Composing with Textures: A Proposal for Formalization of Textural Spaces

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Abstract: *This paper presents the networks of textures called* textural spaces*. Each textural space provides not only various ways of encoding the organization of the component parts of a texture, but also the quality of their relations.* The textural class space *is the most generic description of a texture as it divides the components into two abstract structures:* line *and* block*. This basic components are defined by the quality of their appearance and functionality, determined by the uniqueness versus multiplicity of the sounding components therein. The second textural space, called* ordered partition space*, consist of ascribing an integer partition to specify the number of components within a textural class. Finally, the* partition layout space *provides the most refined description of a texture among all textural space since it considers the internal order of the components according to their registral placement or timbre distribution. After presenting the various concepts and operations, the paper concludes with a discussion about the potential creative application of them, and how they can be decoded into a musical score.*

Keywords*: Musical Texture. Textural spaces. Music Composition. Music Analysis. Theory of Integer Partitions.*

I. Introduction

Music texture is one of the most important aspects of the creative process, not only for allowing the unfolding of the music form through its transformations but also for organizing the role of each component within the compositional structure. Despite its importance, there is a lack of systematic studies on texture, principally if compared to other musical parameters. In fact, many recent composers do not explicitly relate the textural organization of their music to any systematic approach. Since their music encompasses various textures, possibly, these are conceived intuitively as the outcome of the manipulation of other musical parameters.

Perhaps this is due to the diffuse and sometimes elusive definition of the term texture in music, often used rather loosely to describe both the organization of simultaneous voices or instrumental part (texture as a structure) and the overall quality of a piece defined by its instrumental techniques and combinations considering its sonic perspective (texture as sonority) [8, p. 93]. Therefore, a more precise definition shall be provided.

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Following Wallace Berry, by music texture we refer to both the quantitative and qualitative aspect of music [3]. As a quantitative aspect, it refers to the number of simultaneous *threads of sounds*[1]. The qualitative aspect consists of examining the relations between and among the threads of sounds to set the components of the texture (layers or strata).

An important part of the compositional process consists of organizing the textural components considering the hierarchical sense between and among them, their chronological and diachronic presentation, the way they are combined, and the like. Yet, texture is not a matter of style although various music styles can be associated with a specific texture or group of textures. A texture became recognizable by its morphology, i.e., the mutual relations of its internal components rather than their particular characteristics. Hence, a given texture can be found in different works.

Most of the studies on texture have been devoted to an analytical perspective as an alternative to the traditional methodologies based on pitches and rhythm, and their relations with the music form. Apart this, the creative use of texture is most often confined to the pre-selection of textural configurations or generalized concepts of textural behavior, without necessarily presenting any theoretical discussion that justifies such choices. Therefore, the selection results of the composer's personal experiences and creative ideas.

Another common compositional approach to texture consists of describing either the generic organization of parts through conventional labels of classification such as *monody*, *homophony*, *polyphony*, and *heterophony*, or some technical procedures involved in the compositional process that have been developed since twentieth century, such as *micropolyphony*, *pointillism*, *stratification*, and *sound-mass*. This demonstrates the lack of a clearer and more precise refinement for using texture as an autonomous element during the creative process.

In Brazil, there was an special interest in the subject, mainly in the last decades.[2] Some of these authors were concerned not only with the analytical potential of texture formalization, but also with the possible application to musical composition, especially within the scope of the MusMat research group, to which the present author belongs.

Departing from this background, this paper introduces the concept of *textural spaces*, a formalization of different networks of textures. For each textural space, a topological study is developed to examine the connections between textures in each space. This paper concludes by presenting some of the potential application of the textural spaces within the compositional process, including a brief discussion on how they can be decoded into a music score.

II. Textural Spaces

The *textural spaces* are out-of-time networks that list all possible textural structures connected or related by some operations.[3] All possible textures can be encoded within any textural spaces since they differ from one another in their level of details and by the involved operations that connect their elements, although the principle of these operations is common among them. Thus, the textural spaces are subordinated to each other. The details of each textural spaces are related to how a given texture can be evaluated either by examining the score, to observe the way its components are combined regardless their specific nature (actual pitches and durations), or by its

[1] The term "threads of sounds" (or simply threads), a musical allusion to the threads that constitute the weft of a fabric, refers to the number of concurrent sonic events that produces a linear continuity. For example, a four-part polyphony consist of four threads, and a six-note chord comprises six threads.

[2] See, for example, [16], [30], [1], [31], [15], [10], [29], [17], [4], [21], [7], [27], among others.

[3] The textural spaces are, at a certain level, based on Morris' *pitch spaces*, in which the possible relations among the various concepts of pitches are mapped into an exhausted taxonomy ([25]). This proposal is a further development of a previous work [22].

aural perception. Once we understand the aspects involved in all spaces, we are able to discuss their potential to coordinate the texture throughout the creative process.

i. textural class Space (tc-space)

A given texture can be described as the organization of simultaneous threads of sounds into *lines* (L) and *blocks* (B). Each thread is differentiated from the others by its own unique features, but if two or more threads share some characteristics, such as rhythm, note onset times, or register placement, or they have a similar timbre, melodic contour, dynamic, and so on, they can be grouped into a block. Otherwise, each thread not related by similarity with the others constitutes a line. Thus, the difference between blocks and lines is defined by their *thickness*, that is, the number of threads thereof, so that a line could be understood as a block with a single thread, and, in the same way, a block may have two or more threads to differentiate from a line[4].

Each line and block constitutes what we call *textural part* or simply *part*, a layer or strata within a musical texture (musical weft).[5] The morphology of a given texture concerns the *multiplicity* of its textural parts and its correspondent thickness. For instance, a passage of music might have three textural parts: one block and two lines, and the block is formed by four threads. Another passage also may have three textural parts, but three blocks, with two of them having three threads and the other one with five threads. The multiplicity of each textural part can be expressed by a superscript positive integer in the form of $[L^x B^y]$, where x and y are positive integers greater than or equal to zero. For example, $[L^0 B^3]$ stands for three blocks, with the same thickness or not, and $[L^2 B^0]$ indicates two lines. When the superscript is 1, it is omitted, provided that $[L^1]$ and $[B^1]$ are special cases denoted, respectively, by $[L]$ and $[B]$. Also, in order to facilitate the notation, when x and y are equal to zero, we may omit the correspondent part. Thus, $[L^0 B^3]$, for example, is rewritten as $[B^3]$ and $[L^2 B^0]$ is rewritten as $[L^2]$.

The typology of Ls and Bs consist of the most general textural description since it is grounded only in the relative distinction of the textural parts without discerning the exact thickness of the blocks. If the actual multiplicity is ignored for purposes of argumentation, we can preserve the superscript as x and y only to indicate any integer greater than or equal to 2. This general representation of texture using Ls and Bs is called *textural class*. A textural class encompasses all textures that share a similar organization of parts, ignoring the actual thickness and multiplicity thereof. Since texture is not a matter of style and does not depend on the nature of musical materials that differentiate one piece from another, the same textural class can be used in different ways. For example, all textures with a single line and two or more blocks of any thickness are members of the textural class $[LB^y]$. Altogether, there are eight possible textural classes:

1. A single line ($[L]$);

2. A single block ($[B]$);

3. A single line and a single block ($[LB]$);

4. Multiple lines ($[L^x]$);

[4] Thickness can also be defined by the span in the register considering the number of threads at intervals other than unison. Thus, a line could have more than one thread if all of them are in unison and they are perceived as a single and thin unity. In this case, the difference between lines and blocks would be related to the aural perception, taking into account the qualitative observation of the threads' organization instead of the simple calculation of their amount. We have left out this perspective to keep the discussion basic.

[5] The organization of the threads into lines and blocks shall be contextual and argumentative; each analyst or composer can determine his/her own criteria.

5. Multiple blocks ($[B^y]$);
6. Multiple lines and a single block ($[L^x B]$);
7. A single line and multiple blocks ($[LB^y]$);
8. Multiple lines and blocks ($[L^x B^y]$).

All textural classes derive from the classes L and B by combining both themselves and with each other. The set of all textural classes constitutes the simplest textural space called *textural class space* or *tc-space*. Given the large number of all possible textures, the formulation of a textural class space is useful because it defines equivalence relations to group textures with a similar morphology into a finite number of classes. Each textural class can be associated with one of the conventional labels of texture (monophony, polyphony, heterophony, and homophony), but providing a more detailed (and even clearer) description of the organization of their component parts. Indeed, the classes are apart from any aesthetical-stylistic association, which increases their creative potential in various ways.

According to the quality and the number of the parts, we can group the eight textural classes into four different types. First, considering the classes formed by only one type of textural part, we have the first two types *monopart* and *polypart*. Whereas the monopart type encompasses classes with a unique part ($[L]$ and $[B]$), the polypart includes classes with a noted multiplicity ($[L^x]$ and $[B^y]$). Now if we consider the combination of both types of textural part, we have the last two types: *isopart* and *heteropart*. In the isopart classes, the parts have a balanced distribution of multiplicity, that is, both or neither parts are multiple ($[LB]$ and $[L^x B^y]$).[6] In contrast to isoparts, the multiplicity of the heteroparts is restrict to one of the parts ($[L^x B]$ and $[LB^y]$).

All types can be related to monoparts by a given derivation process, which may allow a smooth change between them. Polypart classes constitute an expansion of monoparts to a polyphonic context, given that monoparts are included in the polyparts. Another way to think about the isopart classes is to note that $[LB]$ and $[L^x B^y]$ correspond, respectively, to the union of the two classes within monopart and polypart classes. Lastly, the unbalanced multiplicity of heteroparts outcomes from the combination of a monopart with a polypart class of a different nature (L and B).

Figure 3 summarizes the formation process of all textural classes from the combination of monoparts (ordinary circles) and polyparts (dashed circles). Each line indicates the union of classes to produce the other textural classes, also revealing the present of derivative relations between the four textural types mentioned above. Lines within the same type (dotted lines) indicate the formation of isoparts, while the cross relations (double lines) show the formation of heteroparts. The relation among the four types (and, consequently, among all textural classes), at some level, explicits the complexity hierarchy of them.[7] This hierarchy takes into account the abstract nature of the classes, in which the actual number of threads involved is undefined. Otherwise, the most complex texture of n number of threads would be $[L^x]$ (maximum of threads and parts). Nevertheless, we are not able to determine whether a multiple part has two or more elements, nor the exact number of threads within a block. Thus, the definition of relative complexity shall consider the minimum number of parts to constitute a class, which enables $[L]$ to be the simplest class, while $[L^x B^y]$ is the most complex, given that it is formed by at least four textural parts including blocks and lines (the higher number among all classes).

[6] Obviously, this balance is not necessarily concrete since, in tc-space, the actual multiplicity is not precisely defined.

[7] Complexity here is based on the relation between the number of threads and the way they are organized. The greater the number of threads and parts, the greater the complexity. In this case, a single line is simpler then any block [21][19][18].

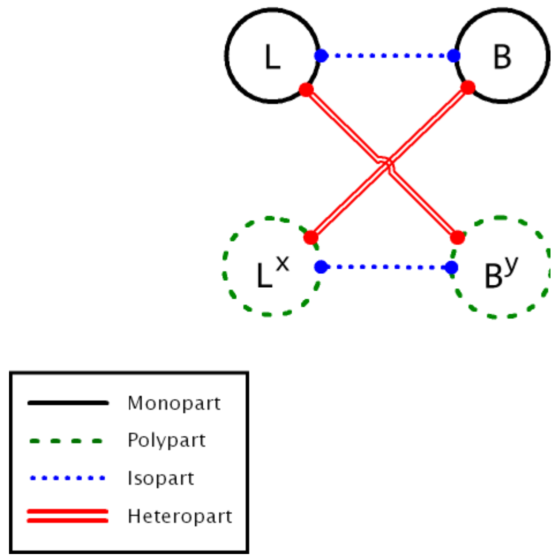

Figure 1: *Textural Classes: process of formation and the relation among the four types.* monoparts *(ordinary circles),* polyparts *(dashed circles),* isopart *(formed by the union of dotted lines), and* heteropart *(the union of double lines).*

To see how textural classes are defined within a piece, and the way they interact to each other, let us examine an excerpt of Mozart's *Eine kleine Nacthmusik*. The annotated textural classes below the score are defined by the rhythmic coincidence of the threads according to the vertical alignment thereof so that, the threads that share the same rhythmic are in collaboration to assemble a block (indicated by blue squares), and the threads with no rhythmic coincidences constitute lines (red squares - Figure 2a).[8] Each instrument of the string quartet corresponds to a single thread since there are no double stops. The excerpt is made out of only three different classes ([B], [B^y], and [LB]), that are combined to produce a textural sequence of five classes. From an imitative process, the monopart [B] is duplicated producing a polyphony of blocks (polypart class [B^y]). The ornamental variation on the rhythm in the first violin divides the first block in such a way that the first violin becomes an isolated line, while the second violin merges to the block of the viola and cello, forming the isopart class [LB]. Given that each class spans a whole measure, the brief use of class [B] within measure 13 can be understood as an deviation of [LB] (a *neighbor texture*[9], a possible clue for the block in measure 14 (or even an anticipation of it).

The graphic in Figure 2b, called *textural flow*, provides a temporal representation of this sequence to facilitate its overall observation.[10] By examining the curve in the Textural Flow, we notice that the first two classes are, respectively, the simplest (class [B]) and the most complex (class [B^y]) textures of the excerpt. Therefore, the interval between them is the greatest possible leap, but due to the melodic content of the parts in the imitative procedure, the textural gap is

[8] A given part is differentiated from the other by comparing their threads. All threads within a textural part are in *contraposition* relation with the threads of the other parts. The opposite relation of contraposition, called *collaboration*, determine whether the threads may form a block or not. Both concepts are based on Berry's independence and interdependence relations (see [3, p. 185]).

[9] For a further discussion regarding structural and ornamental functions of texture, see [23] and [21]).

[10] textural flow is a two-dimensional graphic that includes in the x-axis the duration of each class, and in the y-axis, all classes hierarchically ordered according to a relative complexity. This graphic enables the observation of both the dominance zones of a given textural class by the formation of plateaus, and the unstable or ornamental character of a class by its peaks.

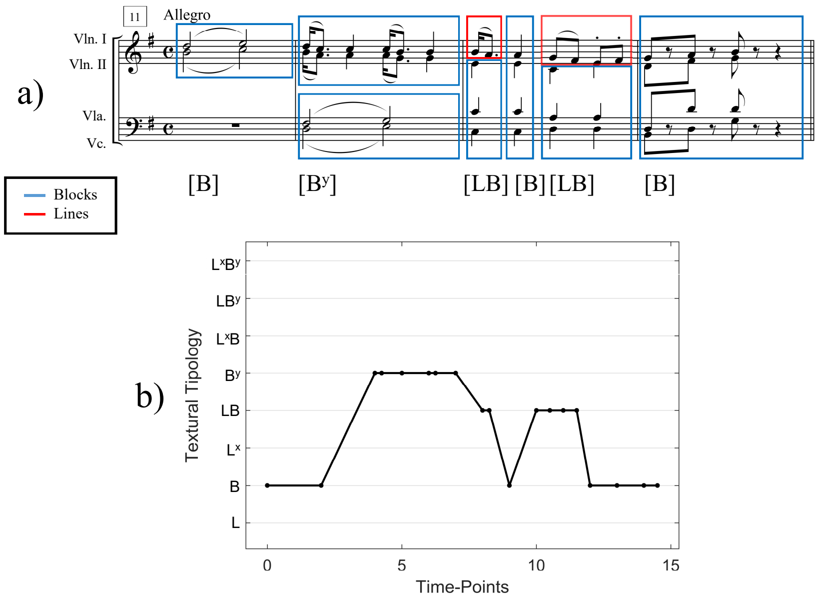

Figure 2: *Textural analysis of Mozart's* Eine kleine Nacthmusik*, K. 525, mm. 11-14 (1787): a) Annotated score with the correspondent textural class; b)Textural Flow graphic presenting the textural classes over time.*

attenuated. Moreover, although with different thickness, the blocks are associated with the same class, which implies that the overall movement of the texture describe and arch-shape trajectory departing from class $[B]$.

Operations on tc-space

Once we have defined the textural classes, we shall now discuss the relational topology of them. The inclusion of a single thread in a given texture may affect either the number of parts, or the thickness of them. This alteration depends on whether the new thread is in collaboration or in contraposition relation with the others. For example, adding a thread in textural class $[L]$ may result in $[L^x]$ or $[B]$. However, textural class $[B]$ can either turn into $[LB]$ or remain the same as a thicker block. In this case, the class would be preserved as $[B]$. Thus, the discussion of how a texture class can be related to another shall concern the changes in the *textural dimensions* (multiplicity and thickness).

Two basic operations may be defined regarding each dimension: *layering* (Y) and *shifting* (F).[11] These operations can be understood as functions for which the output is a textural class different from that of the input, but preserving some features. Therefore, the operations reveals possible adjacencies within tc-space. Both operations involve necessarily a change in the number of threads in the texture in either way increasing or decreasing it, that is, their application demands the inclusion or exclusion of threads in the textural class.

The layering operation refers to the inclusion or exclusion of a textural part, noted as $*Y_n$, where "*" is either a + or – symbol, which indicates the orientation, and n correspond to either

[11] Both operations are based on the relational topology of partitions, called *partitional operators*, proposed by Gentil-Nunes [10, 45-50].

L or B.[12] Thus, $+Y_n$ denotes the inclusion of a new block or line in the class; $-Y_n$ denotes the exclusion of a line or a block from the class.[13] For example, given the textural class K, $+Y_L(K)$ is the addition of a single line to K, while $-Y_B(K)$ removes a block from K. If K is equal to LB, then $+Y_L(K)$ is equal to $[L^x B]$, and $-Y_B(K) = [L]$. Henceforth, we shall use Y to refer to the notation $*Y_n$.

Given the relative description of the parts, in which the actual thickness is not defined, Shifting operation (F) refers to the transformation of a block into a line, and vice versa. Thus, $+F$ operating on a textural class K (that is, $+F(K)$) increases the thickness of one of any line, changing it into a block, whereas $-F(K)$ decreases the thickness of one of any blocks, turning it into a line. For instance, given the textural class $[LB]$, $+F([LB])$ indicates that the $[L]$ in the class becomes a Block, resulting in the class $[B^y]$. Similarly, $-F([B])$ is equal to $[L]$. Henceforth, in general F will denote $+F$ or $-F$.

When a part has multiplicity, operation F has more than one possible output. As the exact number of multiplicity for each part is not specified, we shall consider all possible values. For a multiplicity factor of two, the minimum necessary to form a multiple class, we have one possible output for F, whereas for any number equal to or greater than 3, F produces a different outcome. Therefore, except by $[L^x B^y]$, where one of the outputs corresponds to itself, all multiple class have two possible outputs under F. For example, $+F([L^x])$ is equal to $[LB]$, for $x = 2$, and to $[L^x B]$, for $x \geq 3$. Likewise, $-F(LB^y)$ is equal to $[L^x B]$, for $y = 2$, and to $[L^x B^y]$, for $y \geq 3$. If the multiplicity factor is equal to or greater than 3, F operation produces a redundant output to Y operation. Certainly, both composer and analyst are free to choose which operation (Y or F) is more appropriate to connect the classes according to any of their purposes.

Toward a more refined organization of the tc-space, providing a general view of the operations that connects all textural classes, we propose the *textural class lattice* (TCL - Figure 3). The nodes of the graph are the textural classes, and the edges are operations that can connect the nodes. Each edge represents a different operation and the signs (+ or –) are reflected in their orientations. Double red lines indicate a relation between a pair of textural classes under the operation Y_B. Reading upward from left to right indicates the positive operation, and the opposite direction denotes the negative operation. Dotted green lines stand for Y_L operation, with upward from right to left designating $+Y_L$, and downward from left to right the $-Y_L$. Blue lines denote operation $+F$ from left to right (also considering the upward movements), and $-F$ from right to left (including downward movements).

Having the monopart classes, the simplest among the types, in the bottom of TCL reveals that all other classes derive from them through Y and F. Each monopart is connected to its correspondent polypart class by Y operation, which delimitates a spatial distribution of TCL based on the amount of Ls and Bs. Bottom-up and top-down movements, respectively, increase or decrease the number of textural parts (and, consequently, the number of threads within the class), while horizontal movements change the quality of the parts from lines (more to the left) to blocks (more to the right). Isopart classes balance lines and blocks in the middle of TCL. Except by isoparts, F operation connects both classes within all classes, but polyparts require a succession of F, with the isopart $[LB]$ as an intermediate point.

Let us return to Mozart's example (Figure 2) to examine the operations involved between the classes. Given that all classes have the single block $[B]$ in common, we can relate them to the first block in the violins through Y operation. First, the imitative process itself, under a textural perspective, consists of a positive Y operation to include a duplicated textural part. Thus, the first

[12] Y_L correspond to Gentil-Nunes' partitional operator named *revariance*, while Y_B can be interpreted as the *concurrence* (see [10, 46-49]).

[13] The operation $-Y_n\ (K)$ is context-sensitive because K must have a textural class n in it. For example, $-Y_L([B^y])$ is not defined, since $[B^y]$ has no line to be eliminated. The same principle can be observed in $-Y_B([L])$.

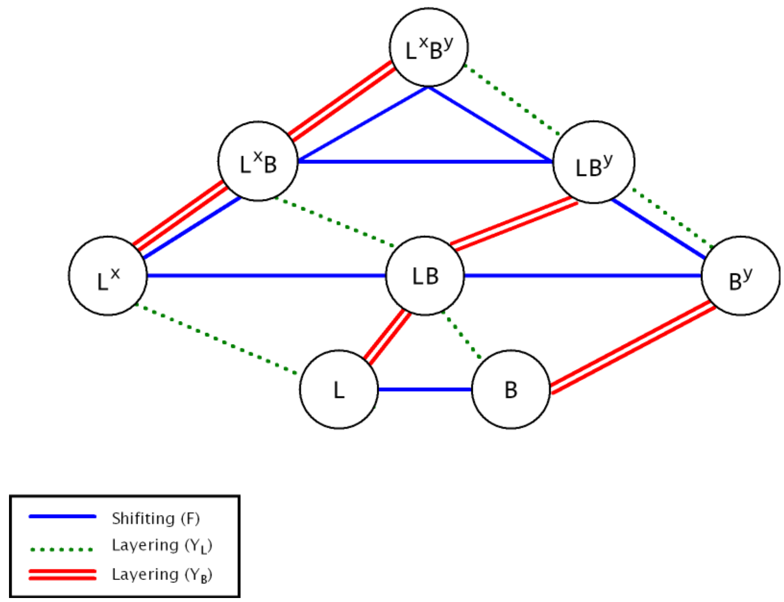

Figure 3: *textural class lattice: all textural classes connected by layering (Y) and shifting (F) operations.*

gesture corresponds to $+Y_B$ operation to connect classes $[B]$ to $[B^Y]$. The next linear transformation involves either splitting the block into two lines and merge one of the lines to the other block, increasing its thickness; however, in Tc-space, classes $[B^y]$ and $[LB]$ are related to $-F$ operation since the change in the thickness of the block does not affect the class representation. [14] A non-linear transformation reveals that class $[LB]$ derives from the initial block by $+Y_L$, a variation within the Y operation used in the first transformation. Indeed, the next two classes in the sequence successively articulate both Y_L operations (+ and -) to produces the oscillation between $[LB]$ and $[B]$ in the last measures.

ii. Unordered-Partition Space (up-space)

For a more detailed description of a texture we can ascribe an integer to express the actual amount of threads of each textural part[15]. While L stands for the integer 1, $[B]$ encompasses any number greater than or equal to 2, that is, according to the number of threads $[B]$ may refer to 2, 3, 4, 5 and forth on. In the same way, $[L^x]$ corresponds to x realizations of 1, whereas $[B^y]$ stands for a set of y integers greater than 1, which may include blocks with equivalent thickness or not. For example, for $y = 2$, that is, $[B^2]$ in a set of six threads only two combinations are possible: $[2, 4]$ and $[3, 3]$. This numerical representation is called *integer partitions*. Each number within a partition is also called textural part.

A partition can be defined as the various ways to represent a positive integer n through the sum of other positive integers [2, p. 1]. Each partition encodes a specific textural configuration in such a way that the texture of a given piece is represented as a sequence of partitions. Giving that the partition set for n is finite, using partitions allows the composer to have available all textures

[14] Another possible interpretation would consider a composite operation, such as $+Y_L$-Y_B or $-F$-Y_B, but, when it is possible, we prefer to use a single operation to simplify the process.

[15] Such methodology was first proposed by Wallace Berry [3] and further developed by Pauxy Gentil-Nunes [13][10] [9] through the approximation with the Theory of Integer Partitions, an important area of additive number theory developed by many important mathematicians, such as Euler, Hardy, Ramanujan, among others.

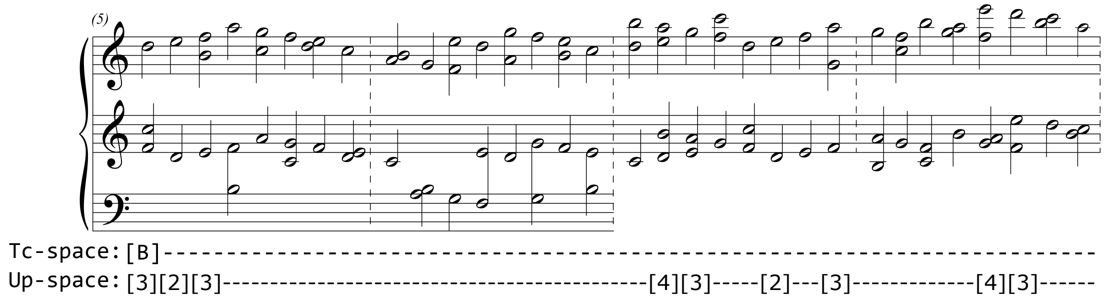

Figure 4: *Textural class [B] and Partitions encoding the thickness variations in the blocks of Ligeti's Étude 15: White on White, mm. 5-8 (1995).*

for the n number of threads, a significant tool for the compositional process.[16] As an example, four threads has five possible partitions: [4], [1, 3], [2, 2], [1, 1, 2], and [1, 1, 1, 1].[17] These partitions are members, respectively, of the following textural classes: $[B]$, $[LB]$, $[B^y]$, $[L^xB]$, and $[L^x]$.

Within a partition the order of parts is irrelevant and, by convention in this work, the parts are presented in an increasing order. For a more concise presentation, the partitions are notated with square brackets in an abbreviated version, where the multiplicity is expressed by an index in a similar way than in textural classes. Moreover, in order to eliminate eventual notational ambiguities and excessive use of spaces, each part is separated by a comma or by the indexes of the previous part. For example, partitions [1, 1, 1, 2] and [1, 2, 2, 3, 3] are, respectively, written as $[1^32]$ and $[1, 2^23^3]$.

The partitions form a partially ordered set that constitute the second textural space, called *unordered-partition space* (*up-space*).[18] Compared to tc-space, up-space provides a more refined description of texture since the actual thickness and multiplicity of each part is depicted by an integer number. Moreover, partitions specify the textural morphology allowing the measurement of nuances among textures within the same textural class through the difference of their subtle internal variations.

For instance, in Ligeti's *Étude 15: White on White* (Figure 4) the chords produce a static texture made up of a sequence of various blocks. Tc-space simplifies the description of the texture ascribing the monopart class $[B]$. Thus, this abstraction does not regard the number of pitches within each chord as relevant. In up-space the texture is more detailed, being encoded through a varied progression that encompasses three partitions: [2], [3], and [4].[19]

Operations on up-space

Similar to tc-space, the operations that connect partitions in up-space concern changes in the multiplicity and thickness of a partition, but in a more specific way. Instead of the abstraction L and B, we use a positive integer to depict the exact number of threads we are including or

[16] This possibility is one of the most important contribution of Gentil-Nunes' Partitional Analysis to music composition.

[17] The sum of the threads of a partition corresponds to what Berry calls *density-number* [3, p. 188].

[18] By convention in this paper, the term partition refers to unordered partition. When the order is relevant it includes the term "ordered".

[19] We are considering the same onset of each thread as a criterion to determine the textural parts. Nevertheless, other relations can be considered in the analysis. For example, one could argue that the registral placement may emphasize some lines within the blocks, provoking a contraposition relation between them and the rest of the block. Or yet, that each staff should be understood as an individual textural part, and so on. In both cases, the analysis would include other textural classes and partitions.

excluding in the partition. In general, we can propose any relational operation to connect two or more partitions based on a given derivative process. To facilitate the discussion, here, we will focus on two basic operations that when combined are capable to connect all partition set: *layering* (Y) and *dimensioning* (D).[20] In contrast to tc-space, where the recursive application of the operations is limited, in up-space, both operations, when contextually applicable, can produce a path with unlimited chain of partitions provided by their successive iterations.

As in tc-space, layering operation refers to multiplicity either by including or excluding a textural part. The notation $*Y_n$, where "*" stands to either a + or – symbol, indicates the positive integer n that shall be included or excluded in the partition. Thus, the negative layering ($-Y_n$) is also contextual, demanding that the partition has a part equal to the integer n therein; otherwise, the operation would be not defined. Suppose $\{a, b, c\}$ are the parts of partition K. Thus, $+Y_d(K)$ is equal to $[a, b, c, d]$, while $-Y_b(K)$ correspond to $[a, c]$. Henceforth, we shall use Y to refer to $*Y_n$. Y is the inverse function of itself by inverting its signal. To put this another way, if Y is positive, to restore the input, we shall operate a negative Y in the output and vice versa. For example, $+Y_3([1^2 4])$ is equal to $[1^2 3, 4]$, and $-Y_3([1^2 3, 4])$ return the first input ($[1^2 4]$).

Dimensioning, a substitute of shifting in tc-space, indicates the increment or decrement of the number of threads of a textural part being noted as $*D_n$, where "*" is either a + or - symbol and n is the number of threads to be included or excluded from a part. Thus, $+D_2(K)$ (we say a positive dimensioning) consist of adding two threads in any part of K, increasing its thickness. Similarly, $-D_3(K)$ (a negative dimensioning) concerns the removal of three threads of any part of K, decreasing its thickness.[21] For example, $+D_1([1, 2, 3])$ stands for three possible outputs: $[2^2 3]$, $[1, 3^2]$, and $[1, 2, 4]$, while $-D_2([1, 2, 3])$ has a single output ($[1^2 2]$). Henceforth, we shall use D to refer to $*D_n$. As Y operation, D is also the inverse function of itself in an opposite direction, but, depending on the number of parts, its reverse operation may also produce outputs other than the input.

Note that the cardinality of n in Y and D refers to the distance between the input and the output partitions. This distance is related to the relative complexity of them [21]. The higher the cardinality of n, the more distant they are to each other. In the same way, if n is equal to 1, both operations reveal an adjacent relation between the involved partitions.[22]

When signed positively, both operations derive from the inclusion relation, in which the input partition is contained in the output. Also, while the recursive application D in any partition with two or more parts will necessarily produce bifurcation within the path, Y gives rise to a single and linear path. Using *Young's diagrams* makes these relations clearer.[23] Figure 5 demonstrates

[20]Despite their conceptual differences, both operations are related in some level to, respectively, Gentil-Nunes' *revariance* and *resizing* [10, pp. 45-50].

[21]Obviously, in the negative dimensioning, the altered part shall not be less than or equal to n because it would imply in its deletion, an effect that concerns the multiplicity instead of thickness.

[22]One of the main conceptual difference to Gentil-Nunes' proposal is that his partitional operators invariably considers one thread at a time for both transformations (resizing and revariance). Although the result of D for n greater than 1 can be equal to n successive applications of resizing, it does not cover all resizings' possible outputs because the n threads of D are necessarily included or excluded in the same part while resizing allows a balanced distribution of them among the parts. In Y operation, if n is greater than 1 it correspond to the composite operation of revariance and resizing instead of the n operations of revariance. However, although Y can be understood as operating on both textural dimensions, we consider the output as the insertion of a part of any thickness, a change in the multiplicity of the partition. Furthermore, while Gentil-Nunes' aim was to provide a topological study of partitions through their adjacencies to guarantee a consistent theoretical foundation, we are assuming a creative perspective for the operations, so that adding or excluding multiple threads simultaneously in both operations constitute a simple compositional procedure to alter the partition, regardless whether the output is adjacent to the input or not.

[23]Young's diagrams are visual ways of representing a partition. Each square corresponds to a different thread. Then, the partition is defined by the square's arrangements so that side-by-side squares stand for the thickness of a part and overlappings are related to their multiplicity [2, pp. 6-7].

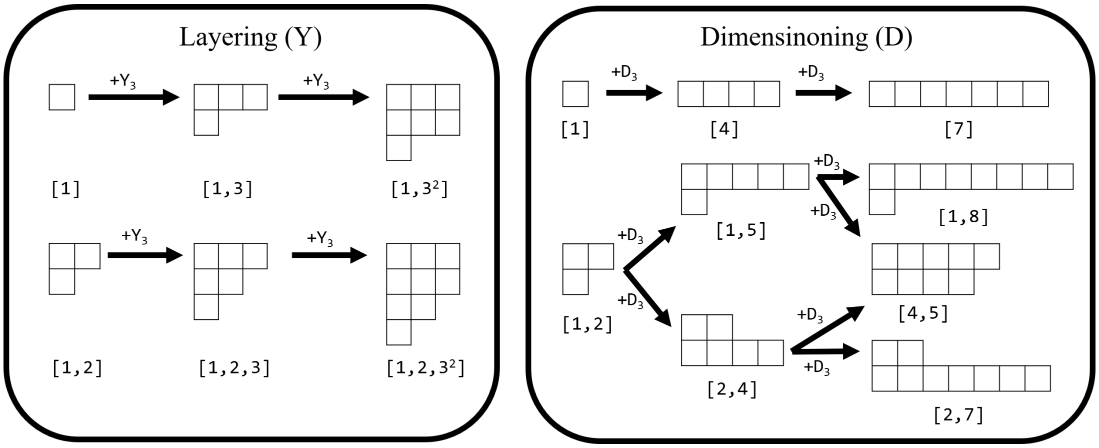

Figure 5: *Young's diagram showing the paths produced by Y (for $n = 3$) and D (for $n = 3$) applied to partitions* $[1]$ *and* $[1, 2]$*.*

through Young's diagram the recursive application of $+D$ and $+Y$ for $n = 3$ departing both from a single-part partition ([1]) and from a multiple-part partition ($[1, 2]$). In multiple parts, the path of D includes bifurcations. Each partition can be included within the following one, which illustrates the inclusion relation commented above.

From an exhaustive taxonomy of the textural configurations, as well as a topology of the partitions operators, Gentil-Nunes [10, pp. 50-51][8, pp. 97-98] proposes the *Partitional Young Lattice* (PYL), based on Young's Lattice (Figure 6).[24] Each square represents a partition (or a pair of partitions as in density-number equals to 6) in such a way that partitions placed side-by-side in horizontal share density-number values. Moreover, similar to TCL, the more to the right are positioned the partitions, the more massive they are (more blocks). Conversely, the more to the left, more polyphonic are the partitions (more lines). Above each partition is stated its equivalent textural class for purpose of comparison. The color of Ls (blue) and Bs (red) expresses the division mentioned above. Any upward movement is expansive, increasing the number of threads of the partition. The number in parenthesis below each partition stands for the pair of indices called *agglomeration* and *dispersion*, whose functions are presenting the number of, respectively, collaboration and contraposition relation between all of threads within the partition, analyzed in pairs (see [10, pp. 33-38]). Each index increases or decreases according to the horizontal position of the partition since they are related to the polyphonic and massiveness degree. The edges that connect the partitions correspond to both operations (Y and D) for n equal to 1 to show the adjacencies of them. Partitions that do not share edges have no definite co-relations, due to the partially ordered set property of partitions.

In the partitions of number 6, the two pairs of partitions within the same square share the same pair of indices (agglomeration and dispersion). Thus, both partition can be understood as equivalent, due to their internal organization of threads, in which they hold the same relations of collaboration and contraposition despite the difference of their component parts.[25] This property, called the *h-relation*[26], becomes more recurrent, including more partitions within the indices, as

[24] Young's Lattice consist of a representation of the partition's lexical set using Young's diagrams, organized by the inclusion order, that is, bottom partitions are included in upper partitions.

[25] The musical impact of this equivalent is left for a forthcoming work.

[26] The "h", which stands for homoindex, is an allusion to Allen Forte z-relation, where two pitch-class sets not related by transposition or inversion produces the same interval-class vector [6].

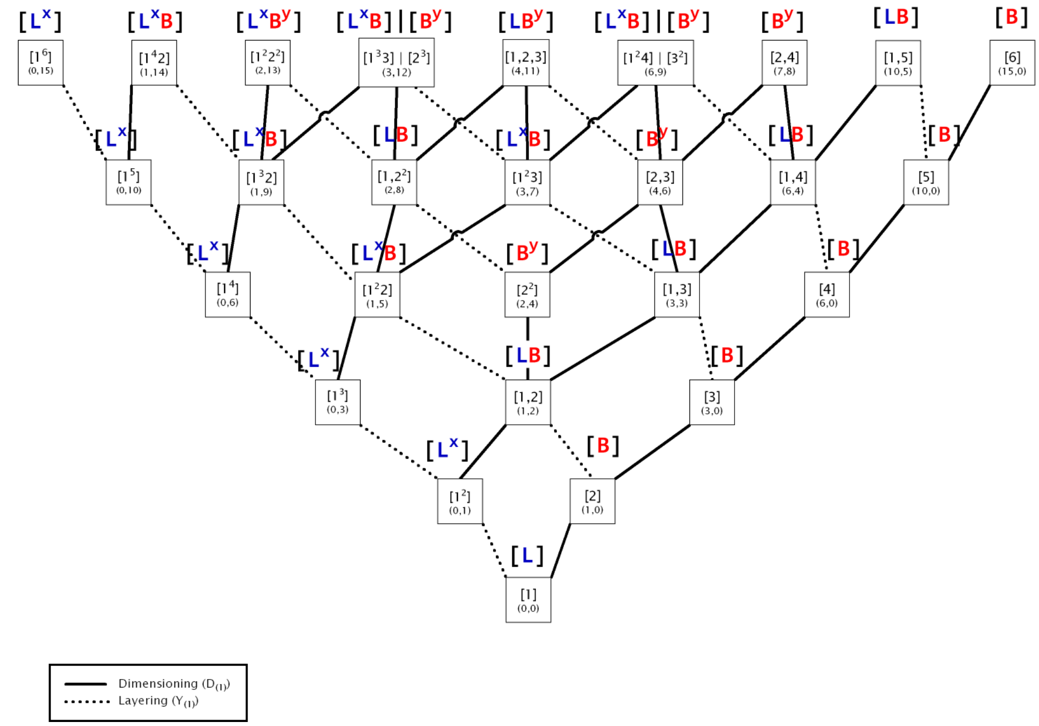

Figure 6: *Partitional Young Lattice: partition lexical set for density-number = 6 connected by layering (Y_n - straight lines) and dimensioning (D_n - dotted lines) for n = 1. (adapted from [10, p. 51]).*

the density-number increases. If we examine the partitions of the number 15, for example, within the 176 partitions there are 42 pairs of indices that include h relations from two to six partitions h per index. Thus, most of the partitions of number 15 are h-related to another [21].

Now, let us examine how the operations allow us to understand the way partitions evolve from one to another. In Debussy's *Prelude VIII*, *La Fille aux Cheveux de Lin*, the texture can be encoded by partition sequence: <[1][1,4][1][5][6][5][2^2][1,2][1,4][6][7][1]> (Figure 7).[27] The excerpt consists of two arc-shaped gestures around the line (partition [1]). The first arc (mm. 1-4) is a simple deviation of the line through the inclusion and exclusion of the block [4] (a $+Y_4([1])$ followed by $-Y_4([1,4])$). For the second arc (mm. 4.3-7) Y and D are combined to build other curves within the arc in a more elaborate gesture.

First, the line becomes the block [5] by including four threads to increase its thickness (noted as $+D_4([1])$ = [5]). Given the number of threads involved, this transformation can be understood as a variation of the Y in the first arc but assuming a collaborative relation with the line instead of contraposition. After that, D produces a small arc or oscillation in the block [5] through insertion and exclusion of a single thread thereinto ($+D_1(5)$ and $-D_1(6)$). Furthermore, the transition from partition [5] to [2^2] demands, necessarily, the combination of Y and D with opposite signals, so that the block's thickness is decreased by three threads ($-D_3$), and the part {2} is added to the partition ($+Y_2$). The following partition [1,2] outcomes from the exclusion of one thread ($-D_1([2^2])$). Finally, except by partitions [1,4] and [6] that involves both operations Y and D, the closure of the arc concerns only thickness transformation of the parts. Note that the maximum of threads within a partition (7) occurs in the last part of the second arc, which is suddenly decreased to form the

[27] The criterion to ascribe each partition considers the evaluation of rhythmic coincidence.

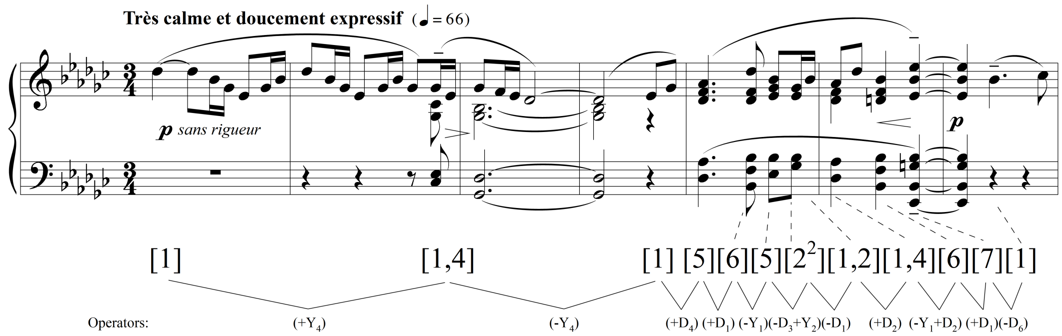

Figure 7: *Partitions that encode the texture sequence of Debussy's Prelude VIII, La Fille aux Cheveux de Lin, mm. 1-7 (1910), and the operators that connect them.*

line [1], resulting in the largest gap between any consecutive partitions. Given that the first pitch in partition [1] is also within the chord in partition [7], this transformation could be understood as a "filtering process", in which all but one pitch is preserved to produce the next partition. Perhaps, this procedure is a strategy to smooth the textural gap in the chain by approximating the musical content of them.

Another way to think about the texture of this piece is to consider each piano hand as autonomous. Thus, the texture depends on the way the threads are divided into each staff of the score according to the rhythm coincidence, so that each hand produces a individual part and the partition is the union of them. The combined sequence $(<[1][1,2^2][1][2,3][3^2][2,3][2^2][1,2][1,2^2][3^2][1]>)$ reveals that the left hand is more stable for alternating among blocks with two, three, and four threads. In contrast, the right hand combines three parts ([1], [2], and [3]) to produce a more elaborated sequence. This combination does not exceed three threats at a time.

Of course, the description through partitions do not cover every aspect of texture; many attributes thereof cannot be encoded into a partition. This is especially clear considering that within a partition the order of the parts is irrelevant, then any characteristic of a texture that involves a vertical spacialization, for example, is out the context of partitions. Figure 8 presents an example of this limitation. The partition $[2^2]$ is defined by the timbre in such a way that either *pizzicato* (blue squares) or *arco* (red squares) assembles one of the blocks 2. Also, the pitch content helps to demarcate the blocks. All pizzicato parts are formed by members of trichord 3-5[016] while the arco parts comprise members of trichord 3-3[014][28]. Although partition $[2^2]$ expresses the general organization of the parts, it does not describe the multiple combinations among the instruments as an elaboration within the partition. Thus, up-space omits some textural data, which can undermine an analytical inference. Surely, under the creative perspective, the lack of details allows various compositional fruition, but, at same time, by refining the texture, the design of a composition can involve more advanced connections. In order to develop this approach, we shall include a vertical ordering to the parts within a partition as a way to depict either their registral placement or timbre distribution.[29] Thus, the last textural space concerns the ordering within partitions.

[28] See [6].

[29] Berry also discusses the relations between texture and register, but his focus is on the actual vertical span of textures, that is, the interval among their uttermost part, and the way such space is fulfilled by the threads (called as *density-compression* [3].

Figure 8: *Different organizations of threads within partition [2^2] according to combinations of* pizzicato *(blue squares) and* arco *(red squares) effect.*

iii. Partition Layout Space (pl-space)

The spatial factor of texture is an important aspect to differentiate the realization of the same partition within two or more different contexts without necessarily dealing with their actual musical content.[30] We can order the parts within a musical partition by either register placement or timbre. If we consider the register, each textural part is ordered in the present paper according to their spans as a result of the unfolding of the music materials into a sounding medium. Else if we consider timbre, then the order can be read top-down from the score disposition. Once we identify whether a part or thread is higher, equal or lower than another through register or timbre, we can ascribe a *partition layout*.[31]

A partition layout consists of a more precise way of encoding a texture by expressing details regarding the internal organization of the partition's component parts. To depict this order we propose two types of components within partition layouts related to a specific notation: an integer number in the form of a *ordered partitions* and threads or grouping of threads, noted as a *thread-word*.

An *ordered partition* can be defined as a partition wherein the order of parts matters[2, p. 54].[32] Hence, a given partition can provide different ordered partitions according to the possible permutation of its parts. For each positive integer n there are 2^{n-1} possible ordered partitions. For example, the number 4, in addition to its five partitions, presents three ordered partitions, resulting in eight configurations: $<4>$, $<1,3>$, $<3,1>$, $<2^2>$, $<1^2 2>$, $<1,2,1>$, $<2,1^2>$, and $<1^4>$.

[30]By spatial factor we are not referring to what Berry calls *texture-space*. While Berry's concept involves the measurement of the actual registral boundaries of texture, we are concerned to the relative vertical distribution of the parts.

[31]This comparison among the parts to define their relative position is similar to the process involved within the contour spaces ([25]).

[32]In mathematics, an ordered partition is called *composition*, but in the present work that word is avoided for the sake of clarity (composition is already used as a musical concept).

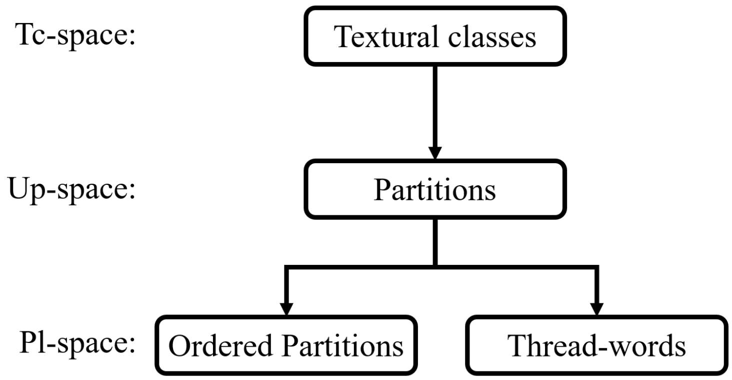

Figure 9: *Inclusion relations among the classes within textural spaces.*

Note that the applicability of ordered partition demands a stratified textural organization, that is, the overlapping of layers (or strata) shall be well-defined; the forming threads of a block shall not be vertically interpolated with any other part. However, most often, the parts may cross the vertical boundaries, connecting apart threads into the same textural part. An ordered partition cannot express a texture whose threads are "interwoven" in the register.

The actual registral placement of non-contiguous organizations (interpolated parts) are described by an organization called *thread-word*. A thread-word is a representation in which each part of a texture is identified by a letter in such a way that each individual thread receives a different letter, and threads that assemble a block are ascribed with the same letter. Thus, the number of letters indicates the cardinality of texture (density-number), the number of occurrences of a given letter is associated with the thickness of that part, and the number of parts is deduced from the variety of letters. For example, a thread-word $<a, b, c, d>$ stands for partition $[1^4]$, given that there are three different letters (threads) without duplication while a thread-word $<a^2b^5>$ indicates two blocks {2} (noted as $\{a^2\}$) and {5} (noted as $\{b^5\}$). The use of a thread-word allows us to observe interpolated part, as in thread-word $<a, b^2, a^2>$. In this case, the five threads are grouped in an interwoven texture with two distinct parts (indicated by letters a and b). This texture is a layout of partition $[2, 3]$ in which part {2} (indicated by b^2) is interpolated with part {3} (the sum of components $\{a\}$ and $\{a^2\}$).

Both proposals (ordered partitions and thread-words), as sub-classes of partitions, provide further information for up-space. Similarly, partitions can be understood as sub-classes of textural classes. Also, textural classes are class containers that encompass partitions in up-space. For example, textural class $[LB]$ comprises, among others, partition $[1, 2]$, and all its possible partition layouts (ordered partitions ($<1, 2>$, $<2, 1>$) and thread-words ($<a, b^2>$, $<a, b, a>$ and $<a^2b>$). Figure 9 shows this relation of textural spaces from the most superficial (tc-space) to the most specific (tl-space).

Although we could use Berry's methodology, in which the textures are written vertically, with their parts stacked according to their vertical position [3], either ordered partitions and thread-words are written horizontally within "< >", following the ordering from the highest to the lowest registral placement or timbre disposition of the parts, to clarify and facilitate the visualization (and notation) of the parts' ordering. Thus, in a thread-word, the first letter is always "a", that correspond to the upper thread in texture. For each new part is ascribed another letter following the alphabet. When there are two equal contiguous letters, we use the abbreviated

Figure 10: *Layering (Ordered Partitions encoding the texture of Bartok' string quartet No. 2, Op. 17, mm. 1-4 (1920).*

form with an exponent.[33] For example, partition $[1,4]$ provides two ordered partitions $<1,4>$ and $<4,1>$ to indicate the exact position of the line (part $\{1\}$) and block $\{4\}$. In addition to these two layouts, thread-words include other three organizations: $<a,b,a^3>$, $<a^2b,a^2>$, and $<a^3b,a>$. The threads ascribed by "a" assemble the block $\{4\}$, while "b" refers to the line.[34] Surely, the register evaluation shall not be totally strict since any eventual crossing parts would be computed as a different ordered partition. The set of all partition layouts (ordered partitions and thread-word) forms the *partition layout space* (*pl-space*), the most precise encoding of a texture among all textural spaces.[35]

Figure 10 shows an example of ordered partition providing an analysis of an excerpt of Bartok's *String Quartet No. 2*, Op. 17 (1920). The sequence starts with the cello solo presenting partition [1], followed by the block [2] assembled by second violin and viola, and, finally, another line in the first violin [1]. In this case, both ordering criteria (register and timbre) provide an equivalent result. The partition chain $<[1][1,2][1^22]>$, even though encoding the threads organization of texture, it ignores the spatial organization of threads, an important detail of the texture. Thus, by examining the ordered partition, we notice that the lines are in the uttermost voices, enclosing the block. Also, the chain $<1><2,1><1,2,1>$ indicates that the texture is made up of the accumulation of new parts. That is, each partition is contained in the following one, preserving their registral (or timbric) disposition. This process expands upward the vertical space of texture, a property that is not encoded by the representation of unordered partitions.

To see how the information provided by thread-words can be essential for the understanding of textural functionality, let us examine an excerpt of Varèse's *Octandre* (Figure 11). The texture can be encoded by three partitions ($[3,4]$, $[7]$, and $[1,7]$) that set up two pitch and dynamic contents according to the number of threads therein. Within eight threads, the clarinet holds a line that is apart from the block of the other instruments through rhythmic and dynamic opposition. The seven-thread textures derive precisely from the pause of the clarinet, as we can observe in the

[33] Attention that in a thread-word the exponent refers to the thickness of the component part instead of its multiplicity. Thus, $<a^3>$, for example, expresses a block with three contiguous threads instead of three lines, which would be noted as $<a,b,c>$.

[34] We are not considering the redundancies between ordered partitions and thread-word, as, for example, $<a^4,b>$ or $<a,b^4>$. See section III for a discussion about the possible realizations of a partition.

[35] A step farther than pl-space would result in either a redundancy of the music score itself or the description of the particularities within each part (their musical content).

Figure 11: *Layering (Varèse's Octandre (1923), mm. 50-54: unordered partitions and thread-words encoding the texture.*

score. By examining partitions $[1,7]$ and $[7]$, these relations also become clearer. Partition $[3,4]$ indicates a split variation of the block [7]. If we use the thread-words, more details regarding the texture will be explicit.

While partition [7] does not enable multiples realizations, partition $[1,7]$ is depicted as $<a^2b, a^5>$ according to the score order. This representation reveals that the clarinet is within the block, dividing the seven threads into two groups (2 and 5, as indicated by the exponents). Without accessing the score, the thread-words indicate the relation between the cardinalities through the clarinet, given that $<a^7>$ and $<a^2b, a^5>$ have the same number of threads assembling the part $\{a\}$ without the interpolated part $\{b\}$, a possible exclusion of it from the texture. Furthermore, while partition $[3,4]$ does not present any relation with other but the number of threads (in this case, 7), $<a^3b, a, b^2>$ shows the amalgam of threads in both blocks, suggesting a mix between the interpolated $[1,7]$ and the block [7]. In fact, by comparing both structures, this relation becomes still clearer. Both organizations have an isolated thread in the center being surrounded by a group of combined threads. That is, while part $\{b\}$ divides $\{a^2\}$ and $\{a^5\}$ in $<a^2b, a^5>$, the combined parts $\{a\}$ and $\{b\}$ are surrounded by $\{a^3\}$ and $\{b^2\}$ in $<a^3b, a, b^2>$. This shows that thread-words can give access to critical information that in some cases can be more useful than the simple enumeration of partitions.

Operations on pl-space

Operating on pl-space demands the observation not only about thickness and multiplicity of the parts, but also about their order. Thus, in addition to operations Y and D, inherited from up-space, pl-space includes two new operations to modify the order, called *mirroring* (M) and *permutation*

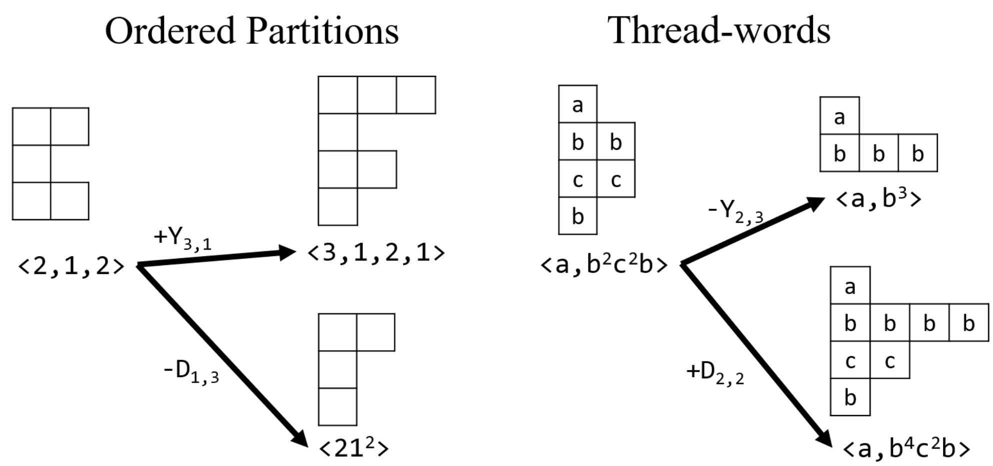

Figure 12: *Ordered Young's diagrams demonstrating Y and D operations including order in ordered partitions and thread-words.*

(T). Both operations do not have a + or - signal because no thread is added or removed from the partition layout.

Concerning order, each position within an ordered partition is numbered from 1 up to the number of parts or threads in the partition layout. Let us say that $\{x, y, z\}$ are the components of a partition layout K. The parts x, y, and z are either an integer number or a letter that refers to a thread. The part $\{x\}$, $\{y\}$, and $\{z\}$ are, respectively, in positions 1, 2, and 3. Thus, any operation involving the order of parts of K will include a reference to position 1, 2, or 3. In thread-words, the position refers to the letters regardless to their exponents in such a way that the third position of $<a^2, b^4, c^3>$ is $\{c^3\}$. Obviously, these operations are only effective in partition layouts with two or more components.

Both operations from up-space (layering and dimensioning) can be applied to pl-space to alter a specific component by appointing its position. The notation is $*X_{n,m}$, where X is either Y or D operation, "*" stands for either + or - signal, n indicates the number of threads to be included or excluded from the partition layout, and m refers to the exact position of this part. Thus, $+D_{n,2}$ applied to a partition K (noted, $+D_{n,2}(K)$) is equal to the inclusion of n threads in the part located in position 2, increasing its thickness, while $-D_{n,4}(K)$ consist of the exclusion of n threads of the part in position 4. Similarly, $+Y_{n,3}$ includes a part with n threads in position 3, and $-Y_{n,5}$ excludes the component in position 5. Note that in both partition layouts, Y operation requires that n shall be equal to the component to be excluded. Also, in thread-words, -Y shall not operate in parts that is a member of a thicker part because it would affect the thickness instead of multiplicity. Similarly, $*Y_{n,1}(K)$ will either increase or decrease the thickness of the first component by n threads. It is important to remember that in -Y, n shall not be greater than or equal to the part to be altered because it would affect the multiplicity. For instance, if we operate $+Y_{3,1}$ and $-D_{1,3}$ on ordered partition $<2, 1, 2>$ the outcomes are, respectively, $<3, 2, 1, 2>$ and $<2, 1^2>$. Note that in Y the part $\{3\}$ is inserted in the first position. Thus, the first position of the input becomes the second position of the outcome. In the same way, $-Y_{2,3}(< a, b^2c^2b >)$ is equal to $<a, b^3>$ and $+D_{2,2}(< a, b^2c^4b >)$ stands for $<a, b^4c^2b>$. For a better visualization of these operations, we can use an ordered version of Young's diagram in which the vertical position reflects the order of parts. In order to depict the thread-words, we include a letter inside the square to indicate the corresponding threads.

Mirroring is the operation that flips the order of the components parts in such a way that the bottom (or the last, considering the horizontal notation) becomes the upper (or first) and vice versa. This operation is a one-to-one mapping of components around an inversional axis, that is, each component of a side of the axis is swapped with its corresponding position in the other side. If there is an odd number of component parts, the axis is the median component. If there is an even number of components, then the axis point is located between the two median components, causing their swapping along with the other ones. For example, given two ordered partitions $K = \{3, 1, 2, 1, 5\}$ and $J = \{2, 3, 1, 3\}$. The mirroring of K (noted as $M(K)$) consist of swapping all parts around part {2} so that $I(K)$ is equal to <5, 1, 2, 1, 3>. Similarly, $M(J)$ swaps the parts around the axis formed by parts {3} and {1}, producing the ordered partition <3, 1, 3, 2>.[36] In the same way, in thread-words, $M(< a, b^2a, c^2 >)$ is equal to $<a^2b, c^2b>$ (a rewritten version of $<c^2a, b^2a>$).

Permutation noted as $T_{n,m}$, where n and m indicate two different positions in the partition layout, consisting of interchange (or swap) the positions of two components. Let us say that the partition layout K encompasses the parts $\{x, y, z, w\}$, where x, y, z, and w are either an integer or a letter ascribing a thread. Then, $T_{1,4}$ means that the part in position 1 and 4 shall permute their position to each other in such a way that $T_{1,4}(K) = <w, y, z, x>$. For example, $T_{2,3}$ <1, 2, 5, 2> is equal to $<1, 5, 2^2>$. Similarly, the outcome of $T_{1,4}$ $<a^2, b, a, b^3>$ swaps the components $\{a^2\}$ and $\{b^3\}$, producing $<b^4a^3>$; however, when $\{b^3\}$ assumes the first position, it shall become an "a". Thus, the outcome is rewritten as $<a^4, b^3>$. If the partition layout has only two parts, then permutation is equal to inversion.

III. Compositional Perspectives of Textural Spaces

Up to now, we have concentrated on the definition of textural spaces, and the basic operations associated with each one of them. This definition is important to establish the theoretical ground necessary to understand the multiple features of texture. Now, we will discuss some aspects of how these spaces can be articulated within a composition, highlighting part of their compositional potential. Also, we will discuss the way a texture can be decoded into a music score.

In general, the combination of parts (lines and blocks) in any space suggests a hierarchy of the type *figure/ground* [26]. Blocks tend to be in the background, supporting the lines in the foreground. However, this logic was often inverted, principally during the twentieth century, as a replacement to the traditional melodic composition. Blocks that were usually treated as accompanying textures became the main prominent musical idea.

In tc-space, due to the linear presentation of solely one part, monopart textures has a sense of a narrative analogous to a single speech. This lack of interactions with other parts assigns to the texture a secondary role, giving focus to other musical parameters. In fact, monopart classes may be (and often are) used as a compositional strategy to highlight a melodic idea, a particular rhythm, a harmonic sonority, etc.

On the other hand, overlapped parts in other types include a contraposition relation among the threads, which may affect the overall complexity. Thus, when a texture has concurrent parts it shifts from a narrative to a creative structure, assuming an important role to unfold the music form, a ploy often used by twentieth-century composers as an alternative to overcome pitch dominance. Thus, a textural sequence built up only from monopart textures may limit the creative use of texture. Its importance, then, is decreased in the compositional process, whereas using other types of tc-space in combinations (including monoparts) or isolated may offer various compositional

[36] Although there are similarity between M operation and the retrogradation of pitch domain, we are considering the vertical alignment of the parts instead of their linear order. Moreover, a retrogradation operation can be applied to a chain of partitions or partition layouts to reverse the sequence of adjacencies, an operation that is out of the scope of this work.

solutions to unfold the music form. The same principle can be applied to the other spaces. In fact, by using up- or tl-space, the textural progression can be more creative and imaginative, allowing the design of a more sophisticated textural structure that includes multiple relations between and among their components. Thus, the importance of texture within the compositional process may be increased.

i. Defining a texture

The composer might ask "how to choose which textural space or which textures to use in a composition?" or "how to define which criteria to use to select textures?". To answer those questions we can consider connections between textures through operations, parsimony between them, the potential of musical implementation of these textures, among others.

Both TCL and PYL constitute a *compositional space* in textural domain.[37] This means that they can be observed as a game board ([8]) whereby the composer can produce various patterns of movements within a given number of threads by following the edges that connect textural classes and partitions to produce chains of textures. By defining a starting and an ending point, the composer may have a finite number of paths to connect them, considering, for example, the most minimal connection (the shortest path) or the path with more intermediate textures between them (the longest path). If two partitions within the path are connected by a single edge, then the movement between them is parsimonious, that is, the transformation from one to another can be smooth, a property that may be convenient within the same formal section. On the other hand, non-contiguous partitions that are connected by multiple operations may create contrast, which can be a divisory point in texture for a formal delimitation.

Although the operations we discussed so far allow us to transform a partition into another parsimoniously, these operations can be quite abstract since they do not explicit the potential process involved from each autonomous part or thread to evolve toward the next texture. Within tl-space, Y and D operations include the position of the part to be altered, which details the process involved. However, other potential transformations between successive textures can be acknowledge in this diachronic perspective. Thus, we can examine the role of each component in the unfolding of the next partition through the way they are connected (a "texture-leading"). The potential transformation according to the texture-leading of parts depends on whether the number of parts and threads will be altered or not. The preservation of the number of parts necessarily involves one of the following transformations:

1. An increment of decrement of the thickness of one or more components (D operation);
2. The reorganization of its internal components (M and/or I operations);
3. The combination of either M and/or I with D operation to change the order and thickness at once.

However, to increase or decrease the number of parts, in addition to the application of Y operation to insert or exclude one or more components, the transformations involve *merging* or *splitting* of two or more parts within the texture combined to a change in the thickness. Merging or splitting a component can be understood as the combination of Y and D operations in opposite signals (i.e., $+Y$-S or $-Y$+S) in such a way that the number of threads is not altered (density-number).[38] When two or more parts are combined to produce a new part whose the cardinality

[37] Compositional Space is an out-of-time set of musical objects related by a given criterion. Such a structure provide the foundation for a compositional designs or an improvisation ([24, p. 336].

[38] This composite operations of Y and D is similar, at a certain level, to Gentil-Nunes *transference* ([10, pp. 47-48]).

is the sum of them, we say they were merged. When a part is divided into two or more parts, we say the part was splitted.[39] Both merging and splitting, when applied in isolation have an invariable density-number, that is, the number of threads in the texture remains the same. Other transformations may include the combination of two or more aforementioned operations. All of these linkage strategies are essential to composition (and analysis). In fact, although these procedures necessarily affect the texture, they are understood as a response to the manipulation of the other parameters (mainly pitch and rhythm).

Another way of creating a parsimonious texture-leading consists of examining the possible intersections between two consecutive textures, that is, one or more common part that they share that can be preserved from a texture to another. This intersection can be minimum, when all parts but one are different in both partitions (their intersection is equal to one), or maximum, if all parts of the first texture are contained within the next one. Two consecutive textures that have intersections of their component parts allow the composer to coordinate their connection in a smooth way.[40] A common use of intersection in musical repertoires involves Y operation, in which a complex texture is constructed by cumulative superposition of simple (or less complex) parts all contained therein.[41]

In pl-space the intersection of two consecutive partition layouts, in addition to the simple common-part procedure, may regard the ordered intersection. That is, if at least one part or thread of any partition layout is equal to the other, they can be arranged in such a way that the common element between them can be maintained in both textures in the same vertical position (registral span).

Using thread-words, intersections can be either literal, when the exact number of threads of a part is preserved into the subsequent texture, or abstract, if the threads are embedded in thicker parts of the other. In both cases, we shall not compare the letters but the number of their threads and their positions. For example, thread-words $<a, b^2a>$ and $<a, b^4c^2a>$ can be literally intersected by threads "a", considering that there are two of them in both textures, and by part $\{b^2\}$.[42] Moreover, part $\{b^2\}$ can also be embedded into part $\{b^4\}$ (an abstract intersection).

For a total intersection between two distinct partition layouts, we shall consider their alignment to examine whether it is possible to connect sequentially their components or not. Figure 13 shows an example of this embedding property by using ordered Young's diagrams. The shaded squares in $<1, 2, 3, 2, 1, 2, 1>$ and in $<a^2b, c^2, b, a^3>$ express, respectively, that $<2, 3, 1^2>$ and $<a, b, c^3>$ are totally contained into them. Attention must be paid to the fact that, although other literal and abstract intersections could be considered in other positions within a thread-word, only the case in question allows the total intersection since each part shall be ascribed by a different letter regardless the number of squares.[43]

The choice of a texture may consider its degree of *entropy*. According to *information entropy* ([32]), the complexity of a process can be measured by how much information is produced (or allowed to be produced) thereby. Thus, the higher the number of potential outputs of a process, the more complex it will be due to its high degree of unpredictability (entropy). In the same

[39] A precise formalization of merging and splitting, as well a further reflection of their effects are out of the scope of this work.

[40] Of course, only the intersectional property between textures does not guarantee the parsimonious sense since it also depends on the musical content of both textures (pitch, rhythm, dynamic, setup of timbre, registral placement, etc.).

[41] This staggered entrance of parts (or threads) is usually associated to the beginning of a piece or a section, as, for example, in the exposition of a fugue. In jazz music, such a procedure is called *pyramid presentation* (see [28, p. 17]).

[42] Note that in this case, we understand that $\{b^2\}$ is rewritten as $\{c^2\}$ because of the another part interpolated $\{b^4\}$.

[43] There are some ways to calculate if two ordered partitions have the embedding property between them. A further discussion about it would be more complex, and, therefore, it is out of the scope of this paper. For more information, see [14][5].

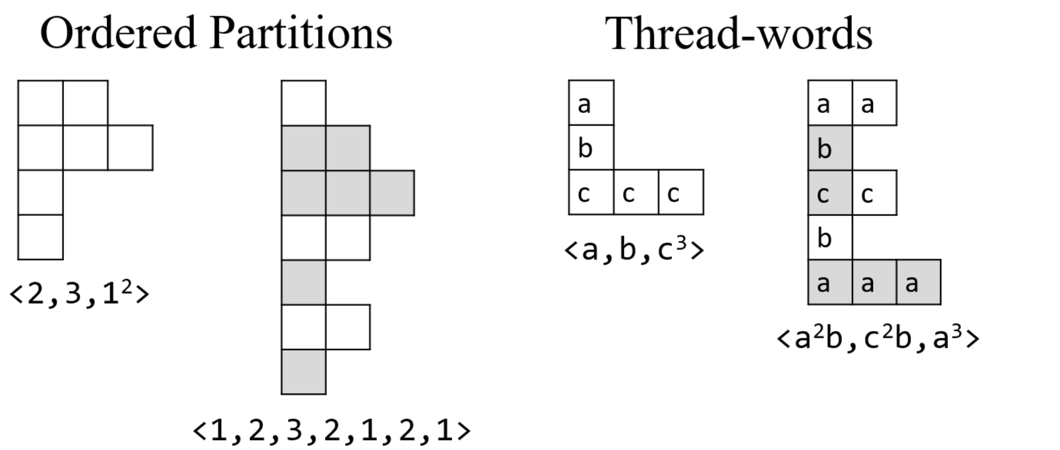

Figure 13: *Layering (Ordered Young's diagrams demonstrating the intersection property in ordered partitions and thread-words.*

way, a process that encompasses a small number of potential outputs are more predictable and, consequently, simpler. Within musical texture, we assume that the entropy level refers to the number of the potential implementations of a partition, that is, how many partition layouts a partition can produce. This is a significant aspect to be considered during the compositional work, after all, the more implementations a texture provides, more liberty the composer may have to explore creative solutions.

The entropy is therefore conditioned to the number of different partition layouts a texture can have. For example, partition $[1^3]$ has four ways to be implemented according to the internal organization of its threads: $<a,b^3>$, $<a^3b>$, $<a,b,a^2>$, and $<a^2b,a>$).[44] The thread-words that refer to the same partition are called in this paper as *anagrams*. Table 1 presents the number of anagrams for all partitions with one up to four threads. Note that any partition with a single part has only one possible anagram because within a block the contiguous threads cannot be differentiated. Similarly, given that the first component of any thread-word is always "a", the permutation between lines in the first position are also redundant since any line placed in the first position will be rewritten as "a". Also, the letters are included in an increasing order. Thus, any partition with multiple lines also has only one anagram as the form $<a,b,...,z>$, where z stands for the number of lines therein, ascribed according to the alphabet order. Moreover, threads that assemble a block are understood as an internal repetition within a thread-word; therefore, the repetitions shall be eliminated from the number of possible permutations.

Furthermore, the entropy is also affected by relation between the density-number of the partition and the total number of available threads to decode it. That is, if the density number of a partition is lesser than the number of available threads for the composer, the entropy is higher, even in partitions with a small number of anagrams since the composer can decide which threads will be involved in the decoding process. For example, suppose that we have three threads available $\{a\}$, $\{b\}$, and $\{c\}$ to decode partition [2]. Although partition [2] has just one possible anagram, within three threads the composer have three possible ways to decode this into the musical score: $\{a+b\}$, $\{a+c\}$, and $\{b+c\}$.

[44] Note that $<a,b^3>$ and $<a^3b>$ correspond, respectively, to the ordered partitions $<1,3>$ and $<3,1>$.

Table 1: *Anagrams of partition lexical set for $n = 4$.*

Partition	Density-number	Number of Anagrams
[1]	1	1
[2]	2	1
$[1^2]$	2	1
[3]	3	1
[1, 2]	3	3
$[1^3]$	3	1
[4]	4	1
[1, 3]	4	4
$[2^2]$	4	3
$[1^2 2]$	4	6
$[1^4]$	4	1

ii. How to decode a texture into the score?

Once the composer decide which textures to use, the next step may involve its musical decoding (or realization). The musical realization of a texture can direct the composer's goals, allowing not only the approximation to different aesthetics and creative goals since the materials to be project into the textural structure is not defined. Mapping some of these possible realizations can help us to understand the musical implications related to a texture.[45]

In order to consider multiple possible decoding of a texture, we shall consider both the size of the window of observation and the possible partition used as a reference, called *referential partition*. Furthermore, the way in which a partition is decoded into the score is intrinsically related to the manipulation of musical parameters in the compositional process. Thus, departing from the standard ways of composing musical texture and form involved in composition, we initiate this discussion by defining a *standard mode* to decode a texture.[46]

By defining a standard mode of decoding enable us to understand the potential variations within a same texture. A standard realization is the most elementary and objective way to articulate a texture in music. One simply articulates the textural parts in some obvious musical way, such as a group of voices, a short uniform passage, or a block of sound. In analysis, standard realization is defined by the smallest possible *window of observation*[47] to encode a texture, with no global considerations. Thus, this mode implies a series of constraints.

First, all threads and parts of the partition shall be articulated simultaneously and continuously.[48] Either each part has the same onsets or some parts tie a note over a onset in another part. Any pause would result in a new partition, as a subset of it. Second, the chosen criterion for grouping the threads shall be preserved throughout the partition's duration. Otherwise, any change in one of the threads could produce either their contraposition or a collaboration with other threads, forming a new partition. Third, doubling threads by octaves, unison, or other

[45]Obviously, although we are proposing a formalization of the creative process within the textural spaces, the formation of a texture, in the current practice, can be combined with other aspects of musical realization, in a less systematically application.

[46]In order to facilitate the discussion, our discussion here will concern only the up-space, but all involved principles can be adapted to the other the textural spaces according to the composer's goal.

[47]The window of textural observation is an important feature of textural morphology that enables to assemble the threads in a temporal and abstract space, that can be as simple as a single simultaneity to large sections, as components of the same texture [11].

[48]Although we are considering the partitions to discuss the standard realization of textures, the discussion can be extended to textural classes and partition layouts since they are intrinsically related to each other.

Figure 14: *My Sagração de um Fauno na Primavera (2016/2019), mm. 5-9, coding the partition sequence <$[1^3][1,5][3^2][1^2 3][1,2,3][1,3][3][2][1]$> into the score.*

intervals alters the partition because it affect the thickness of the parts. In this case, a textural realization with larger ensembles shall consider doubling as part of the partition, which will result in partitions with a large number of threads and parts.

Standard realization is a good (and even didactic) way of composing from small textural units. For example, in the creative process of my composition, *Sagração de um Fauno na Primavera* for flute, bassoon and piano, the partition chain was articulated by standard realization considering rhythm alignment as the main criterion to define the collaboration and contraposition relations. Each pause or rhythmic coincidence was placed to articulate a new partition; therefore, the music is textually diverse, with multiple changes in a short temporal span. The squares encompassing the partitions below the score reveal the size of each window of observation (Figure 14).

To preserve the sense of good continuity between consecutive partitions, I applied some of the aforementioned strategies. First, when two consecutive partitions have intersections, I preserve the common part (or parts) in the same combinations of thread. For example, the bassoon line is sustained in the pair $[1^3]$ and $[1,5]$ (m. 5) and in the set <$[1^2 3][1,2,3][1^2 3]$> (mm. 6-7). Similarly, the piano's block [3] (mm. 6-7) is also preserved throughout the sequence <$[3^2][1^3][1,2,3][1^2 3][1,3]$>.[49]

Second, the potential of briefly merging parts to produce a block. As an example, see the bassoon line (mm. 5-7), in which the independence of the line is momentarily discontinued (m. 6) to form the partition $[3^2]$ through the rhythmic coincidence with piano right hand.

Third, consider different blocks of successive partitions as variations of the same threads through *D* operation. Thus, if a block has its thickness increased, we can simply add threads to it. Conversely, if the thickness is decreased, the threads can be removed from it. For instance, in the chain <[3][2][1]> (mm. 5-6), the decreasing process results from the suppression, respectively, of piano and bassoon.

Now that we have defined a standard mode, we can propose other modes of encoding a texture by considering some local deviations of the standard mode according to a given criterion. In this paper, we propose other four modes that do not necessarily constitute an exhaustive taxonomy since other modes can be presented according to other perspectives for decoding a texture into a

[49]Note that although the pair [1,3] and [3] (m. 8) could be also executed by the piano, I decided to vary it to gradually prepare the entrance of the solo flute (m. 9).

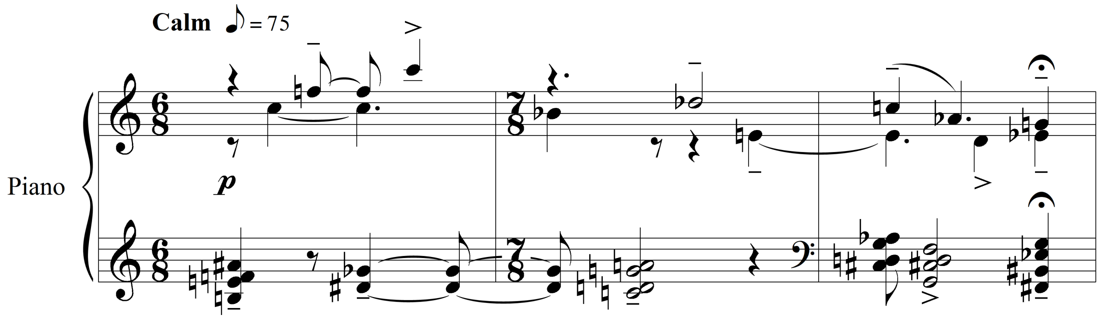

Figure 15: *Partitional complex* {$1^2 4$} *decoded into the score.*

musical score: *partitional complex*, *evolving realization*, *colorization*, and *montage*. Each one of these modes is related to a musical domain involved during the creative process. Note that a deeper elaboration of these modes, as well as the inclusion of examples of their effect, is out of the scope of this paper since our goal here is to initiate the discussion on how texture can be decoded in the musical score.

Partitional complex is an elaboration of a standard partition, proposed by Gentil-Nunes and Ramos (see [12] and [8]), to include local deviations from the referential partition. In some cases, rests in the parts introduce subsets of the partition, called *subpartitions*, and in other cases the changes in differentiation criterion between the parts produces merged parts, called *subsums*. Let parts $\{a, b, c\}$ be the parts of a partition K. The partitional complex enables that a textural sequence may include any of the six subpartitions of K ($\{a\}$,$\{b\}$,$\{c\}$,$\{a, b\}$,$\{a, c\}$, and $\{b, c\}$), or any of its four subsums ($\{a + b\}$,$\{b + c\}$,$\{a + c\}$, and $\{a + b + c\}$), or the combinations of subpartitions and subsums, called *subpartitions of subsums* ($\{a,(b + c)\}$,$\{b,(a + c)\}$,and $\{c,(a + b)\}$).

All subpartition, subsum, and subpartitions of subums are subordinate to partitional reference K, forming a hierarchical organization where textural changes are understood as oscillations within K. Obviously, the window of observation of supartition, subsum, and subpartitions of subums is minimal and necessarily shorter than the window of observation of the whole partitional complex K. For example, the partitional complex $\{1^2 4\}$[50] has four different partitions: the referential partition itself ($[1^2 4]$), two subpartitions ([1] and [4]), three subsums ([2], [5], and [6]), and two subpartitions of subsums ($[2, 4]$ and $[1, 5]$).[51]. The use of this partitional complex is shown in Figure 15. Note that according to standard mode each supartition, subsum, and subpartitions of subums would be interpreted as a different referential partition instead of the unfold of the referential partition $\{1^2 4\}$ within a partitional complex.

Evolving realization concerns the presentation of blocks in the score under a temporal perspective. In addition to standard mode, where a block is formed by vertical alignment of its threads according to a given collaborational criterion, evolving realization may consider the potential of a block to be temporally constructed by successive superposition of its constituting parts. Thus, the block is articulated through the cumulative sustained sound of threads as an outcome of their static motion.[52] In evolving realization, the block is the referential partition (or one of its parts, in the case of a block within a partition with multiple parts). Note that the block is not understood as block until it is fully constructed; or one can say it is retrospectively referential. Thus, evolving realization demands a wider window of observation to consider the entire unfolding of the block

[50] We note a referential partition of a complex within "{ }" to differ from a regular partition.
[51] Of course, we are not considering the redundancies caused by part {1}.
[52] This procedure is known as an orchestration technique called *pyramid construction*.

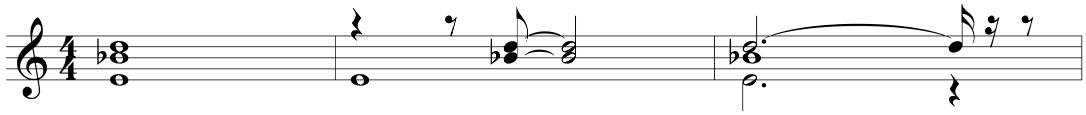

Figure 16: *Three ways to decode partition [3]: 1) standard mode, 2) evolving realization (construction), and 3) evolving realization (dilution).*

and therefore it is the most abstract mode of decoding a texture.

Through evolving realization any block can be accomplished from the cumulative superposition of the partitions of its cardinality. Given a referential partition [3], for example, while standard realization and partitional complex would present it as a simultaneous [3], evolving realization would include other two possible partitions by the superpositions of the threads: $[1,2]$ and $[1^3]$. In this case, both partitions are understood as an unfold of referential partition [3]. Another possible application of evolving realization concerns the opposite sense, in which a simultaneous block is "diluted" by gradually removing their component parts. Figure 16 provides three examples of decoding partition [3] in both standard mode and evolving realization. In standard mode, [3] is decoded according to the note onset of the threads. All threads are simultaneously articulated. For evolving realization, first the block is constructed by the superposition of threads in a static motion, then the block is diluted by excluding the its threads. Note that if we analyze these two evolving examples through standard mode, the sequence would be interpreted as, respectively, the partition chain <$[1][1^2]$> and <$[1^3][1^2][1]$> since, according to the onset note, the threads are in contrapositional relation throughout the whole sequence. However, in evolving realization, both sequences are understood as a temporal elaboration of the referential partition [3] considering the motion criterion to assemble the block.

Other parameters such as timbre and dynamics, in addition to pitch and time, also may affect the decoding of a referential partition. This may occur in two ways: 1) a referential partition may be defined by timbre differences including octave doublings; or 2) differences that are noted in the score may give the impression that a referential partition is changing when it is not. Both cases correspond to what we call colorization mode, an encoding process that has much to do with orchestration and the way in which scores are written. Thus, colorization can therefore be interpreted either with smaller and/or wider windows of observation. A small window shows the doubling of notes within partitions (case 1 above). A wider window of observation can reveal relationships among different partitions within a passage (case 1 or 2 above). For example, in Figure 17, despite the fact that the score presents an elaborated texture with the combination of multiple parts, according to colorization, the whole sequence is understood as a creative unfolding of a single line (partition [1]) as a way to embellish this referential partition through unisons, doublings, and superposition.

Finally, the montage mode considers the various ways to connect two adjacent textures. After decoding them, the composer may decide how they can be linearly presented, considering three different cases: 1) simple juxtaposition of both, with no overlapping and possibly a rest between the two partitions; 2) overlapping or elision of the two textures or any of their parts (considering a wider window of observation); and 3) juxtaposition with *trigger parts* connecting them. Trigger parts are parts or partitions that are added to the point of juxtaposition of two partitions to highlight the change of one partition to the next. Triggers can either lead to the point

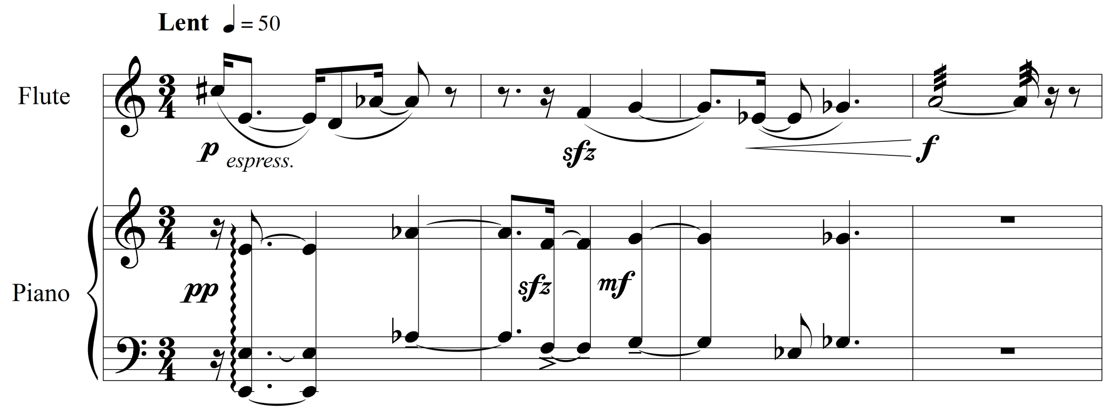

Figure 17: *Colorization mode applied to decode the referential partition [1] into the score.*

of juxtaposition (producing the effect of an upbeat), or they can occur at the point of juxtaposition (producing the effect of a downbeat). In the case of a simple juxtaposition (case 1 above), each texture is understood individually without affect their presentation in the score, but in the last two cases presented above, we may ignore the texture produced by either their overlapping or by the inclusion of a trigger part. For example, in the sequence of referential partitions <[1, 2][4][1]> (Figure 18) the rising gesture of the flute (partition [1] in m. 2) from partitions [1, 2] to [4], rather than configurating a new part to change partition [1, 2] to [$1^2$2], consists of a trigger part to connect both partition with an upbeat effect. In this case, the part has an ornamental effect. Note that the pauses in partition [1, 2] are understood as an articulation under the partitional complex. Moreover, the line [1] of the bassoon that starts at the end of the block [4] (m. 3) is understood as an overlapping of partitions [4] and [1].

Figure 18: *Montage realization decoding the partition chain <[1, 2][4][1]> into the score.*

IV. Conclusion

The formalization of the textural spaces can constitute a refined tool for textural analysis and composition. Each space presents a specific level of details in the process of textural encoding, which is directly related to the degree of liberty a texture can provide within the compositional process. As an analytical tool, textural spaces, rather than just depicting the way the threads of sounds are organized into textural parts (with more or fewer details), provide a transformational perspective through the employment of operations, allowing us to comprehend how a texture is connected to another. Moreover, they can be used as a pedagogic tool for training aural perception of textures with different level of details.

A discussion on some creative potential of the spaces has been provided as a way for helping to fulfill the lack of a systematic compositional approach concerning organization of textures. In fact, the preliminary discussion of the creative application of textural spaces requires a further study in order to understand their potential within a composition to adjust the concepts and to improve their tools.

In this sense, textural spaces constitutes the first step toward the formalization of a *textural design*, an adaptation of the Robert Morris's *Compositional Designs* [25], in which pitches of a composition are pre-defined and organized into an abstract structure that is meant to be implemented as a composition according to the composer's aims. Thus, by using an abstract structure (matrix, array, graphs, and the like) of two or more dimensions that may contain textural classes, partitions, and/or partition layouts, may allow a composer to better understand his/her own creative process, enabling the discussion of the compositional process itself from a textural perspective. The formalization of such proposal is in course.

Finally, the definition of a standard mode of realizing a texture allows us to compare the possible deviation of a given texture in a more sophisticated and creative way. This can provide a more substantial information about the textures, which may impact the criteria for selecting them taking into account the various ways to decode them along a compositional work. Moreover, the formalization of evolving, colorization, and montage modes can provide new perspectives for the study of orchestration through a textural approach [20]. A further formalization of all modes of textural realization, their possible impacts in the creative process, and their potential of combination within a composition, is also in course.

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