A few more Thoughts about Leibniz: The Prediction of Harmonic Distance in Harmonic Space

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Abstract: *This paper shows how, over 300 years ago, Gottfried Wilhelm Leibniz envisioned James Tenney's theory of harmonic distance in harmonic space. A description of Tenney's theory is followed by an analysis of musical ideas contained in letters from Leibniz to Christian Henfling. The analysis compares and contrast Leibniz's ideas to Tenney's ultimately showing that they are practically identical.*

Keywords*: Gottfried Wilhelm Leibniz, James Tenney, harmonic distance, harmonic space, complexity.*

I. Introduction

i. The current discourse on Leibniz's writings about music

Gottfried Wilhelm Leibniz was a polymath of epic proportions. He lived in a time where the notion of the specialist had not become ubiquitous as it is today. His objects of study were the world and most everything in it. As such, it should be of no surprise that he also thought and wrote extensively about music.

While many consider the *Monodology* his most seminal work, Leibniz does not have a single, large-scale text that has singularly defined him historically (say, for example, on the order of Baruch Spinoza's *Ethics*). The *Monodology*, which is indeed an incredibly profound treatise despite its brevity, hardly encompasses the scope of Leibniz's intellectual curiosity.

Most of Leibniz's writings exist in the form of letters and those on music are no exception. History has favored his mathematical and philosophical work whereas the music related texts have only been discussed by a handful of researchers. Most notably, the published findings of Walter Bühler [1, 2] and Andrea Luppi [3] are an invaluable resource for entry into Leibniz's musico-intellectual world. Bühler and Luppi have done extraordinary work citing and providing detailed explanations of Leibniz's music-related texts.

Bühler and Luppi's analysis of Leibniz's musical ideas are contextualized within a music-theoretical discourse that has persisted from Leibniz's time to today. Theories of functional tonal harmony, tuning, and aesthetics developed by Leibniz's immediate contemporaries (for example, Johann Joseph Fux, Jean-Philippe Rameau, and Andreas Werckmeister) continue to serve as a foundation for music analysis. Given that Leibniz lived in a time of incredible musical development, exploration, and experimentation, it is easy to underestimate the progressiveness of Leibniz's own musical ideas. Leibniz was by no means just a curious observer. Rather, he was an ardent contributor to the rapidly evolving discourse.

Bühler and Luppi have done great service bringing many of Leibniz's musical ideas to light. But, as mentioned above, their work is situated within a specific theoretical and musicological context which does not extend to more recent trends in music. As with other intellectual domains, I posit that Leibniz was even more forward-thinking with respect to music than some of his immediate contemporaries. He predicted musical ideas that have yet to enter the theoretical canon. Musical ideas explored by composers of *our* time, not *his*.

ii. Genesis of an unlikely discovery: connecting Leibniz to Tenney

2016 marked the 300th anniversary of Leibniz's death. Through the suggestion of Greg Chaitin, with whom I have been close friends with for many years, Ugo Pagallo invited me to participate in the 'Leibniz's Vision' conference which was hosted in Turino, Italy 300 years to the day of Leibniz's death. I still sometimes smirk at the thought that the conference would have been better served had Bühler or Luppi been invited. After all, they are the world's leading experts on Leibniz's writings about music. I, on the other hand, am just a composer that got to Leibniz rather backwards via my connection to Greg Chaitin and an ongoing interest in algorithmic information theory and complexity.

Originally Ugo and I envisioned my contribution to the conference less as a scholarly one and more so as a commission to write a piece celebrating Leibniz. Still, after receiving the invitation, I completely immersed myself in Leibniz's writings looking feverishly for references to art and music. In the early stages of my research, well before I became familiar with the work of Bühler and Luppi, I wrote a piece titled *preliminary thoughts* [7]. In an homage to the fact that Leibniz was a fervent letter writer, *preliminary thoughts* is a 'musical letter' to Greg Chaitin discussing my preliminary reactions to the ideas and writings of Leibniz which I thought directly related to music; specifically, combinatorics, harmony, aesthetics, structure, epistemological vs. practical limits, free will, and even love with respect to creativity. In the piece, a reading of the text of the letter sounds against a minimal guitar part that continually repeats a set of 6 tones with ever changing durations between the articulations of the tones. The guitar part and the reading of the letter are juxtaposed with random flickerings of computer-generated tones and noises.

My idea at the time was that, after doing further research, I would integrate more 'conclusive thoughts' into a new piece that I would eventually perform in Turino. However, our initial thoughts are often the most poignant. While the text of my *preliminary thoughts* is rather informal (like much of the writings of Leibniz), I think it is actually quite comprehensive with respect to addressing many ways in which Leibniz predicted musical ideas well beyond those that have entered into the current canon of music theory.

As the anniversary neared, I struggled to come up with ideas for another, more conclusive piece to present at the conference. In the meantime, I had written an orchestra piece, *essay on the art of combinations* [8], integrating ideas on combinatorics, expressed by Leibniz through music, in his dissertation *On the Art of Combinations*. The score of the piece includes a short essay on how I incorporated Leibniz's ideas. Even though writing the piece was an integral part of what I now affectionately refer to as 'my year with Leibniz', it was still a project ancillary to the conference.

In the weeks preceding the conference, while on tour in Europe and still unsure of what I would do for the conference, I had the opportunity to visit the Leibniz archive. Thanks to the library staff, particularly Michael Kempe and Werner Ganske, as well as the research by Bühler and Luppi, I was able to go directly to the source.

In particular, the correspondence between Leibniz and and Christian Henfling caught my attention as many of the figures and diagrams within looked uncannily similar to figures and diagrams in the writings of one of my mentors, James Tenney. Tenney, like Leibniz, was incredibly

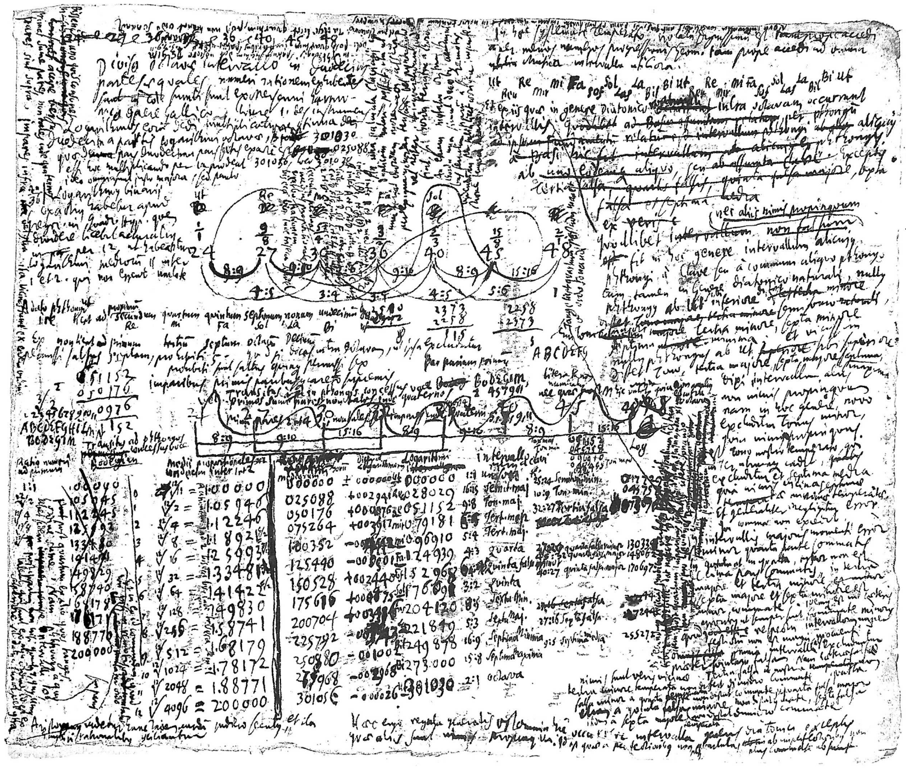

Figure 1: *Example page of one of Leibniz's letters to Henfling.*

prolific and in my humble opinion, one of the most important composers and theorists of the last 100 years.

One of Tenney's primary interests was creating and defining a phenomenologically-based theory of music analysis. His seminal treatise, *Meta ≠ Hodos* [4], written as a graduate student, applies concepts of gestalt psychology to musical analysis. Later, Tenney would combine his interest in phenomenology with an ever-growing interest in harmony. In a series of papers, "The structure of harmonic series aggregates" [5] and "John Cage and the theory of harmony" [6], Tenney defines the concept of "harmonic distance", which is a metric in what he calls "harmonic space". Harmonic distance is essentially an integer complexity function used to create a notion of distance between the frequencies of two tones, often referred to loosely as the level of 'consonance' or 'dissonance'.

Leibniz's letters are difficult to read and decipher (especially for someone whose Latin and French are non-existent). Some of the pages of Leibniz's letters to Henfling are filled with calculations, tables, and diagrams to the extent that there almost seems to be more ink than whitespace (for example, see Figure 1). It might be easy to quickly dismiss Leibniz's music-theoretical attempts as turning music into numbers without taking perception into account. However, after closer examination, I began to realize that over 300 years ago, Leibniz predicted Tenney's theory of harmonic distance in harmonic space.

After making the connection between Leibniz and Tenney, I finally (less than two weeks before the anniversary of Leibniz's death) had more conclusive thoughts and knew for sure what I would do for the conference. I would divide my time between a lecture and a performance: the first part giving my recent findings and the second giving my *preliminary thoughts*.

Ultimately, I think my invitation to the conference turned out to be rather fitting. After all, the central theme was about Leibniz's 'vision' and how it still resonates today. And while I am not a Leibniz scholar, my vantage-point as a working artist/composer helped reveal that Leibniz actually predicted ideas that, as I mentioned previously, are only now being developed more thoroughly. Ideas that have yet to broadly reach the academic world. In fact, it almost seems rather serendipitous. The connections I make in *preliminary thoughts* and Leibniz's prediction of Tenney's theory would have easily gone overlooked if Ugo had not taken a chance in inviting someone like myself, who, in the influence of Leibniz, happily traverses intellectual domains with abandon. Admittedly, I was among such luminaries at the conference that I felt both humbled, honored, and definitely out of place! I am grateful to both Greg and Ugo for including me.

What follows is a more thorough description of harmonic distance in harmonic space followed by an analysis of Leibniz's letters to Henfling demonstrating how he predicts Tenney's theory. The text of my piece *preliminary thoughts* is provided as an appendix.

II. Leibniz's prediction of harmonic distance in harmonic space

i. James Tenney's definitions

The fundamental tenet of Tenney's theory of harmonic distance is that harmonic relations between pitches can be modeled by a multidimensional space with metrical and topological properties that reflect how the human auditory apparatus perceives relations between pitches. In the model, pitches are represented by points in a multidimensional lattice where the dimensions correspond to the prime factors required to specify the frequency ratios of the set of pitches with respect to a given reference pitch. Tenney's own examples are provided in Figure 2 and Figure 3. Note that, as in Figure 3, Tenney often "collapses" (or omits) the 2-dimension as it represents intervals of an octave. Omitting the 2-dimension eliminates duplication of pitch-classes and allows higher dimensions to be more easily plotted.

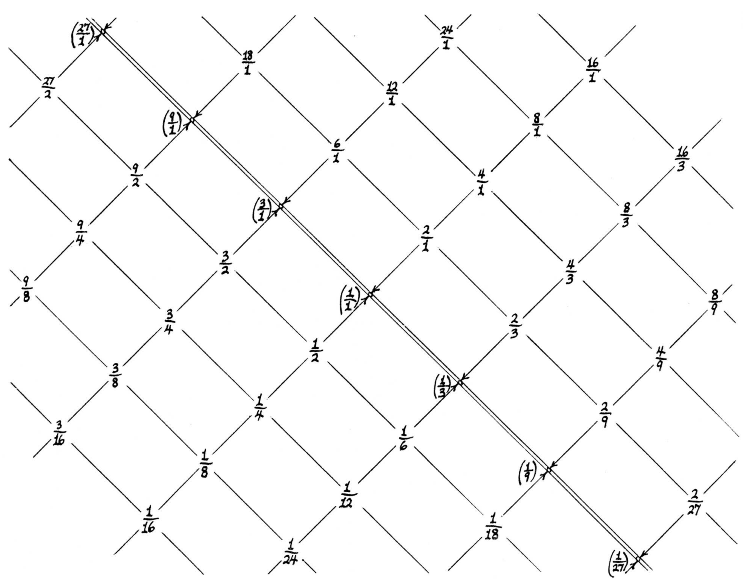

Figure 2: *Harmonic lattice in 2,3 harmonic space from Tenney's "John Cage and the Theory of Harmony". [6]*

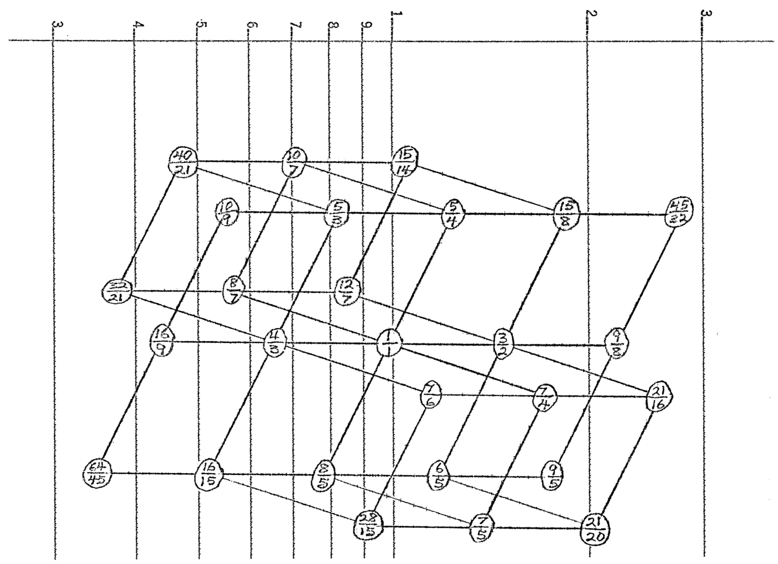

Figure 3: *Harmonic lattice in 3,5,7 harmonic space with the 2-dimension collapsed from Tenney's "The structure of harmonic series aggregates". [5]*

The perceived harmonic distance between two pitches is the distance of the shortest path between the corresponding points in harmonic space. Because harmonic space is a lattice, harmonic distance is a non-euclidean 'city-block' metric.

Tenney's mathematical formulation is beautifully elegant: $HD(a, b) = \log_2(ab)$ where a/b is a frequency ratio such that a and b are coprime.

It is clear to see that harmonic distance is a wonderfully concise quantification of a walk in harmonic space that weights the size of the prime factors because for $ab = 2^i 3^j 5^k \ldots$, $\log_2(ab) = i \log_2(2) + j \log_2(3) + k \log_2(5) + \ldots$

Note that the collapse of the 2-dimension, as in Figure 3, is somewhat deceiving because information encapsulated in Tenney's harmonic distance function—specifically movement in the 2-dimension—is lost. The collapsed visualization implies that a step in harmonic space is the logarithm of a prime over the highest power of two less than the given prime: $p/2^{\lfloor \log_2(p) \rfloor}$. However, the steps in actual harmonic space are always strictly the logarithm of a prime and the harmonic distance function always computes the steps in *all* dimensions. This is why, for example, 3:2 and 4:3 do not have the same harmonic distance. This can be seen by comparing the number of steps from 1:1 to 4:3 in Figure 2, which includes the 2-dimension, in comparison to Figure 3, which does not.

As mentioned in the previous section, Tenney's harmonic distance function is an integer complexity function based on the number, size, and exponents of the prime factors needed to represent the frequency ratio between two pitches. I often explain Tenney's harmonic distance function in the context of computational complexity: that Tenney's formulation of harmonic distance quantifies the amount of time it takes to compute the prime factors of a number. That is, assuming a Leibnizian digital philosophy where the brain is a sophisticated computer, two pitches are perceived as being more closely related (and therefore also closer in harmonic space) because it takes less time for the brain to compute the frequency ratio between them.

ii. Leibniz's vision

What first caught my attention in Leibniz's correspondence to Henfling is an identity function expressing when two pitches are the same (see Figure 4). The equivalence is written in terms of the prime factors and their respective exponents. Written below the identity function is a set of numbers plotted in a 2-dimensional table where the vertical and horizontal dimensions are powers of 2 and 3, respectively. At this point, the connection to Tenney's concept of harmonic space became clear as Leibniz's table is equivalent to a harmonic lattice in 2,3 harmonic space.

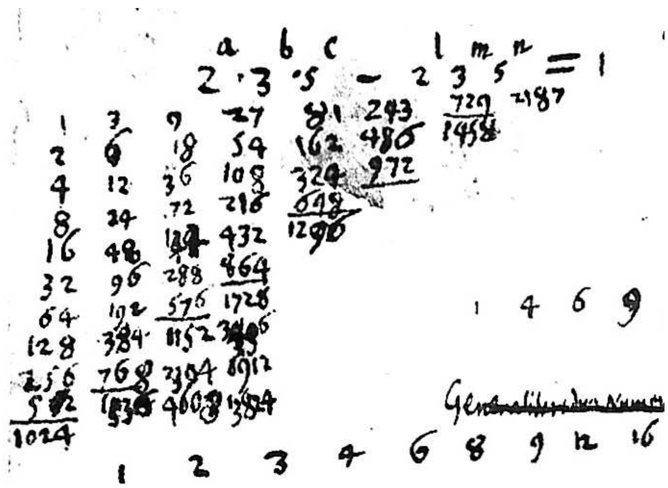

Figure 4: *Leibniz's identity function and 2,3 harmonic space.*

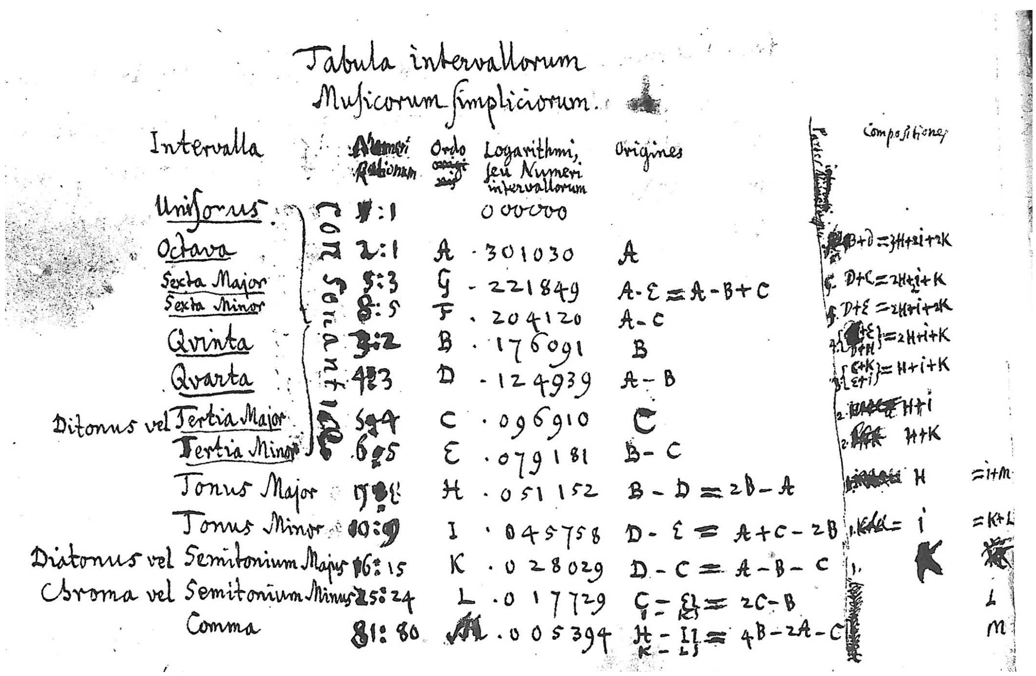

Figure 5: *Leibniz's Tabula intervallorum Musicorum Simpliciorum.*

But the connection did not stop there. A following page in the correspondence includes a table Leibniz titles "Tabula intervallorum Musicorum Simpliciorum" (see Figure 5). Each row of the table corresponds to a musical interval listed in order from big to small with respect to the perceptual size of the interval (the logarithm of the interval). If Leibniz's prediction of harmonic space suggested by Figure 5 was not enough, his Tabula intervallorum seems to be a near verbatim expression of Tenney's harmonic distance function.

Of particular interest are the 3rd and 5th columns: "Ordo" and "Origines". The 1st, 2nd, and 4th are "Intervalla" - interval name, "Numeri Rationum" - frequency ratio, and "Logarithmi seu Numeri intervallorum" - logarithm to the base 10 of the frequency ratio, respectively. In the Ordo column, Leibniz assigns each interval an alphabetical index. While the ordering function is not explicated, I believe Leibniz means 'order of consonance'. He also groups the first 8 intervals and labels them as "Consonantie" (or the consonances). The Origines column expresses each interval as the sum of the 3 most consonant intervals: A - 2:1, B - 3:2, and C - 5:4 - the octave, perfect fifth, and major third, respectively.

Again, what is not completely clear is how Leibniz derives his Ordos. Perhaps it was empirical. However, we arrive at Tenney's harmonic distance function by substituting the terms xA, yB, and zC by $x\log_2(2)$, $-y\log_2(2)+y\log_2(3)$, $-2z\log_2(2)+z\log_2(5)$, respectively; combining like terms; then replacing the resulting coefficients with their absolute values (as shown below). With two exceptions (marked with an asterisks in Table 1), the fact that Leibniz's Ordo corresponds to the order of the intervals sorted by Tenney's harmonic distance function follows.

$$\begin{aligned} & xA + yB + zC \\ \text{substitute} & \\ & x\log_2(2) + (-y\log_2(2) + y\log_2(3)) + (-2z\log_2(2) + z\log_2(5)) \\ \text{combine} & \\ & (x-y-2z)\log_2(2) + y\log_2(3) + z\log_2(5) \\ \text{replace} & \\ & |x-y-2z|\log_2(2) + |y|\log_2(3) + |z|\log_2(5) \end{aligned} \tag{1}$$

Table 1: *Leibniz's Origines expressed as Tenney's harmonic distance.*

Freq. ratio	"Origines"	Harmonic Distance	
1 : 1	identity/unity		
2 : 1	A	$\log_2(2)$	$= 1$
3 : 2	B	$\log_2(2)+\log_2(3)$	$= 2.58496$
5 : 4	C	$2\log_2(2)+\log_2(5)$	$= 4.32193$
4 : 3*	$A-B$	$2\log_2(2)+\log_2(3)$	$= 3.58496$
6 : 5	$B-C$	$\log_2(2)+\log_2(3)+\log_2(5)$	$= 4.90689$
8 : 5	$A-C$	$3\log_2(2)+\log_2(5)$	$= 5.32193$
5 : 3*	$A-B+C$	$\log_2(3)+\log_2(5)$	$= 3.90689$
9 : 8	$2B-A$	$3\log_2(2)+2\log_2(3)$	$= 6.16993$
10 : 9	$A+C-2B$	$\log_2(2)+2\log_2(3)+\log_2(5)$	$= 6.49185$
16 : 15	$A-B-C$	$4\log_2(2)+\log_2(3)+\log_2(5)$	$= 7.90689$
25 : 24	$2C-B$	$3\log_2(2)+\log_2(3)+2\log_2(5)$	$= 9.22882$
81 : 80	$4B-2A-C$	$4\log_2(2)+4\log_2(3)+\log_2(5)$	$= 12.6618$

Computing the necessary replacements in order to arrive from Leibniz's formulation to Tenney's harmonic distance function is rather straightforward. B - 3:2 is the equivalent of one step forward in the 3-dimension and one step back in the 2-dimension (by back, I mean its reciprocal, $1/2$): $3/2 = 3/1 \times 1/2$. 5:4 is the equivalent of one step forward in the 5-dimension and two steps back in the 2-dimension: $5/4 = 5/1 \times 1/2 \times 1/2$. Similar to Tenney's collapsed representation, Leibniz is incorporating movement from the 2-dimension in the other dimensions/terms in order to bring all the frequency ratios within an octave (a ratio between 1 and 2). The replacements above expand out the movement in the 2-dimension encoded in the other dimensions. The fact that Leibniz does not completely remove the 2-dimension (by maintaining the term A - 2:1) is the very reason that it is possible to show that his formulation was similar to, if not completely the same as, Tenney's harmonic distance function. By doing so, Leibniz uses steps that are within an octave (that is, logarithms of primes over the highest power of two less than the given prime) while maintaining a way to compute the movement in the 2-dimension needed to specify the ratio. The very movement, as explained in the previous section, that is lost in Tenney's collapsed visualizations yet preserved in his harmonic distance function. Figures 6, 7, and 8 demonstrate Leibniz's Origins in what I call a Leibnizian harmonic lattice (where a step is $p/2^{\lfloor \log_2(p) \rfloor}$ with exception of the term A - 2:1), in one of Tenney's 3,5 lattices with the 2-dimension collapsed, and finally in actual 2,3 harmonic space.

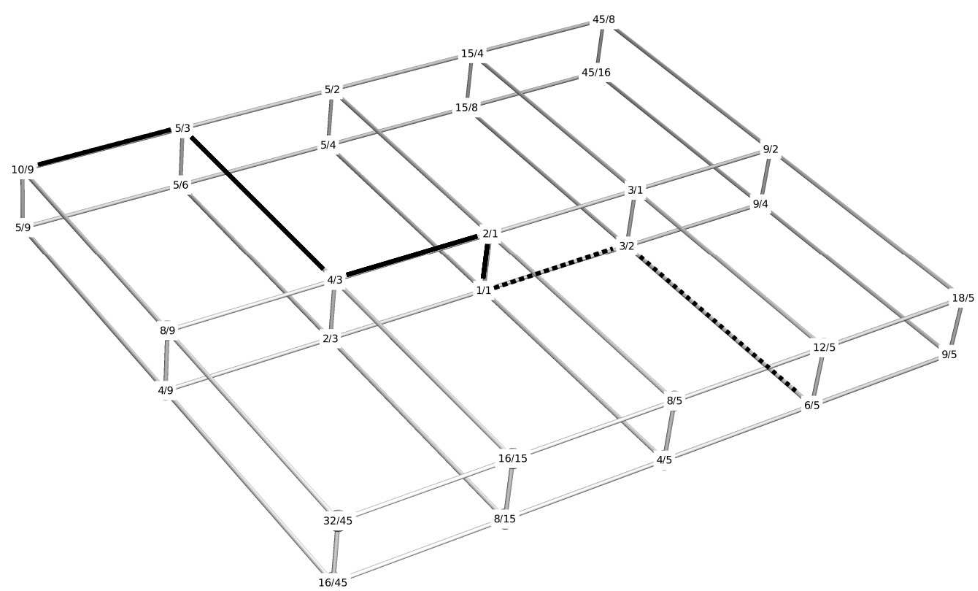

Figure 6: *Leibniz Origines for 10:9 (solid line) and 6:5 (dotted line) as paths on a Leibnizian harmonic lattice.*

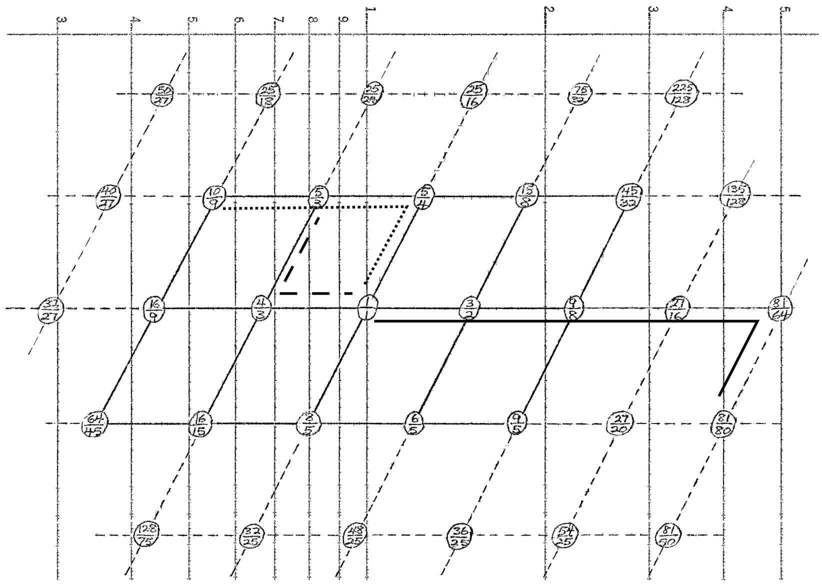

Figure 7: *Leibniz Origines for 81:80 (solid line), 10:9 (dotted line), and 5:3 (dashed line) as paths on Tenney's harmonic lattice in collapsed 3,5 space from "The structure of harmonic series aggregates". (Note that the A terms from Leibniz's Origines have been omitted.)*

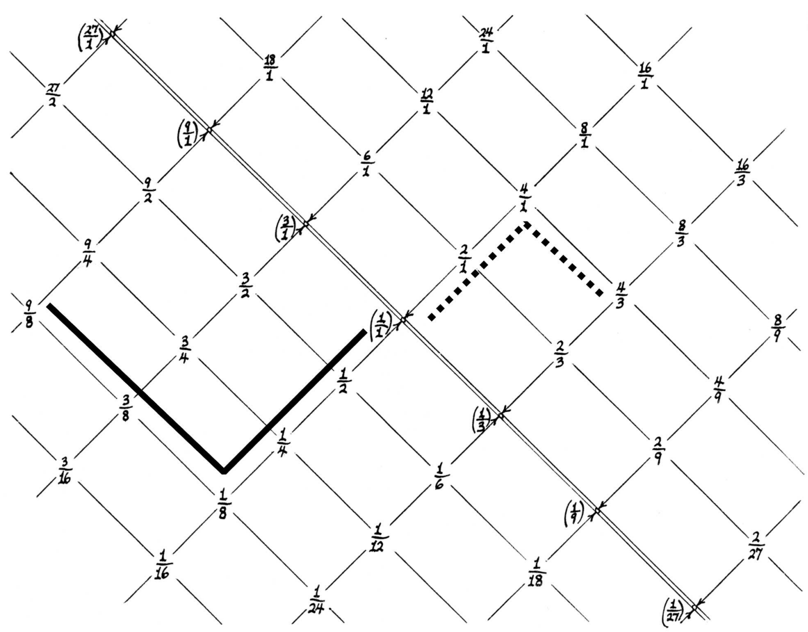

Figure 8: *Leibniz Origines for 9:8 (solid line) and 4:3 (dotted line) expanded out as paths on Tenney's harmonic lattice in actual 2,3 harmonic space from "John Cage and the theory of Harmony". (Note that the numbers of steps in each dimension is equal to the coefficients in the long form expression of the harmonic distance function in Table 1.)*

III. Conclusion

This is a very specific case of Leibniz's vision over 300 years ago that applies to music and theories being advanced today. The text of my piece *preliminary thoughts* gives other examples, but in less depth. With Leibniz, you always get the sense that you are only scratching the surface. Apart from the correspondence with Henflig, Leibniz wrote about music in *The Art of Combinations* and to several others such as Christian Goldbach, Christiaan Huygens, Joseph Sauveur, and Agostino Steffani. Again, I refer the reader to the work of Bühler and Luppi for a comprehensive overview.

It is unlikely that all of Leibniz's writings about music have been accounted for. The Leibniz archive contains copies of letters that he wrote and kept, but copies that he did not keep likely ended up solely in the possession of the correspondee. If someone were so diligent as to look into the archives of those with whom he corresponded, I imagine that there would be more intellectual treasures to be found, especially with respect to music.

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preliminary thoughts

The following text is a 'musical letter'. The full score which includes performance instructions, the musical notation for for the guitar part, the code for the computer generated accompaniment, and a Spanish translation of the text by Nicolás Carrasco Diaz is available at:
http://www.unboundedpress.org/scores/preliminary_thoughts_score.pdf

Dear Greg,

As I mentioned in prior correspondence, in consideration of the upcoming celebration of Leibniz on the 300th anniversary of his death, I have immersed myself in his work; reading and rereading his texts as much as time allows. His oeuvre is so voluminous, that I fear even by the time we meet in November, I will have only scratched the surface.

I have been enjoying the fact that much of Leibniz's writings are in the form of letters. They are less precious, less formal in that way. As I prepare to write the piece for the celebration in Turin, I thought it would be nice to set my correspondence with you to music. As musical letters or studies of sorts. Ideas not yet fully formalized but worth expressing; both the text and the accompanying music.

I write this letter as an exposition of my preliminary reactions in hopes that the very articulation and expression of these thoughts will aid in their future formulation albeit as naive as they may be in their current state.

In Leibniz's writings, I have found several cogent threads that intrinsically (if not explicitly) relate to art and music. I will group them as follows even though they are all interrelated: combinatorics, harmony, aesthetics, structure, epistemological vs. practical limits, and free will.

1) Combinatorics

I found Leibniz's dissertation entitled *On the Art of Combinations* of particular interest. Perhaps because it is an early work; laden with mistakes yet sound in its conception. But more likely because of explicit references to the application of combinatorics to music. Although it was written for his studies in jurisprudence, it is humbling that it can apply to so many other domains.

My composer friend Tom Johnson first showed me the 6th of 12 problems from the dissertation last summer though I was unaware of the source at the time. In the problem, Leibniz tries to count the number of 6 note melodies that can be sung with 7 possible pitches. He classifies them by the number of repeated elements. That is, he was trying to give a solution for the number of tuples and permutations with prescribed repetitions.

Earlier in the dissertation, he also discusses the application of combinations from problems I and II to organ registry and counts the number of possible timbres that an organ with a certain number of stops can sound (i.e., all subsets of the stops). In this sense, Leibniz predicted over 300 years ago musical ideas that are only now being explored by composers more thoroughly. Though there are important precedents. Bell-ringing traditions come to mind and also the music of Bach, of course. I like to think that there was a sort of intellectual resonance between Leibniz and Bach based on the fact that they lived near each other at the same time. I am also curious if Bach might have been alluding to the title of Leibniz's dissertation in the *Art of the Fugue*.

2) Harmony

While I have yet to find a full version of Leibniz's letter to Christian Goldbach, I have found the following translated excerpts:

> "All our usual intervals are ratios based on two of the prime numbers, 1, 2, 3 and 5. If we were endowed with a little more subtlety, we might arrive at the prime number 7. And actually I believe the following ones are also given. Thus the ancients did not openly avoid the number 7. But hardly anybody proceeded as far as the following prime numbers, 11 and 13."

Then later in the letter he writes:

> "I do not believe that irrational ratios are pleasing to the soul in themselves, except when they are very close to the rational ones which give pleasure."

Clearly Leibniz had a keen understanding of musical harmony. These are deep insights rooted in the Greeks yet only revived recently by composers such as Harry Partch and James Tenney. And indeed, as Leibniz predicted, composers are starting to more thoroughly explore harmonies based on higher prime numbers; what Tenney calls extended harmonic spaces with higher dimensions.

The second quotation might refer to the interleaving of dissonances with consonances as is common in chordal progressions within the rubric of functional tonal harmony. However I prefer another interpretation: that Leibniz is suggesting what Tenney calls "tuning tolerance"—the idea that the brain resolves irrational harmonies to the nearest simplest set of frequency ratios.

Admittedly, I have yet to follow this thread in Leibniz's writings to further extent but hope that I can find more texts that refer to harmony and harmonic constructs.

3) Aesthetics

It is hard to fully understand Leibniz's thoughts on the perception of beauty. He often alludes to the concepts of good and bad with respect to music and art, which I disagree with. In my mind, absolute beauty does not exist. People who believe in it are actually referring to status quo bias where the status quo is the current popular opinion. That is, if someone deems something as universally bad, it actually means that it is against the status quo with which they are in agreement. Whether or not, and how, someone appreciates beauty must be subjective even though biases will arise, especially within cultures. I have theorized in the past what can bring about a person's opinions with respect to if and how they appreciate something they perceive and why this can differ from person to person. I can even demonstrate it in terms of Algorithmic Information Theory, but I will leave that for a later time and remain focused for now on where Leibniz and I align.

In both his "Discourse on Metaphysics" as well as "Meditations on Knowledge, Truth, and Ideas", Leibniz discusses the concepts of "clear" and "confused" knowledge. The latter is of particular interest to me. To paraphrase Leibniz with my understanding of the concept: confused knowledge is the ability to perceive something as distinguished from other things yet unable to express the properties which give rise to its distinction. I sometimes tell people that music often interests me when I know that there is some underlying process even though I cannot identify or properly articulate exactly what that process is. I refer to this as the 'incalculability of concept-to-percept-transparency', which is the inability in art to know to which extent someone can deduce the concept of a work from the perception/experience of it.

4) Structure

Leibniz's discussion on the relation of parts to other parts and to the whole (an example of which I will give later with respect to epistemological vs. practical limits) is almost found verbatim in the composer John Cage's definition of structure. However, Leibniz had even more radical thoughts pertaining to structure. As you have pointed out in your writing, Leibniz basically predicts Algorithmic Information Theory with the following quotation from his "Discourse on Metaphysics":

> "If someone traced a continuous line which is sometimes straight, sometimes circular, and sometimes of another nature, it is possible to find a notion, or rule, or equation common to all the points of this line... When a rule is extremely complex, what is in conformity with it passes for irregular... But God has chosen the most perfect world, that is, the one which is at the same time the simplest in hypothesis and the richest in phenomenon."

This statement is essentially synonymous with the fundamental tenet of Algorithmic Information Theory: that you have structure if the computer program that generates a given object is smaller in bits than the object itself. It is this idea perhaps more than the others that I would like to follow as thoroughly as possible in Leibniz's work to better understand its genesis.

5) Epistemological vs. Practical Limits

In the dissertation, Leibniz writes:

> "The concept of parts is this: given a plurality of beings all of which are understood to have something in common; then, since it is inconvenient or impossible to enumerate all of them every time, one name is thought of which takes the place of all the parts in our reasoning, to make the expression shorter. This is called the whole. But in any number of given things whatever, even infinite, we can understand what is true of all, since we can enumerate them all individually, at least in an infinite time. It is therefore permissible to use one name in our reasoning in place of all, and this will itself be a whole."

Similar to how Leibniz was interested in an alphabet of human thought and the lexicon of a universal language, making art is often about defining elements and how they are (or can be in the case of a more open work) arranged. And just as it is inconvenient to enumerate through all subject-predicate pairs for a universal language, so too is it often difficult, if not altogether impossible, to enumerate all possible musics made from a given set of musical elements. I often find that the musical concepts that I envision in the compositional process quickly spiral out of control in the same way that their more abstract mathematical analogs in combinatorics explode exponentially. But where does the inspiration come that guides the artist to limit the material and order it in a particular way? Here Leibniz's faith in God guides him. Much of his work references the perfection of God's creation and the dissertation itself starts with a proof of God's existence. But this is all in search of truth and clearly he is seeding the idea of a universal proof checker. That is yet another thing that amazes me about his thought process. Almost as asides, he invents new fields of mathematics or prophesizes concepts that are only proved or disproved much later.

This rift between the limits of knowledge and the limits of practicality also occurs in Algorithmic Information Theory. Beyond the paradox of not being able to find a minimal program with certainty, just finding a program that outputs a given result at all is exhaustive beyond our computing means today. I dream of a world in which all my ideas would be computable.

6) Free will

The rift I discuss above also gives me a great deal of faith in intuition and inspiration. And that my intuitive decisions are the very computations I am interested in making with machines. But what is choice? Leibniz believes that all true predicates are contained within a given subject. This is yet another idea where Leibniz and I have independently aligned if I interpret his thoughts correctly. I believe he suggests that because you are unaware of the future, despite its containment in the subjects of the world, that whether or not there is free will does not matter. I have referred to this as the 'illusion of choice' in my own writing. And suggest the very same thing I interpret in Leibniz: that in any world, determinate or not, there is no difference between choice and the illusion of choice.

Then finally, there is love, which I believe must be intrinsically linked to art and creativity. I now know how real love is and how inspired I am by my love for others. Just as art is a "confused" knowledge, so too is love. My body and my senses inform me of its presence and of its loss from another, but my mind cannot explain the reasons for these visceral distinctions. I imagine Leibniz has somewhere discussed what I now understand... that all I do is for love... and that every ounce of my creative energy is for that love to be reciprocated.

With Best Regards,

Michael Winter (Los Angeles; January 23rd, 2016)