The Function of Orchestration in Serial Music: The Case of Webern's *Variations* Op. 30 and a Proposal of Theoretical Analysis

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Abstract: *Webern's* Variationen op. 30 *constituted a well-known milestone in the consolidation of serialism as a compositional technique. It has been the target of a large number of investigations focused on the way the composer developed a broader concept. As could it not be otherwise, his orchestral design is also closely tied to his structural concerns. However, it appears to lack a systematization of the composer's orchestration principles. Thus, to propose an analysis of Webern's orchestration, one need to elaborate an* ad hoc *method, virtually starting from scratch. This paper aims to describe the main points of this method in its current experimental stage. At the same time, we point to some conclusions about Webern's orchestration according to his aesthetics.*

Keywords: *20th Century musicology. Webern. Variations op. 30. Textural analysis. Analysis of orchestration.*

1. The *Variations* op. 30 by Webern

Webern's *Variationen op.30* of 1940 (WEBERN, 1956) constituted a milestone in the consolidation of serialism as a compositional technique. For this reason it has been the target of a large number of investigations, seeking the scrutinization of the peculiar way in which the composer absorbed the technique inherited from Schoenberg to develop a broader concept. This includes a rethinking of elements of the German tradition of composition, especially when trying to achieve a synthesis between the principles, formerly mutually excludent, of permanent variation and cyclic form—in this case, which is named by Webern as the *adagio form*—and the parsing of the total chromatic into smaller, musically significant units. This Webern's last row is precisely known by the complex manner with which it operates multiple symmetries. Generated from a motive of four notes, this row is systematically subdivided into tetrachordal subsets (Figure.1), labeled as O1, O2 and O3. However, Kathryn Bailey specifically demonstrates how the *Gestalt* of Webern has also a rhythmic order ([1], p. 224). Indeed, the serial material is supported

*I thank the MusMat research team who provided the English translation and the LaTeX' typeset of this paper and Charles de Paiva (NICS/Mus[3]) for his contribution to the mathematical model and its implementation in the OpenMusic's SOAL library.

by two initial rhythmic motives enunciated in the first three bars and then declined, in order to permeate the entire work in all of its moments. In other words, with Webern the rhythmic patterns acquire as much functionality as the pitch ones.

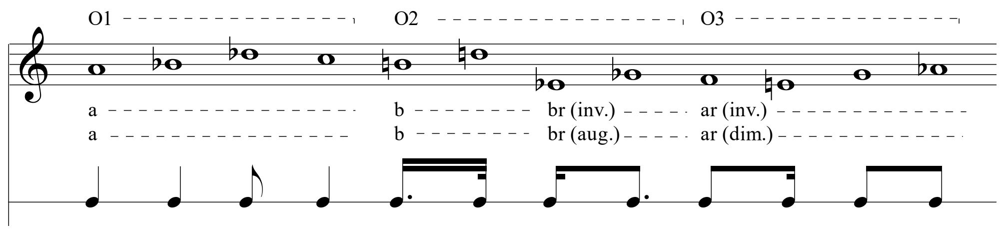

Figure 1: *Original row and basic motives of the* Variations *([1]:224)*

As could it not be otherwise, his orchestral design is also closely tied to his structural concerns, because the orchestral polyphony corresponds "to the extreme point of a motivic-thematic work in which the whole group of voices participates at each moment"([4], p. 31)[1], which necessarily leads to an orchestration with high "timbral chromaticism"([11], p. 426, 434)[2]. However, we conclude that the ways by which the orchestration strategies integrate the process of his compositional project still lack a specific research. Moreover, there is not apparently a systematization of the composer's orchestration principles.[3] There are several reasons for this: first, the composer himself has little reported on this aspect, although, as we can remember, at least considering his orchestration of Bach's *Ricercare*—or, rather, his "analytical instrumentation" ([4])—how this dimension becomes relevant for the consolidation of his serialism. Another reason is the lack of a methodological apparatus that can be compared to the solidity of the tools that a musicologist keeps in hand to turn evident the logical organization of other dimensions, particularly those of pitch and rhythmic patterns, even though Hallis points its relative fragility in the case of Webern.[4] Thus, to propose an analysis of Webern's orchestration is equivalent to elaborate an *ad hoc* method virtually starting from scratch. This paper aims to describe the main points of this method in its current experimental stage.[5] Faced with the magnitude of this task, it is needless to say that we will only come to some insights about the subject. We expect, however, that this first step may encourage the continuation of the project.

[1]Dahlhaus refers to Schoenberg.

[2]This author employs the term "chromaticism" in its etymological meaning, referring thus to the "color" of the sound, that is, to the timbre.

[3]The thesis of Jinho Kim ([11]) is one of the few studies that make some deeper investigation about the question of timbre in the *Variationen*. This author grounds his approach on the question of the setting of relational databases, which is followed by his own interpretation.

[4]"Octatonic collections, the whole-tone scale, symmetry relations, all of them are evident in many of his atonal works [...]. However, his letters, annotations, and conferences do not give support to the idea that he used intentionally them when structured his music " ([9], p. vi).

[5]The publication about the theoretical-analytical model that is presented here is currently being prepared for the Editions IRCAM/Delatour. The procedure described in this article is aided by a computational support developed for this purpose by our research group Mus3: the library SOAL– *SonicObjectAnalysisLibrary*, for the environment *OpenMusic*, proposes a series of functions that incorporate the equations and other calculations explicitly or implicitly described here, and in particular the soal-texture-complexity function that specifically addresses the method of textural analysis (GUIGUE, 2016). Last version at: http://git.nics.unicamp.br/mus3-OM/soal4/tags

2. Seven "Sonic States"

Both the genesis and the Variations' formal framework are well known, including through data informed by the composer himself. In a famous letter addressed to Hildegard Jones, Webern comments: "Here are six notes enunciated in a certain way by succession and rhythm and what follows...it is nothing more than this form, always present !!! [...] All these metamorphoses of the first form give birth to the 'theme'. This new unity, in turn, passes through several metamorphoses, which, merged into a new unity, results in the form of the whole" ([16], preface).[6] Webern organized the overall format of the piece in seven sections (which are intended to meet, as already pointed out, the format of a three-part adagio):

- Introduction (mm. 0-20);
- Theme 1 (mm. 21-55);
- Transition 1 (mm. 56-73);
- Theme 2 (mm. 74-109);
- Recapitulation / "in the manner of a development" (mm. 110-134);
- Transition 2 (mm. 135-145); and
- Coda (mm. 146-180).

However, as properly pointed by Kim, who is based on the same source, "the division of the work into seven sections is inserted not without difficulties or contradictions within the scheme of the form of variation, in the sense that both the coordination of the introduction with the theme, and of the theme with the first variation, are not completely successful" ([11], p. 377). Hence, Kim prefers to support, as Makis Solomos ([13]) had done for opus 21, a structuring based on the "perception of a succession of seven sounding states" corresponding to the seven sections of the macro-form ([11], p. 377).

He describes these states by means of some dimensions that resemble those with which we have approached the notion of sonority as a structuring element of composition ([7]). More specifically, as summarized in Figure 2: the *ambitus*; the relative achronic density (which Kim describes as "the density of notes relative to range"); something that approaches what we define as relative diachronic density (in this case, Kim counts the number of notes per measure[7]). He also evaluates the number of notes per section, the number of different notes per section, and the number of bars. It also addresses the intensity, albeit in a concise way, since it is only limited to the three broad levels ([11], p. 423). In the following chart, we normalize the data provided by Kim to produce a visualization of the configuration of each section, according to some of his criteria. Kim's representation of the evolution of the *ambitus* per section is reproduced in Figure 2, bottom.

These surveys bring statistical data that allow us to observe the low density of sounds per unit of time (*diachronic density*), which would be "compensated" by a much higher *achronic* density, within a generally broad range—where the third section differs from the rest by its somewhat narrower tessiture. But these data are not sufficient for a more effective analysis of the impact of parsing of sonorities on formal configuration. A second step includes an investigation of the orchestration strategies adopted by the composer. Kim analyzes these by means of calculation of *timbral entropy*, assuming that the more the music has timbral information, the more the entropy value rises. This would imply greater activity or complexity, towards disorder. He also collects this information from the mapping of the frequency of appearance of each instrument of the

[6] Declaration that perfectly corroborates Bailey's analytical approach.

[7] See our definition of these two types of density ([7], p. 396,399). See also [8], p. 5, 12), under the name of *spatial-density* and *events-density*, respectively.

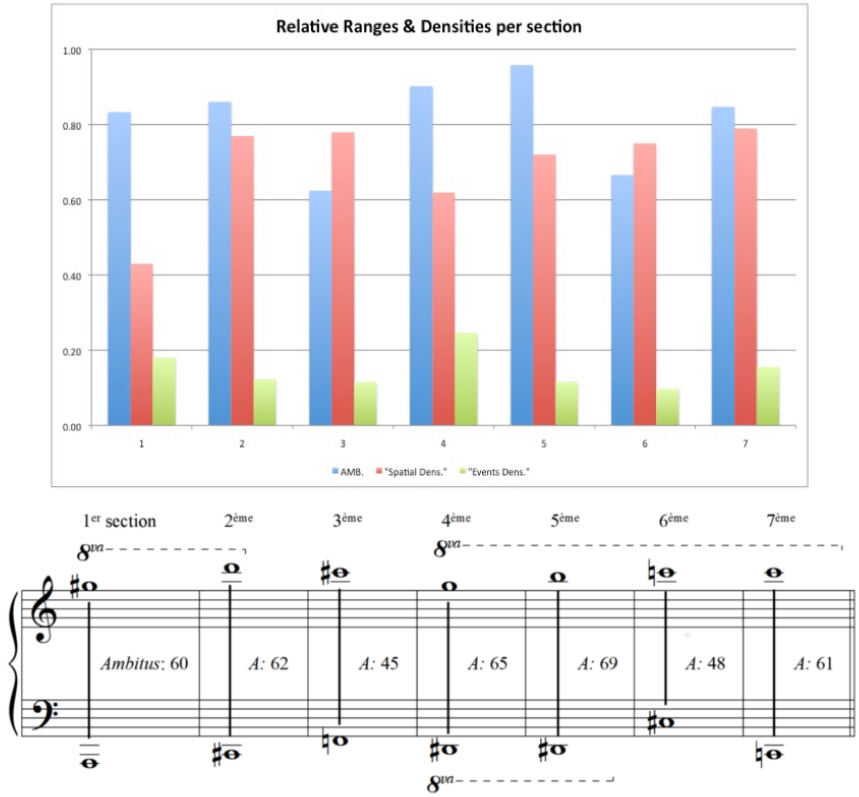

Figure 2: *Some sound characteristics of the* Variations, *section by section. Histograms (top): the* ambitus *(AMB.), blue; the* ***relative achronic density*** *("Spatial Dens."), red; the* ***relative diachronic density*** *("Events Dens."), green; bottom: the ambitus of each main formal section, according to Kim [11], p. 395-396.*

nomenclature (Ibid., p. 310). Kim then applies the entropy-calculation formula (Equation 1) based on Shannon-Weaver model ([12]).[8]

$$P = -(P_1 log_2 P_1 + P_2 log_2 P_2 + P_3 log_2 P_3 + ... + P_n log_2 P_n) = -\Sigma(P_i log_2 P_i) \quad (1)$$

Albeit revealing some aspects peculiar to Webern's aesthetics, the analysis through entropy only suggests tendencies—to *order* and *predictability*, or to chaos and surprise. Thus, it seems us too generic. Moreover, in order to assemble his database, Kim only takes into account the instruments indicated in the nomenclature, thus neglecting the multiple details of timbral variation prescribed by the composer during the course of the work (*mutes*, *pizzicatos*, etc.). We will not, however, discard the entropy information. It will be applied to our method of gathering information about the sounding resources employed, during the retrieving of the instrumental resources data, as it will be described in the next session of this paper.

[8]We retrieved this formula in order to implement the function *relative entropy* in the SOAL library ([8]:22).

Sonic Res.(SRI): bars>	**56**	**58**	**59**	**60**	**62**	**63**	**65**	**67**	**68**	**69**	**70.3**	**71.2**	**72**	**73**
Fl.	1			1		1	1		1	1	1			
Fl. ***flatterzung***														
Ob.	1			1		1	1		1	1	1			
Cl.[B♭]	1			1		1	1		1	1	1		1	1
Bass Cl.				1		1	1		1	1	1			
Hn.[F]														
Hn.[F] ***sord.***	1	1		1		1	1			1		1		
Tpt.[C]														
Tpt.[C] ***sord.***	1	1		1		1	1			1		1		
Tbn.														
Tbn. ***sord.***	1	1		1		1	1			1		1		
Basstuba														
Basstuba ***sord.***				1		1	1			1		1		
Celesta					4				4					
Hp.		4			4				4	4				
Hp. ***harm. fing..***														
Timp.														
Timp. trill														
Vn. I ***solo arco***		1												1
Vn. I ***solo pizz.***													1	
Vn. I ***solo pizz. sord.***														
Vn. I ***div. pizz.***						1								
Vn. I ***tutti arco***								2		2				
Vn. I ***tutti arco sord.***														
Vn. I ***tutti arco harm.***														
Vn. I ***tutti pizz.***														
Vn. I ***tutti pizz. sord.***														
[etc.]														
D.B. ***pizz.***														
n. Sonic Resources (nSR)	9	5	3	8	2	16	8	4	8	11	8	8	2	2
((partition)(crit.))	((3 3 3) (2))	((3 1 1) (2))	((3) (5))	((4 4) (1))	((1 1) (1))	((4 4 4 4) (1))	((4 4) (1))	((2 2) (1))	((4 2 1 1) (1))	((4 4 2 1) (1))	((4 4) (1))	(4 4) (1))	((1 1) (1))	((1 1) (1))
WNR	0,67	0,49	0,34	0,64	0,21	0,85	0,64	0,43	0,64	0,74	0,64	0,64	0,21	0,21
(-a)	-9	-3	-3	-12	0	-24	-12	-2	-7	-13	-12	-12	0	0
(d)	27	7	0	16	1	96	16	4	21	42	16	16	1	1
(-a + d)	18	4	-3	4	1	72	4	2	14	29	4	4	1	1
RVC	0,17	0,03	-0,02	0,04	0,01	0,72	0,04	0,02	0,14	0,29	0,04	0,04	0,01	0,01
SRC	1	0,39	0,25	1	0,16	1	1	0,34	0,62	0,98	0,47	0,41	0,16	0,16

Table 1: *An excerpt of the mapping of* Variations' *sonic resources from mm. 58 to 73 (top line; bar numbers also label the Local Sonic Setups); Left Column:* ***Sonic Resources Index*** *(the full list contains 78 itens); Other columns: The sonic resources content of each* ***Setup****; integer corresponds to the number of simiultaneous notes the instrument plays; Line 'n.* ***Sonic Resources*** *(****nSR****): number of Sonic Resources per Setup. Other lines antecipate analytic data which will be described later in the paper.*

3. An analysis of the instrumental partitioning

3.1. The sonic resources index

The choice of the instrumental colors determined *a priori* by the composer seems to be a good starting point for the analytical process. The procedure consists, in a first quantitative stage, of identifying in the score the complete range of the instrumental sonorities listed by the composer, establishing a Sonic Resources Index (SRI), which is the set of sonic resources that arise along the work (see Table 1, left column). This index has the format of a list that corresponds to the nomenclature of the instrumental parts indicated by the composer, provided in the caput of an orchestral score (**OP**, **Orchestral Part**s), with the addition of all indications and information, textual or symbolic, that aim at producing a differentiated sonority in the instruments or instrumental groups, which are indicated by the composer, either in the caput or inside the score. These correspond, in particular, to *divisi and soli* indications, and modalities of modifications of the sound by mechanical means (mutes or others) or specific expanded techniques (*flatterzunge*, etc.). The **SRI** index then encompasses the universe of tone colors, or Sonic Resources, from which the composer will extract subsets along the work, called **Local Sonic Setups (LSS)**, or, in short, **Setups**.

3.2. Local Sonic Setup and instrumental distribution

A **Local Sonic Setup** is a particular instrumental configuration at any given moment. Composed of one or more **Sonic Resources**, they form the core of the composer's orchestration strategy. A new **setup** is identified each time the composer changes the instrumental distribution, adds or modifies a timbre. The number of sonic resources identified in the *Variations*, starting from an initial nomenclature of 15 **Orchestral Parts**, increases to 78, due to the diversified use of the sonic modulation resources in the strings (*arco, pizz., sordina, sul ponticello, tremolo, harmonics*, and various modalities of *divisi* and *soli*), and the permanent alternation of brasses between normal and muted sounds. A survey of SRs in the *Variations* is presented in Table 2 (left column).

The complete chart can be used to map the distribution of the instruments, therefore allowing inferences about prevalent sonorities. In this work, the three higher woodwinds stand out due to their recurrence (they very often play together, even in unison, as will be seen), as well as the muted trumpet. In fact, the graph (Figure 3) shows clearly how evident is Webern's preferential use of the mutes in the brasses. In contrast, due to the large amount of sonic transformation techniques employed, each **Setup** involving the string instruments has a low reiteration rate.

An integer, corresponding to the number of notes that an instrument plays simultaneously, has been inserted for each **Sonic Resource** (**SR**) active in each Setup , which in turn are summed to reach a global value (see Table 2, line "n. Sonic Resources"). Each Setup is then classified according to the index Weighted Number of Resources (WNR) (Table 1, line WNR) that corresponds to the ratio between the number of SRs it contains and the total number of SRs that could be used. The procedure employed for reaching this number consists of establishing, on the one hand, the amount of instruments prescribed by the composer, ordered by instrumental sections, and, on the other hand, the total number of sonic effects that each instrument is called to perform during the course of the work. From this double list is extracted the minimum value for each instrumental section. Indeed, of the two things one: either the number of sonic effects is smaller than the number of instruments in the section (in this case, the number of SR corresponds to the number of different effects of timbre) or is the number of instruments that is smaller than the number of requested effects. In the latter case it is this number that imposes its own restriction. Table 1 shows the application of this procedure in the work.

Nomenclature	**#instr.**	**#SR**	**Min.**
Fl	1	2	1
ob	1	1	1
cl	1	1	1
Hn	1	2	1
Tp	1	2	1
Tbn	1	2	1
Tub	1	2	1
Cel	1	1	1
Hp	1	2	1
Timp	1	2	1
vln I solo	1	3	1
Vln I Tutti(div.)	2	5	2
Vln II solo	1	2	1
Vln II Tutti(div.)	3	5	3
Va solo	1	4	1
Va Tutti (div.)	3	5	3
Vc solo	1	3	1
Vc Tutti (div.)	3	5	3
CB	1	3	1
TOTAL	**26**	**52**	**26**

Table 2: *Number of Sonic Resources available at once.*

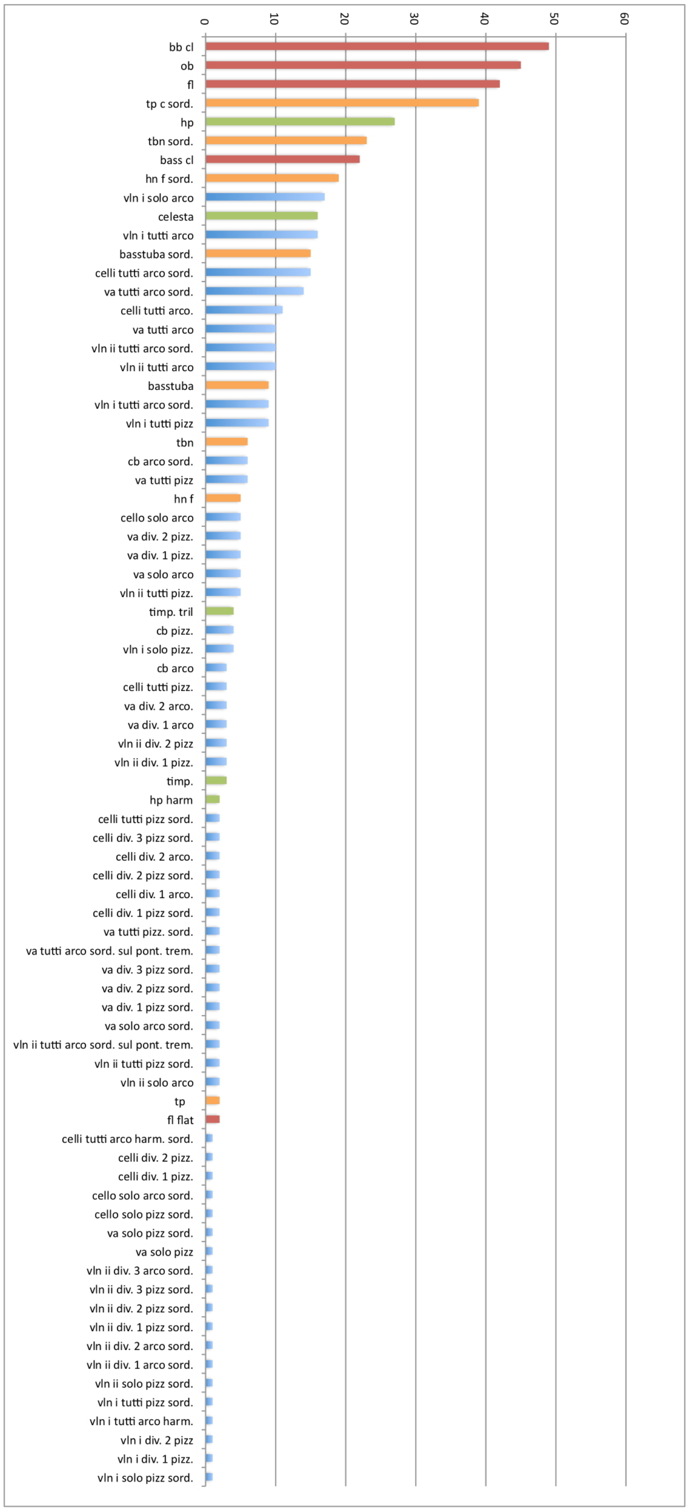

Figure 3: *The* ***Sonic Resources Index*** *of* Variations, *ordered (top-bottom) by recurrence in* ***Setups***

The formula we choose to normalize these two values (the SR number of each LSS and the constant denominator) is logarithmic. The logarithmic curve introduces a compensatory equilibrium, giving more visibility to the setups that use less resources - since we them to be the most usual ones in the work - and to approximate as much as possible those setups that are close to the maximum to the value of 1. Thus, for each LSS, we have one value for WNR (Equation 2).

$$WNR = \frac{ln(nRS)}{ln(nSRI)} \tag{2}$$

The graph (Figure 4) shows the evolution of the WNR in *Variations*, in histograms. We perceive a process of progressive densification when arriving at Transition 1, with one high point in m.63, which accumulates 16 SRs (WNR = 0.85, the largest Setup of the work). This is followed by a sound depression that happens when the composer exposes Theme 2. A second process of sound incrementation takes place from the Reprise, to reach the other climax of the piece, at the end of the Coda, m.168, with its 15 SRs (WNR = 0.83).

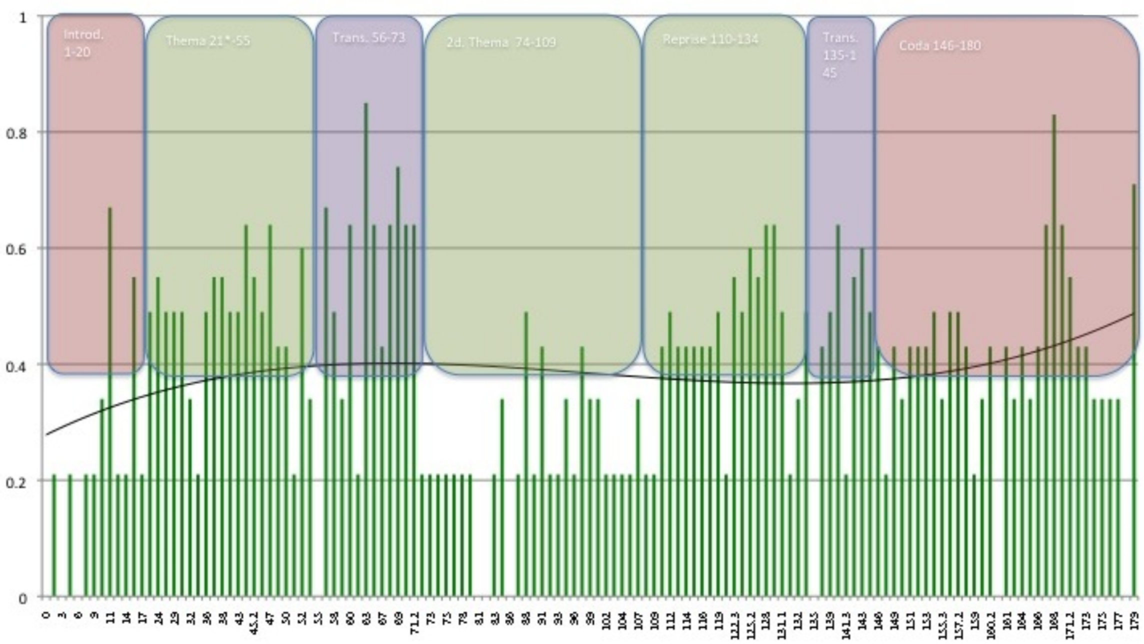

Figure 4: *Evolution of the WNR along the* Variations, ***Setup** by **Setup**. Background colors identify the 7 formal sections.*

However, the numerous histograms of value 0.21 indicate that the most frequent formation is that contains only two instruments. Considering that Webern works with a universe of 78 possible timbres, it is clear that there is an orchestral strategy that considers each sound resource individually, each instrument being able to become an agent that defines the articulation of the work.

Another data, obtained by means of an *ad hoc* pattern-recognition algorithm, showed that 127 of the 137 instrumental combinations are used only once during the course of the work, and the remaining 10 are used twice each. Thus, a secondary application of the principle of non-repetition, which stimulates an investigation into the treatment of sonority from the point of view of Webern's economy, is revealed.

This sound discontinuity would have to be reflected in the calculation of entropy, as this is a way of evaluating the rate of unpredictability of occurrence of data - a more obvious result of the principle of non-repetition. In fact, this calculation reveals an interesting behavior, as seen in the reading of the graph (Figure 5), which summarizes the average number of SRs used per section, in absolute numbers. It confirms the processes we have already discussed regarding the initial accumulation followed by a central depression at the moment of Theme 2 (Figure 5, histogram 4). The calculation of the entropy average, in turn, in the lower graph, brings another image of the orchestral structuring, which can be synthesized by a bipolarization between two states: a relative state of order - Introduction, Theme 2, Transition 2 [9] - destabilized by another, of relative chaos, this reaching its peak at the end of the piece. In fact, in the comparative observation of our survey of SRs used, respectively, in Theme 2 (mm.74-109) and Coda (mm.146-179 – Figure 6. See also Figure 7), one can verify that, in this last section, in spite of accelerated changes of Setups (it reaches three changes in a single measure: 160), the composer does not repeat any configuration. A strategy of *fuite en avant* that asks the listener for a continuous updating of his perception of the work.

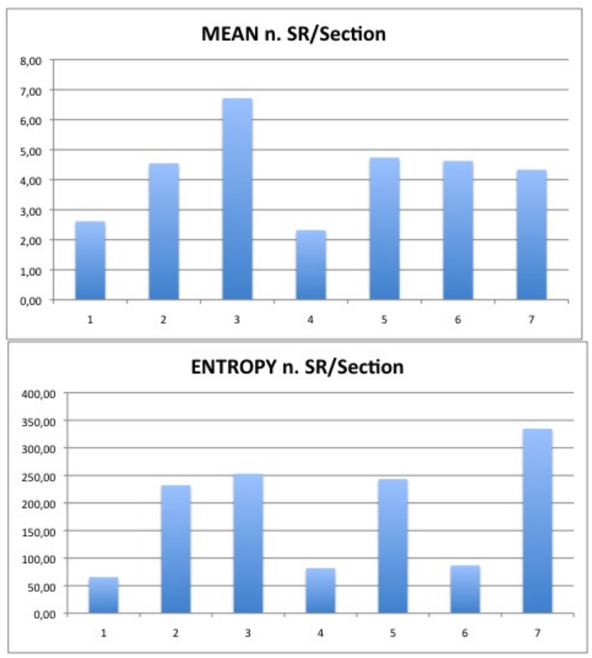

Figure 5: *Top: mean number of* ***Sonic Resources*** *per Section; bottom: entropy value of the number of* ***Sonic Resources*** *per Section*

This effect, however, is counterbalanced in some way at the primary level, where the rhythmic motive values are increased (Figure 7). The rate of motivic changes, therefore, is slower than that of the sound settings. In other words, there is a compensatory gap between the respective dynamics of these two levels of articulation.

Theme 2, on the other hand, maintains similar instrumental formulas (the relative constancy of the flute and clarinets in the first half generates a certain sound stability that highlights motives), less resources involved (less information, therefore, for the listener) and a little slower change rate (some **Setups** last even more than one bar). The entropy rate translates in some way an

[9] The structural weight of this section may have to be relativized because of its brief duration (10 bars only).

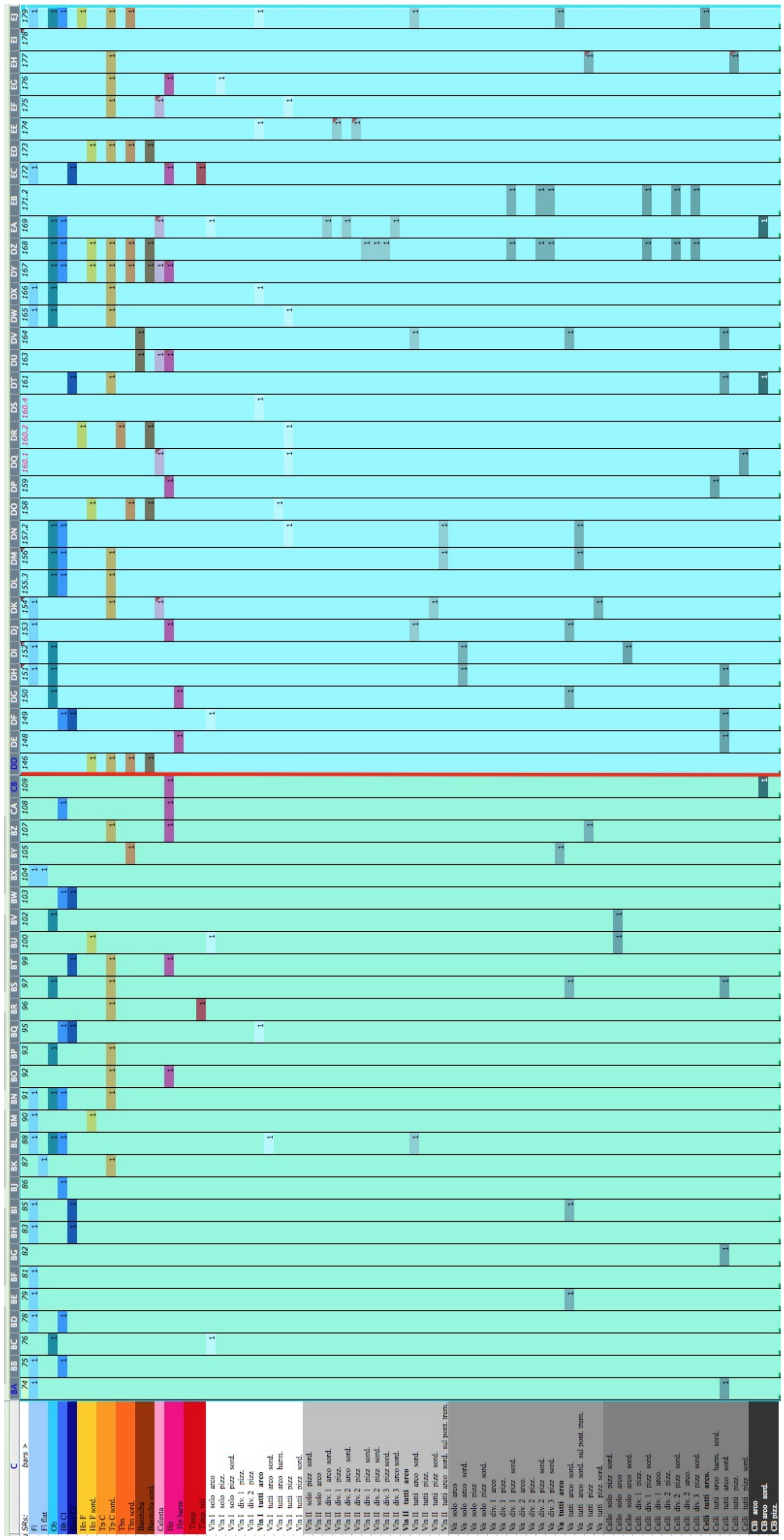

Figure 6: *A comparison of the orchestration of Theme 2 (left side, bars/setups 74-109) and Coda (right side, bars/setups 146-179)*

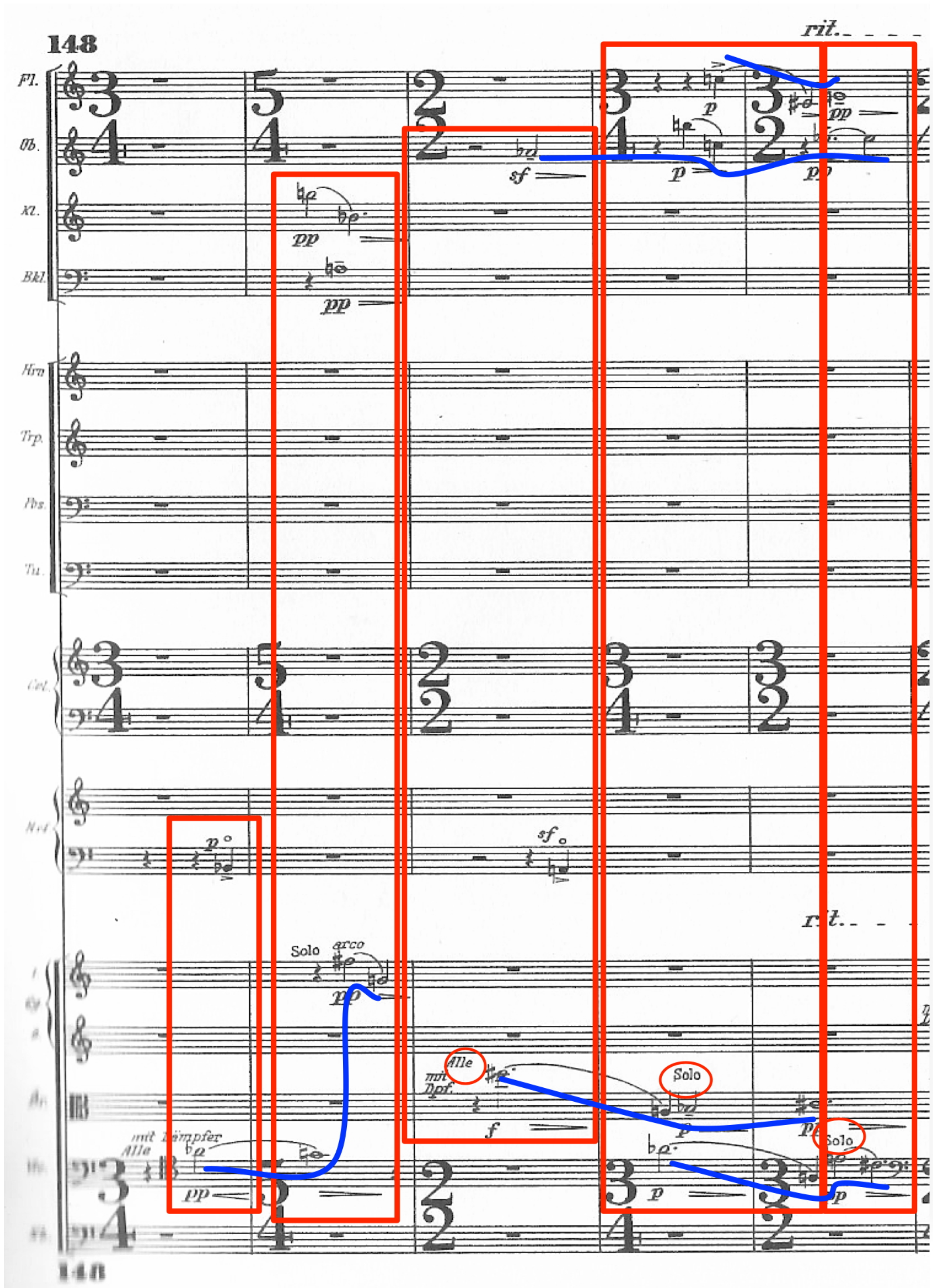

Figure 7: ***Variations***, *mm. 148-152 (apud Universal Edition, p. 25). Red squares identify the sequence of Local Sonic Setups (circles draw attention to tutti/solo permutations), while blue lines help to follow the subsets (tetrachords) of the serie.*

orchestration strategy directly linked to the expression of different and contrasting instances of the formal structure. But before moving further on these considerations, it is necessary to incorporate a second aspect of orchestration.

3.3. Instrumental Density and Textural Complexity—Relative Voicing Complexity

The impact the manipulation of orchestral resources can have on the formal dynamics in the time axis, which we have just shown, is affected by the way with which the composer organizes them into more or less autonomous streams. This distribution characterizes what is conventionally called texture, a dimension that signals the composer's personal style of orchestration, since it reveals his or her way of negotiating instrumental individualities and more or less stratified sonic masses. In terms of orchestral writing, Webern is known for his economy and pointillism, which our analysis highlighted. The expectation, then, is to be, in most of the time, in front of fine and transparent textures.

To support this hypothesis, we developed an algorithm, *Relative Voicing Complexity* (RVC), based on *Partitional Analysis*, a theory proposed by Pauxy Gentil-Nunes ([5]), which refines the methodological proposal of Wallace Berry ([2], p. 184). The more the instrumental parts are *agglomerated* - that is, the more they form homophony and/or homorhythmy - the more the texture becomes "simpler", and the reverse when dispersed. In the model proposed by Gentil-Nunes, the possible partitions for any integer are arranged in a vector format departing from the agglomeration units. In this way, a set of five instruments, for example, presents a lexical-sum of 18 partitions, which can be represented as follows:

lex(5) = (5), (4.1), (3.2), (3.1.1), (2.2.1), (2.1.1.1), (1.1.1.1.1), (4), (3.1), (2.2), (2.1.1), (1.1.1.1),
(3), (1.1.1), (2), (1.1), (1)

in which the partition (1.1.1.1.1) indicates that all "voices" are independent, as in a polyphony, thus qualifying the texture as "complex", and 5, that all instruments play a chord or are in unison, with a texture resultant described as "simple" ([5], p. 16). Partition (1.4), for example, would typically indicate a soloist accompanied by a four-voice homophonic harmony. Figure 3, line ((Partition) (crit.)), shows the format we use so that the parsing analysis is intelligible for OpenMusic and SOAL, encoded in Common Lisp [10].

The calculation of agglomeration and dispersion indices from the partitions survey and their vectorialization starts from the counting of the total number T of the binary relations between the nSRs (the *n* sound features of a *setup*), e.g., the two-by-two combination of nSR, according to a formula borrowed from Tucker's combinatorial analysis (Equation 3; see [6], p. 2 and [14], p. 181).

$$\begin{aligned} T_2 \colon \mathbb{N}^* &\to \mathbb{N} \\ n &\mapsto \frac{n(n-1)}{2} \end{aligned} \tag{3}$$

This function (Equation 3) allows us to enumerate the agglomeration and dispersion relations of each partitioning. Indeed,

> when Berry attributes these indices to the musical text, he is implicitly dividing the set of total relations (T) into relations of contrast and identity, since the constitution of

[10]Cf. [8] for a more detailed explanation on the format.

> the real components is done in terms of relations of identity and the differentiation is accomplished through relations of contrast. From this observation, we can infer that the sum of the relations of identity and contrast in a given textural configuration will always be equal to T. ([6], p. 3).

The agglomeration index *(a)* corresponds to the sum of all binary combinations of the sound resources of each real component (Equation 4), where r is the number of real components and r is the number of sound resources of each real component separately ([6], p. 4 - Equation 4)

$$\begin{aligned} a\colon \mathbb{N}^r &\to \mathbb{N} \\ (a_0 \ldots\ a_{r-1}) &\mapsto \sum_{i=0}^{r-1} T_2(a_i) \end{aligned} \tag{4}$$

In practice, it is enough to apply to each real component the equation T.

The dispersion index (d) is the result of the difference between T and (a) ([6], p. 4).

$$\begin{aligned} d\colon \mathbb{N}^r &\to \mathbb{N} \\ (a_0 \ldots\ a_{r-1}) &\mapsto T_2(\rho) - a(a_0\ \ldots\ a_{r-1}) \end{aligned} \tag{5}$$

A pair of indices a and d is then obtained. The visual arrangement in the form of an indexogram contributes to the interpretation of the dynamics of the textural configurations in the time axis. The indices a and d are symmetrically displayed around zero, by inverting the sign of a. This inversion has another virtue: when added to d, it forms the sum I, then

$$\mathcal{I}(a_0 \ldots a_{r-1}) = (d-a)(a_0 \ldots a_{r-1}) \tag{6}$$

This produces a synthesis evaluation that shows the tendency of the texture, either towards agglomeration (when the sum is negative) or dispersion (positive sum). The values also provide a dynamic curve of this trend. In other words, if, as we have argued above, it is the dispersion rate that determines the complexity of a texture, the integration of the calculation of its agglomeration rate allows for a finer calibration.

To place I on a single axis of relative complexity ([7], p. 40 *et seq.*), what we call *Relative Voicing Complexity*, we normalize I by dividing it by the T value of the setup that has the highest number of sound resources, that is, by the largest number of binary relations possible in a given set of setups. A configuration whose dispersion index would be equal to this number would, in fact, represent the greatest possible complexity in the context. The RVC index is then obtained (Equation 7)

$$\mathrm{RVC}(a_0 \ldots a_{r-1}) = \frac{\mathcal{I}(a_0 \ldots a_{r-1})}{T_2(\rho_{\max})}. \tag{7}$$

In Table 1, one can observe the results of these equations for the mentioned section of Webern, respectively lines -a, d and d - a, corresponding to I.

Partitional Analysis, however, does not stipulate *a priori* the criteria to determine the lexicon of a given partition. Still based, in part, on Berry ([2], p. 193), we have observed that the dispersion of the voices in the textures of *Variation* are generated by:

1. Heterorhythmy: i. e., divergence or asynchrony between rhythmic structures;

2. Heterodirectionality: voices progress into different directions.

We believe that the first agent has more impact on the perception of a sound fission than the second. The second is chosen when the first is idle or when it appears as predominant in the given context. Thus, the instrumental **Setups** are weighted not only by the degree of agglomeration/dispersion of their component parts, but also depending on the method by which this is brought about. In Webern, serial assumption turns heterorhythmy into a systemic feature - due, particularly, to techniques derived from the counterpoint and canon - which, through analysis, proves to be a permanently active agent of dispersion [11] . It follows from these same assumption that the probability of occurrence of massive agglomerations is quite low, which in fact is the case, as shown in the indexogram of Figure 8.

In fact, statistically, Webern privileges fine textures or even "null" ones: from 137 **Setups** 42 contain only one or two SRs. In this sense, the second Theme, unlike the first (to which we will return), stands out since it is made up almost exclusively of Setups of one or two instruments, never agglomerated (see Figure 9, left). However, the dispersion index does not decrease in proportion to the increase in sound resources, as often happens in the orchestral writing of the nineteenth century: on the contrary, it remains almost always high, so that even when he uses many instruments, the composer still avoids to agglomerate them. The most remarkable examples of this procedure are presented in mm. 140-141 and 179. If the Setup of m. 179 still agglutinates three woodwinds forming the partition (3.1.1.1.1.1.1.1.1), the Setup of mm. 140-141 does disperse the eight voices. (Figure 9, right). Because of this configuration, these two Setups are qualified with the highest rate of relative complexity of the work.

The agglomeration of the three high woodwinds, by the way, is a very common solution of orchestration that Webern uses to highlight some motive. For instance, it marks, just from the start, the most salient point of the Introduction (mm. 11-12, see Figure 14 at the end of this paper). In this moment, the Setup is partitioned into two instrumental subsets, in which six string voices constitute a chord that punctuates the unison formed by the three woodwinds. The climax of the Reprise, located between mm. 125 and 131, also brings the main motive to the same trio, in similar partitions and high complexity. And it is still a unison of this trio that will give an end to the work, at m. 179 (Figure 9).

In a brief instant, Webern joins the three main instrumental groups in a homophonic texture and calls them to a dialogue: in this way, he provides a specific sonority to the first Transition (mm. 56-63). This sonority is caused by the most unusual (and exceptional) partitions up to this point, like (3.3.3) (m. 56) or (4.4.4.4) (m. 63) (Figure 10).

Another extremely important aspect of textural organization of this piece derives from the systematic outline of the series in tetrachords: for all chordal agglomerations of the work are grouped into four instrumental parts. Mm. 60-63, we just comment, are a good example of how Webern put against each other the homophonic tetrachords, subsets of versions of the series. But even more remarkable in this view is the First Theme, only section of the piece that draws an accompanied soloist type partitions (Figure 11) [12] . This 'classicism' of the thematic presentation - in a direction which clearly refers to a Schoenbergian practice [13] , and, beyond this, of course, the common practice of music from previous periods - is unique to this theme, showing how Webern valued the traditional conventions in moments where he thought they constituted the best

[11] Heterodirectionality was only taken into consideration in m. 9, and especially in mm. 56-58, in which it acts in an ostensive fashion (see Figure 10).

[12] We adopt the convention O, I, R, RI for naming the conventional transformations of the row. If transposed, a letter "T" will be added with a subscript integer corresponding to the transposition applied (in number of semitones). Lowercases (a , b, ...) refer eventually to rhythmic motives according to the nomenclature by Balley ([1]), with the letter r being reserved to label retrogradation. Information about augmentation and diminution were omitted for clarity. Because the characteristic Werbenian circular construction it is evident that there are alternative possible nomenclature for a given pitch group.

[13] Cf. his *Variations* op. 31.

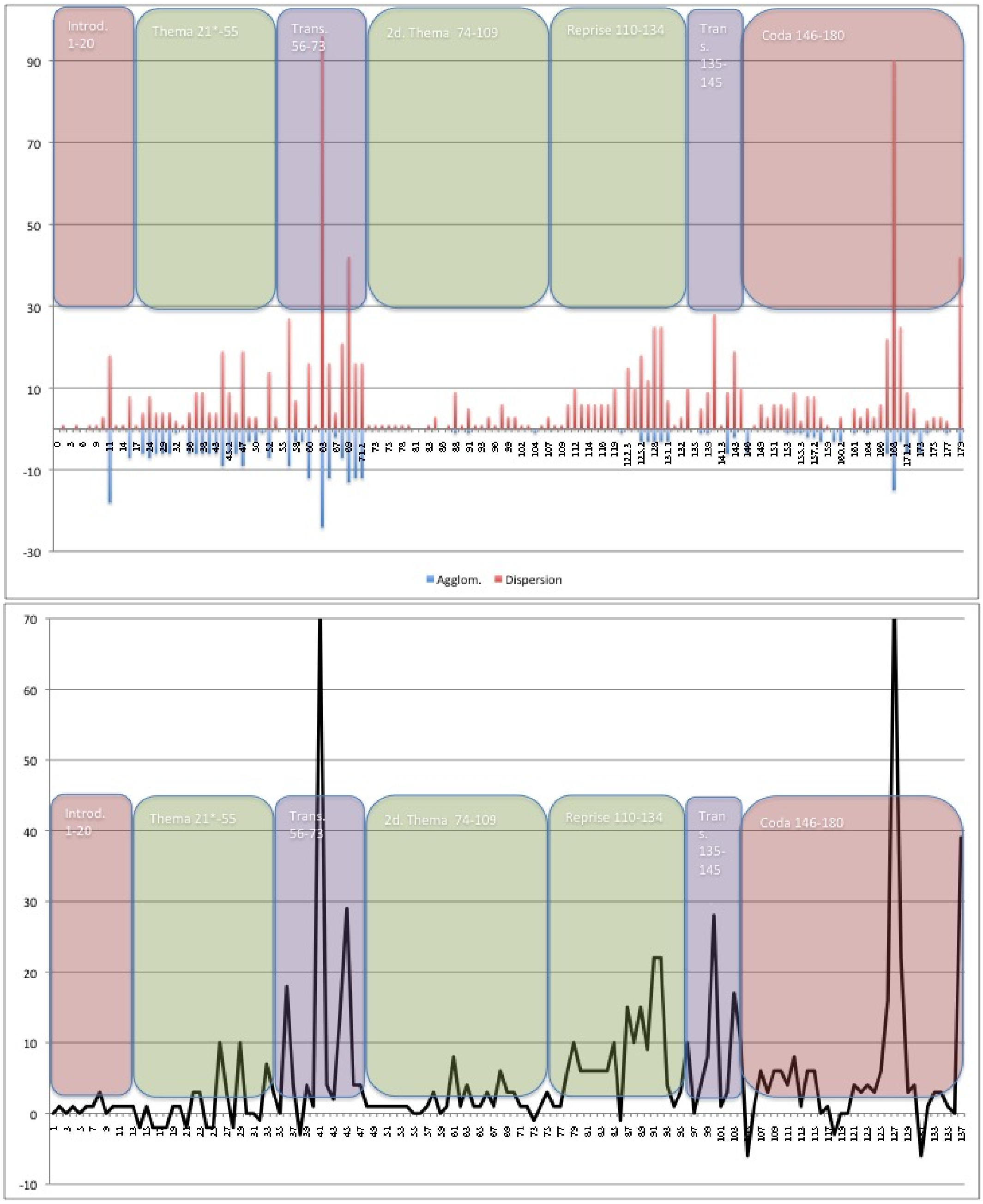

Figure 8: *Top:* indexograms, *with* agglomeration *(blue) and* dispersion *(red) indices, section per section; bottom: Sum* I.

Figure 9: *Comparing textures (mm. 78-81, 140-141, 179-180 (*apud *Universal Edition). The* ***Setups Complexity*** *values will be explained in the last section of the paper. "Part." shows the* partitional analysis*.*

Figure 10: *An excerpt (mm. 56-63) of the Transition 1, where textures present agglomerated subsets (*apud *Universal Edition, p. 10-11).*

solution for the realization of musical ideas. In this sense, he did take care of tuning, as before, in the climax of the Introduction (that we will cover ahead), the two levels: when the goal is to emphasize the thematic focus, the orchestration can return to its historical function.

Figure 11: *Beginning of Theme 1, mm. 9-17 (apud Universal Edition, p. 5).*

The stability of this textural organization is unique: in the beginning of this theme occurs the most extended Setup of the piece, with five-bar long (mm.19-24). This sounding staticity (formed by a muted brass quartet below the solo violin) provokes a sudden break on the constantly changing sonorities that Webern tried to accustom the listener since the beginning of the piece. Undoubtedly, this is a strategy for calling the attention for the *incipit* of the solo violin's theme. It is still maintained in the following three bar-long Setup (in which the brasses are substituted by the woodwind quartet, with the violin being reinforced by the second violins group), and in a lesser extent in some other moments in the same section. The structure of this "harmonic accompaniment" deserves to be examined, because of its exclusivity. It is formed by a sequence of twelve tetrachords, shown in reduction in Figure 12, which distributes the chords along the horizontal plan, proportionally according to their approximate durations.[14]

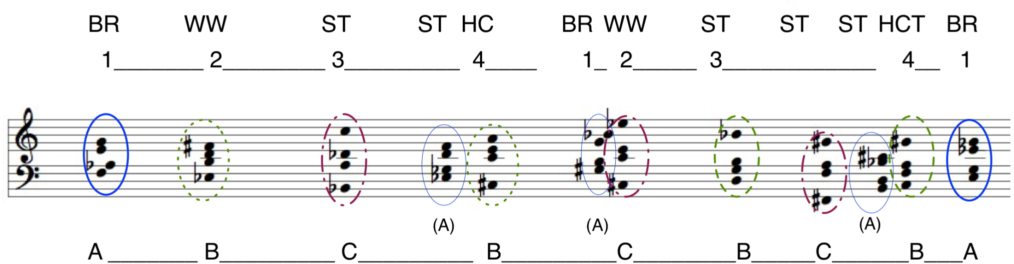

Figure 12: *Harmonic sequence of Theme 1 (mm. 21-55).*

The tetrachords are reduced to three types of intervallic combination. Type A is formed through superimposition of intervals [3 6 3].[15] Type B includes a perfect triad [16], and type C forms the most dissonant superimposition [11 4 11]. As one can observe in Figure 12, Webern adopts a regular chord sequence ABCBCBCBA.[17] The last A chord is identical to the first one, transposed a semitone lower.

This material is reused in the remaining passages in which chords are present. The Transition initiates just after the Theme, with a restatement of chord A in its original version (m.56, strings - Figure 10). The same chord is also present in mm.69 and 168, as it will be later detailed (Figure 15).

Back to the theme, now considering it under the perspective of orchestral distribution, we obtain another sequence, which is expressed in Figure 12 through numbers: "1" represents the brasses (BR), "2" the woodwinds (WW), "3" the strings (ST), and "4" the combination harp + celesta (HC) + timpani (HCT). This time the sequence is still more linear: < 1 2 3 4 1 2 3 4 1 >. We observe that the return of the A chord at the end corresponds to the returning of the same sound Setup (brasses). Such stability is undoubtedly the most notable aspect of this theme structure. Moreover, Webern's music genuinely unfolds through striking adjacent contrasts of instrumentation, as it will be seen below.

[14] Durations are roughly calculated in number of eigth-notes, from one chord to the following.
[15] That is, superimposition of two minor thirds at the distance of tritone.
[16] The triad is successively presented in root position, first, and second inversion.
[17] Due to their very short duration, the "passing" type-A chords of mm.30-1 (strings), 36 (brasses), and 46-7 (strings) are not considered.

3.4. SRC—Local Sonic Setups Relative Complexity

Needless to say, these two sets of data, which we have just explored in terms of their structuring function – the number of sound resources used (WNR) and the way in which they interact (RVC) – are absolutely interdependent. Indeed, the number of voices in which a texture can stratify depends, of course, on the number of instruments involved. At the same time, it is more likely that a small number of instruments will generate more polyphony than the mass of a large orchestra. Therefore, the complexity of voicing (RVC) makes perfect sense as a qualitative modulator of the quantities of resources identified in each Setup.

In the practice of the experimental method we apply, RVC weights multiply those of WNR, in a kind of metaphor of the frequency modulation process, or, describing more specifically: the result of this multiplication is added to the value of WNR. The weight of the modulator may eventually be adjusted, up or down. For this analysis, we leave this weight neutral (= 1). The result sets what we will call *Local Sonic Setup Relative Complexity*, simplified acronym SRC. From which,

$$SRC = WNR.RVC_p \tag{8}$$

In which p is the weight (in %) of the RVC modulator.

The reader will notice that the procedure described does not take into account the duration of the setups. In effect, these are segmented on the basis of their sound configuration. Therefore, the setups go on as long as the sound configuration does not change. This procedure can generate segments of very dissimilar durations. Furthermore, there is no doubt that the time factor can be decisive in the appreciation, or sensation, of the relative impact of a structural unit on the whole. However, we have decided not to take this parameter into account in this essay, although we plan in the future to incorporate it into our model. For, since the Webernian agogic is extremely dynamic and flexible in its prescriptions, it makes it innocuous or inaccurate to deduce realistic temporal proportions from the score. It is therefore necessary, in order to incorporate this dimension, to enter into the study of the recorded performances of the work, from which the interpretative solutions adopted for the management of time can be extracted, a task that would exceed the scope of this article.

3.5. Orchestration as structuring agent

We will close this essay with more global analytical considerations, woven from the analysis of the relative complexity of setups. The SRC is represented in Figure 13 in the form of a sequence of histograms, section by section. Contains trend lines (3rd or 6th order polynomial functions).

We have already identified, based on our previous observations, an orchestral script elaborated in order to characterize each section by some kind of sonority, through different instrumental distributions. In Figure 13, the green histograms table (MEAN SRC / section), averaging the relative complexities of **Setups** by section, shows a general tendency of economy of resources - the highest average does not reach 50%—with the "chamber-like" treatment (even soloistic) that we have already discussed, in the Introduction (Figure 5, histograms 1) and in the second Theme (Figure 5, histograms 4). The latter forms a sound depression between the two adjacent sections, which correspond to the denser orchestration moments. In this sense, this section constitutes an axis after which the sections follow in inverse order of complexity of the sections that precede it. At this level of abstraction, however, the sensitive impact of a cycle like this remains extraordinarily diluted. There are other impacts on the surface that most efficiently capture the attention of the listener.

It is interesting to observe that the Introduction and the transitional sections are developed through great contrasts of sound complexity: the musical time is very bumpy, the sound renovation

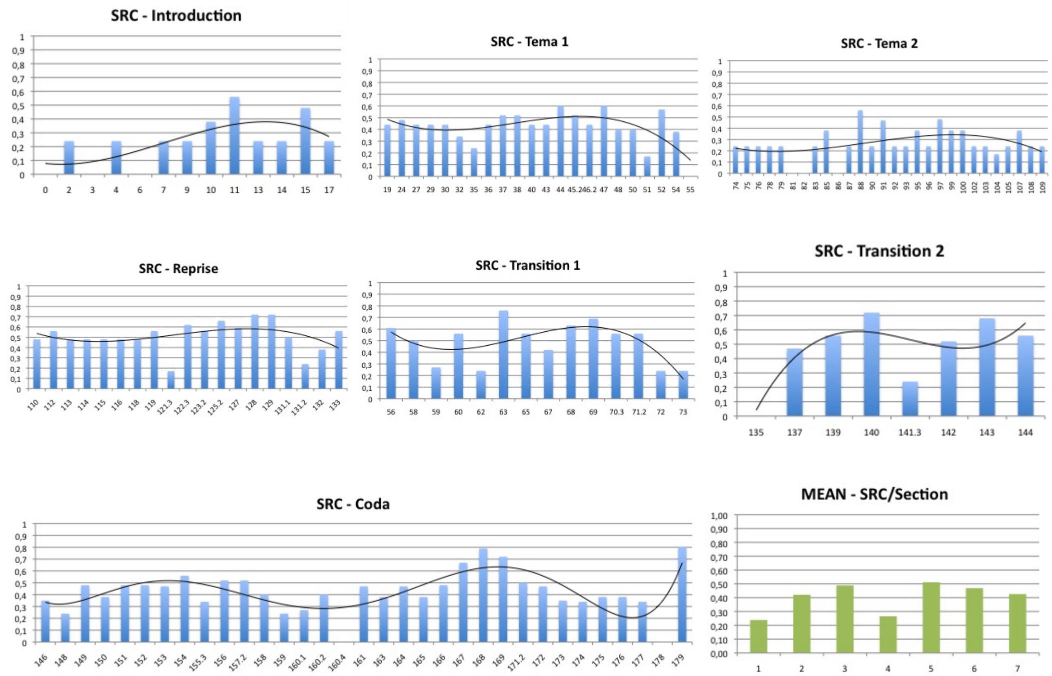

Figure 13: *Histograms of the sequence of **SRC** (**Local Sonic Setups Relative Complexity**) for the whole work, section per section. Green histograms: The mean of **SRC** values per section.*

is permanent, the sonority of the work remains in constant instability, moving abruptly from solos to duos and suddenly to almost *tutti*, and so on. However, at the same time, it is revealed, through the trend lines, a certain directionality in the succession of sonorities, markedly in the Introduction, which obeys a classic format of growth of complexity followed by resimplification, by the structural parallelism between Theme 1 and the Transition that follows it (we have seen that they also share similar chords and textures), in Transition 2 with its central sound reduction, and by the double focus of growth in the Coda, terminated by abrupt contrast from the silence.

As a more detailed example, Figure 14 shows the heart of the Introduction (mm. 6-17). The excerpt is constructed from the cross-overlapping of tetrachords of the four forms of the series supported by elaborations of the two rhythmic motifs a and b (as indicated by colored circles, symbols, and arrows). Although sound transformations are concomitant with the rapidity of tetrachordal changes, a complex dialectic is established between the primary level of organization of these tetrachords and the articulation of orchestral timbres. We note that, in mm. 7-8, *I3ar* and *RI3ar* are divided into 2 successive Setups, which have in common the sonority of the 1st violins and as complementar sonority the Harp, for the first, and the double basses, for the second, in a kind of sound symmetry. We note that it is the same tetrachord (the third) and that I and R have the same rhythmic motion (ar) in phase shift. The following 3 Setups (mm.10, 11, and 13) support two complete forms of the series, Inverted and Retrograde, in alternating distribution between strings, brasses and woodwinds. Different from what usually happens in the rest of the work, in this section the two planes are in synchrony. In other words, the sonorities are exchanged at the same time as the tetrachords, and the complexity curve of the Setups accompanies and sustains the formal logic of tension, climax and rest. In effect, the section culminates with the Setup of

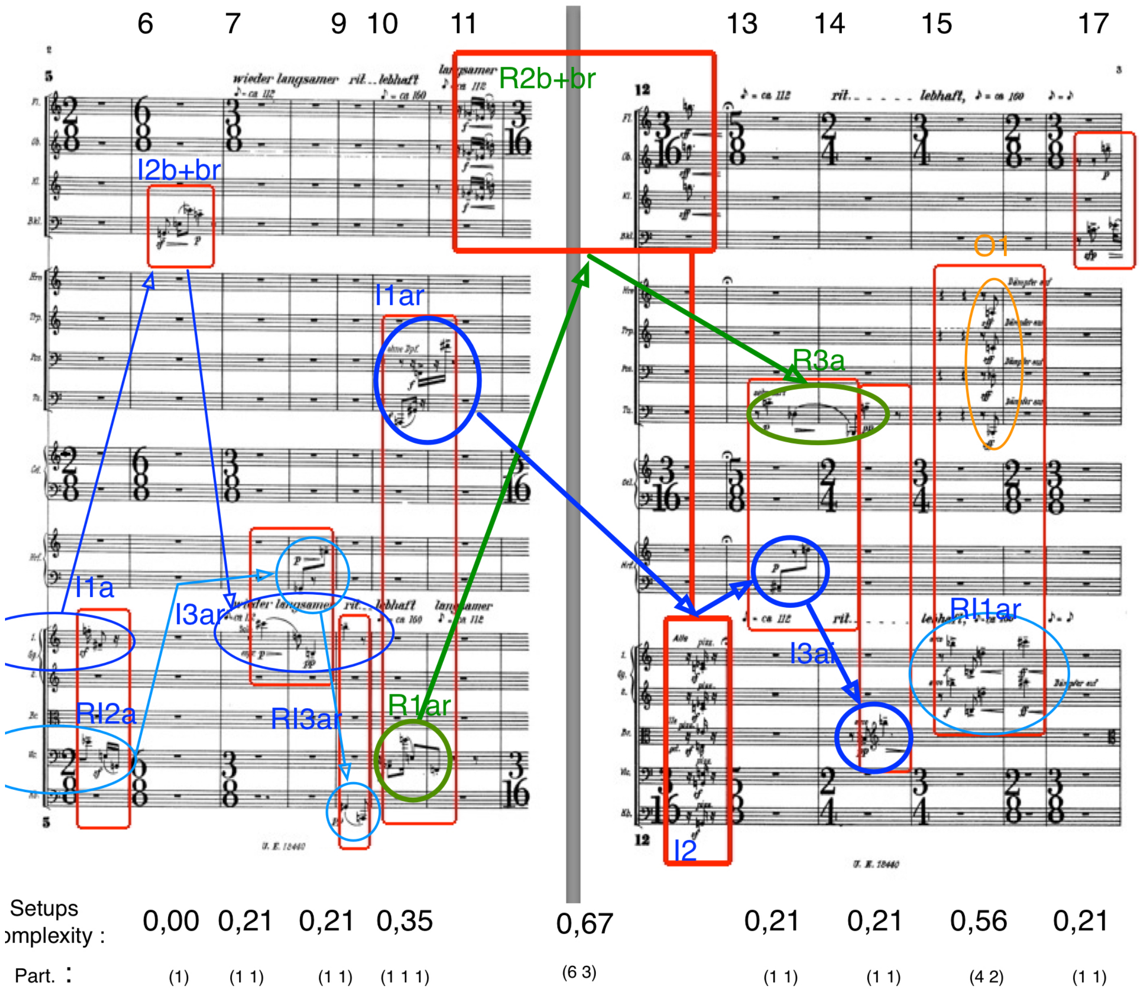

Figure 14: *Examples of the dialectic between serial organization and orchestration, mm. 5-17 (apud Universal Edition, p. 2-3).*

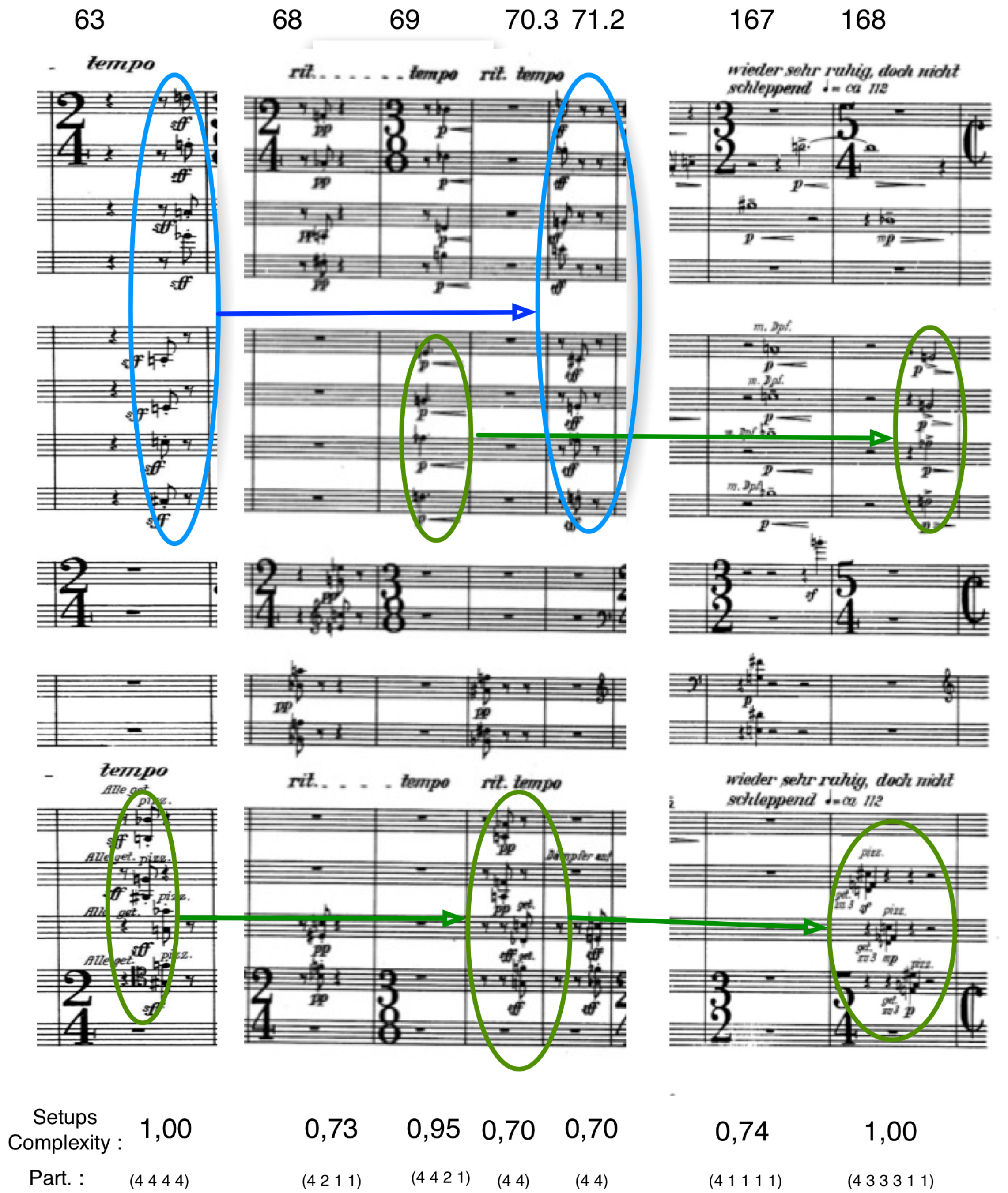

Figure 15: *Most dense* ***Setups*** *of the work. Circles and arrows emphasize similarities of material: mm. 63, 68-71, 167-168 (apud Universal Edition).*

mm. 11-12, which for the first time requests 11 SRs in strong *tutti*, opposing a striking unison in the woodwinds - an orchestral solution structuring in this piece, as we have already mentioned - to a *pizzicato* in the strings. These convergences are precisely reserved for moments in which it is necessary to perceive some structural framework: we have already observed, that they focus, essentially, on the exposition of Theme 1.

Transition 1, on the other hand, concentrates two exceptionally dense **Setups**, since they involve 16 and 11 sound resources, respectively (mm. 63 and 69, Figure 15). They constitute moments of greater sonic impact. This impact will be far fetched at the end of the Coda, in a point also culminating, as if it were a reminiscence: it is m. 168, requesting 15 SRs. In fact, there are many points in common between the three most salient sound objects of the work. We have already pointed out the most apparent: the repetition of chord A in mm. 69 and 168 - which consolidates its structural function. In the case, it is played in the two occasions by the brasses, but the second time, the dynamics is reversed (from p growing to p decreasing): a subtle mirroring, indeed. In addition, the strings always work as a sound punctuation, by means of *pizzicato* or short chords. Woodwind quartet and brass quartet answer each other (in inverted order in mm. 70.3-71), with brief chords of which the *secondo* one forms a species of echo to the first one. The harp always intervenes with tetrachords, with emphasis on that intervallic structure of type C, the most dissonant (in mm.68 and 167).

4. Conclusion

In short, we have tried, through these analytical tools still in the stage of experimentation, to show how Webern's serial economy interacts in the plane of the composition of the orchestral sonorities. We have only approached here the orchestration in its symbolic stage: what the composer lets us know, by means of the score, of his intentions at this level. A second essential step would be to evaluate the results produced by analyzing the sound footprints of the work. Only then could we validate, in the perception level, the sound rhetoric predicted by the composer. In our discussion about Debussy, we had shown that raw materials, motifs, or cells tended to "manifest as amorphous elements, the scourge of the sound atmosphere", diluting "within the systems of articulation of sonorities" ([7], p. 96). We would be inclined to say that Webern radicalizes this logic by freezing the integrality of his primary material in a single intervallic organization (the series) and in two rhythmic patterns. It is not difficult to reach the conclusion that the extraordinary dynamics of this work lies in the ways in which the composer uses this static material to configure sound units that are renewed every moment: we have shown here many examples of the procedures adopted. If Webern's music resists to abstraction by maintaining an organic bond with nature, inherited from romanticism, as argued by Julian Johnson ([10], p. 212 *et seq.*), the art of organizing sounds through the manipulation of instrumental resources, then becomes the locus of the Webernian aesthetics.

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