Discrete and Combinatorial Mathematics, Geometry and Mathematics of Continuous Functions Used in Some of my Compositional Projects

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***Abstract**: This paper intends to demonstrate the different ways many of my compositional projects used mathematical tools, from the pre-compositional stage through a final product done with sound synthesis. These tools are of diverse nature, depending on the theoretical needs of the problem faced. In some cases, the project employed discrete and combinatorial mathematics. In other cases, geometry was a useful tool to visualize rhythmic manipulations. Irrational numbers were the basis of a non-conventional tuning proposition. Continuous functions, like "sine", are at the core of digital sound synthesis and, in a particular project, served to the design of a digital filter.*

***Keywords**: Algorithmic composition. Inharmonic tuning. Fractal diminution. Set similarity. Cyclic rhythm. Digital filter.*

I. Introduction

Many music compositions of the 20th Century have benefited a great deal from Mathematical, Physical or Technologica l knowledge and many continue to do so nowadays. We will concentrate the approach of this paper in the contribution of three different branches of Mathematics: Discrete and Combinatorial Mathematics, Euclidian Geometry and all kinds of Continuous Functions Mathematics.

We may distinguish two main general compositional approaches used in this period. The first one prolongs the validity of the very ancient idea of reducing the complexity of the musical phenomenon to a symbolic representation called the "note" which embraces some of the predominant characteristics of sound to human perception: pitch, duration, dynamics and timbre. This approach allowed the development of musical notation. It still represents, to most composers, their daily tool for music conception and representation. The second approach, that had only subsidiary relevance until the 19th Century, depends on the possibility of dealing with the internal characteristics of the sound. Some, as [7], say that is music composed with the sound itself. Some, as Landy [8], call it "organized sound" and do not even defend that we need to call them "music" anymore. Of course, we are talking of sound products in which the author intentionally explores the internal qualities of the sound evolving in time. Therefore, they belong to the realm of sound design, electronic, concrete, acousmatic or electroacoustic music, or whatever other name is used to identify music to which the concept of "note" is, at most, of secondary importance.

The mathematical tools of discrete and combinatorial mathematics, and geometry, apply mostly to music that continues to use traditional notation, while the mathematics of continuous functions holds the conceptual basis for music that, besides employing technological means to generate sound, treats the sound from inside out.

I might use compositions of most of the established composer and the major names of the 20th Century to demonstrate my point but I choose to use my own compositions in order to state my personal view of how important I consider the influence of mathematical thinking in my compositional trajectory. The selection of cases intends to illustrate the use of different mathematical tools, notwithstanding that more than one may have contributed to develop each particular compositional project.

II. Pitch numerical representation allowing a processual form

It may seem a problem of nostalgic self-indulgence to resort to one of my first attempts in music composition to illustrate how the elementary idea of representing the chromatic scale with numbers emerged to me. Indeed, the circumstance around this report is what makes it interesting. It was the year of 1970 and I was seventeen years old. I had just started to attend college classes and one of the required freshman courses was "Introduction to Computer Programming". The instructor taught us the Fortran 1.0 computer language. We had to punch cards and stay on line to run our codes on the only IBM mainframe computer available in the school, a device that filled a large room. We could not enter the room, only glimpse through a door window. Besides the scheduled homework, we were supposed to come up with real world problems that a computer program might solve. The teacher used to say that the computer was a solution in search of problems. I was already interested in music composition, following whatever reached me of the European avant-garde music. This means that I had some information about basic concepts of dodecaphonic and aleatory music.

During that year, among other projects, I devised the idea of composing automatically a short piece of atonal music with the aid of a computer program. The name of resulting piece of music was *Three Episodes for piano*. Its definitive version dates 1974. The first problem I had to face in that project was how to represent the notes of the chromatic scale with numbers. My first attempt was to assign ten pitches to the numbers 1 to 10, substituting 0 for 10 to deal only with single digits. Therefore my numeric code was: $1 = C$, $2 = C\sharp$, $3 = D$, $4 = D\sharp$, 5= E, (...) through $9 = G\sharp$ and $0 = A$. As I was missing numeric representations for $A\sharp$ and B, I circumvented the problem with a systematic rotation of the numeric correlation assignment to include all the pitches.

I keep to this day a print out of the output, but unfortunately, the code itself was lost. It reads like that:

4	1	7	8	3	0	9	5	2	6
5	8	5	1	3	9	4	7	8	0
3	3	6	4	2	3	1	5	8	5
6	9	0	6	5	4	6	3	3	8

etc...

The idea of the program was to calculate the numbers of each new line adding the adjacent numbers of the previous line. For instance, the second line is based on the first: $4 + 1 = 5$,

$1 + 7 = 8$, until the last, which should turn around to the beginning and retrieve the first element, $6 + 4 = 10$, however making $10 = 0$. Insofar, when the sum exceeded 10, the program kept only the last digit. For instance, $9 + 5 = 14$, but 14 was replaced by 4.

Soon I realized that I did not need to restrain myself to single digits. I could operate the same reduction used for numbers above 10, using the concept of base 12. For a new programming attempt, I chose a more practical correlation that begun with the association $C = 0$, as practiced nowadays. Therefore, a second program using the principle of mode 12, processing a new sequence of twelve numbers without repetition, yielded the following result:

10	4	1	7	8	3	0	9	5	2	6	11
2	5	8	3	11	3	9	2	7	8	5	9
7	1	11	2	2	0	11	9	3	1	2	11
8	0	1	4	2	11	8	0	4	3	1	6
etc..											

These attempts of 1970 precede the publication of Forte's pioneer book on musical set theory [4]. Took me almost two decades to acknowledge the development of a set theory of music in other part of the world. For sure, I had assumed that it might be happening, so intuitive the approach seemed to me. The only problem was that, at that time, before the internet, information reached Brazil much slower than today.

The more interesting aspect of that first attempt was how it allowed the generation of pitch data by numeric manipulation. One cannot add pitches, unless numbers replace them. The purpose was to build a machine that makes music using a process that only stops when it reaches a certain condition, for instance, the completion of one hundred loop cycles. At that time, I was only vaguely aware of the concept of "music as process" and the major trend it represented. Still years later, when critics commented the first performance of the piece, referring to it as piece of serial music, I thought they were mistaken because I did not follow the rules of serial music. For me, then, serial music was dodecaphonic music with its principles extended to others parameters. I realized that the pitch generation of that piece was somehow unpredictable and therefore closer to stochastic music. One thing particularly pleased me: the process allowed pitch repetition, a negative imperative to Schoenberg. All I knew at that time, concerning serial music, followed the teachings of Krenek (1940). We knew very little details about the explosion of the series promoted by Boulez and Stockhausen techniques, but the ear guided me to obtain similar results, although the technique used was somehow original.

Figure 1: *Coelho de Souza's* Three Episodes for piano*, mov. 3, m. 14-16.*

Even though Figure 1 does not show an analysis of the pitch generation, it illustrates the style of the music produced by the numerical process above described.

As a corollary to this line of reasoning, we may question what would be the meaning of negative numbers in this context. In fact, the model is consistent because the chromatic scale supports a symmetrical reflection:

...	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
...	G	$A\flat$	A	$B\flat$	B	C	$C\sharp$	D	$D\sharp$	E	F	$F\sharp$	G

On the other hand, what happens with actual pitch frequencies? For instance, consider $A0 = 27.50Hz$ as the low A in the scale above. Calculating the descending pitches according to the tempered tuning, we obtain:

$$
\begin{aligned}
A(0) &= \text{27,50 Hz} \\
A\flat(0) &= \text{25,96 Hz} \\
G(0) &= \text{24,50 Hz} \\
G\flat(0) &= \text{23,12 Hz} \\
F(0) &= \text{21,82 Hz} \\
E(0) &= \text{20,60 Hz} \\
E\flat(0) &= \text{19,44 Hz} \\
D(0) &= \text{18,35 Hz} \\
D\flat(0) &= \text{17,32 Hz} \\
C(-1) &= \text{16,35 Hz} \\
B(-1) &= \text{15,43 Hz} \\
B\flat(-1) &= \text{14,56 Hz} \\
A(-1) &= \text{13,74 Hz}
\end{aligned}
$$

Therefore, the pitches plunge into a sub-sonic frequency realm, asymptotically tending to zero. There are no negative frequencies and even if we forcefully assign a negative value to the frequency of a pitch, in physical terms this will be the same sound of the equivalent positive frequency with a 180 degrees inverted phase. Therefore, this physical reality impairs the dualistic principle used by Hugo Riemann to justify his Theory of Functional Harmony because its postulate requires the existence of an inverted harmonic series. He missed that we can draw pitches in a linear scale that supports negative numeric values, but these pitches map frequencies into a logarithm curve that asymptotically approaches zero, never assuming negative values or any symmetrical shape.

III. Tuning with the Golden Section

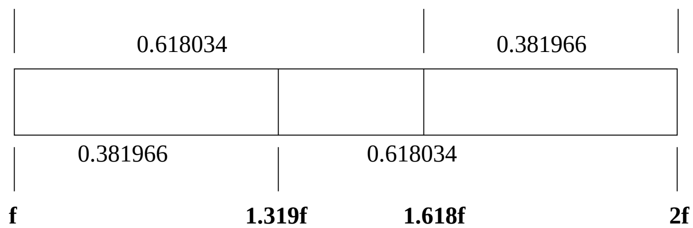
From my first compositional project, I jump now to my most recent project that is an opera named *The Machine of Pascal in Pernaguá*. For this project, I have also rescued from my memory the generative operations described above and used them to compose some of the scenes. This is also a reason to mention the procedure in this report. Unfortunately, I could not find a short good example of the method taken from the opera therefore I resorted to that old but clearer example.

Going now to the point, one of the main scenes of the drama depicts an hypothetical ability of Pascal's machine to produce music. The story is set in the 18th Century but, obviously, the Pascal's machine is a metaphor of today's computer. The music associated with the machine has characteristics that go beyond human motor control or limits of instrumental performance. One of such aspects is the microtonal tuning used by the computer-generated music of this scene. It implements a scale division inspired by the golden section.

The purpose of this tuning model is to obtain inharmonic relations between pitches. As the basic generative proportion is an irrational number, we do not expect to obtain any of the harmonic relations, known since Pythagoras, based on integer numbers.

$$\varphi = (1 + 5^{1/2})/2 = 1.6180339887\ldots \tag{1}$$

In this construction, we also applied two principles very fond to mathematical reasoning: the principle of symmetry and the principle of self-similarity. Applying symmetry, reflecting the division around the middle point, we obtain a first step of the division:



The following steps are recursive applications of the golden ratio, dividing each remaining segment into three parts that replicate self-similarly the scheme above. For a matter of clarity, we present the results in a vertical table instead of horizontally, as above.

The first column of Table 1 shows a linear division of the octave applying a nested golden ratio proportion. The next column shows the linear increments: adding the values of the first and second columns, we obtain the next line of the first column. However, we know that the human hearing is not linear, but logarithmic. The next column shows a similar division of the octave with logarithmic scaling. The fourth column shows the logarithmic increments: multiplying the values of the third and fourth columns, we obtain the next line of the third column. The fifth column depicts the tempered division of the octave. Of course, there is no perfect equivalence with, neither the first, nor the third column, but we emphasize in bold italic that the values of the tempered fourth and fifth degrees are very close of those in the golden rate division column. The small discrepancy of values is not only a matter of accuracy. We tried to proof a mathematical equivalence and performed a more precise evaluation of the results too. We find out that values are indeed not equal, but only a coincidence up to a certain degree of precision. However, we cannot perceive the difference between these pitches because they are within the JND (just noticeable difference) limit.

The marks on sixth column show the nesting branches. Centered "x"s indicate the first division step. A left positioned "x" shows the second branch e right positioned "x" the third interaction. The last column applies the results to the interval A2-A3. Notice that we obtain a microtonal scale divided in 25 intervals. There are three kinds of intervals that on the second column are identified by the increments 0.022. 0.034 e 0.056. Notice that the sum of the first two equals the third. Therefore. for practical purposes. we might reduce the division to 21 intervals of only two sizes.

These frequencies can be considered the fundamental of a complex note but also harmonic partials of inharmonic sounds. They can disperse in many octaves or concentrate in clusters. A digital synthesis program can implement any kind of pitch combination and their relative weight. That is what we have done in the above-mentioned opera scene.

Table 1

linear proportion	linear increment	logarithmic proportion	logarithmic increment	tempered d = 2 1/12	fractal nesting	pitches for A2 = 110.0
1.000 f	+ 0.056	**1.000 f**	x 1.0396	**1.000 f**	x	**110.0 Hz**
1.056 f	+ 0.034	1.040 f	x 1.0238		x	**114.4 Hz**
1.090 f	+ 0.056	1.064 f	x 1.0396	1.059 f	x	**117.0 Hz**
1.146 f	+ 0.034	1.106 f	x 1.0238		x	**121.7 Hz**
1.180 f	+ 0.022	1.113 f	x 1.0154	1.122 f	x	**122.4 Hz***
1.202 f	+ 0.034	1.150 f	x 1.0238		x	**126.5 Hz**
1.236 f	+ 0.056	1.178 f	x 1.0396	1.189 f	x	**129.6 Hz**
1.292 f	+ 0.034	1.224 f	x 1.0238		x	**134.6 Hz**
1.326 f	+ 0.056	1.253 f	x 1.0396	1.260 f	x	**137.8 Hz**
1.382 f	+ 0.034	1.303 f	x 1.0238		x	**143.3 Hz**
1.416 f	+ 0.022	***1.334 f***	x 1.0154	***1.335 f***	x	**146.7 Hz**
1.438 f	+ 0.034	1.355 f	x 1.0238		x	**149.1 Hz***
1.472 f	+ 0.056	1.387 f	x 1.0396	1.414 f	x	**152.6 Hz**
1.528 f	+ 0.034	1.442 f	x 1.0238		x	**158.6 Hz**
1.562 f	+ 0.022	1.476 f	x 1.0154		x	**162.4 Hz***
1.584 f	+ 0.034	***1.499 f***	x 1.0238	***1.498 f***	x	**164.9 Hz**
1.618 f	+ 0.056	1.534 f	x 1.0396		x	**168.7 Hz**
1.674 f	+ 0.034	1.595 f	x 1.0238	1.587 f	x	**175.5 Hz**
1.708 f	+ 0.056	1.633 f	x 1.0396		x	**179.6 Hz**
1.764 f	+ 0.034	1.698 f	x 1.0238	1.681 f	x	**186.8 Hz**
1.798 f	+ 0.022	1.738 f	x 1.0154		x	**191.2 Hz***
1.820 f	+ 0.034	1.765 f	x 1.0238	1.782 f	x	**194.2 Hz**
1.854 f	+ 0.056	1.807 f	x 1.0396		x	**198.8 Hz**
1.910 f	+ 0.034	1.879 f	x 1.0238	1.888 f	x	**206.7 Hz**
1.944 f	+ 0.056	1.923 f	x 1.0396		x	**211.5 Hz**
2.000 f	-	**2.000 f**	-	**2.000 f**	x	**220.0 Hz**

IV. A process of fractal rhythmic diminution

The above mentioned section of the opera also uses another mathematical procedure that departs from a rhythmic motive based on seven beats, irregularly divided with four kinds of durations: a half note, two quarter notes, a dotted quarter note and three eight notes, assembled in a sequence that produces syncopation, as displayed in the first line of Figure 2.

Figure 2: *Fractal rhythmic diminution (Coelho de Souza's The Machine of Pascal in Pernaguá).*

Each line represents a new level of self-similar diminution of the original line. In the second level, within the duration of a half note, a tuplet of seven eight notes (replacing four notes) reproduces the rhythmic proportions of the original measure. Pitches are the same seven pitches but on a different permutation. The process continues towards three more levels of diminution, two of them displayed in Figure 2. The last one, not shown in Figure 2, replaces the three eight notes by a 7:8 diminution. This musical process was inspired by a visual graphic, proposed by Peano. to generate the design of a snowflake, as we can see in Figure 3. Although not identical, these two processes of fractal self-similar diminution exhibit certain common features.

The main differences results from the asymmetrical internal structure of the first musical level, which induces, by self-similarity, a quite chaotic rhythm, as the process continues through the other levels. The snowflake design, on the other hand, appears to be much more regular. In fact, we listen to that fragment of music almost as a random sequence of events, like a Brownian movement, although, in a deeper level of perception, we realize that there is a coherent structure resulting from the multi-level rhythmic self-replication.

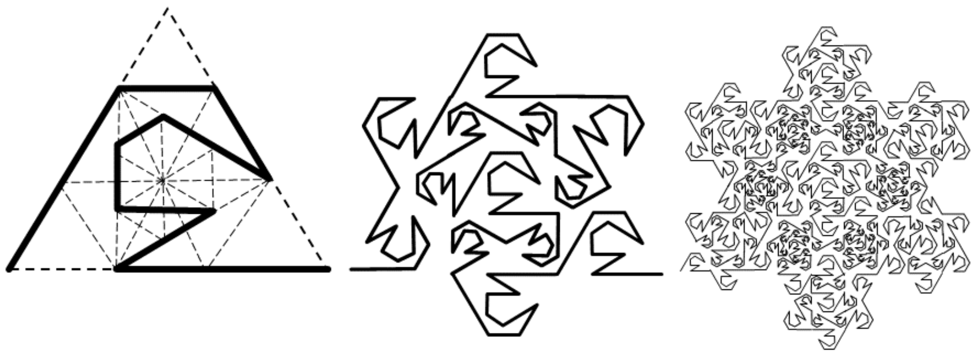


Figure 3: *Snowflake curve depicted as a process of fractal diminution ([11, p. 190]).*

V. A particular set property

The principle of *permutation* is another tool of combinatorial mathematics that I have used since 1984 when I composed *Rébus* for piano solo. Applying it to pitches yields trivial results but applying it to intervals allows us to study an interesting similarity relation that is not part of the standard Set Theory proposed by Allen Forte [4]. Forte devised a property related to the interval content of sets that he calls the *Z relation*. This property measures the similarity of two sets based on the identity of their interval vector, which counts the number of occurrence of each interval between the pitches of the set. Forte was able find some pieces in which the segmented sets exhibit the Z relation, but they are very likely to have occurred by chance, not intentionally by the composer.

The relation that I have proposed is different from the Z relation, as I demonstrated in an article [2]. It starts with the CORD vector, proposed by Soderberg [10], that lists the intervals of a set class. Going a step further, I have proposed a PCORD set that rearranges the intervals of the CORD vector to normal order, or actually, without any additional transformation, to its prime form.

This proposition differs from Forte's Z relation because it aims to be not only an analytical tool, but also a generative model. Based in a single PCORD, we can generate sets of different set classes. These sets have a second degree of structural similarity although to standard set theory they seem to be unrelated.

If we segment this music grouping the pitches of each measure in the treble clef and the bass clef. and reduce these sets to their prime form. the result will be:

Measures $1-2$:
Treble clef: $\{C\sharp.F\sharp.D.A\sharp.G\sharp\} \rightarrow$ set class (01468)
Bass clef: $\{B.C.D\sharp.F.A\} \rightarrow$ set class (02368)
Measures 3-4:
Treble clef: $\{B\flat.A.G.D\flat.E\flat\} \rightarrow$ set class (02368)
Bass clef: $\{E.G\sharp.F\sharp.C.B\} \rightarrow$ set class (01468)
Measures 5-6:
Treble clef: $\{D.F.C.A\flat.E\flat\} \rightarrow$ set class (02358)
Bass clef: $\{G.A.C\sharp.E.F\sharp\} \rightarrow$ set class (02358)

Figure 4: *Fragment (reduced) from Coelho de Souza's Concerto for Percussion (w. in progress).*

At a first approach. it seems that three different unrelated set classes have been used in these six measures of the piece: (01468), (02368) and (02358). If we calculate the CORD vector of these sets classes. reorder them to their prime form to calculate PCORD form. we obtain:

(01468) $\rightarrow$ [[1322]] $\rightarrow$ ((1223))
(02368) $\rightarrow$ [[2132]] $\rightarrow$ ((1223))
(02358) $\rightarrow$ [[2123]] $\rightarrow$ ((1223))

Therefore. we realize that this entire passage has been generated from a single PCORD. namely ((1223)). In the above mentioned article [2]. we listed all the set classes that are related by PCORD similarity. for each cardinality. In Table 2 we reproduce only the list of cardinality 5 because the sets used in the music of Figure 4 all the sets as based on five pitches. As expected in the column of PCORD ((1223)) we find the three set classes used in that fragment: 5-30 (01468), 5-25 (02358) and 5-28 (02368).

VI. Geometric representation of rhythm

The music of Figure 4 allow us to approach another mathematical tool that can be used to generate or analyze music: the depiction of rhythm by geometry means. This is particularly efficient when the rhythm presents cyclic features. That is the case of the music of Figure 4. We may calculate the rhythms based on the smallest division value. in this case sixteenths:

Measures 1-2 (right hand): 3 - 3 - 3 - 3 - 4
Measures 3-4 (left hand): 2 - 3 - 3 - 3 - 4 - 1
Measures 5-6 (right hand): 1 - 3 - 3 - 4 - 3 - 2

Table 2: *List of PCORD x Set Classes with Cardinality 5*

PCORD	**((1111))**	**((1112))**	**((1113))**	**((1114))**	**((1122))**	**((1123))**
Set Classes	5-1 (01234)	5-2 (01235) 5-3 (01245)	5-4 (01236) 5-6 (01256)	5-5 (01237) 5-7 (01267)	5-9 (01246) 5-10 (01346) 5-Z12 (01356) 5-8 (02346)	5-Z36 (01247) 5-14 (01257) 5-16 (01347) 5-19 (01367) 5-Z18 (01457) 5-11 (02347)

((1124)	**((1133))**	**((1222))**	**((1223))**	**((1233))**	**((2222))**	**((2223))**
5-13 (01248) 5-15 (01268) 5-Z17 (01348) 5-20 (01568)	5-Z38 (01258) 5-21 (01458) 5-22 (01478) 5-Z37 (03458)	5-24 (01357) 5-23 (02357)	5-27 (01358) 5-29 (01368) 5-30 (01468) 5-25 (02358) 5-28 (02368) 5-26 (02458)	5-31 (01369) 5-32 (01469)	5-33 (02468)	5-34 (02469) 5-35 (02479)

Although a certain degree of similarity in these rhythmic patterns induce us to suspect the existence of some hidden consistency. the numerical strings do not allow an immediate realization of some intentional process. On the other hand. it is clear to the ear that there are cyclic rhythmic patterns driving the discourse. When we represent these three rhythmic patterns in a circle. assigning striking points to sixteen possible positions, we obtain a much clearer visualization of the displacement procedure (Figure 5)

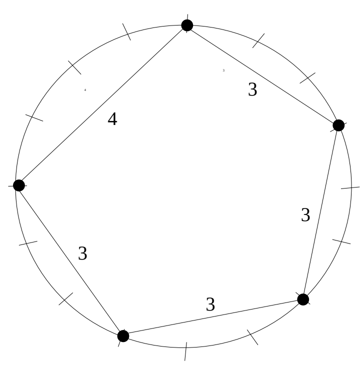


Figure 5: *Rhythm pattern of measures 1-2.*

This representation allows us to realize that there is only one rhythmic pattern altogether in the passage. Actually a single pattern is rotated at $+45°$ and $-115°$. as shown in Figures 6 and 7. so we have it starting at a different point of 16 points grid cycle at each two bars. A similar linear representation is possible and in that instance. instead of rotation. the procedure to consider is displacement. Another instance of geometric representation is the well know method of representing the twelve pitches in a circle. as the hours in a clock. This is a standard procedure of the set class theory. found in any textbook on the subject. Although we have used this kind of representation to illustrate principles of symmetry in my own compositions. we chose to

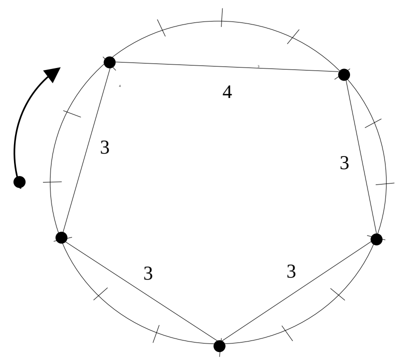


Figure 6: *Rhythm pattern of measures 3-4.*

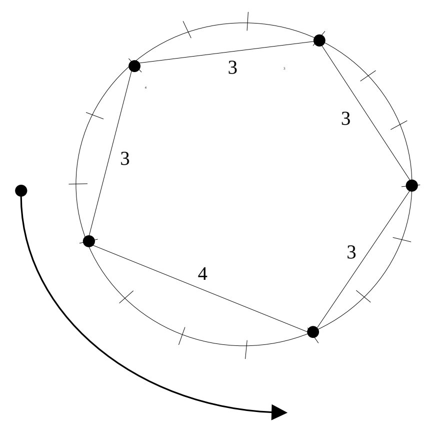


Figure 7: *Rhythm pattern of measures 5-6.*

illustrate the use of geometry as a tool for music composition. using cyclic rhythms because it is a less known problem. although some recent releases like [12] are quickly becoming popular. Another reason is that for rhythm the number of points represented in the circle is variable. and not a fixed clock face. We might also bring about the subject of geometry representation used in the neo-Riemannian theory. especially the Riemannian Tonnetz (see [5]). besides other achievements like those proposed by [13]. I did not show these devices because I have not used them in my music. In this paper. I voluntarily limited myself to tools that I have employed in my own compositions to demonstrate how mathematical tools can be helpful for establishing a pre-compositional background for building a personal style.

VII. A mathematical filter used to transform a sound signal

So far. we have examined cases where the mathematical tools described belong to chapters of discrete and combinatorial mathematics (see [6]) or geometry. I have promised to approach also examples of the mathematics of continuous functions. Indeed. I might collect a bunch of examples from my compositions and. fortunately. the mathematical foundation of them certainly belong in one of the two volumes of the all-encompassing book on the subject written by Gareth Loy [9].

Nevertheless. since I am trying to demonstrate that my fellow composers can easily understand these mathematical tools. I will restrain myself to only one example because the mathematics involved in the continuous functions usually depends on the knowledge of calculus and other high-level mathematics.

The following *Csound* program. besides the usual opcodes offered as presets by the program. uses a transformation of the wave signal applying directly to it a sine function multiplication. We highlight that line of the code with boldface. As we know sine waves are continuous functions. and according to Fourier's theorem. we can analyze any sound wave as a sum of harmonic sine waves. if we found the appropriate variables. In this case. however. the sine wave works as a kind of mathematical filter.

This *Csound* experiment is based on a standard two-stack frequency modulation design. but with the above mentioned transformation. we tried in this project. to perform a filtering that somehow works like the waveshaping technique. however done with a brute-force mathematical function instead of the usual tables. Most of the *Csound* opcodes are based in continuous functions but the composer does not have to deal directly with them. For more information on computer music synthesis see [3] and on *Csound* programming see [1].

```
;"EXPERIMENT 1.orc"
;instrument with time variable timbre

sr = 44100
kr = 4410
ksmps = 10
nchnls = 2

instr 1
  idur = p3
  iamp = p4
  ipitcar = p5
  iratefreq = p6
  iindex = p7

;transient for the attack
  ifrtr = cpspch(ipitcar)
  ikftr = .975
  imfrtr = ikftr*ifrtr
  kamptr1 expon iamp.idur.0.1
  kamptr2 oscili kamptr1.imfrtr.2.0
  aout1 oscili kamptr2.ifrtr.2.0

;filter units
  kamp = 1
  ifrcar = cpspch(ipitcar)
  ifrmod = ifrcar*iratefreq
  idev = iindex*ifrmod
  kamp2 linen kamp..50*idur.idur..50*idur
  amod oscili idev.ifrmod.1.0
  afreq = (ifrcar)+(amod)
```

```
  aosc oscili kamp2.afreq.1.0
  afilt1 = sin(aosc*3.14159/2) ;ïČ§ sine filter
  again = ((1/(aosc+1.5))-1.2)*1.25
  kvar line 0.idur.1
  aout2 = ((afilt1*kvar)+(again*(1-kvar)))*2*iamp
  kenv linen 1..05*idur.idur..05*idur
  aout2 = aout2*kenv

;triangle
  ktrian linen 0.5*iamp..01*idur.idur..95*idur
  aout3 oscili ktrian.ifcar.2
  aout4 = (aout1+aout2+aout3)*.75
  outs aout4.  aout4
  endin

;test score "EXPERIMENT 1.score"
  f01 0 512 10 1
  f02 0 512 10 1 0 .1111 0 .04 0 .0204 0 .01234 0 .00826

;a sequence of notes with harmonic relations of modulation

;instr start dur amp pitch fm/fc I=d/fm
  i01 0  6 10000 8.09 1   3
  i01 3  6 10000 8.03 2   3
  i01 6  6 10000 7.11 2   4
  i01 9  6 10000 7.05 1.5 3
  i01 12 6 10000 8.06 1.5 4

;a sequence of notes with inharmonic relations of modulation
;instr start dur amp pitch fm/fc I=d/fm
  i01 0  1 10000 8.09 1.54  3.04
  i01 2  1 10000 7.11 1.55  4.05
  i01 4  1 10000 8.03 2.56  3.06
  i01 6  1 10000 7.06 2.57  4.07
  i01 8  1 10000 7.09 1.25  5.08
  i01 10 1 10000 8.01 1.414 3.14
  e
```

VIII. Conclusions

We tried to demonstrate. describing a varied set of examples assembled from my own composition projects. how mathematical tools can be useful to shape a style. from the pre-compositional stages through a final sound synthesis stage. In some cases. these tools use discrete mathematics. for instance. in set theory that employs note representation by numbers. or in combinatorial mathematics applied to algorithmic composition based in set manipulation. We can also resort to irrational numbers to implement non-conventional tunings other than tempered or traditional tunings based on integer proportions.

On the other hand. the universe of continuous functions is at the base of human hearing. as far as logarithmic functions explain the perception of pitch and sound dynamics. However. when we jump into the world of direct manipulation of the sound. the mathematics of Fourier transforms. Hilbert transform. convolution. filters (simple. FIR. IIR or Z transf). resonance. acoustic systems modeling (with finite differential equations). and also techniques of sound synthesis (like AM. FM. vocal synthesis. physical modeling. etc). dynamic spectra (Gabor. short time Fourier transform). sound vocoder. and so on. are matters in which the deep understanding of their mathematical foundations enhance the use of their capabilities.

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