An Information Theory Based Analysis of Ligeti's *Musica Ricercata*: Movements I and II

Adolfo Maia Jr.
State University of Campinas (UNICAMP)
adolfo@unicamp.br
Orcid: 0000-0001-7224-8310

Igor L. Maia
Federal University of Minas Gerais (UFMG)
imaia@ufmg.br
Orcid: 0000-0001-9936-5023

Abstract: *In this work we present an analysis of* Musica Ricercata *I and II from the point of view of Information Theory and Complexity. We show that some patterns can be recognized and quantified using Information Theory methods such as Shannon Entropy and Statistical Correlation. In addition, we study some of Ligeti's techniques of texture formation and their time evolution. The analysis of Movement I is more concentrated on rhythmic aspect whereas for the case of Movement II, we study phrase variations along time with their timbre variations, as well some slight modifications. For both movements a suitable alphabet of symbols and coding is introduced in order to get quantitative results for comparison and analysis. We also present some comments on the power and limitations of the approach of Information Theory for the analysis of scores. Some suggestions for further work and potential applications in music composition are also briefly discussed.*

Keywords:*György Ligeti. Musica Ricercata. Information Theory. Computer Music Analysis.*

I. Introduction

> I am in a prison: one wall is the avant-garde, the other wall is the past, and I want to escape.
>
> *György Ligeti* [1]

Formal and mathematical methods have a fertile role in twentieth-century music. Since the pioneer works by Xenakis and the computer experiments of Hiller in the middle of last century with his *Illiac Suite*, this kind of formal approach has attracted many researches and composers and nowadays is a consolidated area of research in music composition and analysis. Mainly due to the increasing power of computer processing since that time, besides the development of new and efficient software, sophisticated mathematical, statistical and computer tools have been extensively used in areas such as Computational Music Analysis, Computer-assisted Composition and Sound Synthesis. In this work we are most interested in the application of the Mathematical Theory of Information to music analysis. The book edited by [2] brings contributions of several authors in the area. In general, computer methods aid the researcher to develop quantitative methods which allow a broader vision of the work under study. A recent example is the work by Jacoby *et al* on chord categorization in functional harmony using several kinds of representations [3].

This is in line with the increasing number of different styles in contemporary music. Since much of the traditional methods of analysis (Riemann, Schenker et al) are not always suitable to study the music of the 20th century and later, we think that mathematical and computer approaches can be an important tool to music analysis of the 20th and 21th centuries. Although abstract methods of mathematics can be applied to many different areas and models, the fact that music of those centuries have a huge number of different aesthetics, methods and styles, for sure, none mathematical/computer tool is comprehensive enough to encompass analyses of many different works. So we must be humble and to live with the reality that a method can be useful for a small set of works or perhaps for just a single work and that adaptations or extensions are needed most of times. Within this framework we analyse only the two first movements of György Ligeti's *Musica Ricercata* from the point of view of the Mathematical Theory of Information. In the spirit of the above comments we also discuss in some lenght the power and limitations of Information Theory in musical analysis and composition.

Musica Ricercata (MR, for short) was composed as a music problem-solving: how to compose a piece with "minimal resources" and "maximal results". It deserves attention not only due to its musicality and craftsmanship but also because it points to some strong characteristics in Ligeti's later music such as textural structures and processual procedures [4]. The title has a double meaning of ricercare as a musical form but most importantly as "researched music" [5] and, in fact, it was a very personal experiment on chromaticism [6]. Ligeti was always fond of science and, particularly, beautiful mathematical structures like Fractals and mechanical processes.

> Quelques-unes de mes oeuvres n'auraient pas été possibles sans la connaissance de la théorie du chaos. Je ne vois pas de limite de principe entre l'art et la science. Leurs méthodes sont quelque peu différentes: la science est orientée vers la réalité et développe à partir de là des hypothèses; l'art ets plus libre, il n'est pas soumis aux prémisses du monde réel. Et pourtant, l'art n'est pas arbitraire, car s'il a une certaine de liberté, il est cependant lié à l'histoire et à la société [7].

Within this collection of small pieces we chose to analyze the first two ones, since they have a small number of pitches and techniques, besides, many pattern repetitions, such characteristics which make them amenable to apply Information Theory methods as well as Complexity Theory using computer codes of musical information [8].

The freedom from the domain of tonality, which Ligeti imposed to himself, implied in a search for new methods of composition. About the gross structure of these two small first movements of MR, Ligeti wrote:

> Ainsi, le style de ces deux compositeurs [Bartók et Stravinski] est perceptible dans mes pièces pour piano, bien que j'espère que certaines d'entre elles - par exemple la dernière ou les deux premières - fassent apparaître déjà un style personnel. ... Le statisme des trois premières pièces constitue une particularité stylistique qui devint ensuite

> caractéristique des "veritables" compositions de Ligeti apparues dans la seconde moitié des années cinquante [7].

The close interest Ligeti had with mathematics, and machinery repetitive patterns like clocks, were influential to a number of formal procedures he used in his compositions, mainly to those so called "Pattern-Mecannico" [9]. The quote below clearly express his thought about his method of composition

> Although I am an artist, my working method is that of a scientist active in basic research rather than in applied science. Or of a mathematician working on a new mathematical structure, or of a physicist looking for the tiniest particle of the atomic nucleus. I do not worry about the impact my music will make or what it will turn out to be like. What interests me is to find out the way things are. I am driven by curiosity to discover reality. Of course, there is no reality in art the way there is in science, but the working method is similar. Exactly as in basic research where the solution of a problem throws up innumerable new ones, the completion of a composition raises a host of new questions to be answered in the next piece [10].

Ligeti's approach in MR, as well as in other later works, is quite evident as an experiment of a minimal (simple) formal model and artistic craftsmanship. In our opinion, he seems fulfilling a famous quote attributed to Albert Einstein which appears, among many other sources, in a 1950 Roger Sessions article in the N.Y. Times *How a "Difficult" Composer Gets that Way*:

> A also remember a remark of Albert Einstein, which certainty applies to music. He said, in effect, that everything should be as simple as possible as it can be, but not simpler ... I try only to put into each work as much as myself possible [11].

In this paper we make a partial analysis of Movements I and II of *Musica Ricercata* through some methods from Information Theory and Statistics also following the Einstein's premise Sessions prescribes. That means we search for patterns which can be, in some way, analyzed through Information Theory but also taking into account the expression "but not simpler" as related to the creativity of the composer, those other aspects which are not grasped by our code.

There exist many different measures of informational content of a string of symbols. We can mention, besides Shannon entropy, Kolmogorov Complexity, Self-correlation, Lempel-Ziv-Welch compression, among many others [12]. Many of them were applied to music encoding and extracting of quantitative information from a musical work. However, there is no common sense about what is more suitable for music or even for different styles of music. So, plenty of room for research.

However, as any area of science, the Information Theory has its limitations. Shannon Entropy returns a numerical value from a bunch of data (in our case, musical data) which in itself hasn't much utility. However, in the case of comparison of data sets, it indeed can have guide us in the analysis of related structure patterns of such data. For example, Information Theory can be used to compare variation of musical parameters in a corpus of music. For the case of *Musica Ricercata* an obvious approach is to compare patterns of right and left hands, or sections of the same hand, etc. Nevertheless, it is important to stress that Shannon Entropy does not take into account time order of the elements in a sequence. This is evident from the definition of entropy in Equation 1, which uses only probabilities which, in turn, are defined just by counting symbols with no order taken into account. So, Information Theory is not a well-tailored tool for a detailed analysis of a piece, since time order of symbols is very important for composition or analysis of a piece. In section III we make some additional account about these limitations.

We used MATLAB® as our mathematical tool for calculations and graphics.

II. Shannon Entropy

Shannon's Information, or Entropy, is based on the probability that a given symbol appears in a given sequence. Formally, if x is a sequence of symbols from an set commonly named alphabet $A = \{a_1, a_2, \ldots, a_N\}$ and p_i is the probability to find the symbol a_i in this sequence. The Shannon Entropy of the sequence x is given by the formula of Equation 1.

$$H(x) = -\sum_{i=1}^{N} p_i \log_2 p_i \tag{1}$$

Observe that the crucial problem here is to define probabilities for each symbol. In general, there isn't a unique way to do it. For example, to get the entropy of a written text in a language, it is necessary to count the relative frequencies of all symbols of the alphabet of the language, as appearing in the text. In the same way we can count frequency of words (higher structures) or even phrases. In general this is done by counting and calculating relative frequencies of all possible combinations of two letters, three letters, etc. These frequencies define the probabilities for symbols (first order) and higher structures, or words, (higher orders). So we have a distribution of probability for all symbols of the alphabet in that language. The entropy of the language is the entropy of the distribution of probability. Once we have the probabilities, we can use them to calculate, by plug them in the Shannon's formula, the entropy of any text in the language. This is pretty the same for a score where the musical symbols and higher structures as chords, motives, dynamics, etc., can also be counted and calculated their entropies. For the sake simplicity, we consider only the entropy taking into consideration only probabilities for symbols (first order).

In this work we are also interested with the time evolution of parameters such as rhythm and pitch patterns and their sequences of symbols in the score. So we prefer calculate entropy for short sequences of symbols representing rhythmic or pitch patterns in a bar and then to analyze the time evolution of the entropy along the bars. So, the probabilities we work are defined by the relative frequencies of symbols in short musical segments such as bars and we study the time evolution of these bar entropies along time, or, along bars. The definition below goes in this direction.

Definition: Let $A = \{a_1, a_2, a_3, \ldots, a_R\}$ be an alphabet and $s = [s_1\, s_2\, s_3\, \ldots s_N]$ a finite sequence of symbols of A, that is, for any $1 \leq i \leq N$, $s_i = a_k$, for some k, $1 \leq k \leq R$. Denote n_k the relative frequency of the element a_k in s, the probability of this element is defined as shown in Equation 2.

$$p_k = \frac{n_k}{N} \tag{2}$$

Formally, we're defining a *Distribution of Probability* X for the symbols of the alphabet A. So, in fact, we should write $H(X)$ for the entropy associated to the probability distribution X. We wrote $H(s)$ here because the distribution X is defined, as mentioned above, by relative frequency of symbols of sequences s, where s is the code sequence of a bar or a small set of bars.

As an example let's take $A = \{0, 1\}$ as an alphabet and consider the following sequence:

$$s = [0\,1\,1\,0\,1\,0\,0\,1\,0\,1\,1]$$

Denoting $p_1(s)$ and $p_2(s)$ the probabilities of symbols 0 and 1 in the sequence s, it's easy to see, from Equation 2, they read: $p_1(s) = 5/11$ and $p_2(s) = 6/11$. Taking these values into Equation 1 we get the entropy of the sequences (Equation 3).

$$H(s) = -\frac{5}{11}\log_2\frac{5}{11} - \frac{6}{11}\log_2\frac{6}{11} \approx 0.99 \tag{3}$$

Observe that order is not taken into consideration to calculate entropy. So, for example, the sequence $s' = [1\,1\,0\,0\,1\,1\,0\,0\,0\,1\,1]$ has the same entropy of sequence s.

Clearly the above method of getting probability and calculate entropy can be applied to any musical parameter: pitches, durations, rhythm accents, rests, dynamics, and so on. Such general scheme seems an impressive tool for analysis or even composition but that is not necessarily true. Some limitations of the approach are shown in section III.

A note on units in Information Theory: in base 2, entropy measures the quantity of "bits per symbol" in a sequence. Take, for example, an alphabet with only two equally probable symbols $\mathcal{A} = \{x, y\}$. A bit can be viewed as a box in which we can write x or y. So, we have 2 possible "symbols per bit" which implies that the probability to find any of them is its inverse $\frac{1}{2}$ "bits per symbol". Thus, the information for symbol x, for example is $-\log_2 p(x) = -\log_2 \frac{1}{2} = -(-1) = 1$ bit. Other numerical basis can be used as well which define other information units. One can go from one representation to another through a constant of translation relating the logarithm of both basis.

III. Limits of Information Theory in Music

Edgar Varèse defined music as organized sound [13]. However, from a physical-mathematical point of view, sound is a multi-parameter physical phenomena and is detected in the human brain after a series of complex signal transformations (filters) through the ears. So any attempt to code music is very restrictive and the organization Varèse meant refers just to the surface of a thick structure [14]. Hopefully, Varèse definition is, in general, enough for music analysis. So, Information Theory is used here in the Varèse sense, that is, as time organization of symbols which represent sounds to be played. Taking this into account, we must describe the limits of our analysis. Firstly, it must be stressed that many aspects of performance and musical gestures are not grasped by our analysis, since we are most interested in pitch and duration only. So, we have, as any mathematical approach, a kind of reductionist analysis which, nevertheless, can shed some light on the overall construction. For example, the *accelerando* in the *Misurato* section, bars 6 to 13, is easily represented by our coding shown in section IV. However, the *accelerando* in bars 76 to 80 is only partially represented since there is not a well-defined way, with only our alphabet, to represent tuplets in $\frac{4}{4}$ signature. In order to do that we would need another symbol or perhaps a set of symbols. In fact, the increase in the number of symbols allows a better representation of the score, but the price for this is that the Information value is more difficult to interpret since it is a result, or sum, of many variables. For example, we did not represent harmonic notes (diamond shaped), since an additional symbol would be needed to represent the information that the note is a "harmonic". So, this kind of effect is not grasped by our coding of *Musica Ricercata* and thus we have only a partial information on the timbre actually indicated in the score. In general we can say that in Theory of Information the rule of thumb is "what isn't coded does not exist".

Another important point to mention here is about tempo. This is a very important aspect of musical material but our coding isn't able to give any information about it. In Information Theory, the calculations are about sequences of symbols, in general not directly related to the speed of time. In other words, if the same message, including music, is sent quickly or slowly, the entropy is exactly the same. Just to add a new symbol for tempo has not much utility since, in general, it would appear once or, at most, few times in a code of a musical work. On the same premise, articulation also is not represented by our code.

Likewise, Information Theory can be useful also to give some clues of the compositional process behind the musical surface in different contexts. Nevertheless we think that, in general, its use as a compositional tool is a poor one. In fact, since Xenakis and Hiller, many formal and mathematical approaches for composition as, for example, Stochastic Processes, Markov Chains associated to different statistical distributions, have been far more effective to composition than

Information Theory. In short, you need firstly an object to get information from it. Perhaps, it's better to compose music through other formal devices such as, for example, Combinatorics, and to use Information Theory as music analysis tool. Although Information Theory hasn't a high performance for music composition, since it doesn't take into account ordering in sequences but only symbols counting, there is plenty of room for creation of motives and phrases and textures, following prefixed entropy values, or just making permutations of the elements of a sequence of symbols keeping entropy fixed. Also, in composition this could be used for comparison of different sketches of a work in progress or even to compare them with material from other composers works.

Information Theory approach can be used, theoretically, for the analysis of any kind of music. Nevertheless, clearly it works better for music with repetitive structures or with small variations of them such as, for example minimalist music [16].

IV. Coding Musical Data as Symbolic Sequences in MR1

In this movement Ligeti uses only two pitch classes, that is A and D (this last pitch-class only at the end of the movement to create a cadence). All the pitches used in this movement can be seen in Figure 1.

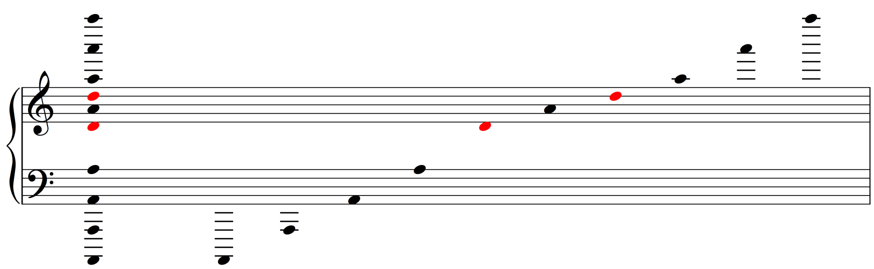

Figure 1: *The set of notes used in Movement I of Musica Ricercata.*

With such meagre resources, which Ligeti forced on himself, the other musical parameters like rhythm and timbre gain prominence. Nonetheless, Ligeti is able to find a fine balance between repetition and variation. Shannon Entropy gives an estimate of this balance. Nevertheless other statistical and complexity measures can be associated to symbolic sequences, such as Kolmogorov Complexity Correlation [15].

We show below how we code the musical material of Movement I of *Musica Ricercata*. Needless to say that a useful general code for analysis and composition is not yet available and our method is also plagued with limitations, mainly those ones of complementary performance indications as, for example, articulations, pedals, *crescendi*, etc. Therefore, although we intend to get the greatest possible generality, each code depends strongly on the piece or set of pieces under study. In order to describe our coding we firstly observe that, out of the four final bars which is a cadence in D from A, all material is obtained from the pitch class A. Each bar is coded as a set of two strings of integers, one string for each hand. The octaves range from A1 to A8. An isolated note is coded by its octave position, from 1 to 8. A chord is coded by the position of its two notes, that is, a pair of two integers in the interval from 1 to 8. Rest is denoted as the negative integer **-1**. In a string, the number of numerical symbol 0 after a positive number (isolated note), or pair of positive numbers (chord), denote duration of the note, or chord, in beats. We reserve the numerical symbol **9** to code any other needed parameter. So the set of symbols read as shown in Equation 4.

$$S = \{-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9\} \tag{4}$$

With this set of symbols and the simple algorithm defined above, a typical bar as number 33 in Figure 2 is coded as two sequences, one for each hand

$$\begin{aligned} lh\{33\} &= [2 \quad 0 \quad 3 \quad 0 \quad 2 \quad 0 \quad 3 \quad 0 \quad 2 \quad 0 \quad 3 \quad 0 \quad 2 \quad 0 \quad 3 \quad 0] \\ rh\{33\} &= [5 \quad 0 \quad 0 \quad 4 \quad 0 \quad 5 \quad 0 \quad -1 \quad 0 \quad 6 \quad 0 \quad 5 \quad 0 \quad 6 \quad 0] \end{aligned}$$

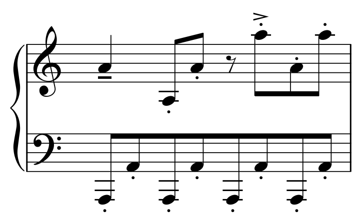

Figure 2: *Musica Ricercata, Movement I, Bar 33. Reprinted by permission of ©SCHOTT MUSIC, Mainz – Germany.*

For a simple example of calculation, using our symbols and coding algorithm, consider the right hand of bar 6 which is coded as:

$$rh\{6\} = [-1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

It uses only two symbols and the size of the string is 9. The relative frequency (probability) of each symbol is $p_1 = p(-1) = 1/9$ and $p_2 = p(0) = 8/9$. The entropy is then given by Equation 5.

$$H(rh\{6\}) = \sum_{i=1}^{2} p_i \log_2 p_i = -\frac{1}{9}\log_2(\frac{1}{9}) - \frac{8}{9}\log_2(\frac{8}{9}) \approx 0,503 \neq 0 \tag{5}$$

As a second example, observe that the left hand of bars 6 and 7 seem equivalent since both have 2 notes and 6 rest beats but in different positions. Our representation can differentiate them and, in fact, they have different entropies. Their codes read:

$$\begin{aligned} lh\{6\} &= [2 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 2 \quad 0 \quad -1 \quad 0] \\ lh\{7\} &= [-1 \quad 0 \quad 0 \quad 2 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 2 \quad 0 \quad -1 \quad 0] \end{aligned}$$

and their entropies are given, respectively, by $H(lh\{6\}) \approx 1.25$ and $H(lh\{7\}) \approx 1.33$). The difference comes from the additional symbol -1 in $lh\{7\}$.

V. Information Based Analysis of MR1

Below we make a comparative analysis from the point of view of Theory of Information through the calculation of Shannon Entropy for several sections along the work. We've calculate separately entropy coefficients of left and right hands.

The piece starts with an Introduction section indicated as *Sustenuto* from bars 1 to 4. Bar 5 is just a rest, functioning as a bridge for the next section.

1. **Bars 1 and 2**

   Clearly the two gestures in these bars are equal and, of course, their information measure as well. For example, $H_{left}\{1\} = H_{left}\{2\} = 1.55$, and the entropy ratio between right hand and left hand is equal to 1 (Equation 6).

$$C_{left-right} = \frac{H_{right}}{H_{left}} = 1 \tag{6}$$

   As mentioned in the Introduction, our alphabet is not rich enough to take into account the "tremolo" effect of this section.

2. **Bars 3 and 4**

   In this subsection there is only one chord on the right hand and a harmonic resonance on the chord A2-A3. Our code isn't able to get information from the harmonic chord. So we can only add it as a normal chord or simply ignore it in our analysis. Either way the information doesn't correspond to a more realistic representation of the real sound. A better solution which does approximate to the real sound is to assign new symbols for harmonic notes and chords (resonance). The downside for this is to increase the number of symbols which are not usable in our case since harmonics just appear in these bars and again in the final 4 bars of the cadence to D.

3. **Bar 5**

   This bar is just a pause to start of the next section, *Misurato*.

4. **Bars from 6 to 13**

   This subsection is the beginning of *Misurato* section which lasts from bars 6 to 59. It is played only with the left hand and is clearly an *accelerando* with asymmetrical rhythmic figures. The first 4 bars show an increasing number of notes, but their position do not seem to obey any specific order. From bar 10 to 13 a new pitch is added, namely, A3 and the number of notes continues to increase until bar 13, which contains no rests. The time evolution of the entropy along this subsection is given by the Figure 3.

5. **Bars from 14 to 58**

   This is the longest subsection which Ligeti keeps a more or less definite pattern. The left hand is an ostinato along the entire subsection, while the right hand develops an increasingly complex patterns including more notes in different octaves as well as chords. Figure 4 shows the initial bar of this subsection.

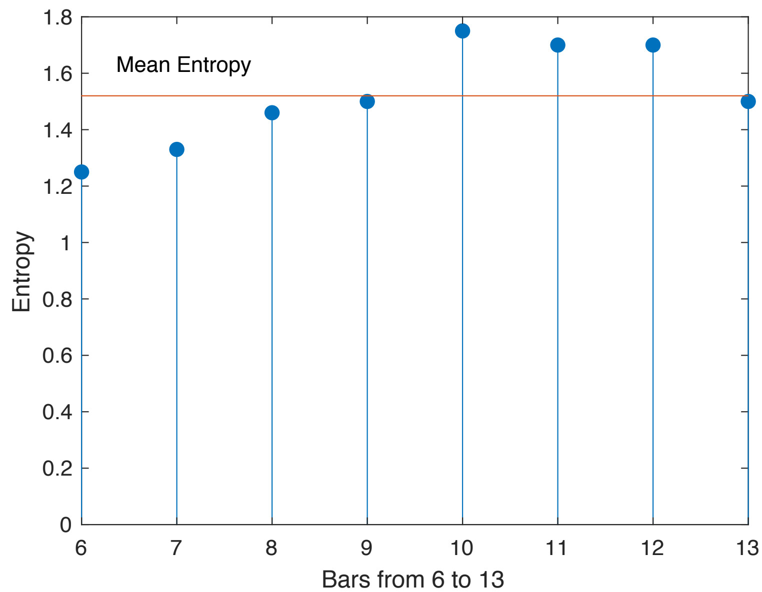

Figure 3: *Musica Ricercata, Movement I, Time evolution of Entropy of the left hand: Bars 6 to 13.*

Figure 4: *Bar 14 of Movement I of Musica Ricercata. Reprinted by permission of ©SCHOTT MUSIC, Mainz – Germany.*

Clearly, the repetition of the same pattern in all bars of the left hand implies a constant entropy along it. So, it's more interesting to study the entropy variation of the right hand along this segment, although one can argue that is just the variation of the right hand against an *ostinato* (constant) background that strikes as new for the human ear. The behavior of the right hand against the *ostinato* can be seen in Figure 5 in terms of Shannon entropy.

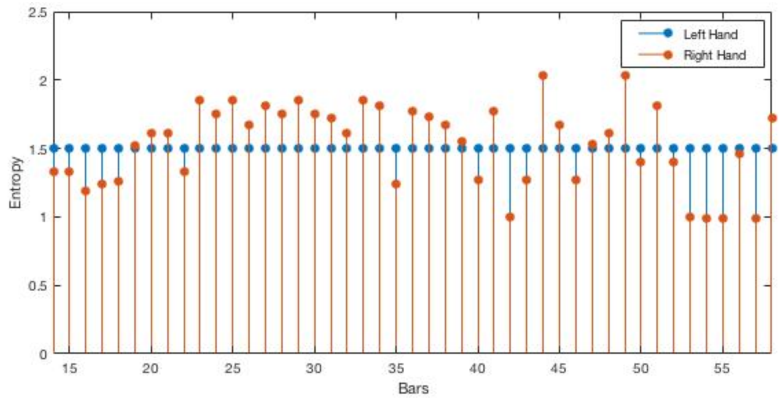

Figure 5: *Musica Ricercata, Movement I, Time evolution of Entropy for Bars 14 to 58.*

The ratio between the left and right hand entropies, for Bars 14 to 58, can also be seen in Figure 6.

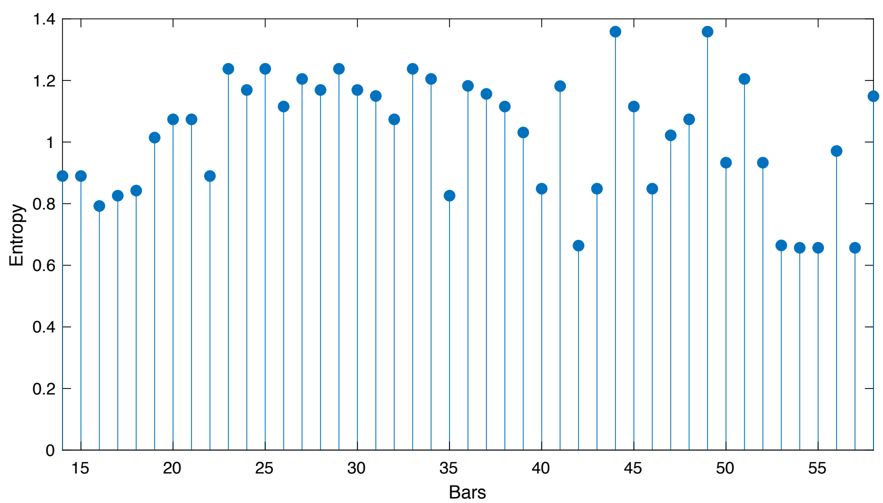

Figure 6: *Musica Ricercata, Movement I, Time evolution of the ratio between Left and Right Hand for Bars 14 to 58.*

6. **Bar 59**

   In this bar Ligeti keeps the same pattern of the previous bars but changes the time signature to $\frac{3}{4}$. The left and right entropies of this bar read $H_{left}\{59\} = 1.50$ and $H_{right}\{59\} = 1.56$.

7. **Bars from 60 to 65**

   These bars are the beginning of the section *Prestissimo* returning to signature $\frac{4}{4}$. The very similar rhythm patterns with the same notes lead to pretty the same entropy for left and right hands as shown in Figure 7.

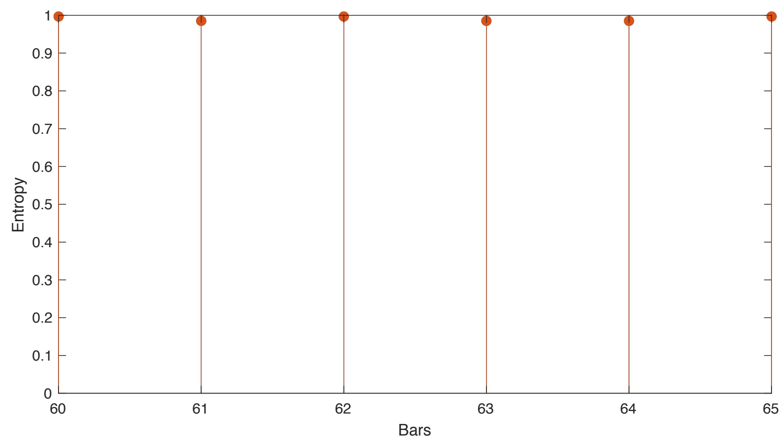

Figure 7: *Musica Ricercata, Mov. 1, Time evolution of Left and Right Hand Entropies for bars 60 to 65.*

8. **Bars from 66 to 80**

This is an increasingly fast (*accelerando*) section preparing to the final cadence in D. It is not difficult to see that due the absolute parallelism between left and right hands, their entropy curves are the same. That is just case as shown in Figure 8.

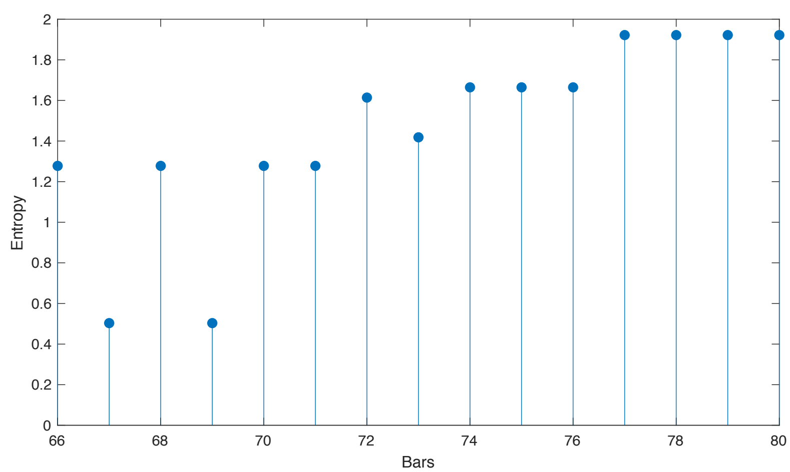

Figure 8: *Musica Ricercata, Mov. 1, Time evolution of Left and Right Hand Entropies for bars 66 to 80.*

Observe, as we have mentioned regarding to other parameters, our code is not able to see the clef changing in bar 68. So, this does not affect the calculation of the entropy. On the other hand it is possible, in this case, due to the parallelism, to measure the *accelerando* through our code simply counting the number of positive single or pair of numbers (notes or chords) per bar and taking into account that the time signature is $\frac{4}{4}$.

9. **Bars from 81 to 85**

   After a pause, bar 81, Ligeti makes a cadence in D. The basic notes are D3 and D5 late accompanied by harmonic chords D3-A4 and A6-D6. Our code isn't able to take into account both the addition of note D and the information "harmonic". So we must analyze this section in a different basis. Clearly this is a completely unexpected event, if we take into account all the previous bars where we have just A in different octaves, which is the only pitch information. So it is not a fault of our code, but simply the event implies an extension of our alphabet. If we keep the alphabet, this section cannot be coded and so there is no sense to calculate entropy for it. However, if we can consider bar section 81–85 as a new unique event (a unique sound event), extending our alphabet to include the pitch D, the Shannon entropy, restricted to it, is rigorously zero.

VI. Analysis of Movement II

MR2 is essentially melodic. The pitch-class set of this movement is $\{E\sharp - F\sharp - G\}$, the most compact of pitch-class sets Ligeti used in MR. Much as in MR1, the resources here are very limited. In a piece with 33 bars, the third pitch, in fact, the same note $G5$, is introduced only halfway, in bar 18.

In his Masters dissertation, D. Grantham makes a very good analysis of Musica Ricercata [6]. In particular, his analysis of MR2 is extensive in the description of the general structure of the movement. Here we present a different approach, based on Information Theory which can complete his one. Firstly we make some observations on the movement's structure.

Again, as in the MR1, Ligeti is able to find a fine balance between repetition, symmetry and variability in his two-note melodic construction along the movement. The melodic construction is based on a four-bar key phrase which can be thought as two question-answer bar motives. Figure 9 shows the key phrase.

Figure 9: *Initial Phrase of Movement II of Musica Ricercata. Reprinted by permission of ©SCHOTT MUSIC, Mainz – Germany.*

Along the piece, slight variations of these two one bar motives appear in both hands, however with different vertical constructions, such as chords and against contrasting textures. Also it's worth to say that dynamics, most of times, changes wherever a new phrase starts which in turn is correlated also to changing in timbre. Overall taking into account, Ligeti gets a fine variability of material even using so restrictive pitch material.

In order to facilitate our quantitative analysis, we use here a simple code alphabet: for the pitch classes $\{E\sharp = 0, F\sharp = 1, G = 2\}$. The overall structure can be divided in the following sections:

Section 1: bars 1-16

The section can be thought as somewhat indolent and mysterious melody in tempo ♩= 56 composed of a sequence of variations of the above two one bar motives of Figure 10. These variations has a periodic behavior. The time signature has a periodic behavior each 4 bars as $\begin{bmatrix} 5 & 5 & 4 & 6 \\ 4 & 4 & 4 & 4 \end{bmatrix}$.

In fact, taking into account that each motive, comprising one or two bars, ends at an agogic (duration prolongation) accent of two or three beats, the sequence of motives, using the above code, is given by Table 1 in pairs.

Table 1: *Code of bars 1-16*

Bar Number	*Type*	*Bar Code*	*Signature*
1-2	Q-A	$\|011001\hat{\mathbf{1}}\|\|100110\hat{\mathbf{0}}\|$	$(5,4)-(5,4)$
3-4	Stat	$\|01101001\|\|100110\hat{\mathbf{0}}\|$	$(4,4)-(6,4)$
5-6	Q-A	$\|011001\hat{\mathbf{1}}\|\|100110\hat{\mathbf{0}}\|$	$(5,4)-(5,4)$
7-8	Stat	$\|01101001\|\|100110\hat{\mathbf{0}}\|$	$(4,4)-(6,4)$
9-10	Q-A	$\|011001\hat{\mathbf{1}}\|\|101001\hat{\mathbf{1}}\|$	$(5,4)-(5,4)$
11-12	Stat	$\|01101001\|\|101001\hat{\mathbf{1}}\|$	$(4,4)-(6,4)$
13-14	Q-A	$\|011001\hat{\mathbf{1}}\|\|100110\hat{\mathbf{0}}\|$	$(5,4)-(5,4)$
15-16	Stat	$\|01101001\|\|100110\hat{\mathbf{0}}\|$	$(4,4)-(6,4)$

In this table the duration accentuation is in bold face with a hat. We defined two types of the motive: *Q-A* means question-answer motive and *Stat* means statement motive as described below. As in MR1 Ligeti explores timbre with different uses of pitch classes of the pair $\{E\sharp, F\sharp\}$, such as, for example, right hand melody in bars 1-4, chords in both hands in bars 5-8, left hand melody in bars 9-12 and so on.

It is easy to check, from Table 1 that, except for bars 9-10, all pairs of bars with time signatures $\begin{bmatrix} 5 & 5 \\ 4 & 4 \end{bmatrix}$ are symmetrical under the exchange symbols $0 \leftrightarrow 1$. However bars 9-10 are symmetrical under the exchange of the two first digits. These pairs of bars sound as a kind of question-answer pattern, mostly due to the fluctuating sequence of notes ending with the “cadence" $F\sharp$ in the first bar of the pair and $E\sharp$ in the second one. Now the pairs with signature $\begin{bmatrix} 4 & 6 \\ 4 & 4 \end{bmatrix}$ seems to work as a two bar reinforcement of the previous two bars. As the previous case, except the bars 11-12, all other pairs are the same two-bar motive and don't have any apparent symmetry between the bars in each pair. Nevertheless, the first element of all pairs has symmetry $0 \leftrightarrow 1$ between the its half parts.

Once described the overall structure of the section we are interested to compare it with its information content. Much as we did in the case of MR, our approach here is just to calculate the Shannon Entropy defined in Equation 1 for each of the two-bar motives coded in Table 1 and plot its time evolution. We do not take into account rhythm, but only the pitch classes. So the alphabet $\{0, 1\}$ only represents the change of pitch. The relative frequencies of symbols for *Q-A* and *Stat* motives are given by Table 2.

From Table 2 we get the Relative Probabilities (Table 3).

Since the alphabet has just two symbols $\{0, 1\}$, Table 3 shows that they are fair balanced along the section and the Shannon Entropy for the eight motives gets almost constant:

$$H = [1.000 \quad 0.997 \quad 1.000 \quad 0.997 \quad 0.985 \quad 0.997 \quad 1.000 \quad 0.997]$$

So, from the point of view of pitch class, as expected, MR2 has a very low variation of information.

Table 2: *Relative Frequencies of Q-A and Stat Motives*

Relative Frequencies			
Q-A motives		*Stat* Motives	
7	7	8	7
7	7	8	7
6	8	7	8
7	7	8	7

Table 3: *Relative Probabilities of Q-A and Stat Motives*

Relative Probabilities			
Q-A motives		*Stat* Motives	
0.50	0.50	0.53	0.47
0.50	0.50	0.53	0.47
0.43	0.57	0.47	0.53
0.50	0.50	0.53	0.47

Section 2: bars 18-24

What do you do with just one pitch? In music, a lot, mainly rhythm patterns, but from the point of view of Theory of Information, given the alphabet, we use in MR2, with just 3 symbols in which $G = 2$, the entropy is exactly zero. In other words, in this section information on durations of notes is clearly detected by human ear, but our code cannot do it. This section, with time signature $\frac{4}{4}$, consists only of the accented note $G5$ played in *accelerando*. Ligeti's notation shows, in tempo of ♩ = 126, groupings of accented notes of shorter and shorter durations along the bars.

In the beginning of the section, bar 18, Ligeti wrote "tutta la forza" with dynamics ***ff***. This short section had an intentionally extramusical motivation

> . . . Ligeti revealed that, as he composed the piece, the reiterated Gs had symbolised for him "a knife through Stalin's heart" [5].

In order to quantify the *accelerando* we use the concept of *Symbol Rate* from Information Theory which means the number of symbols per time unit which, in our case, we take the bar. In this section each symbol $G5$ corresponds to an event (note attack) from the right hand. So, the pace of attacks per bar along this section reads $1 - 1 - 2 - 3 - 4 - 5 - 96$, which shows a steady linear growth with a big discontinuity in the last bar where also Ligeti's indications "Senza Tempo" and "Rapido" leave some openness to the details of performance. Here we have "in nuce" the first example of the fuzzy zone of transition from discrete to continuum in time almost in the limit of human performance. This is to be compared with the last bar where we find the reverse transition, that is, from continuum to discrete. As it is well-known, Ligeti would explore these artifact much later in his 1968 piece *Continuum for Harpsichord*. Nevertheless, along all this section the Shannon Entropy is zero, since only one symbol $G5$ was used. This show that Shannon Entropy has some limitations when applied to some kinds of physical or artistic information, such as, for example, identical pulses from a source, or in our context, the same pitch from a musical instrument, since the human perception in fact gets information such as rhythm, although just one symbol is used. Another example is the obvious physical information (rhythm) coming from the use of odd n-tuples that get fractional durations for notes. That is not coded as a sequence of symbols. It is possible circumventing this problem extending the coding by choosing a very short time unit. However this will lead to a very artificial and long string representation of note durations which has not simple musical correspondence.

Section 3: bars 25-28

In this short section, spanned 3 bars, Ligeti superposes the two materials presented before: it starts with the initial melody by octave chords in the left hand and, halfway the bar 25, it is strongly contrasted, now on the right hand, with the octave $\{G4 - G5\}$, with $G4$ a half note, while $G5$ is articulated as the same sequence in the previous bar in tempo "rapido" on the right hand, but now with some notes of the sequence accented just playing the octave $\{G5 - G6\}$. These artifacts get a higher timbre complexity, as well rhythm variations. This pattern is kept until bar 28.

Section 4: bars 29-32

This four-bar section has the same initial melody line, a reaffirmation of the mysterious and somewhat nostalgic mood, but now with a stronger emphasis, since it is written with parallel octaves in both hands. Again it is a way Ligeti uses to get timbre variability. The code of this section is shown in Table 4.

Table 4: *Code of bars 29-32*

Bar Number	*Type*	*Bar Code*	*Signature*
29-30	Q-A	$\|0110011\hat{\mathbf{1}}\|\|1001100\hat{\mathbf{0}}\|$	$(5,4)-(5,4)$
31-32	Stat	$\|01101001\|\|1001100\hat{\mathbf{0}}\|$	$(4,4)-(6,4)$

Clearly the Entropy is the same of the bars 1-4, that is, $H = 1$.

Section 5: bar 33

On the left hand, a half tone descending "cadence" $\{F\sharp \rightarrow E\sharp\}$ in octave chords is played twice, while above it, on the right hand, the $G5$ "rapid" train of notes in *perdendosi* and continuously decelerating. The half-tone descending cadence works like a tension-release which is confirmed by the "perdendosi" sound attenuation (Figure 10).

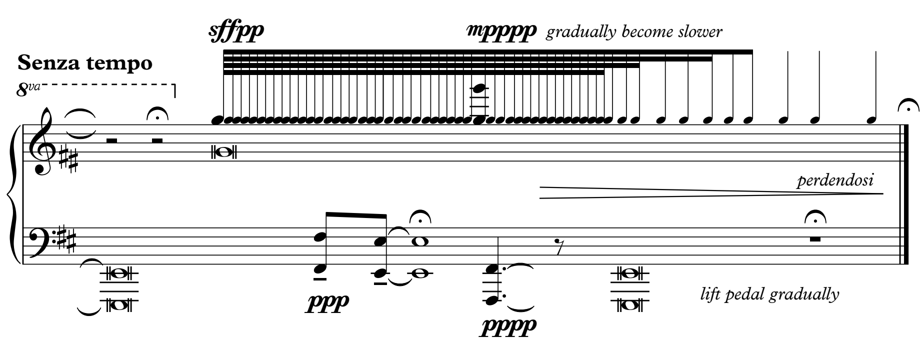

Figure 10: *Final Bar of Movement II of Musica Ricercata. Reprinted by permission of ©SCHOTT MUSIC, Mainz – Germany.*

VII. Conclusions

Musica Ricercata is nowadays, as several other of Ligeti's works, very famous. Although a work of Ligeti's early years as a composer, and tending to a prescient minimalism, some structures and indications in the score of *Musica Ricercata* make it a piece with some characteristics which are beyond the mathematical approach of Information Theory.

Of course, mainly for 20th-century music, any kind of musical analysis can be only partial and Information Theory based analysis is not different in this aspect. However, this work on the first two movements of *Musica Ricercata* is an example in which Information Theory can be useful for analysis of non-tonal music, since its quantitative approach explicitly shows the relative complexity of different musical structures and gestures and in which tonal aspects are not relevant.

As we mentioned in III, Information Theory based analysis depends strongly on the chosen variables for analysis. In this work, while our analysis of MR1 is concentrated on rhythm patterns of pitch octaves, which implies in some timbre variations, that one of MR2 explores more the pitch content.

It is worth to stress that we can make analysis of combined symbols from two or more alphabets with different distributions of probability. In this case, other Information Theory parameters are useful such as Joint Entropy, Conditional Entropy, Mutual Information and others [17].

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