative harmony between music and biochemistry, displaying the magnitudes of atomic Larmor frequencies (the angular frequency of atoms). Since hydrogen has a Larmor frequency (Lf) of approximately (radians converted to) 42.5761 MHz, this measurement can be symbolised as a musical diatonic pitch high E (i.e. E + 1/3of a tone). Congruently, whether carbon has Lf of 10.7058 MHz, then it also can be symbolised as a pitch high E + 1/3, although *two octaves* below the frequency of hydrogen. Thus, the H-C interval (i.e. generator-first subharmonic) should define a proportional arrangement with the following elements participating in biochemistry, as suggested in Figure 2.

The study of proportionality in organic chemistry is a consolidated field at least since Jacob Berzelius' (1779-1848) times. But what is relevant for the present study is rather the affinity between models of harmony: one in musical practice, another one in cardiorespiratory quasiperiodicity, and the mentioned one in biochemistry; all of them hypothetically leaded by the H-C interval, as suggested in Figure 1–2 and Table 2.

According to these diagrams, which compare musical and biochemical harmony, the interval between *generator* H-C, and its *major seventh*, iron, has an interesting congruence with the *diazeuctic feature* of the diatonic/chromatic proportion (12/8 : 3/2), and the *fifths cycle*. In simple words, the relationship (H-C : Fe) locks the cycle of *biochemical harmony* in a an arrangement similar to musical ratios 12/8 : 3/2 : 2/1. These proportions are equally related to the optimization of the topological features of both, music and biochemistry. Optimal two-dimensional geometry can be measured in graphene carbon hexagonal lattice, analogous to the Euler-Riemann lattice of tonal classic harmony, the *Tonnetz* (see Figure 3). Unlike most of chemical elements tending to perform linear bonds with other elements, carbon may perform complex periodic compositions with special properties, including graphene periodic tiling and variations upon this regular tiling in two dimensions and emerging complexity in three dimensions, as in *fullerene manifolds* topology. Analogous self-organization economy in music includes circle mapping in the torus of phases (a product of circles is a torus), as employed by [31, p. 105], and later by [1] and [62], among other recent research exploring the Fourier space in a musical context.

Bell Telephone Laboratories reported, in 1938, an "objectionable" and "burning" noise that "results in resistance, volume efficiency and carbon noise characteristics which [...] are essentially independent of [their] angular position." Along the 20th century's second third, this noise became to be known as 1/f "fractional noise" expressed on a log-log scale, usually measured in electric circuits. In recent years, 1/f circuit noise has been reduced by the employment of silicon dielectric materials, and is investigated in low-temperatures carbon dispensed semiconductors [36], although this line of research and its applications still in experimental stage

The two typical crystal structures of carbon in two dimensions: graphite simple hexagonal, and face-centred diamond-cubic, are analogous to tonal music self-structuring: the graphite simple-hexagonal in relation to the Tonnetz, and the cubic one as described by [19]. Three dimension analogies of carbon are also meaningful in music. This conception may accept *forced coupling* as physical emulation of harmonic fields, as studied in dynamical systems applied to music, as well as harmonic segmentation (*graphene zigzags*) and self-containment as occurs in fullerenes [see [15, pp. 48-50]], providing a more complete analogy with music.

Table 2: Mathematical and musical values of biochemical harmony. Columns from left to right: musical ratios ordered by numerator size, starting at 1; next column lists corresponding classical denominations of musical intervals, followed by their analogies to elements participating in biochemistry, including their atomic Larmor frequencies (*converted from radians to megahertz), and an approximation to diatonic intervals, in the rightmost column.

ratio	harmonic interval	associated chemical element	Larmor frequency* $(x10^6Hz)$	Approach to diatonic orbit
1/1	generator	Hydrogen	42.5761706239	$\mathrm{E}^{\uparrow 1/3}$
2/1	diapason'	Carbon	10.7058112488	$E^{\uparrow 1/3}$
3/2	diapente (fifth)	Potassium	1.98680993149	$\mathrm{B}^{\uparrow1/3}$
5/3	major sixth	Phosporus	17.235193094	C‡
5/4	major third	Sulfur	3.2681491128	G‡
6/5	minor third	Nitrogen	3.07672499835	G
8/5	minor sixth	Selenium	8.1199364118	С
9/7	septimal major third	Lithium	6.265495351128	$\mathrm{G}^{\uparrow 1/4}$
10/9	minor whole tone	Calcium	2.86540299085	F♯
11/10	neutral second	Oxygen	5.7717722319	F♯
12/7	septimal major sixth	Silicon	8.4587326212	$\mathrm{C}^{\uparrow 1/4}$
15/8	major seventh	Iron	40.06158770288	$\mathrm{D}\sharp^{\uparrow1/4}$
36/35	quarter perfect tone	Zinc	2.663870321	$\mathrm{E}^{\uparrow 1/4}$
64/63	Archytas' comma	Magnesium	2.606304759	$\mathrm{E}^{\uparrow 1/8}$
\downarrow				



Figure 2: Left: Harmonic profile of the main elements participating in biochemistry (those included in Table 2, above), ordered as musical harmonic intervals. The highest peak represents the hydrogen-carbon "generator" interval in Lf scale 0-45 (×10⁶ Hz). Iron, with position 12, is represented both by its major seventh (broken lines) and minor second values respect to hydrogen. The complete length of the graph in its horizontal extension represents the interval of the biochemical octave. Right: A more complete picture of the elements following the periodic table's order (from left to right), where peaks represent Larmor frequencies (Lf) of atomic isotopes ranking from Hydrogen (¹H) to Uranium (²³⁵U). The highest peak in leftmost area in the graph represents Lf of ³H, Tritium, extremely rare on Earth. The next higher peak, at position 14, is iron (symbol Fe in Figure 1).



Figure 3: Left: a hexagon lattice of tonal harmony can be rolled to form a torus, nesting and connecting the cycles of tonal functions [31, p. 105]. The torus is useful to map the space of musical intervals of optimal cardinality 12, in all of their possible harmonic relations and cyclical concatenation (diatonicity). Right: a hexagon lattice of graphite can be rolled to form a carbon tube with electrical conduction. Other shapes of carbon in two and three dimensions (including cube and truncated icosahedron) are also analogous to musical harmony.

IV. DISCUSSION

The harmonic distribution of intervals in the Arnold tongues follows a Farey tree consisting of a self-structured sequence of ordered proportions. A Farey sequence of order n is "the set of irreducible fractions between 0 and 1 with denominators less than n, arranged in increasing order" [33, p. 22]. We may theorize that human utterances and music following Zipf's law may be described as a set of *environmental resonances*; more precisely thermodynamic systems dissipating energy through a fluid medium (the air itself), ⁵ following a Farey tree self-hierarchisation. ⁶

Harmonisation is a generalised feature of music in any cultural practice, not only in the most noticeable way as multiparametric proportionality of intervals (*resonance*), but as *phase synchronization* in acoustics and psychoacoustics. Acoustic synchronization, as explained in [43, p. xviii], is due to reciprocal influence and adaptation of mechanical systems to their interactions:

Our surroundings are full of oscillating objects: violins in an orchestra, chemical systems exhibiting oscillatory variation of the concentration of reagents, a neural center that controls the contraction of the human heart and the heart itself [...] All these and many others systems have a common feature: they produce rhythms. Usually these objects are not isolated from their environment, but interact with other objects, in other words they are open systems. [...] This interaction can be very weak, sometimes hardly perceptible, but nevertheless it often causes qualitative transition: an object adjusts its rhythm in conformity with the rhythms of other objects.

Psychoacoustic synchronization is a complex, multi-layered and *transductive* phenomenon (i.e. that involves and correlates chemical, electrical and mechanical synchronization). Distinct systems can be analogous among them, replicating resonance patterns from their smaller layers to larger ones. Actually, societies can massively synchronize-mostly unaware-within a variety of physio-psychological phenomena [43, pp. xvii, 129]. Long term influence of this synchronization is hard to track in culture, nevertheless music seems to map resonance and synchronization over historical periods and trends. Accordingly, vast samples of music and speech from different contexts share features of spontaneous measurement and recurrence due to common physiological grounds and environmental conditions. Cardiorespiration, metabolism, nervous and cellular cycles, within their own feedback loops, express and transform the self-sustained and self-oscillatory characteristics of biochemical bonds.

The "preceptive rules" of musical traditions — somehow equivalent to syntax in speech — would be rather a cultural formalisation of "spontaneous" practices resulting from the evolution of systems within a specific context in resonance with common biochemical and physiological bases. This is a key concept for developing a hypothesis on the role of carbon in living organisms evolution, with its electrochemical features as precondition for the individual-context relationship where music and language arise.

Within this framework, caution is in order to avoid oversimplification, so it is crucially important to notice emerging stages and degrees of complexity, from the alluded carbon atomic correlations, to carbon-related functions among coordinated individuals and societies. Dynamical systems modelling of music does not look uniquely for specific objects, but particularly for systems

⁵For a physical modelling of this phenomenon in fluid mechanics, see [30, p. 44].

⁶A remarkable antecedent of this concept is Charles S. Peirce's (1839–1914) hierarchical structure of rational intervals, homologous to the Stern-Brocot tree (a comprehensive Farey binary sequence), in order to formulate a generalised sequentiation for the structure of human thought [41, pp. 277-280]. More recently composer and theorist Ervin Wilson (1928 -) adopted this model to construct a harmonic system on the grounds of the natural harmony first put forward by Novaro. Wilson (1994) emphasizes this musical model by its comparison with "growing systems [...] from crystals to living organisms" [61].

of relations. In this sense, dynamical systems "predicts that the perceived dynamics of tonal organization arise from the physics of non-linear resonance. Thus, non-linear resonance may provide the neural substrate for a substantive musical universal, [...] offering a direct link to neurophysiology" [29, p. 209]. We may add, *a direct link from music to physiology and biochemistry*.

Although direct analogy between chemical and acoustic intervals cannot be exact, because the former may be of *atomic*, and the latter of *molecular nature*, a generalised analogy is preserved in the context of biochemistry; namely, the leading and self-organizing resonance of the hydrogen-carbon interval, towards emergent multi-scalar complexity. Even when a huge amount of identical hydrogens may interact with other atomic forces spinning around their z-axis in different frequencies, *loss of phase coherence* and fall out of synchrony do not happen, since in many synchronizing interactions "phase differences don't have time to accumulate, so our signal [may] stay nice and strong despite the changing frequencies" ([17, pp. 115-116], within a biomolecular context).

Of course, many questions remain unanswered from this first-approach theorizing on a *biochemical harmony*. How a first, primary hydrogen did establish its current electromagnetic behaviour, is a question that also seems to implicate an intricate relationship in the universe's emergence of carbon. A question that goes much further from the initial purpose of the present study. Even "clear" assumptions from modelling a *biochemical harmony* entails very intriguing "findings". Two examples of this, in the context of Table 2, are:

- 1. Weather the iron ratio (15/8) is *responsible of magnetising the tonal octave*, as it produces a harmonic loop successively leading to a self-similar return to the H-C generator interval (1/1:2/1), then producing the *spiral of tonal harmony*, so invoked by scholars since Aristoxenus commentators up to Athanasius Kircher, and materialised in its modern physical modelling by Augusto Novaro [38].
- 2. Weather arsenic appears as an exact diminished fifth "in conflict" with the harmoni c hydrogen-carbon interval. In music theory this could be easily identified as the tritone "classical conflict" or 64/45 interval, disputing tonal orientation with perfect 3/2 consonant interval (see both intervals vicinity in Figure 1).

Furthermore, the *Arnold tongues interpretation* of musical harmony is useful not only as a set of classical proportionality, but particularly in terms of *music as an open system*. Then *probability*, a concept repeated in the initial pages of this contribution, does mean a guideline conducted by harmonic attractors (i.e. higher hierarchies in the Arnold tongues), which also generates intervallic possibilities in a discontinuous-dense set (the tongues' lower limit). Under this approach, a consistent theory of music is pending, from standard probability to fuzzy logic and uncertainty analogous to *quantum circles* as explained in [18], related to factual musical performance.

V. CONCLUSIONS

Whether the electronic behaviour of carbon sets preconditions for dynamical systems in terms of cycles of bioacoustic recurrence, such behaviour may have an effect in emergent patterns through human biology and socialisation, without disregarding cultural "development". This would explain, at least in part, how phase synchronization is expressed in music, speech and culture, as carbon-based correlated phenomena.

The *carbon hypothesis of music* (CHM) allows us to propose a dynamical and organic definition of music, as the set of psychoacoustic analogies of the human body, both individually and collectively, where the physical context dialogues with components of our evolutionary and actual existence. Besides, CHM also may be helpful to understand why music is a so common practice — if not obsessive — in human societies, independently of epochs and cultural contexts.

Music inherits and reflects psycho-physiological synchronization, where cardiorespiration acts as a leading force, with implications for the rhythms of brain/mind emerging complexity. In this sense, cyclical electric patterns of biochemical networks in quasi-periodic couplings, are statistically self-similar in respect to the quasi-periodic cycles of music. Such self-similarity is structurally related to the functional participation of carbon structures in psychoacoustic systems from the middle ear to the most complex brain electrical processing; but also to a smaller scale of *carbon noise*. This is how [55, p. 58] find $1/f^{\sim 1}$ noise ubiquity as a statistic footprint of carbonic self-similarity in speech (under Zipf's law) and music (e.g. in Riemannian aggregates), and later detect $1/f^{\beta}$ noise in DNA sequences, following "long-range fractal correlations as well as prominent short-range periodicities" [56, p. 7].

CHM allows us to answer the question of the *evolutionary motivation for vocalisation* in humans, transforming cycles of exhalation in potential socio-acoustic codification. Instrumental and progressively abstract music would originated from this, following the *law of least effort* and Weber-Fechner constraints. Then vocalization and music are literally an *expression* of the aerobic-carbonic biological dynamics.

Nevertheless, not only speech and musical vocalisation are expressive or communicational phenomena shaped by the *law of least effort*: since the carbon aerobic relationship strongly contributes to shape the brain/mind rhythms of communication and emerging analogies, such relationship also should involve symbolic and acoustic, verbal, non-verbal, and spatial epistemics (for instance, dance, walking, gesturing, and wider range proxemics). Human individual and collective bodies hear and respirate, and thus we are *physically synchronized* (as understood in [43]) with our communities and with the rhythms of our own culture and environment.

Finally we conclude that a new conception of music is needed in order to reflect the symmetrical pace from the conventional idea of music as a result of "high developed societies and culture", to the basic idea of music as the plural manifestation of a biochemical harmony in many other organisms than humans, including plants, fungi and bacteria. This post-anthropocentric definition of music should also stimulate other conceptual decentralisation processes in current musicology.

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