Musical Time: A Gestural Construction

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Abstract: *We propose a mathematical construction of musical time, which is derived from mathematical gesture theory and its application to free jazz. The mathematical construction makes usage of the projective limit of diagrams of gestures.*

Keywords: *Musical Time. Gesture Theory. Improvisation. Performance.*

I. Time in Philosophy, Physics, and Music

Saint Augustin, in his confessions, states those famous words:"For what is time? Who can easily and briefly explain it? Who even in thought can comprehend it, even to the pronouncing of a word concerning it?" Time is a mysterious concept, and it has a huge impact on culture and science, see for example [4]. Philosophers have discussed time, spent many thoughts and words, but never came up with a unified understanding. Not even the time's ontological status has been clarified. Between Immanuel Kant's "Die Zeit ist kein diskursiver, oder, wie man ihn nennt, allgemeiner Begriff, sondern eine reine Form der sinnlichen Anschauung."[1] [6] and Jean Wahl's "Si le temps est qualité, il est qualité de ces événements qui se prédèdent ou succèdent ou sont contemporains les uns des autres."[2] [13, p. 308] there are fundamental discrepancies. We are not time experts in philosophy, but may refer to an excellent review [12] of those ideas, especially with regard to Paul Valéry's reflexions on time in his *Cahiers*. Let us just recall that Valéry in these writings states that time may have multiple dimensions, a thesis that has become virulent in contemporary theoretical physics in the works of Stephen Hawking [5] and Itzhak Bars [2], where time is viewed as a complex number, adding an imaginary coordinate to the usual real value.

On could argue that physics has a better concept of time since it is a basic parameter for many physical concepts: velocity, acceleration, kinetic energy, Lagrange function, etc. But beyond these basic concepts, time is also a divergent concept when one compares its role in General Relativity to that in Quantum Mechanics. In Einstein's approach, time is absorbed in a geometric space-time, which receives its curvature from the distribution of gravitational masses. In Quantum Mechanics, time is not observable. It does not correspond to a self-adjoint linear operator on Hilbert space, whose eigenvectors generate experimentally measurable quantities. Perhaps this ontological difference is one of the reasons for the present failure of a physical Theory of Everything.

In music, time seems to be a crucial variable that is at the origin of this "art of time". In this paper, we will investigate the role of time as a musical reality that differs from its philosophical

[1] Time is not a discursive or, as they say, general concept, but a pure form of sensual point of view.
[2] If time is quality, it is quality of these events that succeed or precede one another or are simultaneous.

or physical phenomenology. In fact, music shears a more constructive appoach to time. Musical time is not given *a priori*, as it is conceived in physics, but much more the result of musical creativity. The famous conductor Sergiu Celibidache, in a fascinating video about his rehearsal of Gabriel Fauré's *Requiem* with the London Symphony Orchestra [3], states that "we make time." The insight upon musical time construction could—this is our strategic hope—help generate a better understanding of time in philosophy and physics. Recall for this hope that music (and its theory) has played an important role a predecessor of other sciences. The typical example is the Pythagorean *tetractys*, which was effectively a first musical model of a physical "world formula".

i. Our Contribution to Time in Music

In this paper, we shall develop a model of musical time that is deduced from the gestural generators of musical activity. In this approach, we shall model time as a kind of "harmony among gestures", a construction that results from the intimate collaboration of musical gestures as they appear in the performative interaction of musical processes. We apply the mathematical theory of musical gestures in the framework of the category of gestures as developed in [7, 8, 11]. More precisely, we shall give a temporal interpretation of projective limits of gestures, also in view of the work of Juan Sebastian Arias [1] on topological properties of gesture spaces.

II. The Distributed Identity in Performance Is a Time Phenomenon

Our setup starts with an analysis of what was called "distributed identity" in [9]. In that model, they investigated the question of quality in free jazz performance. This is important since free jazz has no score-related abstract templates: it can only be qualified when the process of improvisation establishes a coherence of interaction in the making. Let us review this approach.

i. Distributed Identity in Free Jazz

In [9, Ch. 9.2, Ch. 11], they exhibited a phenomenon of success when the interction of musicians reached a state, where the passionate engagement is at a level, where their efforts generate a mutual understanding that flips the passionate activity into a shared stability. The music is no longer played, but plays upon the involved musicians. It was described as an axis around which the musicians rotate, becoming components of a rotational energy, which means that a higher stability of motion is achieved, this was called "distributed identity". In this model, time was not explicitely included, the model was built from an interaction of expressive gestures in performance.

ii. The Case of Score-Driven Orchestral Performance

This model seems to be limited to free jazz, but it is well known that also in traditional Western score-driven performance, the phenomenon of a distributed identity is characteristic for a successful performance. The classical example is the performance of a string quartet, which from its very beginning was conceived as a dialogue of educated persons. The interaction of voices is a basic criterion for a successful quartet performance. The interaction of voices is much more than an abstract contrapuntal architecture, it is a substantial exchange of musical gestures that transcends the score's mechanism. This is also, *mutatis mutandis*, the case in any collaborative arrangement of voices within or among instruments.

III. The Gestural Theory of a Distributed Identity

In the model of [9], the mathematical theory of musical gestures was used. We now review that approach and develop a mathematical architecture for a temporal category.

i. Morphisms of Hypergestures as Causal Units

In that model, the musicians' activities are described by hypergestures, i.e., gestures of gestures of...gestures. They are the elements of the topological hypergesture spaces $\Delta_n @ \Delta_{n-1} @ \ldots \Delta_1 @ X$, where X is a topological space (called "body") and the Δ_i are directed graphs (digraphs in short), the "skeleta" of these hypergestures. Recall from [7] that the gestures g with skeleton Δ and body X are digraph morphisms $g : \Delta \to \overrightarrow{X}$, where $\overrightarrow{X}$ is the digraph of continuous curves $c : I = [0,1] \to X$, its arrows, together with their projections to initial and terminal values as tails and heads. The set $\Delta @ \overrightarrow{X}$ of gestures from Δ to X defines a topological space, and it can be shown [1] that this topology is homeomorphic to the compact-open topology of the continuous function space $|\Delta| @ X$[3], where $|\Delta|$ is the topological space associated with digraph Δ, when its arrows are turned into real line interval I copies.

In [9], the musical dialogue between such (hyper)gestures δ_1, δ_2 is then described intuitively as "throwing a gesture δ_1 to a gesture δ_2". See Figure 1 for the example of a jazz trio. In this example, every musician's gesture lives in a space of hypergestures. Its digraphs may also be permuted in their order, and according to the Escher Theorem [7, Proposition 3.1], these permutations don't change the hyergestural spaces up to homeomorphisms. These permutations signify musically, which digraph skeleta are understood as being more on the internal or external aspect of the hypergestural constructions, see [9, Ch. 9] for the technical details behind this musical understanding of hypergestures. In Figure 1, the permutations are signaled by short red arrows, while the long arrows between different musicians signal the permutation that is selected for a throwing activity.

In [9, Ch. 9.2], the mathematical restatement of the intuitive (musi)causal "throwing gestures" from (hyper)gesture δ_1 to a (hyper)gesture δ_2 is given by a morphism of gestures. A morphism $f : \delta_1 \to \delta_2$ is defined as follows. If $\delta_1 : \Delta_1 \to \overrightarrow{X_1}, \delta_2 : \Delta_2 \to \overrightarrow{X_2}$, a morphism f is defined to be a pair $f = (h,k), h : \Delta_1 \to \Delta_2, k : X_1 \to X_2$, where h is a digraph morphism and k is a continuous map, such that the diagram

$$\begin{array}{ccc} \Delta_1 & \xrightarrow{\delta_1} & \overrightarrow{X_1} \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle \overrightarrow{k}} \\ \Delta_2 & \xrightarrow{\delta_1} & \overrightarrow{X_2} \end{array}$$

of digraphs commutes. Here, the map $\overrightarrow{k}$ is the evident digraph morphism induced by k. Such a morphism is also associated one-to-one with the commutative diagram of topological spaces

$$\begin{array}{ccc} |\Delta_1| & \xrightarrow{|\delta_1|} & X_1 \\ {\scriptstyle |h|}\downarrow & & \downarrow{\scriptstyle k} \\ |\Delta_2| & \xrightarrow{|\delta_1|} & X_2 \end{array}$$

Refer also to [1] for this fact.

[3] We denote by $A@B$ the set of morphisms from A to B in a determined category.

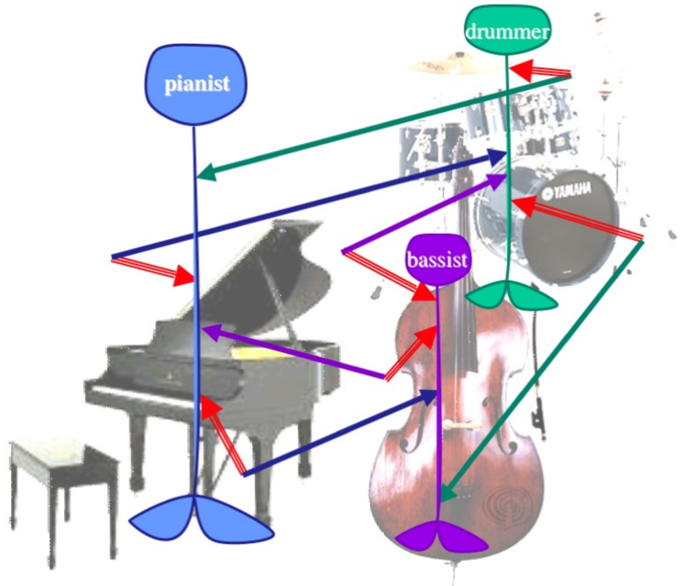

Figure 1: *The musical dialogue between such (hyper)gestures is described intuitively as "throwing gestures" from musician's to musican's gestures. Here in a jazz trio.*

ii. The Diagram of Distributed Identity

This setup reinterprets the "throwing" actions as a diagram $\mathcal{D}$ of gestures δ_i in the category *Gesture* of gestures, which define the present orchestral setup (in free jazz or elsewhere).

In this approach the morphisms $f_{i,j,k} : \delta_i \to \delta_j$ between the given gestures play the role of the musicians' understanding of their relationships to their fellow musicians. Be aware that there might be several morphisms between the same couple of gestures. There might also be endomorphisms $f_{i,i,k} : \delta_i \to \delta_i$, which describe the dialogue with oneself, which Cecil Taylor stressed in his discussion of lonely years without gigs, playing alone at home and listening to oneself. This setup is not thought to happen within physical time, it is part of the imaginry reality of the musicians' artistic presence. We don't discuss this aspect here, but refer to [10, Ch. 2.3], where a precise space of artistic presence was discussed.

IV. Time as a Projective Limit Structure

So far the idea of a distributed identity in [9] was not made explicit in mathematical terms. They establish a diagram $\mathcal{D}$ in the category *Gesture* of gestures, but the criterion of a "rotational axis" is not made precise. To put it into critical words: Why would such a diagram guarantee any success, in free jazz, say?

We now want to make this criterion more precise: What would be a precise mathematical statement of the existence of such a "rotational axis"? It must be a structure that emerges from the diagram $\mathcal{D}$, but is not automatically realized by that diagram.

i. The Projective Limit of Communicating Hypergestures

Given a diagram $\mathcal{D}$ in a category, it is in general not true that the projective limit **Limit**$\mathcal{D}$ of the diagram exists. Let us look at the situation in the category *Gesture*:

Theorem 1 *In the category Gesture, a diagram $\mathcal{D}$ has a projective limit* $\mathbf{Limit}\mathcal{D}$*. It is the projective limit of the domain skeleta, being mapped into the projective limit of the bodies' digraphs by means of the canonical morphism of projective limits. Moreover, if one views the gestures and their morphisms* $\delta_i : \Delta_i \to \overrightarrow{X_i}$ *within* $\mathcal{D}$ *as being represented by corresponding continuous maps* $|\delta_i| : |\Delta_i| \to X_i$*, then the projective limit is the projective limit of this diagram of continuous maps, i.e., a continuous map* $\mathbf{Limit}|\Delta_i| \to \mathbf{Limit}X_i$*.*

The theorem's proof is straightforward in view of the functorial correspondence between digraph and topological space representation of gestures, see also [1].

The critical point of this projective limit construction is that the limit might be empty if the "throwing morphisms" are not sufficiently compatible. For example, if two musicians interact with two morphisms $f_1 : \delta_1 \to \delta_2, f_2 : \delta_1 \to \delta_2$ such that no two skeletal points are mapped to each other, the limit will be empty. This example makes clear that the existence of points in $\mathbf{Limit}\mathcal{D}$ means that there is a mutual understanding among these musicians. Clearly, the limit will not be empty if the diagram's directed graph has no two morphisms ending on the same codomain.

A first simple example of a potentially non-empty limit is the situation, where we only have arrows from the musicians' gestures to the conductor's gesture in an orchestra. Here the morphisms ending in the conductor's gesture must end on common points of the conductor's gestural skeleton. This is also what one understands when agreeing that the orchestra follows the conductor's gestures. A similar situation is derived from musicans playing according to a metronome's gestures.

In any case, the idea of a rotational axis for a distributed identity will now be made precise in the sense that

the rotational axis of a distributed identity of a gestural diagram $\mathcal{D}$ *is the set of points in* $\mathbf{Limit}\mathcal{D}$*. This is equivalent to having a non-empty* $domain(\mathbf{Limit}\mathcal{D}) = \mathbf{Limit}\ domain(\mathcal{D})$ *of the topological spaces of the diagram's skeleta. And this again is equivalent to the existence of gestures* $I@\mathbf{Limit}\mathcal{D}$*, or gestures* $\Delta@\mathbf{Limit}\mathcal{D}$ *for any skeleton* Δ *(empty skeleta are not allowed).*

ii. Temporal Gestures in a Projective Limit

We now use the projective limit construction (well, its existence) in the category *Gesture* as a point of departure for a temporal interpretation of a distributed identity's rotational axis. What has been achieved when we have a gesture $\delta : \Delta \to domain(\mathbf{Limit}\mathcal{D})$? We have a continuous map from $|\Delta|$ to the domain limit of the diagram's skeleta qua topological spaces. This means that the connecting morphisms enable points that correspond to each other within the skeletal parameters.

This situation means that the local skeletal parameters of musicians' gestures are now connected by morphisms and generate limit points, i.e., they define a "harmony of local gestural parameters" of the distributed identity. We understand this as a construction of a "global gestural parameter", a parameter of mutual congruence in the gestural interaction. Such a global gesture is what we now define as being a musical time of $\mathcal{D}$. It means that the totality of the 'orchestra' shares gestural parameters that are exchanged via the diagram's morphisms between the musicians' gestures.

V. Conclusion

The present construction of a temporal parameter of shared gestures makes use of the projective limit of a diagram of gestures that describes the musical interaction of an orchestral setup (in any style of music, not only free jazz). This approach confirms Jean Wahl's claim that time is only generated from an environment of events, but it also uses the mental construction of a projective limit, which is not, as such, derived only from these events, and in this sense refers to an *a priori*, but not identical to Kant's approach. It is not a point of view, it is a mental—and this must be stressed: a musically conceived—construction.

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