Axis of Contextual Inversion

Ciro Visconti
Escola de Comunicação e Artes da USP
ciroviscon ti@gmail.com

Abstract: *contextual inversion operations are commonly associated with neo-Riemannian transformations, but the labels* ***P****,* ***L*** *and* ***R*** *and their obverse versions* ***P'****,* ***L'*** *and* ***R'*** *only map sets with at least one common pitch-class. This article shows how contextual inversion operations can be mapped by axes in a similar manner to I_n-operations, and how the positions of these axes can also be labeled. The advantage of this approach is that it will make the representation of any contextual inversion operation between members of any set class possible, which will be useful for both musical analysis and pre-compositional processes. The theoretical concepts developed in this article will be demonstrated in analyses of passages of works by Webern, Stravinsky and Villa-Lobos.*

Keywords*: Contextual inversion. Neo-Riemannian Theory. Graph Theory. Voice-leading.*

I. Introduction

My interest in contextual inversions started when I was trying to wholly adapt Cube Dance[1] to trichords besides sc. (037) and sc. (048) and I ended up finding some difficulties in labelling the connections between the sets. The sets in Cube Dance are organized in voice-leading zones, meaning that the entire graph is within a clock face and a single or a group of trichords are positioned on the radius next to the number relative to their sum class[2]. Richard Cohn calls these numbers as "voice-leading zones" ([3, p.102]). On the original Cube Dance, augmented triads, members of sc. (048), are on voice-leading zone 0, 3, 6 and 9 and consonant triads, members of sc. (037), are on voice-leading zone 1, 2, 4, 5, 7, 8, 10 and 11. Two members of sc. (037) are always connected by neo-Riemannian P or L transformations in two adjacent voice-leading zones of Cube Dance, which means that these sets are both related by contextual inversion and by parsimonious voice-leading. In order to substitute the original sets on cube dance for any type of trichord I had to give up the parsimonious connection between all members of the same set class, since the consonant triads are the only set class in which a member can be connected to two others by contextual inversion and parsimonious voice-leading. In many of the Cube Dances I have built, members of the same set class were connected in two adjacent voice-leading zones, even though they were related by contextual inversion, not sharing a common pitch-class, which means that there is no transformation label for these connections. To overcome this problem I began working on a way to label the contextual inversions in which it was not necessary for the sets to share any pitch-class, which resulted in the theoretical concepts described in this article.

[1] The original Cube Dance is a graph created by Jack Douthett and Peter Steinbach ([4, p.254].

[2] "Sum Class" is a concept created by Joseph Straus: "Two pitch sets are equivalent as members of the same sum class if their pitch integers have the same sum; two pitch- class sets are equivalent as members of the same sum class if their pitch-class integers have the same sum. ([10, p.279]).

The difference between I_n and contextual inversion operations will be discussed in section II, this question will be clarified in the comparison between I_n-operations and neo-Riemannian transformations **P**, **L**, **R**, **P'**, **L'**, and **R'**. In section III it will be argued that transformations and contextual inversion are not necessarily the same. Some authors, like Jack Douthett and Peter Steinbach, consider that the transformations can occur between any two parsimonious sets, even if they are not members of the same set class. Others, like Joseph Straus, take an opposite approach and consider that all transformations are contextual inversions and therefore they must occur between two members of same set class, even if they are not parsimonious. However, the association of contextual inversions with neo-Riemannian transformations generates labels only for those contextual inversions between sets that share pitch-classes.

In Section IV, contextual inversions will be represented by axes in a similar manner to In-operations, but with the difference that they will move according to the pitch-classes of the pair of sets they connect. Also, in the same section, it will be shown that there are twelve contextual inversion axes and that they will also be labeled by letters. Section V will show examples of the usefulness of labels for musical analysis. There will be shown passages in which the sets are related by contextual inversion in Webern's *Concerto for Nine Instruments, op. 24*, in Stravinsky's "Musick to Heare," from *Three Shakespeare Songs* and in Villa-Lobos *Etude 10 for Guitar*. All appendixes for this article are available as supplementary material at https://axesofcontextualinversion.com

II. I_N-operation *versus* Contextual Inversion

Inversion, as well as transposition, is an operation that relates two sets with the same interval content and therefore belonging to the same set class. Following Joseph Straus's approach to this matter ([9, p.53]), we will represent the operation of inversion as I_n, where I stands for inversion and n is the index number (or index of inversion) that is the result of the sum (in mod 12) of the two pitch-classes related by inversion. Thus, for example, I_9 maps C onto A (0 + 9), E onto F (4 + 5) and B onto B♭(11 + 10); I_4 maps D onto D (2 + 2), F♯onto A♯(6 + 10) and A onto G (9 + 7). Inversions with any index number may be represented by axes in the clock face. If the index number is even, the axis crosses two opposing pitch-classes, if the index number is odd, it crosses the point halfway between pitch-classes. The axes for all index numbers are shown in Figure1.

However, contextual inversions are a different type of operation and they are not represented by the axes shown in Figure 1. According David Lewin "'contextual' inversion operation is not defined with reference to any pitch-classes whatsoever" but rather "with respect to a 'contextual' feature of the configuration (s) upon which it operates" ([5, p.7]). The difference between the inversion and the contextual inversion can be clarified in the following examples with the consonant triads: I_7 maps C [0,4,7] onto Cm [0,3,7] and I_1 maps A [9,1,4] onto Am [9,0,4], although in both cases the triads are built over the same root;

I_{11} maps C [0,4,7] onto Em [4,7,e] and I_5 maps A [9,1,4] onto C♯[1,4,8], although in both cases the triads roots are separated by 4 semitones;

I_4 maps C [0,4,7] onto Am [9,0,4] and I_{10} maps A [9,1,4] onto F♯m [6,9,1], although the triads, in both cases, are relative;

I_0 maps C [0,4,7] onto Fm [5,8,0] and I_6 maps A [9,1,4] onto Dm [2,5,9], although in both cases the triads roots are 5 semitones apart;

I_8 maps C [0,4,7] onto C♯m [1,4,8] and I_2 maps A [9,1,4] onto A♯m [t,1,5], although in both cases the triads roots are one semitone apart;

I_2 maps C [0,4,7] onto Gm [7,t,2] and I_8 maps A [9,1,4] onto Em [4,7,e], although in both cases the triads roots are 7 semitones apart.

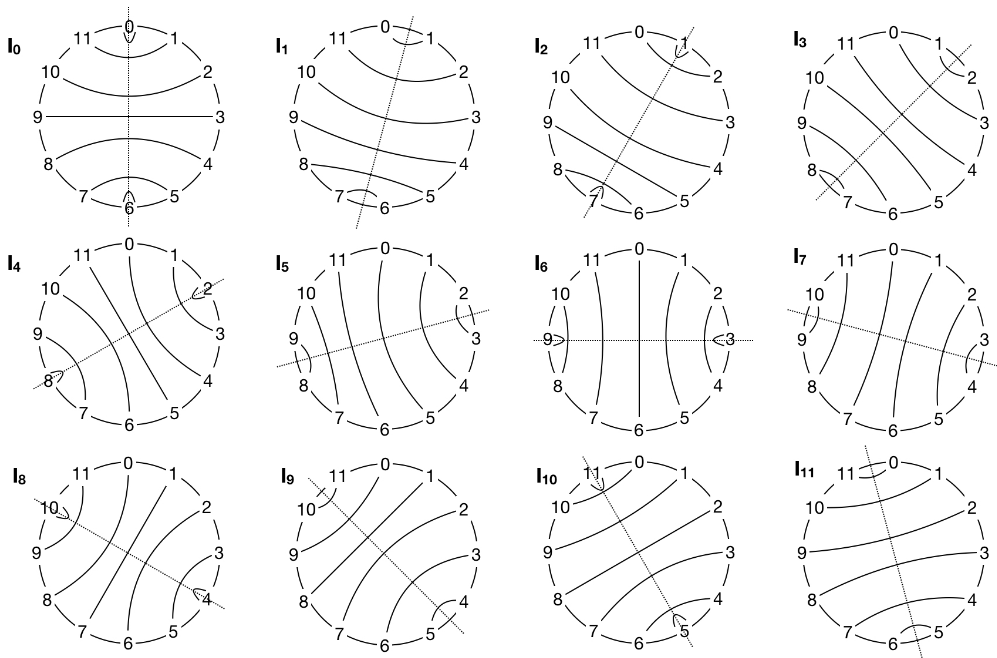


Figure 1: *axes of inversion operations.*

One can note in these six examples that the context between the both pairs of consonant triads does not change, but the index number does so every time. Since connections between consonant triads are a great deal in neo-Riemannian theory, it provided transformational labels for all these six contextual inversions: **P**, **L**, **R**, **P′**, **L′** and **R′**, respectively[3]. **P** maps two parallel triads because it is the contextual inversion that preserves ic 5 with the remaining note flipping around it (Figure 2a); **L** maps a major onto a minor triad preserving ic 3 with the remaining note flipping around it (Figure 2b); **R** maps two relative triads because it is the contextual inversion that preserves ic 4 with the remaining note flipping around it (Figure 2c); **P′** maps a major onto a minor triad flipping ic 5 around the remaining note (Figure 2d); **L′** maps a major onto a minor triad flipping ic 3 around the remaining note (Figure 2e); **R′** maps a major onto a minor triad flipping ic 4 around the remaining note (Figure 2f). Figure 2 shows all these contextual inversions in the consonant triads of the six previous examples.

The transformations used by the neo-Riemannian theory shown in Figure 2 are a nice example of contextual inversion and how it differs from the I_n operations whose axes were shown in Figure 1. However, in the following section, we will show how not all the possibilities of contextual inversions have a transformation label in neo-Riemannian theory and that these two concepts may not be associated when they relate sets of cardinality greater than 3.

III. Transformation *versus* Contextual Inversion

Section II shows how the neo-Riemannian transformations can relate to contextual inversion, but not all authors employ this approach. Douthett and Steinbach, for example, use **P***, **L*** and **R***

[3] The labels **P′**, **L′**, and **R′** is from Robert Morris and he refers to them as "obverse operations”. ([6, p.185]). Cohn uses labels **S** and **N** instead **P′** and **L′**, respectively ([3, p.61-64].

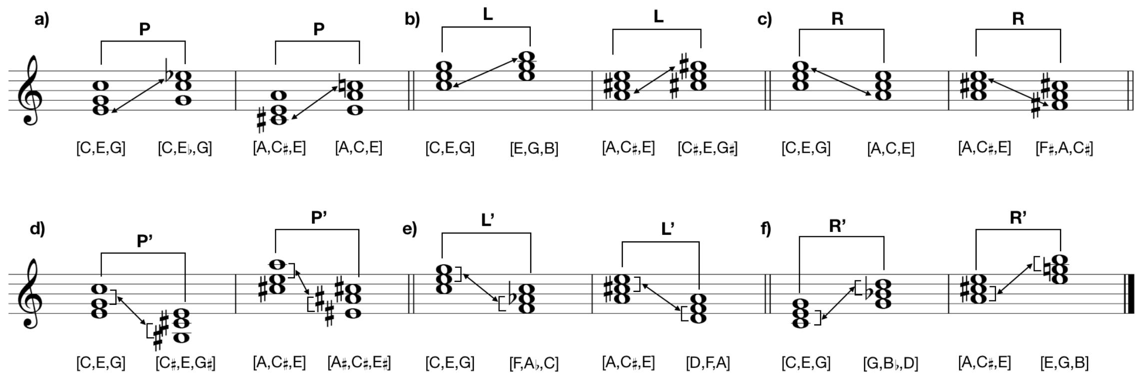


Figure 2: *the contextual inversions of the six main transformations of neo-Riemannian theory.*

(an adaptation of **P**, **L** and **R**) for seventh chords connections that are not related by contextual inversion:

> In the case of seventh chords there are two Parallel* transformations. The transformation $\mathbf{P^*_1}$ exchanges the half-diminished and minor seventh chords that have the same root, and $\mathbf{P^*_2}$ exchanges the minor and dominant-seventh chords with the same root (...). For seventh chords there are two Leittonwechsel* transformations; $\mathbf{L^*_1}$ exchanges root-distinc $P_{1,0}$-related[4] half-diminished and minor seventh chords, and $\mathbf{L^*_2}$ exchanges root-distinc $P_{1,0}$-related dominant and minor seventh chords (...). the Relative* transformation **R*** exchanges two seventh chords[5] that are $P_{0,1}$-related. ([4, p.250]).

It can be noted in this adaptation of the **P**, **L** and R* transformations for seventh chords that the authors are more interested in the parsimonious voice-leading between the chords than in the contextual inversion, since **P*** and **L*** relate members of two different set classes, namely sc. (0258) and sc. (0358). In fact, **P**, **L** and **R** connect two consonant triads that, besides being contextual inversions, are parsimonious, that is, they have two pitch-classes in common and the remaining pitch-class move by a half or a whole step. However, parsimony among members of the same set class is a rather rare feature found only in few classes of trichords, pentachords, heptachords and nonachords, so in order to adapt the labels of transformations for all remaining set classes it is necessary to choose between parsimony and contextual inversion. Straus approaches the problem differently from Douthett and Steinbach. He adapted the 6 transformations labels used in the neo-Riemannian theory for all trichords classes ([8, p.53-56]). He shows how **P** is a contextual inversion that retains the dyad with the largest trichord interval and flips the remaining pitch around it; **L** is a contextual inversion that retains the dyad with the smallest trichord interval and flips the remaining pitch around it; **R** is a contextual inversion that retains the dyad with the second largest trichord interval and flips the remaining pitch around it. He also shows the obverse operation **P’**, **L’** and **R’** and provides a table with the 6 transformations for all the 12 trichords classes ([8, p.56], table 1). This approach can be easily transported to set classes with greater cardinalities. Figure 3 shows how the 6 neo-Riemannian transformations can relate members of sc.

[4] The authors use $P_{m,n}$ notation for parsimonious chords with same cardinality. P stands for parsimony, m is the number of pitches that move by semitone and n is the number of pitches that move by a whole step. Thus, $P_{1,0}$-related chords have all pitches in common, except one that moves by semitone; $P_{0,1}$-related chords have all pitches in common, except one that moves by a whole step.

[5] In this case, both chords are members of sc. (0258).

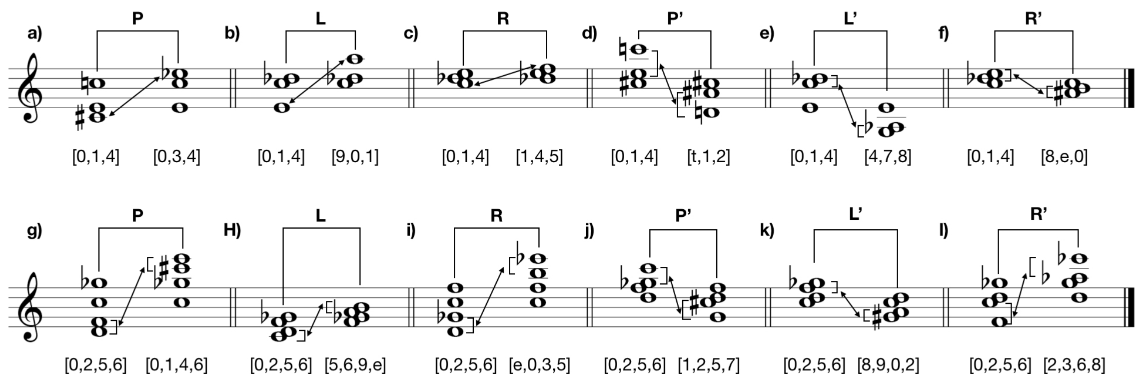


Figure 3: ***P**, **L**, **R**, **P′**, **L′** and **R′** between members of sc. (014) and sc. (0146).*

(014) and members of sc. (0146). For members of sc. (014): **P** retains ic 4 and flips the remaining pitch around it, **L** retains ic 1 and flips the remaining pitch around it, **R** retains ic 3 and flips the remaining pitch around it, **P′** flips ic 4 around the remaining pitch, **L′** flips ic 1 around the remaining pitch and **R′** flips ic 3 around the remaining pitch. For members of sc. (0146): **P** retains ic 6 and flips the remaining dyad around it, **L** retains ic 1 and flips the remaining dyad around it, **R** retains ic 5 and flips the remaining dyad around it, **P′** flips ic 6 around the remaining dyad, **L′** flips ic 1 around the remaining dyad and **R′** flips ic 5 around the remaining dyad.

One can see in Straus's approach that he is interested in contextual inversion rather than in the parsimonious voice-leading, since none of pairs of sets shown in Figure 3 are parsimonious. The advantage of this approach is that it settles the neo-Riemannian labels to any set of any cardinality, however those labels are limited to pairs of sets that have common pitch-classes to each other. In other words, following this approach, all transformations are contextual inversions, but the inverse is not true because the context of the inversions in **P**, **L**, **R**, **P′**, **L′** and **R′**, are given by an axis that must be a dyad or a pitch-class common to both sets. Therefore, there are contextual inversions that cannot be labeled as transformations because they connect sets that do not have pitches in common.

One example of contextual inversions mapping two sets with no common pitches can be show in the Anton Webern's *Concerto for nine Instruments, Op. 24*. Figure 4 shows nine tone rows used in the first 16 measures of the first movement, they are related by four basic operation of twelve-tone system: prime (P), inversion (I), retrograde-prime (RP) and retrograde-inversion (RI). The initial row (Figure 4a) is P_{11}, and the following rows are RI_2 (Figure 4b), RI_1 (Figure 4c), P_0 (Figure 4d), I_0 (Figure 4e), I_5 (Figure 4f), RP_3 (Figure 4g), RP_4 (Figure 4h) and I_6 (Figure 4i). Figure 4 also shows how each row can be segmented into four trichords members of sc. (014)[6] and how the pitches sequence in each trichord also follows operations P, RP, I and RI, as already has observed by Milton Babbitt ([1, p.90-91]). Furthermore, the members of sc. (014) in all tone rows are related by inversion (the index numbers of all inversions are also shown in Figure 4) where the pitches of the first and last pair of trichords in each row always make one of the four hexatonic collections[7], and pitches of the middle pair and the pair with last and first set always make a chromatic hexachord.

It may be noted that inversion index numbers (I_1, I_3, I_5, I_7, I_9 and I_{11}) that map all trichords do not show the uniformity of the criterion according to which their ordering was chosen in each

[6] This segmentation follows the instrumentation choice used by Webern.

[7] In Figure 4, the hexatonic collections are labelled following Straus's notation: $HEX_{0,1}$ [0,1,4,5,8,9], $HEX_{1,2}$ [1,2,5,6,9,t], $HEX_{2,3}$ [2,3,6,7,t,e] and $HEX_{3,4}$ [3,4,7,8,e,0] ([9, p.257]).

row and this is a situation similar to that of the six contextual inversion examples that map the triad consonants shown in Figure 2. But despite the consistency in the connection between the trichords in this example, there is no transformation labels for the contextual inversions mapping the members of sc. (014) because they do not share any common tone. The next section will show how contextual inversions can also be determined by axes, as well as the iI_n-operations, and also can be labelled, as well transformations. In this way, any pair of sets that map onto themselves by contextual inversion, even if they do not share common pitches, can be represented by a label. The Webern's tone rows shown in Figure 4 will be on focus again as examples of axes of contextual inversion.

IV. Axes of Contextual Inversion

Straus states "when a pitch-class set is transposed or inverted, its content will change entirely, partially or not at all" ([9, p.96]). Sets that map entirely onto themselves by transposition are called "transpositionally symmetrical" ([9, p.100]), if they map entirely onto themselves by inversion they are called "inversionally symmetrical" ([9, 107]). According to Straus, there are 14 Tn_n-symmetrical and 79 I_n-symmetrical among the 232 set classes on the Forte List ([9, p.101 and 107]). Since 13 T_n-symmetrical set classes are also I_n-symmetrical[8] , there are 153 set classes which members do not map onto themselves by inversion. These 153 set classes have their members divided into two groups with different OPTC normal forms[9] , following Larry Solomon [7] I will label them as group A and B[10] . For example, sc. (037) is the set class of all consonant triads and they are divided into group A, with 12 minor triads that may be represented by OPTC normal form [0,3,7], and group B, with 12 major triads that may be represented by OPTC normal form [0,4,7]. Members of sc.(014) are also divided into to two groups: A = [0,1,4] and B = [0,3,4], both with 12 members. Any inversion or contextual inversion operation for these set classes will map any member of group A onto a member of group B and vice versa, in this way: I_9 maps C major triad [0,4,7] (group B) onto D minor triad [2,5,9] (group A) and **P** maps B♭minor triad [t,1,5] (group A) onto B♭major triad [t,2,5] (group B); I_7 maps [0,1,4] (group A) onto [3,6,7] (group B) and **L** maps [9,0,1] (group B) onto [9,t,1] (group A). In contrast, members of I_n-symmetrical set classes are not divided into two groups with different OPTC normal forms, since any inversion or contextual inversion operation will map two sets that can also be mapped by inversion. For example: I_0 maps [0,3,6] onto [8,e,2], but these sets are also mapped by T_8; **P** maps [0,3,6] onto [6,9,0], but these sets are also mapped by T_0.

In order to provide positions of axes of contextual inversion it is necessary to determine which one is the fixed point around which a set pair maps themselves by inversion, in other words, it is necessary to determine the context of the inversion. Like the axes of In operations (shown in Figure 1), the axes of contextual inversion operations may cross a pitch-class or halfway between pitch-classes. The difference between these two operations is that the positions of axes in In operations are fixed for each index number (that is, I_0 axis always crosses pitch-classes 0 and 6, I_1 axis always crosses the point between the pitch-classes 0 -1 and 6-7, and so on) while the contextual inversion axes move according to the set pair that is mapped by the operation.

[8] Sc. (013679) is the only one T_n-symmetrical set class that is not I_n-symmetrical.

[9] Callender, Quinn and Tymoczko use the mnemonic OPTIC to refer to octave (O), permutation (P), transposition (T), inversion (I), and cardinality (C) equivalences. The authors explain that there are 32 ways to combine these five equivalences to create different kind of normal forms to represent a set ([2, p.346-348]). OPTC normal forms differ to the prime forms because they do not have the inversion equivalence.

[10] Salomon, in his table of pitch-class sets, labels all the inverses Forte's prime form (which are the OPTC normal form) as "B", so is logical to label the prime form as "A".

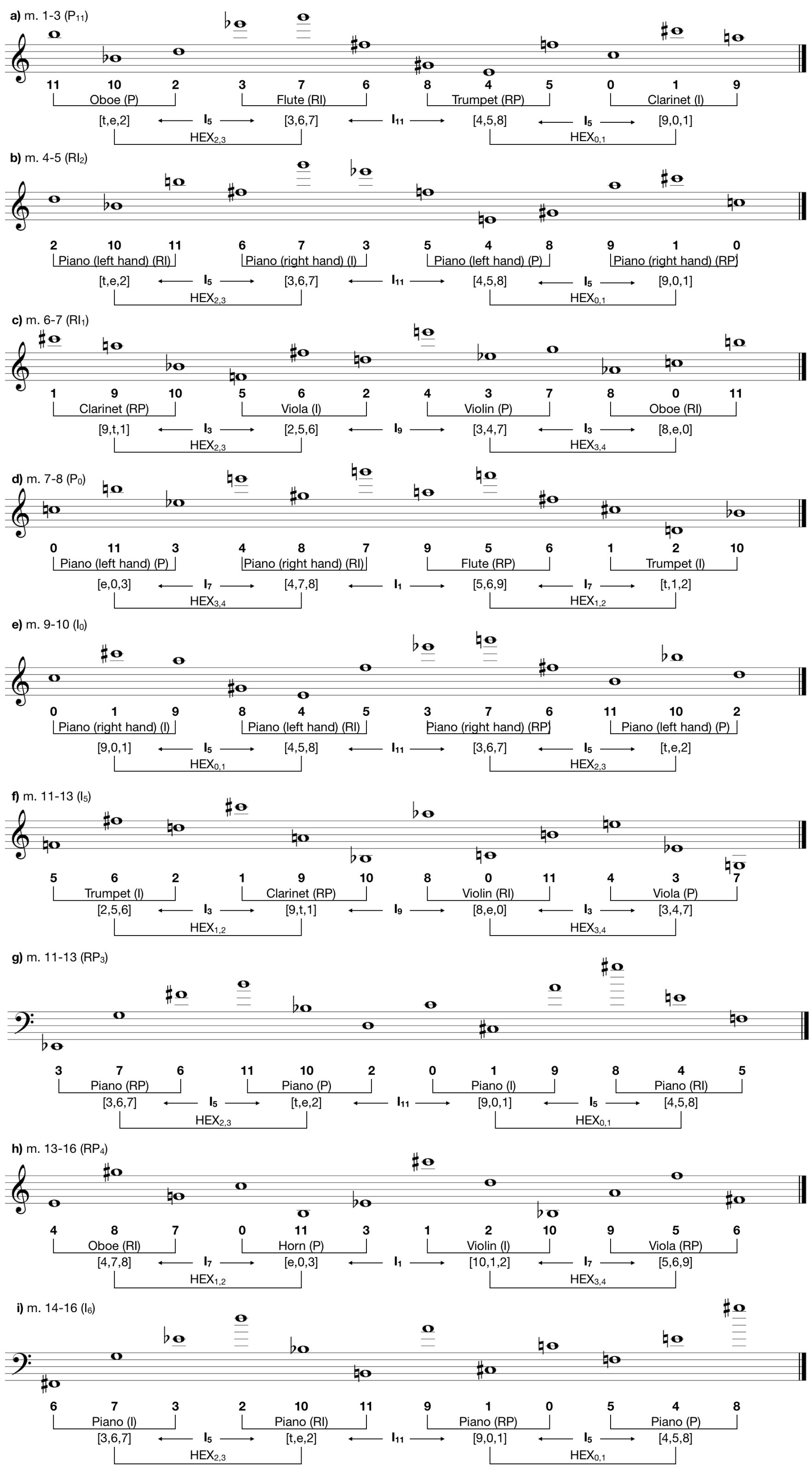


Figure 4: *nine tone rows used in the beginning of the first movement of Anton Webern's Concerto for nine Instruments, Op. 24.*

To determine the position of a contextual axis that maps two sets of same cardinality it is necessary to put both sets in normal form. For sets that are not I_n-symmetrical the position of the contextual inversion axis relative to the first pitch-class of the normal form A is always mirrored to the last pitch-class of the normal form B, this position changes according to the sets. There are 12 different contextual inversion axes that I will label with bold capital letters:

- **A** maps two sets with the axis crossing the first pitch-class of the set in the normal form A and through the last pitch-class of the set in the normal form B;
- **B** maps two sets with the axis crossing the point that is 1/2 semitone over the first pitch-class of the set in the normal form A and the point that is 1/2 semitone below the last pitch-class of the set in the normal form B;
- **C** maps two sets with the axis crossing the point that is 1 semitone over the first pitch-class of the set in the normal form A and the point that is 1 semitone below the last pitch-class of the set in the normal form B;
- **D** maps two sets with the axis crossing the point that is 11/2 semitone over the first pitch-class of the set in the normal form A and the point that is 11/2 semitone below the last pitch-class of the set in the normal form B;
- **E** maps two sets with the axis crossing the point that is 2 semitones over the first pitch-class of the set in the normal form A and the point that is 2 semitones below the last pitch-class of the set in the normal form B;
- **F** maps two sets with the axis crossing the point that is 21/2 semitone over the first pitch-class of the set in the normal form A and the point that is 21/2 semitones below the last pitch-class of the set in the normal form B;
- **G** maps two sets with the axis crossing the point that is 3 semitones over the first pitch-class of the set in the normal form A and the point that is 3 semitones below the last pitch-class of the set in the normal form B;
- **H** maps two sets with the axis crossing the point that is 31/2 semitone over the first pitch-class of the set in the normal form A and the point that is 31/2 semitones below the last pitch-class of the set in the normal form B;
- **I** maps two sets with the axis crossing the point that is 4 semitones over the first pitch-class of the set in the normal form A and the point that is 4 semitones below the last pitch-class of the set in the normal form B;
- **J** maps two sets with the axis crossing the point that is 41/2 semitone over the first pitch-class of the set in the normal form A and the point that is 41/2 semitones below the last pitch-class of the set in the normal form B;
- **K** maps two sets with the axis crossing the point that is 5 semitones over the first pitch-class of the set in the normal form A and the point that is 5 semitones below the last pitch-class of the set in the normal form B;
- **L** maps two sets with the axis crossing the point that is 51/2 semitone over the first pitch-class of the set in the normal form A and the point that is 51/2 semitones below the last pitch-class of the set in the normal form B.

In this way, the context of the inversion is always determined by the position of the axis in relation to one given pitch-class (the first or the last) of a set written in normal form. One can see in Figure 5 some examples of axes of contextual inversion mapping two members of same set classes in clock faces. The pitch-classes of members in the normal form A are connected by lines drawn outside the circle, while the pitch-classes of members in the normal form B are connected by lines drawn inside the circle.

For I_n-symmetrical set classes, the 12 axes of contextual inversion will map members that are

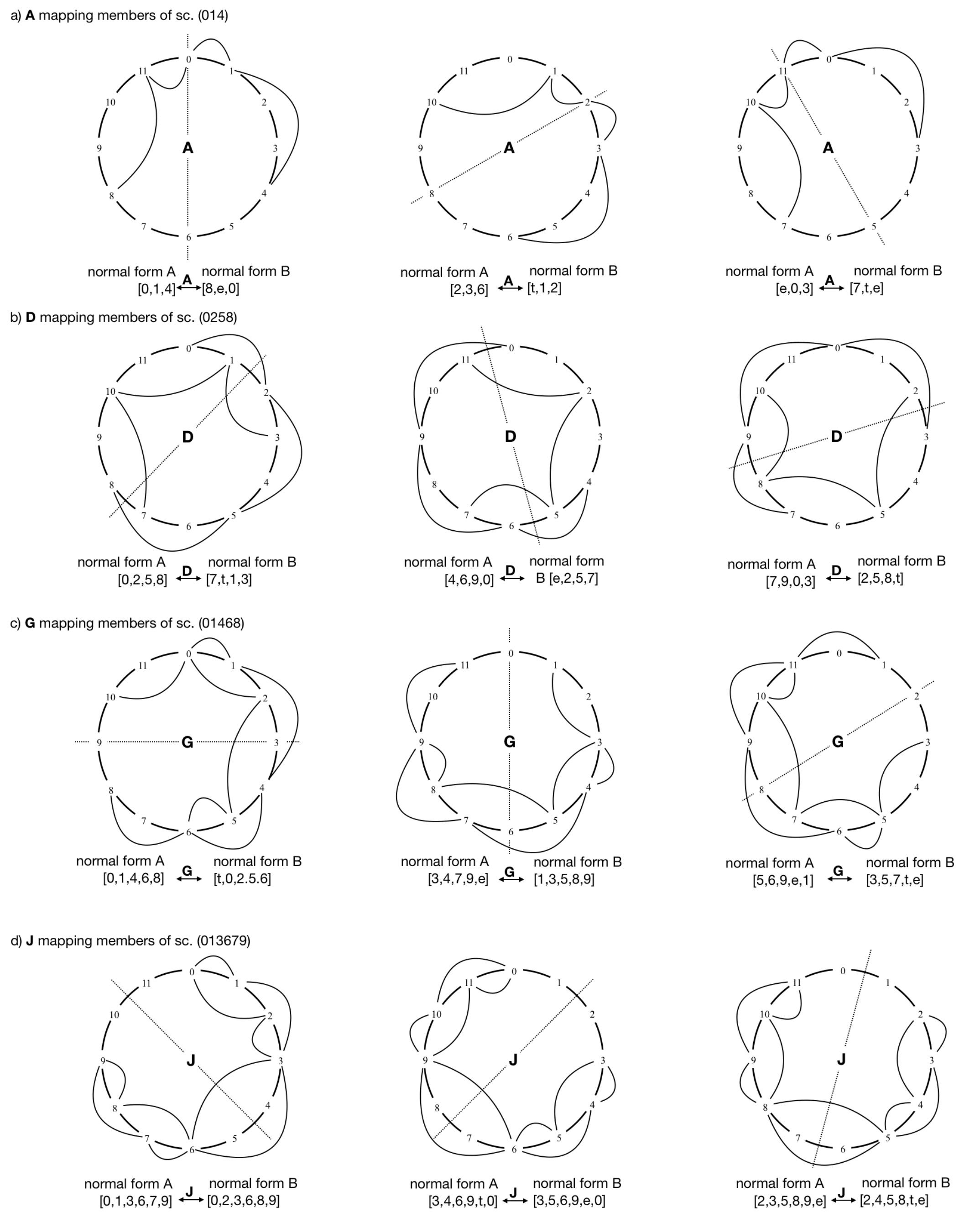


Figure 5: *examples of **A**, **D**, **G** and **J** mapping members of sc. (014), (0258), (01468) and (013679), respectively.*

Table 1: *all members of sc. (014) connect by the axes of contextual inversion.*

[0,1,4]		[8,e,0]	[0,1,4]		[t,1,2]	[0,1,4]		[0,3,4]	[0,1,4]		[2,5,6]	[0,1,4]		[4,7,8]	[0,1,4]		[6,9,t]
[1,2,5]		[9,0,1]	[1,2,5]		[e,2,3]	[1,2,5]		[1,4,5]	[1,2,5]		[3,6,7]	[1,2,5]		[5,8,9]	[1,2,5]		[7,t,e]
[2,3,6]		[t,1,2]	[2,3,6]		[0,3,4]	[2,3,6]		[2,5,6]	[2,3,6]		[4,7,8]	[2,3,6]		[6,9,t]	[2,3,6]		[8,e,0]
[3,4,7]		[e,2,3]	[3,4,7]		[1,4,5]	[3,4,7]		[3,6,7]	[3,4,7]		[5,8,9]	[3,4,7]		[7,t,e]	[3,4,7]		[9,0,1]
[4,5,8]		[0,3,4]	[4,5,8]		[2,5,6]	[4,5,8]		[4,7,8]	[4,5,8]		[6,9,t]	[4,5,8]		[8,e,0]	[4,5,8]		[t,1,2]
[5,6,9]	**A**	[1,4,5]	[5,6,9]	**C**	[3,6,7]	[5,6,9]	**E**	[5,8,9]	[5,6,9]	**G**	[7,t,e]	[5,6,9]	**I**	[9,0,1]	[5,6,9]	**K**	[e,2,3]
[6,7,t]		[2,5,6]	[6,7,t]		[4,7,8]	[6,7,t]		[6,9,t]	[6,7,t]		[8,e,0]	[6,7,t]		[t,1,2]	[6,7,t]		[0,3,4]
[7,8,e]		[3,6,7]	[7,8,e]		[5,8,9]	[7,8,e]		[7,t,e]	[7,8,e]		[9,0,1]	[7,8,e]		[e,2,3]	[7,8,e]		[1,4,5]
[8,9,0]		[4,7,8]	[8,9,0]		[6,9,t]	[8,9,0]		[8,e,0]	[8,9,0]		[t,1,2]	[8,9,0]		[0,3,4]	[8,9,0]		[2,5,6]
[9,t,1]		[5,8,9]	[9,t,1]		[7,t,e]	[9,t,1]		[9,0,1]	[9,t,1]		[e,2,3]	[9,t,1]		[1,4,5]	[9,t,1]		[3,6,7]
[t,e,2]		[6,9,t]	[t,e,2]		[8,e,0]	[t,e,2]		[t,1,2]	[t,e,2]		[0,3,4]	[t,e,2]		[2,5,6]	[t,e,2]		[4,7,8]
[e,0,3]		[7,t,e]	[e,0,3]		[9,0,1]	[e,0,3]		[e,2,3]	[e,0,3]		[1,4,5]	[e,0,3]		[3,6,7]	[e,0,3]		[5,8,9]
[0,1,4]		[9,0,1]	[0,1,4]		[e,2,3]	[0,1,4]		[1,4,5]	[0,1,4]		[3,6,7]	[0,1,4]		[5,8,9]	[0,1,4]		[7,t,e]
[1,2,5]		[t,1,2]	[1,2,5]		[0,3,4]	[1,2,5]		[2,5,6]	[1,2,5]		[4,7,8]	[1,2,5]		[6,9,t]	[1,2,5]		[8,e,0]
[2,3,6]		[e,2,3]	[2,3,6]		[1,4,5]	[2,3,6]		[3,6,7]	[2,3,6]		[5,8,9]	[2,3,6]		[7,t,e]	[2,3,6]		[9,0,1]
[3,4,7]		[0,3,4]	[3,4,7]		[2,5,6]	[3,4,7]		[4,7,8]	[3,4,7]		[6,9,t]	[3,4,7]		[8,e,0]	[3,4,7]		[t,1,2]
[4,5,8]		[1,4,5]	[4,5,8]		[3,6,7]	[4,5,8]		[5,8,9]	[4,5,8]		[7,t,e]	[4,5,8]		[9,0,1]	[4,5,8]		[e,2,3]
[5,6,9]	**B**	[2,5,6]	[5,6,9]	**D**	[4,7,8]	[5,6,9]	**F**	[6,9,t]	[5,6,9]	**H**	[8,e,0]	[5,6,9]	**J**	[t,1,2]	[5,6,9]	**L**	[0,3,4]
[6,7,t]		[3,6,7]	[6,7,t]		[5,8,9]	[6,7,t]		[7,t,e]	[6,7,t]		[9,0,1]	[6,7,t]		[e,2,3]	[6,7,t]		[1,4,5]
[7,8,e]		[4,7,8]	[7,8,e]		[6,9,t]	[7,8,e]		[8,e,0]	[7,8,e]		[t,1,2]	[7,8,e]		[0,3,4]	[7,8,e]		[2,5,6]
[8,9,0]		[5,8,9]	[8,9,0]		[7,t,e]	[8,9,0]		[9,0,1]	[8,9,0]		[e,2,3]	[8,9,0]		[1,4,5]	[8,9,0]		[3,6,7]
[9,t,1]		[6,9,t]	[9,t,1]		[8,e,0]	[9,t,1]		[t,1,2]	[9,t,1]		[0,3,4]	[9,t,1]		[2,5,6]	[9,t,1]		[4,7,8]
[t,e,2]		[7,t,e]	[t,e,2]		[9,0,1]	[t,e,2]		[e,2,3]	[t,e,2]		[1,4,5]	[t,e,2]		[3,6,7]	[t,e,2]		[5,8,9]
[e,0,3]		[8,e,0]	[e,0,3]		[t,1,2]	[e,0,3]		[0,3,4]	[e,0,3]		[2,5,6]	[e,0,3]		[4,7,8]	[e,0,3]		[6,9,t]

also mapped by transposition, meaning that they cannot be expressed by two different OPTC normal forms. Because of that, the position of any axis relative to the first pitch-class will be mirrored to the last pitch-class of the same normal form and this will result, in some cases, in a single axis mapping two different pairs of sets. For example, A maps set [0,1,2] onto [t,e,0], since the axis crosses the first pitch-class of its normal form, but A also maps [0,1,2] onto [2,3,4] because the axis also crosses the last pitch-class of its normal form. In order to differentiate these two connections, I will use A_0 to label the axis between [0,1,2] and [t,e,0], since it crosses pitch-class 0, and A_2 to label the axis between [0,1,2] onto [2,3,4], since it crosses pitch-class 2. For axes that cross points halfway between two pitch-classes, I will use the integers that represent these two pitch-classes separated by a comma after the label letter, so $F_{2,3}$ maps [0,1,2] onto [3,4,5], since the axis crosses halfway between pitch-classes 2 and 3, and $F_{e,0}$ maps [0,1,2] onto [9,t,e], since the axis crosses halfway between pitch-classes 11 and 0. Figure 6 shows some examples of axes of contextual inversion mapping two members of same In-symmetrical set classes in clock faces.

One can see in Table 1 an 2, how all members of same set class connect to each other by the 12 contextual inversion axes. Table 1 shows the axes (labels **A** to **L**) connecting all members of sc. (014), with the sets represented by normal form A to the left and those represented by normal form B to the right of each label. Table 2 shows the axes connecting all members of sc. (0158)[11] . Since this is a In-symmetrical set class, a pair of members can be connected by two different axes, resulting in 7, instead of 12, different labels[12].

V. Analytical examples

In this section, examples of how labels for contextual inversion axis discussed on previous section are helpful for analyses will be discussed. I will show some analytical examples of passages

[11] Access www.axesofcontextualinversion.com to download the tables for all each set-classes.

[12] One can see in Table 2 that C maps the same pairs of sets that A, and the same happens with I and G, J and F, K and E and L and D. All In-symmetrical set classes will have pairs of sets that can be map by two axes.

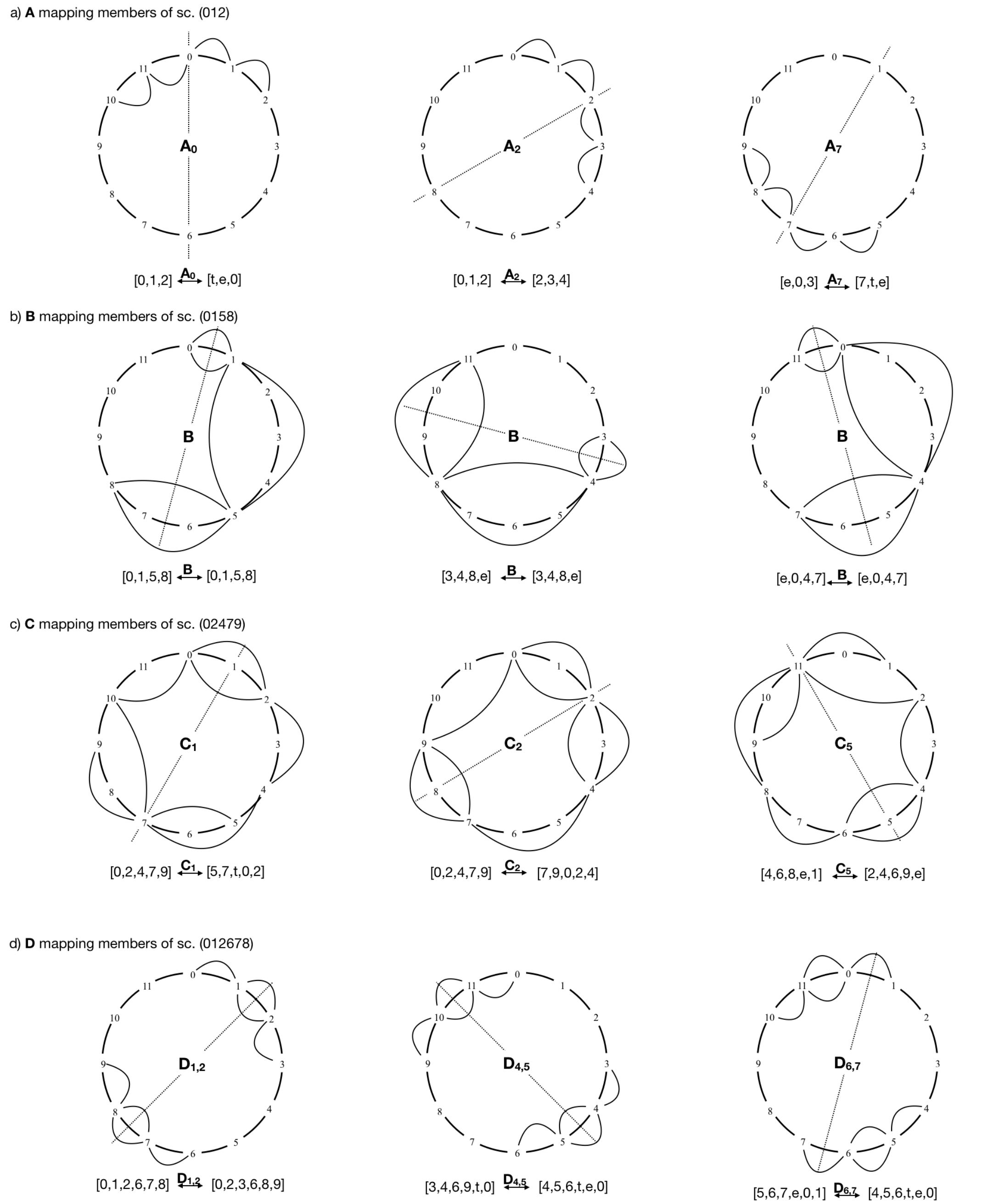


Figure 6: *examples of* ***A***, ***B***, ***C*** *and* ***D*** *mapping members of sc. (012), (0158), (02479) and (012678), respectively.*

Table 2: *all members of sc. (0158) connect by the axes of contextual inversion.*

	A			C (=A)			E			G		I (=G)	K (=E)
[0,1,5,8]	$\mathbf{A}_0$	[e,0,4,7]				[0,1,5,8]	$\mathbf{E}_2$	[3,4,8,e]	[0,1,5,8]	$\mathbf{G}_3$	[5,6,t,1]		
[1,2,6,9]	$\mathbf{A}_1$	[0,1,5,8]				[1,2,6,9]	$\mathbf{E}_3$	[4,5,9,0]	[1,2,6,9]	$\mathbf{G}_4$	[6,7,e,2]		
[2,3,7,t]	$\mathbf{A}_2$	[1,2,6,9]				[2,3,7,t]	$\mathbf{E}_4$	[5,6,t,1]	[2,3,7,t]	$\mathbf{G}_5$	[7,8,0,3]		
[3,4,8,e]	$\mathbf{A}_3$	[2,3,7,t]				[3,4,8,e]	$\mathbf{E}_5$	[6,7,e,2]	[3,4,8,e]	$\mathbf{G}_6$	[8,9,1,4]		
[4,5,9,0]	$\mathbf{A}_4$	[3,4,8,e]				[4,5,9,0]	$\mathbf{E}_6$	[7,8,0,3]	[4,5,9,0]	$\mathbf{G}_7$	[9,t,2,5]		
[5,6,t,1]	$\mathbf{A}_5$	[4,5,9,0]				[5,6,t,1]	$\mathbf{E}_7$	[8,9,1,4]	[5,6,t,1]	$\mathbf{G}_8$	[t,e,3,6]		
[6,7,e,2]	$\mathbf{A}_6$	[5,6,t,1]				[6,7,e,2]	$\mathbf{E}_8$	[9,t,2,5]	[6,7,e,2]	$\mathbf{G}_9$	[e,0,4,7]		
[7,8,0,3]	$\mathbf{A}_7$	[6,7,e,2]				[7,8,0,3]	$\mathbf{E}_9$	[t,e,3,6]	[7,8,0,3]	$\mathbf{G}_t$	[0,1,5,8]		
[8,9,1,4]	$\mathbf{A}_8$	[7,8,0,3]				[8,9,1,4]	$\mathbf{E}_t$	[e,0,4,7]	[8,9,1,4]	$\mathbf{G}_e$	[1,2,6,9]		
[9,t,2,5]	$\mathbf{A}_9$	[8,9,1,4]				[9,t,2,5]	$\mathbf{E}_e$	[0,1,5,8]	[9,t,2,5]	$\mathbf{G}_0$	[2,3,7,t]		
[t,e,3,6]	$\mathbf{A}_t$	[9,t,2,5]				[t,e,3,6]	$\mathbf{E}_0$	[1,2,6,9]	[t,e,3,6]	$\mathbf{G}_1$	[3,4,8,e]		
[e,0,4,7]	$\mathbf{A}_e$	[t,e,3,6]				[e,0,4,7]	$\mathbf{E}_1$	[2,3,7,t]	[e,0,4,7]	$\mathbf{G}_2$	[4,5,9,0]		
[0,1,5,8]		[0,1,5,8]	[0,1,5,8]	$\mathbf{D}_{1,2}$	[2,3,7,t]	[0,1,5,8]	$\mathbf{F}_{2,3}$	[4,5,9,0]	[0,1,5,8]	$\mathbf{H}_{3,4}$	[6,7,e,2]		
[1,2,6,9]		[1,2,6,9]	[1,2,6,9]	$\mathbf{D}_{2,3}$	[3,4,8,e]	[1,2,6,9]	$\mathbf{F}_{3,4}$	[5,6,t,1]	[1,2,6,9]	$\mathbf{H}_{4,5}$	[7,8,0,3]		
[2,3,7,t]		[2,3,7,t]	[2,3,7,t]	$\mathbf{D}_{3,4}$	[4,5,9,0]	[2,3,7,t]	$\mathbf{F}_{4,5}$	[6,7,e,2]	[2,3,7,t]	$\mathbf{H}_{5,6}$	[8,9,1,4]		
[3,4,8,e]		[3,4,8,e]	[3,4,8,e]	$\mathbf{D}_{4,5}$	[5,6,t,1]	[3,4,8,e]	$\mathbf{F}_{5,6}$	[7,8,0,3]	[3,4,8,e]	$\mathbf{H}_{6,7}$	[9,t,2,5]		
[4,5,9,0]		[4,5,9,0]	[4,5,9,0]	$\mathbf{D}_{5,6}$	[6,7,e,2]	[4,5,9,0]	$\mathbf{F}_{6,7}$	[8,9,1,4]	[4,5,9,0]	$\mathbf{H}_{7,8}$	[t,e,3,6]		
[5,6,t,1]	B	[5,6,t,1]	[5,6,t,1]	$\mathbf{D}_{6,7}$	[7,8,0,3]	[5,6,t,1]	$\mathbf{F}_{7,8}$	[9,t,2,5]	[5,6,t,1]	$\mathbf{H}_{8,9}$	[e,0,4,7]		
[6,7,e,2]		[6,7,e,2]	[6,7,e,2]	$\mathbf{D}_{7,8}$	[8,9,1,4]	[6,7,e,2]	$\mathbf{F}_{8,9}$	[t,e,3,6]					
[7,8,0,3]		[7,8,0,3]	[7,8,0,3]	$\mathbf{D}_{8,9}$	[9,t,2,5]	[7,8,0,3]	$\mathbf{F}_{9,t}$	[e,0,4,7]					
[8,9,1,4]		[8,9,1,4]	[8,9,1,4]	$\mathbf{D}_{9,t}$	[t,e,3,6]	[8,9,1,4]	$\mathbf{F}_{t,e}$	[0,1,5,8]					
[9,t,2,5]		[9,t,2,5]	[9,t,2,5]	$\mathbf{D}_{t,e}$	[e,0,4,7]	[9,t,2,5]	$\mathbf{F}_{e,0}$	[1,2,6,9]					
[t,e,3,6]		[t,e,3,6]	[t,e,3,6]	$\mathbf{D}_{e,0}$	[0,1,5,8]	[t,e,3,6]	$\mathbf{F}_{0,1}$	[2,3,7,t]					
[e,0,4,7]		[e,0,4,7]	[e,0,4,7]	$\mathbf{D}_{0,1}$	[1,2,6,9]	[e,0,4,7]	$\mathbf{F}_{1,2}$	[3,4,8,e]					
	B			D			F			H		J (=F)	L (=D)

from Webern, *Concerto for Nine Instruments, op. 24*, first movement, Stravinsky, "Musick to Heare," from *Three Shakespeare Songs* and Villa-Lobos, *Etude n^o 10*. In these examples, the labels for the contextual inversion axis will be essential both to map two sets and to construct graphs that visually represent a sequence of connections.

a) Webern, *Concerto for Nine Instruments, op. 24*, first movement

With these new labels it is possible to analyze any passages which the sets connect by contextual inversion, especially those passages in which the sets do not share any common pitch-class. One example is the nine tone rows of the first sixteen bars of Webern's *Concerto for nine Instruments, Op. 24* shown in Figure 4. As seen in the previous section, there is no transformation to label the connections between the trichords that Webern choose for each tone row, in addition, the index numbers for the I_n-operations that map those trichords do not help to understand why their connection is so consistent. However, using the labels previously presented in this section it is possible to verify that the members of sc. (014) are mapped only by two axes of contextual inversion in all tone rows. Figure 7 shows the sets of the same nine Webern's tone rows discussed in previous section, one can see that all the members of sc. (014) are mapped by **J** and **D**. Since all trichords are written in the normal form, it is easier to notice that some of these tone rows use same sets.

It can seen in Figure 7 how the first pair of members of sc. (014) in all tone rows used in the beginning of *Concerto* is connected by **J**, the second pair is connected by **D** and the last pair is connected by **J** again. That means that **J** connects all pairs of members of sc. (014) that make a hexatonic collection and **D** connects all pairs of members of sc. (014) that make a chromatic hexachord. Since the connection between the last and the first set of each row is also by **D**, all tone rows sets became a cycle. Based on these tone rows it is possible to build six different cycles induced by a <**DJ**> chain with all 24 members of sc. (014). I will call these cycles as Webern's Graphs and each of them will be numbered according to the first pitch-class of the set in its North Pole, thus: WG_0, cycle with [0,1,4], [5,8,9], [6,7,t] and [e,2,3]; WG_1, cycle with [1,2,5], [6,9,t], [7,8,e]

a) m. 1-3 (P_{11})

Oboe)		Flute		Trumpet		Clarinet
[t,e,2]	←— J —→	[3,6,7]	←— D —→	[4,5,8]	←— J —→	[9,0,1]

b) m. 4-5 (RI_2)

Piano (left hand)		Piano (right hand)		Piano (left hand))		Piano (right hand)
[t,e,2]	←— J —→	[3,6,7]	←— D —→	[4,5,8]	←— J —→	[9,0,1]

c) m. 6-7 (RI_1)

Clarinet		Viola		Violin		Oboe
[9,t,1]	←— J —→	[2,5,6]	←— D —→	[3,4,7]	←— J —→	[8,e,0]

d) m. 7-8 (P_0)

Piano (left hand)		Piano (right hand)		Flute		Trumpet
[e,0,3]	←— J —→	[4,7,8]	←— D —→	[5,6,9]	←— J —→	[t,1,2]

e) m. 9-10 (I_0)

Piano (right hand)		Piano (left hand)		Piano (right hand)		Piano (left hand)
[9,0,1]	←— J —→	[4,5,8]	←— D —→	[3,6,7]	←— J —→	[t,e,2]

f) m. 11-13 (I_5)

Trumpet		Clarinet		Violin		Viola)
[2,5,6]	←— J —→	[9,t,1]	←— D —→	[8,e,0]	←— J —→	[3,4,7]

g) m. 11-13 (RP_3)

Piano		Piano)		Piano		Piano
[3,6,7]	←— J —→	[t,e,2]	←— D —→	[9,0,1]	←— J —→	[4,5,8]

h) m. 13-14 (RP_4)

Oboe		Horn		Violin		Viola
[4,7,8]	←— J —→	[e,0,3]	←— D —→	[10,1,2]	←— J —→	[5,6,9]

i) m. 14-16 (I_6)

Piano		Piano		Piano		Piano
[3,6,7]	←— J —→	[t,e,2]	←— D —→	[9,0,1]	←— J —→	[4,5,8]

Figure 7: *all trichords in Webern's tone rows are connect by **D** or **J**.*

and [0,3,4]; WG_2, cycle with [2,3,6], [7,t,e], [8.9,0] and [1,4,5]; WG_3, cycle with [3,4,7], [8,e,0], [9,t,1] and [2,5,6]; WG_4, cycle with [4,5,8], [9,0,1], [t,e,2] and [3,6,7]; and WG_5, cycle with [5,6,9], [t,1,2], [e,0,3] and [4,7,8]. The members of sc. (014) in each cycle are the set's content of 8 different tone rows that are listed below them, once the sequence of sets can start with the member to the North, to the East, to the South or to the West of a cycle and move clockwise or counterclockwise. The matrix with all 48 forms of the tone rows for the Webern's Concerto is shown in the bottom of Figure 8.

One can see in Figure 8 that all nine Webern's tone rows shown in Figure 4 are built with WG_3, WG_4 or WG_5 sets. Both P_{11} and RI_2 start with the set to the South Pole in WG_4 and move clockwise in the cycle; RI_1 and P_0 start with the set to the South Pole in WG_3 and in WG_5, respectively, and move clockwise in the cycles; I_0 starts with the set to the East Pole in WG_4 and moves counterclockwise in the cycle; RP_3 and I_6 start with the set to the West Pole in WG_4 and move counterclockwise in the cycle; I_0 and RP_4 start with the set to the West Pole in WG_3 and in WG_5, respectively, and move counterclockwise in the cycles.

Figure 9 shows a space with these graphs where we can see the sequence of the sets and tone rows of the beginning of Webern's *Concerto* in timeline[13]. The advantage of creating a space for these passage is that some relationships between the sets are easier to understand with this visual feature.

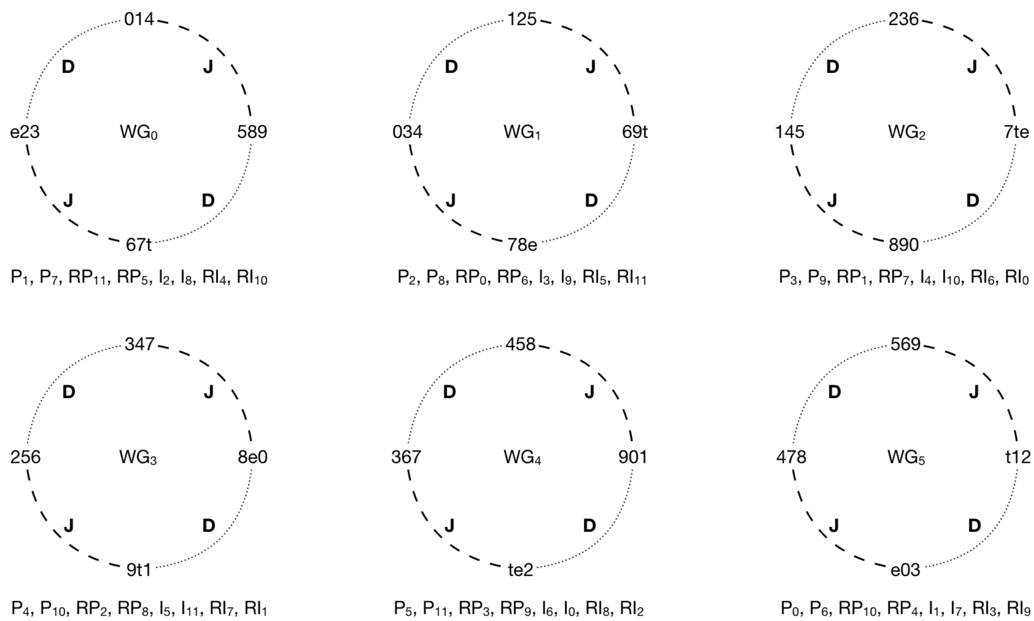
The space in figure 9 shows all the tone rows of the beginning of Webern's *Concerto* divided in three Webern's Graph: WG_3, WG_4 and WG_5. The sequence starts with the oboe playing the member of sc. (014) placed to the South Pole in the first cycle to the left and follows the arrows along all the remaining cycles. Note how the first and the second tone rows use sets in the same cycle (WG_4) in bars 1-5; then in bars 6-10, the three cycles are heard in sequence (WG_3, WG_5 and WG_4); and in the end, they are heard simultaneously in pairs (WG_3 with WG_4 and WG_4 with WG_5). Figure 9 also shows the labels of contextual inversion axes for sets that connect two different tone rows. D also connects last set of first row (P_{11}) and first set of the second row (RI_2), once these two tone rows share a same cycle (WG_4); **E** connects last set of second row (RI_2) and first set of the third row (RI_1); the last and the first sets of next two rows are connected by **B**; the last and the first sets of the fourth and the fifth tone rows are connected by transposition (T_{11}) this is the only exception for the contextual inversion connections in this passage of the *Concerto*; the last and the first sets of next two rows are connected by I; then the second set of the I_0 tone row connects to first set of RP_3 by **K** and its last set connects with the first set of RP_4 by **F**; finally, **D** connects the last and the first set of the final two piano tone rows (they share a same cycle). By observing the paths of tone rows in their cycle, it is possible to divide them into two different groups. The first group has the first four tone rows of these passage as they share a same path in their cycle, starting in the set placed on the South Pole and moving clockwise. The second group has the last four tone rows of these passage as they also share a same path in their cycle, starting in the set placed on the West Pole and moving counterclockwise. Between these two groups is the only tone row that does not share its path with any other, once it starts in the East Pole of the cycle and moves counterclockwise. One can see how this tone row between those two groups also functions as a pivot on the texture change that occurs between the first part of that passage (m. 1 to 8), where each tone row are heard alone without harmonic background, and the second (m, 11 to 16), where two tone rows are heard simultaneously.

b) Stravinsky, "Musick to Heare," from *Three Shakespeare Songs*

Straus also shows how transformations **P**, **L**, **R**, **P'**, **L'**, and **R'** map members of tetrachords

[13] In figure 9, abbreviations for instruments are: ob = oboe; fl = flute; tpt = trumpet; cl = clarinet; pn = piano; va = viola; vn = violin; hn = horn. In the same figure, (LH) and (RH) stands for left hand and right hand for piano, respectively.

a) six cycles with members of sc. (014) induced by <**DJ**> chain.



b) matrix for Webern's *Concerto*

	I_0	I_{11}	I_3	I_4	I_8	I_7	I_9	I_5	I_6	I_1	I_2	I_{10}	
P_0	0	11	3	4	8	7	9	5	6	1	2	10	RP_{10}
P_1	1	0	4	5	9	8	10	6	7	2	3	11	RP_{11}
P_9	9	8	0	1	5	4	6	2	3	10	11	7	RP_7
P_8	8	7	11	0	4	3	5	1	2	9	10	6	RP_6
P_4	4	3	7	8	0	11	1	9	10	5	6	2	RP_2
P_5	5	4	8	9	1	0	2	10	11	6	7	3	RP_3
P_3	3	2	6	7	11	10	0	8	9	4	5	1	RP_1
P_7	7	6	10	11	3	2	4	0	1	8	9	5	RP_5
P_6	6	5	9	10	2	1	3	11	0	7	8	4	RP_4
P_{11}	11	10	2	3	7	6	8	4	5	0	1	9	RP_9
P_{10}	10	9	1	2	6	5	7	3	4	11	0	8	RP_8
P_2	2	1	5	6	10	9	11	7	8	3	4	0	RP_0
	RI_2	RI_1	RI_5	RI_6	RI_{10}	RI_9	RI_{11}	RI_7	RI_8	RI_3	RI_4	RI_0	

Figure 8: *a) Webern Graphs, cycles with members of sc. (014) induced by <DJ>; b) matrix with all form of tone rows for Webern's Concerto.*

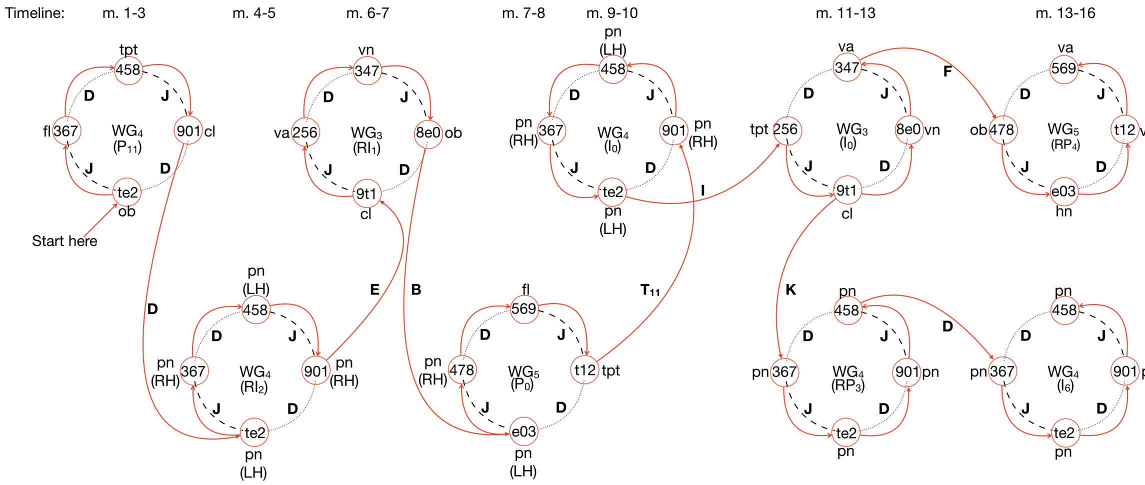


Figure 9: *space that shows the tone rows and the sets of the beginning of Webern's Concerto in the Webern Graphs (see the animation movie at https://axesofcontextualinversion.com).*

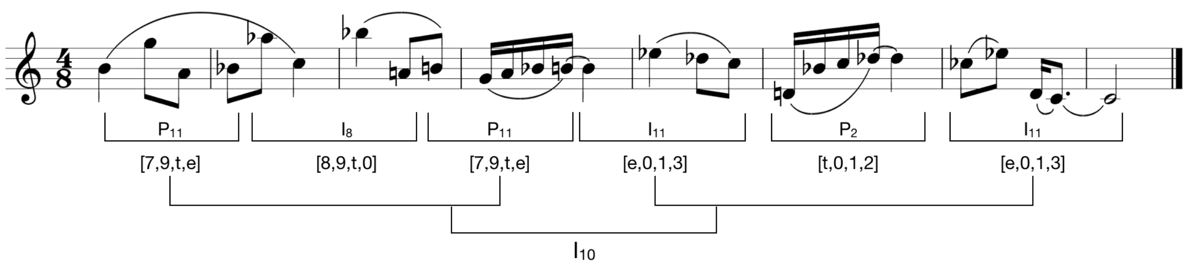


Figure 10: *melody played by flute in the introduction of Stravinsky's "Musick to Heare" (m1-8).*

set classes ([8, p.63-67]). All transformations preserve two pitch-classes in common between the sets. This means that in addition to inversions that do not have pitch-class in common, there are also no transformation labels for inversions that preserve only one pitch-class. In his analysis of Stravinsky's "Musick to Heare", from *Three Shakespeare Songs*, Straus provides a good example of a musical passage with tetrachords of the same set classes connected by contextual inversion sharing only one pitch-class. He explains: "with the exception of a brief diatonic reference in opening and closing measures, this song is based entirely on a four-note series: B–G–A–B♭"([8], p. 67), which is a member of sc. (0124)". Straus segmented the melodic line played by the flute in the introduction (mm. 1-8) in six statements of the series: three forms, P_{11}–I_8–P_{11}, followed by I_{11}–P_2–I_{11} ([8, p.67-69]), the first and the second segments are mapped by I_{10}. Figure 10 shows this melody and its segmentation made by Straus.

One can see in Figure 10 that there are four different members of sc. (0124) in this passage: [7,9,t,e], [8,9,t,0], [e,0,1,3] and [t,0,1,2], they are all related by context inversions that preserve only one pitch-class in common. Since there are no neo-Riemannian's transformations labels for these connections, we will use the labels for axes of contextual inversion presented on section IV in these passage. The clock faces at the top of figure 11 show how **D** maps [7,9,t,e] onto [8,9,t,0] and [e,0,1,3] onto [t,0,1,2], and **A** maps [7,9,t,e] onto [e,0,1,3]. In the bottom of the same figure we can see the sequence of sets of the Stravinsky's flute melody connected by these labels of contextual inversions.

Since **D** and **A** map all sets in these passage, we will build cycles with the members of sc.

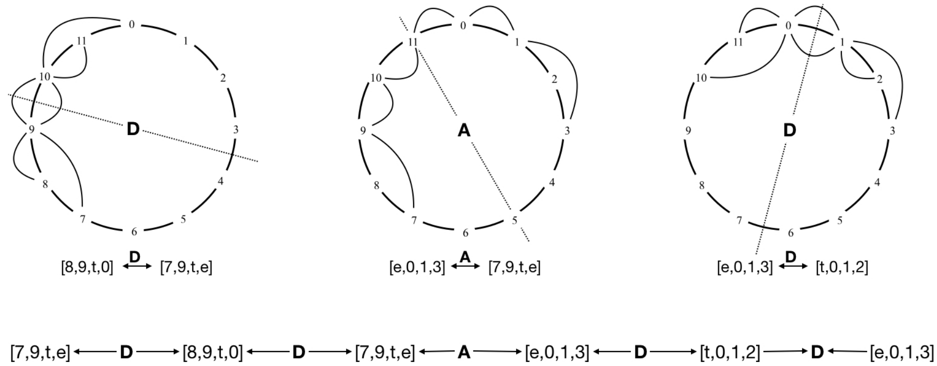


Figure 11: *D and A map all sets of the melody played by flute in the introduction of Stravinsky's "Musick to Heare".*

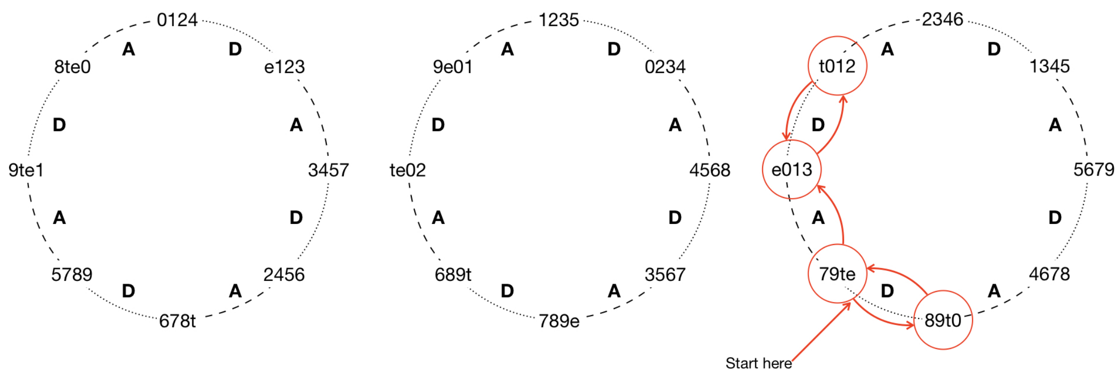


Figure 12: *all three cycles of members of sc. (0124) induced by <**DA**> chain, the connections between the sets from introduction of "Musick to Heare" are shown in the right cycle (see the animation movie at https://axesofcontextualinversion.com).*

(0124) induced by <**DA**> chain in the same way we did for members of sc. (014) in the Webern Graphs (Figure 8). There are three of these cycles, each one with eight members of sc. (0124), they are all shown in Figure 12. All fours sets from Stravinsky's "Musick to Heare" flute introduction are embedded in the third cycle on the right and the path traced by the connection between them starts in the southwestern set of the cycle and follows the neighbouring sets.

c) Villa-Lobos, Etude n^0 10

In the last example of this section I will focus on contextual inversion between sets that are I_n-symmetrical. As has been said previously, inversion or contextual inversion operations map two I_n-symmetrical sets that are also mapped by transposition. In a previous analysis of Villa-Lobos *Etude n$_o$ 10* ([11, p.83-94]), I mentioned the great recurrence of members of sc. (0257) in its section B relating these sets by transposition. I also divided this section into three distinct parts: in the first part (m. 21-42) the texture has two simultaneous layers; the second part has mostly only one layer that only splits itself into two in the five final bars; the third part (m. 57-65) is a retransition to section A' which then returns. For this example I will consider just the first and the second part of section B. In the first part there is a layer with high notes in an ostinato in the rhythm of sixteenth notes and another layer with a lower melody with greater note values. These two layers are heard simultaneously and I will call them as first and second layer, respectively. With exception of the last set in the first layer, all sets are members of sc. (0257), in my previous analysis

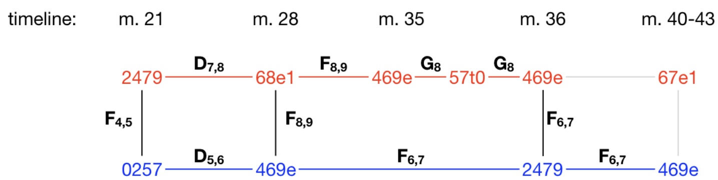


Figure 13: *the sets in first part of Section B of Villa-Lobos's Etude 10.*

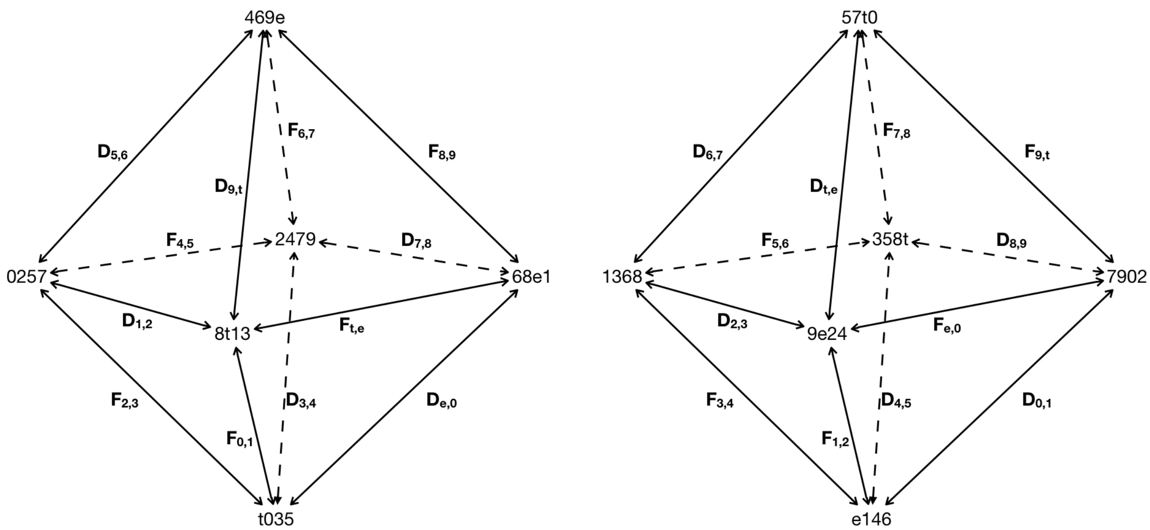


Figure 14: *two octahedron-shaped graphs with members of sc. (0257), each one is the union of two cycles induced by <**DF**> chain.*

I related them by transposition, but now I will focus on the contextual inversion operations that relate them. See in Figure 13 all the sets in both layers of this first part of section B.

All sets of first layer in figure 13 are written in red and all sets of second layer are written in blue. The lines represent the connection between the sets: red lines are the connections between members of sc. (0257) of first layer; blue lines are the connections between members of sc. (0257) of second layer; the black lines represent connections between members of sc. (0257) of different layers that are heard simultaneously; gray lines represent connections of members of different set classes. We can see how almost all sets are connect by **D** and **F** in this passage (with exception of connection between [4,6,9,e] and [5,7,t,0] mapped by G_8) and thus, I will, however, build a graph induced by <**DF**> chain in the same way I did in last two analyses. Once **D** and **F** can connect one member of sc. (0257) to two others, all members will be part of several different cycles. All of these cycles can be joined into a two octahedron-shaped graph that incorporate all members of sc. (0257). See the graphs of members of sc. (0257) induced by <**DF**> chain in Figure 14.

It is possible to trace a path of the connections between all members of sc. (0257) of this passage in this new two graphs in the same way we did with the cycles in previous analyses. One can see in Figure 13 that this type of set is the pitch content of both layers between mm. 21 and 36. In the top of figure 15 we can see how these sets are connected by contextual inversion, this figure has the same color pattern used in Figure 13 red circles indicate sets of the first layer and red arrows indicate connections between these sets; blue circles indicate sets of the first layer and arrows indicate connections between these sets; black line connecting two sets of different layers indicate that they are heard simultaneously. One can see in Figure 15 that all members are connect by **D** or

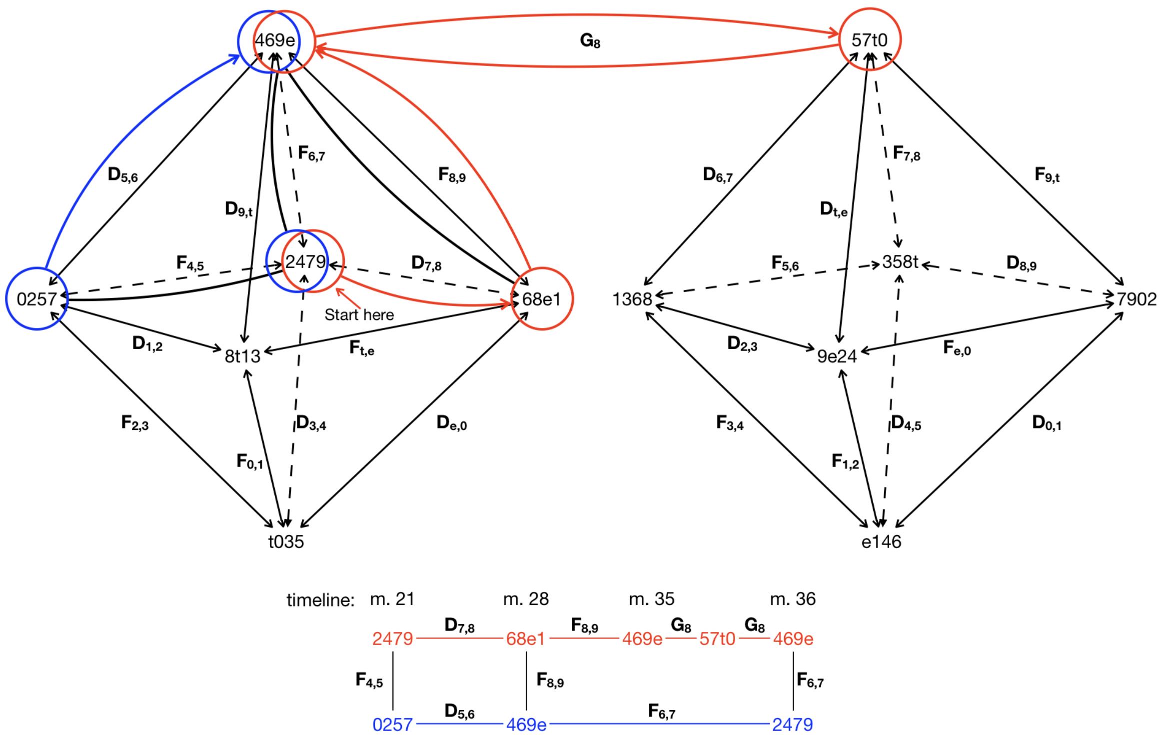


Figure 15: *connections between members of of sc. (0257) between bar 21 and 36 (see the animation movie at https://axesofcontextualinversion.com).*

F and belongs to the left octahedron-shaped graph except [5,7,t,0] that are connected to [4,6,9,e] by G_8, these two sets are in the same position in its graph (north pole). Note that because both graphs are related by T_1, **G** always maps two sets in the same position in each octahedron, since it is the contextual inversion that connects two members of sc. (0257) that are semitone apart.

In the beginning of the second part of section B of *Etude 10*, the texture has just one layer. From bar 48 a sequence of connections between five set class members (0257) begins. Next (m. 52), the texture is divided again into two simultaneous layers with the same two sets that started section B. In the last bar of section B, the texture has a single layer again; notice how the sequence of the latter sets make a palindrome. The path representing these connections over the graph is quite different from that shown in the previous figure, once most sets belong to the right octahedron, in addition, the connections from one graph to the other are mapped by **E** instead of **G**. Figure 16 shows on the bottom all these connections in timeline and on the top their representation on the octahedron-shaped graphs.

VI. Conclusion

Straus states that the separation of parsimonious voice-leading and contextual inversion concepts, which "sometimes has been assumed that they are inseparable (...), has led to research in two profitable directions" ([8, p.84]): (1) an interest in parsimonious voice-leading between members of different set classes; (2) an interest in common-tone contextual inversion between sets that are not parsimonious ([8, p.84]). This article proposed a third direction in this research, an interest in contextual inversion operations between sets that have no pitch-class in common or that are

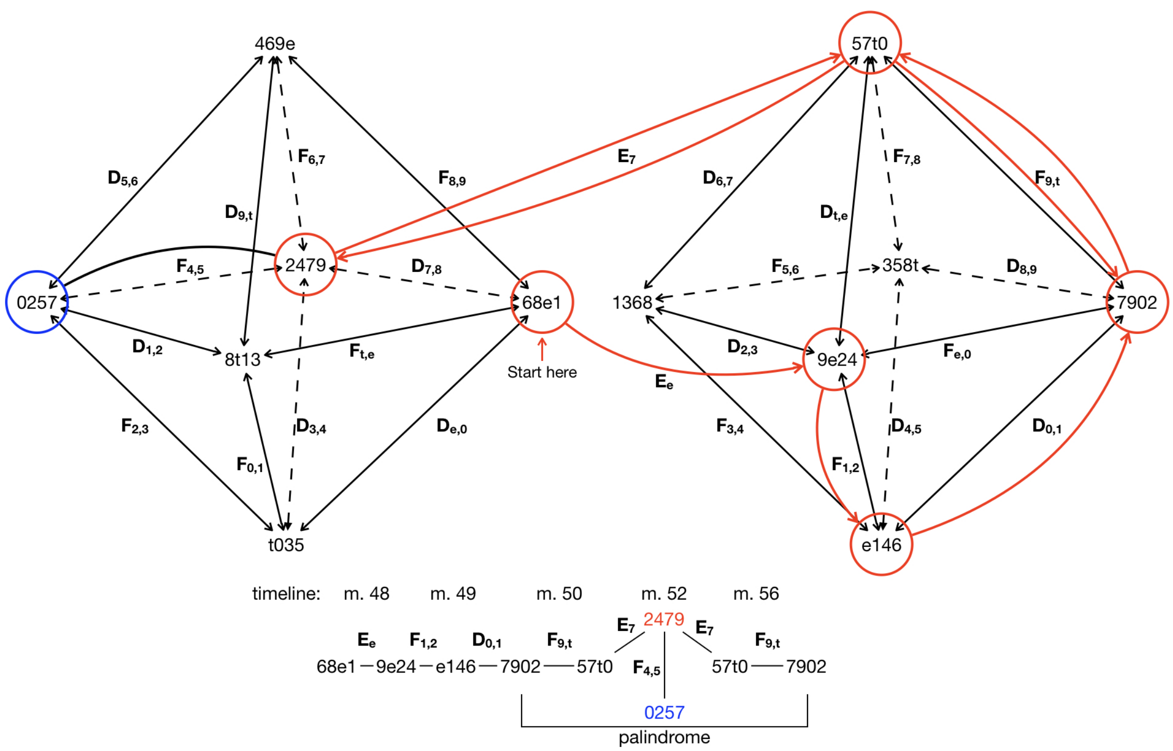


Figure 16: *connections between sets in second part of section B of Etude 10. (see the animation movie at https://axesofcontextualinversion.com).*

not mapped by any of the neo-Riemannian transformations. Actually, the labels for the axes of contextual inversion presented in section IV can also be used in passages where the connections are mapped by neo-Riemannian transformations, since **P**, **L**, **R** and their obverse versions can also be defined by an axis. But in addition, with these new concepts it is possible to go beyond and label any kind of connection by contextual inversion between sets of any cardinality. The analyses in section V have shown that the labels for the axes of contextual inversion are useful not only for designating a connection between two sets, as in Figures 7 and 13, but they can also be used to create new graphs and spaces, as in the Webern case Graphs (Figures 8 and 9), in cycles of members of sc. (0124) induced by <**DA**> chain (figure 12) and in two octahedron-shaped graph that incorporate all members of sc. (0257) (figures 14, 15 and 16). Of course, a wide variety of these graphs can be created with these axis and labels for using by both analysts, especially those focused on post-tonal works, and composers in the pre-compositional stages.

References

[1] Babbit, M. 1955. Some Aspects of Twelve-Tone Composition *The Collected Essays of Milton Babbitt*. Priceton University Press

[2] Callender, C, Quinn, I, and Tymoczko, D. 2008. Generalized Voice-Leading Spaces. *Science*, vol. 320, pp. 346-348.

[3] Cohn, R. 2012. *Audacious Euphony*. Oxford University Press

[4] Douthett, J, and Steinbach, P. 1998. Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition. *Journal of Music Theory*, vol. 42, pp. 241-263.

[5] Lewin, D. 1993. *Musical Form and Transformation: Four Analytic Essays*. New Haven: Yale University Press

[6] Morris, R. 1988. Voice-Leading Spaces. *Music Theory Spectrum*, vol. 20, pp. 175-208.

[7] Solomon, L. *The Table of Pitch Classes*. Available at: <http:// solomonsmusic.net/pcsets.htm>

[8] Straus, J. 2011. Contextual-Inversion Spaces. *Journal of Music Theory*, vol. 55, n. 1 pp. 43-88.

[9] Straus, J. 2016. *Introduction to Post-Tonal Theory, 4th ed*. W. W. Norton & Company

[10] Straus, J. 2018. Sum Class. *Journal of Music Theory*, vol. 62, n. 2 pp. 279-338.

[11] Visconti, C. 2016. *Simetrias nos Estudos Para Violão de Villa-Lobos*. Paco Editorial