# Rhythm and Entropy: The Exile of the Metric in the Dance of Pulsation* 

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#### Abstract

This essay intends to demonstrate that the very system that codified the rhythmic subdivision subsumed to the metronome's beat or pulse might offer, through its own mechanisms, a window to its deconstruction and yet, to a new, intrinsic, development. When metric subdivisions occur that are farther away from the metronomic beat's referential, (as when rhythmic deviations by many sub-ratios accumulate underneath a certain rhythmic figure), the performer experiences a cognitive loss in the sense of immediate metronomic adjacency. New ways to perform a certain rhythmic outcome buried within the grounds of complex subdivisions require mechanisms to momentarily suspend the main, overarching, beat, to impose emergent, micro-metronomes. These devices are codifiers of speeds whose regularity opens up terrain for new, rhythmical deviations and sub-ratios. They also allow the performer to negotiate between rhythms that present diverging metric configurations, linking their speeds, through rhythmic bridges. As the performer reaches these bridges, located at a deeper level of rhythmic subdivision, he/she ought to return to the main metronomic surface using the speed managed within these momentary micro-metronomes. Such performative and cognitive inversion, lies at the center of the Micro-Metric Modulation Theory.


Keywords: Micro-Metric Modulation. Ratios and Sub-ratios. Complex Rhythms. MicroMetronomes. Diverging Metric Configurations. Commutative and Associative Properties of Rhythm.

## I. The Metric Paradigm and the counting of beats

THis paper intends to bring to the fore an unusual aspect of the metric unfolding: the one that is not associated to an immediate correlation to the metronomic pulse. One that suspends the rhythmic anchor of the pulsating metronome that regulates durations and rhythms in order to negotiate new rhythms that are farther away from the metronomic reference, but encapsulated in the inner musical fabric. What I am trying to show is that our usual way to deal with rhythms offers implicit mechanisms that can be of great help to achieve a broader understanding of the way the pulse is relative to its context and not an absolute construct that precedes every performative action. When complex rhythmic situations are at play, performers deviate continuously from the beat in order to go deeper within the interstices of a given rhythm. Consequently, a sudden loss of correlation between macro (metronomic) and micro (ratios and sub-ratios) tempi, starts to tear the musical logic of a predominant (above all) pulse, to entertain proximal references regarding the agency of a neighboring subdivision. It is necessary, at farther deviations from the main beat or metronomic pulse, to create emergent, new sub-metronomes that will function as an instantaneous metric basis to perform a sound rhythmic correlation with the rhythm that immediately preceded it. ${ }^{1}$

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## i. Il Tempo de la figura (after Ferneyhough)

Figures 1 and 2 demonstrate the way a rhythm grid appears to an interpreter firstly anchored in the metronomic pulsation to slowly loose its reference relative to the said pulsation when new subdivisions farther away from the reference beat, start to appear.


Figure 1: Stronger to weaker relation to the Metronome given pulse/speed. However, they keep their tight correlation with the initial metronomic pulse as they present a clear base 2 subdivision with the initial beat/speed

Note at Figure 1 that all the subdivisions are tightly correlated with the pulsation or the feeling of the main beat, which is the metronome. Even when smaller subdivisions are required they keep a strong reference to the metronome marking because a) they are direct subdivisions (halves and halves of halves) of the metronomic beat and b) because they act as a perceptual gestalt, that re-spells the main beat through a regular span (or distance) whose return is clearly expected even when smaller subdivisions are at play. Although rhythmic unfolding within a regular beat might contain irregular pulsations (i. e., a $2 / 4$ bar can also be felt as the conjunction of a $3+5$ rhythmic accentuation of dislocated sixteenth-notes) they are still under the spell of a propulsive metronomic regularity that reaffirms itself at each rhythmic cycle. This is extremely important for performers, since they can coordinate their respective and independent inner rhythms, the micro-fluctuations that their parts might contain, with a projected sum given by the main metronomic beat.


Figure 2: Levels of subdivision: a) Zero or Neutral Level. b) First level of subdivision. c) Second level of subdivision. d) Third level of subdivision. e) Fourth level of subdivision.

At Figure 2, a similar process is at play initially. Even if irregular subdivisions of the main beat (like 3 eighths in the space of 2 eighths) appear, this first deviation is still strongly related to the speed of the metronomic beat. Only when the process starts to acquire a further redundancy and the deviations are compressed towards an irregular part of the figure (like the ones found on the second, third and fourth levels of subdivision) then our notion of a projected beat defaulted by a predictable rhythmic regularity of the figure starts to weaken. As the main metronomic beat suffers an acceleration when novel subdivisions are formed, we are caught between two metric pulls: a suspended beat coming from the metronomic speed, and the inner metric subdivisions of the rhythmic figure. Notice that the metric figure is not a slow accelerando politely written and
following therefore an even scale of speed. It is a metric route that demands specific rhythmic contours. To recap: while the metronomic beat acts as a frame of metric reference towards which all rhythms are subsumed, it is, nevertheless, slowly suspended when new metric deviations are called forth. Thus, in order for the performer to mediate between the idea of an even out acceleration whose purpose is just to cover the space of the metronomic beat, she/he will have to articulate such acceleration according to specific rhythmic demands written within it. This dichotomy between prospective, overarching beat, and the inner complexity of figural speeds, entails a mechanism that must be accounted for when there is the need of any type of rhythmic carving in the path of performing the figure and thus, crossing the span/speed of the metronomic beat.


Figure 3: Differing levels of speed to traverse the second half-note beat of the bar: a) First level of subdivion accelerates from metronomic beat's speed. b) Second level of subdivion decelerates (slower pulse than $1^{\text {st }}$ level). c) Third level of subdivion accelerates. Faster pulse than the second level.

What exactly does that mean for the mind of the performer? It means, as shown in Figure 3 at second half-note beat, that a gestaltian comprehension of the path to be covered can't be solely overcome by the maintenance of the metronomic beat irrespective of the inner mediations of figural demands. Thus, a new concept emerges here where the prospective acceleration must be halted by the understanding of which speed exactly is the performer deviating from. As we can see, the performer can't simply accept that all rhythms accelerate towards a prospective beat in a random, or regular, speed. She/He ought to consider which speeds are being required so that every step is compatible with the inner logic, or rhythmic design, of the figure. Therefore, a new strategy to cross a specific metric figure emerges, one that creates instantaneous spans similar to the metronomic beat, that again, offers to the performer a new metric hierarchy from which a specific rhythmic speed will be articulated and properly fit. Notice too, another tantamount aspect regarding the flexibility of the rhythmic figure within this process. At the second-level of subdivision (4 notes in the time of 5's ratio) the rhythmic figure acquires a distinct metric configuration and it is shown as a quarter-note value instead of an eighth-note. This might seem to contradict somehow, at the level of figural representation, the first ratio layer, which is supposed to be slower than the second. Here, it is necessary to understand that similarities between figural representation and a hierarchy of speeds is illusory within the very system of metric subdivision. That's a lesson hard to be understood by the performer's intuitive grasp of rhythm, many times. It comes from the simple principle that any metric figure maintains its figural identity until it is twice as fast or twice as slower than its current configuration. Thus, a stream of sixteenth-notes, for instance, starting with 4 sixteenth-notes, won't change its configuration till it reaches a speed twice as fast, becoming finally a thirty-second note rhythmic figure. The same process happens in the slower direction. So, for instance, when a triplet that fits a quarter-note's duration is shown as a rhythmic figure comprising 3 eighth-notes and not 3 sixteenths it is because the quarter-note is first divided in half by two eighth-notes, then in three equal parts by 3 eighth-notes, and only
when it has to fit 4 notes it acquires the figural representation of the sixteenth-note since it is twice as fast as the eighth-note. This logic serves for us to understand that when a slower tempo is called forth, as in the second-level of Figure 3, it is a consequence of similar process. It is easy to understand such process just noticing that the 5 eighth-notes (span of the ratio) might only double its figural configuration when reaching a next level of sixteenth-notes. Till then, it maintains the same eighth-note configuration. Only at the point that 10 sixteenth-notes occupy the same space as 5 eighth-notes, a new rhythmic configuration is formed. Thus, 4 quarter-notes belong to a metric hierarchy that precedes the rounding off tempo of 5 eighth-notes, as they come from a slower figural configuration. (Figure 4).


Figure 4: a) Span of the figure (5 eighth-notes). b) It will only change its rhythmical configuration when it reaches a speed twice as fast becoming a sixteenth-note (10/16 bar). Thus, it is possible to see that the 4 quarter-notes placed as sub-ratio (see Figure 3), are simply the twice-slower configuration of the 8 eighth-notes of the 8:5 ratio. c) 4 quarter-notes from a slower rhtyhmic layer forming the ratio 4:5 instead of the 8:5 above

While in Figure 3 a constant negotiation between accelerandos and decelerandos is shown in order to cover the whole span of the beat, at Figure 2 a straight metric acceleration proposes a diverse rhythmic trajectory. Both examples, however, oblige the performer to halt the speeding up strategy according to an emergent metronome found at a deeper level of the rhythmic unfolding.

Thus, as we can observe at the last quarter (beat) of Figure 2, there are four types of metric accelerations being woven by the performer to cross this figure.
a) At the first-level of subdivision the performer is deeply connected with the metronomic beat from which rhythmic deviations are managed;
b) At the second-level of subdivision a new span hierarchy is at place: the performer has to mediate between the span of the metronomic beat, and the span of a part (two-thirds) of the triplet that was just managed at the first level of subdivision. At this point, a new quarter-note emerges. It is the result of the addition of the last two eighth-notes of the triplet. Note that this emergent quarter-note is not subsumed to the metronomic hierarchy's immediate subdivision, and therefore exhibits a different span/speed, even if its figural embodiment has an identical metric/figural correspondence with the metronomic quarter-note;
c) At the third-level of subdivision, the new triplet is found when the performer carves a new span or a new frame of metric reference out of the last two eighth-notes (or legs) of the triplet (located at second level of subdivision) in order to be able to fit its new triplet within these last two eighth-notes or legs of the figure. Again, this is managed through a similar method used above to acquiring a new quarter-note span out of the last two eighth-notes of the second-level triplet;
d) At the fourth level of subdivision the same method ensues. However, at this time, a new metric configuration is called forth. One that requires a quintuplet-sixteenth as the last
rhythmic subdivision before a new prospective beat at the upper or neutral-level of the metronome marking is reached.

The propelling energy of the metronomic span encapsulates a regularity that coordinates all types of subdivisions found within a regular beat. Only when under the spell of a metronomic regularity we manage to pre-calculate the necessary steps we will need to cross it, no matter if these steps are built with regular or irregular subdivisions of the beat. Thus, with the reference of the metronomic beat it is equally possible to envision a strategy, given a certain amount of time or a specific span, for taking for instance, either seven steps (septuplet-sixteenths) or four steps (four regular sixteenths) to effectively cross the proposed beat or the metronomic span.

To make things transparent, under a certain metronome we have regular subdivisions of eighths, sixteenths, thirty-seconds, sixty-fourths or, for that matter, even slower values like the half and whole, notes, as shown in Figure 1. This simple fact underlies a theoretical construct that foments rhythmic contrast and diversity within the combinatorial vectors of the Western Music's rhythmic practice. They act as well as a great cognitive glue to arrange packets of diverse, albeit limited, number of figures in repetitive gestural 'condensates' which reinforces mechanical associations with a cemented understanding of rhythm.

Nevertheless, it is possible to combine these regular subdivisions of the main beat to forge a rhythmic discourse of highly contrasted semantic outcomes. More, these rudimentary tools of treating time as a regular multiple or divisor of the main beat, entail a huge combinatorial output. However, as much as the contrasting potential amounts to a great metric flexibility of the rhythm, they reveal, as well, an underlying understanding of meter that becomes detached from the way Time, as a general and multidimensional phenomenon, behaves and unfolds. In fact, they promote a deviation from the rhythmic units that, in themselves, are just isolated syllables of a yet non-spelled aggregate. In order for these rhythmic units to transpose their mere mechanical and arithmetic relationship they need to create a hierarchy of figures that become associated with gestures that fit the metronomic span. For instance, they can unite to create diverse arrangements of the units used for subdividing time: syncopations, an eighth-note followed by two sixteenth-notes, four sixteenth/thirty-second/sixty-fourth-notes in a row, among innumerable arrangements. They start to point to a state-of-affairs where the very apprehension of time is mediated by the perceptual gestalt of the figure as a unit. As they condense the inner rhythmic flexibility given by isolated subdivisions of the beat (no matter which particular one is being used), a higher order of hierarchy is foregrounded to the performer's perceptual apprehension. Some of these units, when placed together, or adjacent to each other, form gestures that suspend momentarily the counting of micro-rhythms derived from the beat's speed. They enhance the view of a larger or higher gestural coherence, creating a quasi-mnemonic association with a specific rhythmic object. Similar to the way we understand words within a phrase. We are not spelling every letter in order to construct the word. We are apprehending a syntactical rule that underlies such construction or correlation. In fact, seeing a known arrangement of rhythmic units can be compared to dealing with pieces of legos. These small pockets of information become, by and in themselves, the very underlying fabric of rhythmic construct. As we can see, there is a slight departure of the metric subdivision principle, to favor the faster apprehension of information as a whole. One that is build (or cemented) in known monads of gestural and figural detachment. The cognitive apparatus of the interpreter apprehends such new metric hierarchies as a gestalt, and it is brought, by fastidious practicing, to the level of ergonomic memorization. Scales and phrases are formed to guarantee the agreement of a diverse arrangement of these types of second-order metric aggregates. Consequently, an aesthetic is built based in the modular rationalization of information into contrasting and yet, repetitive possibilities. Such state-of-affairs is none but the embodiment of a comprehensive, privileged, view, that divides the world into poles of assimilation
and irrelevance. There is much more in the story/history of rhythmic unfolding than a mere theoretical construct of base two might address in spite of its arithmetical neutrality. And I advance, that no system, no matter how comprehensive, will be able to provide a complete apprehension of time within any graphic or written, representative, model.

## ii. Towards a parametric interchangeability of tempo, meter, figure and duration

After the above intro and a rudimentary demonstration of the evolving state of metric subdivision within Western Music, emphasizing the very system's apprehension and appropriation of time/duration, it is possible, at least tentatively, to peruse a more general understanding of some of the implications of metric subdivisions. Regarding new possibilities of the metric unfolding it is important to understand the contrasting and very practical aspects that differentiate the concept or parameter of span from the concept or parameter of rhythm. Span means, literally, the full extent of something from end to end; or, in other words, the full amount of space that something covers. In musical terms, span might be seen as accretions of rhythms whose durational trajectory traverses a certain amount of time. For our purposes in this paper we will simply state that the accretion of speeds that differ from each other in terms of non-correspondent numerical/rhythmical scales are boundaries that lie outside the scope of a specific, regular, stream or span. A span must have an underlying common denominator speed that is graspable by a specific rhythmic figure. While we can measure the span that a regular pulsation of sixteenth-notes covers (for example, 3, 4 or 7 sixteenth-notes, at Figure 5a), we can't possibly grasp (within the musical framework) the durational span that starts, say, at the third quintuplet-sixteenth of a quintuplet rhythm and stops at the $5^{\text {th }}$ septuplet-sixteenth of a septuplet rhythm (see Figure 5b). The latter does not cohere as a regular totality from which we can calculate a homogeneous quantity having a common, underlying rhythm.


Figure 5: a) Representation of a regular span from which a new subdivision might occur. a1) Metric span coherent to be notated as one isolated, whole, rhythmic figure, a dotted-eighth in this case. We can add any amount of notes from this regular pulsation of sixteenth-notes to create a regular stream/span from which new rhythms can be inferred from. b) Undefined metric span not coherent to be notated as one isolated rhythmic figure. b1) What's the size and figural representation of this span? With which coesive and regular rhythmic figure you can cross this span? Not possible since it accelerates and looses common ground from which you could deviate from. b2) Undefined metric span not coherent to be notated as one isolated, whole, rhythmic figure. While the total of the span can be thought of as an addition of two distinct rhythmic figures they can't cohere into a specific rhythmic unity since they belong to two differing metric hierarchies or two unrelated scales of speed.

The second parametric aspect of time is rhythm. Rhythm cannot be understood as a diverse pull of quantifiable elements forming disconnected aggregates. It is not a mere game of accretions and eliminations whose parametric independence might be measured as an isolated layer of materials. As said before, all elements of duration are interconnected. (We also know that we can define rhythm as the speed of frequencies that, if above a certain acoustic threshold, enables the perceptual foregrounding of pitch, another parametric strata of time.) At a very basic level, rhythm is but the intervallic obviation of flowing time, not its concrete representation. It is possible to demonstrate even rudimentary that rhythm can be understood, at a primary level, as a metric concomitance with the metronomic speed. At that point, the metric parallelism between metronome and rhythmic pulse is leveled. Both are heard as one. The maintenance of musical duration is given by a periodicity that needs to reenact itself to reacquire musical pertinence. In order to re-potentialize the audibility of a repetitive chunk of information that otherwise would fade in the acoustic background, it is necessary the introduction of small disturbances. Therefore, whenever a rhythmic deviation from the beat is introduced we start to hear back the very beat. The idea of rhythm as an intervallic default of the metronomic is more akin of my understanding of the concept of rhythm. And it makes it less petrified and more fluid to the idea of parametric interchangeability. Between the vertical prospectiveness of the metronomic and the horizontal filling of micro-units of time, lies no boundaries. The musical figure might be re-contextualized and detach from the grip of the regular metronomic subdivision when the metric figure itself contributes to create an independent, asymmetric level of perceptual spans that neutralizes, at least temporarily, the regularity of the overarching, metronomic, beat/speed.

Figures 6 and 7a illustrate two instances of parametric interchangeability: one of metric subordination (Figure 5, where rhythm is presented as a polyphonic strand), the other of detachment (between metronomic prospectiveness and rhythm-as-new-metronome), where the eruption of asymmetric figuration starts to impose itself as perceptual irregularities. At Figure 6 two scales of metric units are at play: one (sixteenth-notes) belongs to a regular, repetitive unit, that divides the metronomic beat into small equal parts. Running in parallel and below it (bass line), we find rhythmic figures of differing spans. These figures present either regular or irregular sizes or durations. However, under the performer's perspective, these accretions of notes are not immediately perceived as a metric discrepancy weakening the main beat's coordinates. They are polyphonic strands still subsumed to the underlying stream of sixteenth-notes and therefore seen (or read) as a part of the beat's metric unfolding and regularity.


Figure 6: Polyphonic strand of linked rhythms forming the bass line. They are just regular and irregular spans subsumed to the beat's subdivisions. Because of their strong connection with the Metronome's pulse they are not seen as detached, independent, isolated, rhythms yet.

Notice, at Figure 7a, that when the bass line is written in such a way as to tentatively forge independent, more autonomous rhythmic figures, a new layer of information erupts, one that departs from the immediate tutelage of the counting beat. Their sudden graphic autonomy forming a metric scale that begins at an eighth-note and reaches the doubled-dotted quarter-note shows the totality of a rhythmic figure not obviously subscribed to a part of the beat's metronomic pulsation.

This clearly points out to a privileged stance of parametric interconnectedness: the rhythmic figure carved out of the metronomic pulse might acquire an implicit role as potential new metronomic span. Here, a new paradigm is at play that although not contradicting the pulsation of the main beat or metronome marking, weakens nonetheless, the feeling of regularity and prospectiveness. Thus, they can be suddenly understood as irregular accretions, becoming somehow anti-intuitive for the performer and consequently, not necessarily being felt as part of the metric narrative of the metronomic beat.

b)


Figure 7: a) The rhythmic figure within a specif metronomic marking (bass line) might be seen as a potential new Metronome Marking (MM). b) Below the bass line detached from the sixteenth-notes' context shows a clear independence and can acquire the dimension of durational span itself.

These new durations might be considered as a quasi second-level of rhythmic deviation and are, in themselves, spans that might function as the base or reference for further deviation. In fact, the perception of fertile ground for deviation indicates that an independent terrain is available to insert novel rhythmic variations. They are the result of a cognitive inversion where the inner time of rhythmic units starts to be perceived as nodes of metric coordination, or small metronomic fields. What was initially seen as the very codification of purely gestural affairs, acquires now a privileged, metric, autonomy where new materials will reinstate with devious, clever, features, a subliminar metronomic pulse, reenergizing the underlying pertinence of the metronomic beat and the coherence of the musical.

Again, at Figures 7a and 7b the metronome marking placed below each rhythmic figure indicates, precisely, the amount of time necessary to cross a certain sum of sixteenth-notes under a specific, overarching, metronome marking. Thus, each MM can be understood within two immediate levels:
a) They reflect the speed of the sixteenth-note (taken here as a temporary figural unit) under a certain metronome marking (in this case quarter $=72$ );
b) They can also become in themselves, a sudden metronome, whose figural representation might be equally seen as a new quarter-note if there is a metric modulation to the span shown by them. For instance, a dotted eighth-note under the metronome marking of quarter $=72$, shows a (faster) sum of $M M=96$. Its figural representation is, therefore, of a dotted eighth-note. However, if I want to metric modulate to quarter $=96$, the figural representation of these three sixteenth-notes' aggregate that fill the dotted-eighth under $\mathrm{MM}=72$, ought to suffer a new figural representation when addressed as quarter $=96$ and, under such metronomic beat, it will be seen as an eighth-note triplet (Figure 8).


Figure 8: Metric equivalency between distinct rhythmic figures: a dotted-eighth $M M=72$ is rhythmically identical to a quarter $=96$. Thus, the sixteenth-note and the triplet share a similar rhythmic speed

There are three variables that work together to coordinate rhythmic hierarchies. At the primary level of duration, we will encounter the metronome marking variable, setting the overall speed of the (regular) beat, under which all other rhythmic elements will be subsumed. The second element is the meter (time signature) where a certain amount of rhythmic information will be squeezed into such that it creates perceptible (or countable) frames of reference to the fluency of musical materials. It functions as a type of sieve (filter) where groups of notes can comfortably fit and accommodate a certain number of beats, or part of it. It promotes the understanding of phrasings as well, as it helps to pack rhythm in diverse monads of energy, justifying the underlying waves of the text's semantic fluency and rationalizes the distribution of information. The third aspect is the musical figure (i.e., rhythm) as the residual consistency of periodic quantities whose fast parametric coordination exhibits the most condensed way to join spans of divergent speeds. It works by conjoining or atomizing a diverse array of durations derived from subdivisions of the main beat, forming small groupings of regular or irregular assemblages. These variables, seen initially as independent parameters, are co-dependent and work as the underlying fabric of our apprehension of time and durations. Figure 9 shows the parametric exchange between rhythmic figures.


Figure 9: Parametric exchange between rhythmic figures.
If every span derived from Figure 9 is understood as a new metronome marking, and not
uniquely as a rhythmic figure merely subsumed to the metronomic time, whose intrinsic duration covers a certain amount of sixteenth-notes, then we are forced to reconfigure the two other variables of meter and rhythm in order to create a new environment and presentation for that specific durational quantity. Thus, while the equivalency of durational quantities is maintained across two different rhythmic configurations (or metronome markings), their figural and metric representations are now re-contextualized under a new beat or metronomic speed.

Notice, at Figure 9, how we depart from the very tiny figure of the sixteenth-note under the metronome marking of $\mathrm{MM}=72$ to a first equivalency that transforms the speed of the note (that runs at a speed of 288 beats per minute, which is $4 \times 72$ ) into a new metric coordinate redressed as metronome marking. Not only that, we decided arbitrarily and for the sake of clarity, to suppose that this new metronome is relative to a quarter-note figure. (We certainly could have decided that it belongs to any other figural equivalency: either an eighth-note, a dotted-eighth-note, a whole-note, etc. And this would imply a distinct reconfiguration of meter and rhythm, obviously.) The new metronomic quarter-note that beats at the speed of 288 beats per minute, has to manifest itself under a new metric and figural representation. Having a metronome of 288 implies that the quarter-note is fitted under a specific metric frame that is subsumed to a clear metronomic correlation. Thus, I could choose any time-signature that has the capacity to encapsulate such quarter-note. I can fit it in a $1 / 4,2 / 4,3 / 4,4 / 4$, etc., bar, under $\mathrm{MM}=288$. I choose a $4 / 4$ bar to keep a strong equivalency to the metronome marking's span of 72 where 4 sixteenth-notes suffice to be encapsulated in the total metronomic span. The next figure which transforms the speed of the note into a new metronome, has similar characteristics, as it is strongly related to a regular part of the beat. The same operation ensues ( $2 \times 72$ or $288 / 2=\mathrm{MM}=144$ ). Here, the span acquired is equivalent to the exact span of 2 sixteenth-notes under MM=72. Again, I create a new quarter-note metronome and consequently a new metric frame of reference in order to fulfill an entire beat. My choice of bar is relative to context but is still strongly related to the beat hierarchy as it maintains a regular coordination with it. Again, under MM=144, I choose a $2 / 4$ bar, since a quarter-note in $M M=144$ fills the equivalent span of an eighth-note under $\mathrm{MM}=72$. But since I want to cover the $M M=72$ span $I$ add to the bar one extra quarter-note/beat. Next, I have a dotted eighth-note, an irregular quantity formed by the sum of 3 sixteenth-notes under $\mathrm{MM}=72$. The span of such figure is $\mathrm{MM}=96$ (MM $72 \times 4 / 3$ ). This span can also be converted into a quarter-note durational/metronomic span. Such choice is deliberate in order to exemplify a new exchange that happens this time at the level of the figure itself. When choosing to represent a dotted, irregular figure, into an equivalent one however subsumed to a regular representation (a quarter-note), I am obliged, as well, to reconfigure the rhythmic figure and the time signature in order to fit it within this new metronomic marking. So, if my new quarter-note metronome indicates a speed of MM $=96$ beats per minute, I have to fit 3 notes under this beat. When my metronome marking was $\mathrm{MM}=72$, I needed 3 sixteenths-notes to fill that figure. As my new metronome marking is quarter equals $\mathrm{MM}=96$, the three sixteenth-notes have to change their figural representation in order to acquire a metric correlation with the new metronome. Thus, if three equally spaced rhythms have to fit this new quarter, I can only place an eighth-note triplet as a valid figural equivalency with the speed of three sixteenth-notes under $\mathrm{MM}=72$.

The precedent demonstration brings us home towards the understanding of the equivalencies and discrepancies between rhythms and metric figures. It implies the parametric interchangeability as a temporal given under which the hierarchization of figures and speeds are subsumed to a flexible coordination of variables. When one aspect of the durational hierarchy is changed all others follow suit and exhibit a novel configuration either at the metronomic, metric or rhythmic levels.

Figure 10 illustrates a typical metric modulation between $\mathrm{MM}=60$ and $\mathrm{MM}=75$. To pass from


Figure 10: Realization of sharing possibilities between meter, rhythms and metronomes and calculating the parametric exchange between the components of duration.
the first MM to the next, it is necessary to understand how these two metronomes connect through a common denominator's speed. For that, it is necessary to factor each metronomic number in order to find how they are formed and their numerical similarities. Below, the resulting factorization of each MM: $\frac{60}{75}=\frac{2235}{355}$

Since we are going from $\mathrm{MM}=60$ toward $\mathrm{MM}=75$, we are trying to subdivide 60 into 75 . Or to be more transparent, we are trying to figure what type of operation will enable us to match their speeds. When numbers are factorized they exhibit the building blocks of their total span/speed. For example, 60 is first divided by 2, resulting in $=30$; then it is again divided by 2 , because 30 is still a multiple of 2 and therefore can be divided by it which results in $=15$. At this point the smallest prime factor of 15 is 3 , (not 2 ) which results in $=5$; finally, 5 is only divided by 5 , which brings the division to an end, as it results in $=1$. If you do the same operation for the denominator which is 75 , you will obtain the three lower numbers in the fraction above $(3,3,5)$. At this point we can see how these numbers match into each other. Thus, if a fraction contains 3 at the top and 3 at the bottom these numbers can be scratched off as they cancel each other. The same happens with the 5 located at both, the top and bottom of the fraction. What is left is the most condensed form of the fractional expression between both, numerator and denominator, which is $\frac{4}{5}$ because we can't eliminate the 2's in the numerator as there are no 2's in the denominator neither the 5 in the denominator for similar reason. The 2's in the numerator are then multiplied and become a 4. At the denominator, the only number left is the 5 as the other one was previously cancelled by its counterpart at the numerator. Now, we can clearly see through this fractional expression, that in order for one to go from $M M=60$ to $M M=75$ and even out their speeds, or, in other words, to match them, it is necessary to multiply (or accelerate) MM=60 by the $\frac{4}{5}$ fraction. That amount will suffice to find common rhythmic ground between both metronomic speeds. I want to use the example just discussed to call attention for the final figural representation of both bars shown. If I may use a bit of poetic license to illustrate the operation, I could describe it by saying that an invisible knob was turned so that $\mathrm{MM}=60$ is now suddenly seen as $\mathrm{MM}=75$. What happens next with the other parameters when basic speed information is changed? The metric machine is now obliged to convert meter and rhythm as well in order to convey the same relationships we found previously when our metronomic beat's pulse was at a slower speed. Note that at $\mathrm{MM}=60$ we have a meter/bar of $2 / 4$ (that could be either $8 / 16$ or $4 / 8$ ), and a rhythmic stream of quintuplet-sixteenths. On the side of $\mathrm{MM}=75$ we will encounter a completely different configuration for meter and rhythm. This points already to the fact that the rhythmic figures that exhibit dissimilarities in their presentation or configuration, are not necessarily implying discrepancy of speeds. An eighth-note under a certain metronomic speed or under a sub-ratio (which can be seen as another buried manifestation of a metronome) might be faster than a sixteenth note under a slower metronomic tempo or a comparative rhythmical (sub-ratio) speed.

There are many strategies for sharing rhythms and we could therefore program them in endless ways (Figures 11, 12, 13, and 14).


Figure 11: Micro-Metric Modulation: The exile of the metric in the dance of pulsation
The sharing strategy is a fundamental feature to understand the unfolding of the rhythmic parameter within the Western Music's practice. At Figure 11 we are showing the distributive aspect of factorization. This means that I can express the metronomic speed using any number's product within the smallest prime numbers found within a metronome's factorization. Without changing the total of the factored number it is possible to show any arrangement of numbers that presents the same result, if multiplied. This implies a very important feature inscribed in the factorization: that the amount of numbers found within it reveals the maximum levels of subdivision within which such number can be dismembered. Thus, at Figure 10, the factorization of both metronomes $(M M=40$ and $M M=63)$ shows the smallest prime numbers for each of these speeds: $\frac{40}{63}=\frac{2225}{337}$. Initially, the factorization shows that there are at least four layers of possibilities to account for in the distribution of the metronomic fraction. (These layers by the way, map the numerator into the denominator, as I will soon show.) No matter how we shuffle their order of appearance, we will have a maximum of fractional configurations conditioned by the metronome which presents a larger amount of numbers. To exemplify:
a) Layers of configuration and mapping: The fractional configuration that maps 40 into 63 can be expressed into a maximum of these four levels: $\frac{40}{63}=\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{7} \cdot \frac{5}{1}$. Shuffling imparts no alteration of results for mapping 40 into 63 . Thus, the above fraction could also have an alternate order: $\frac{40}{63}=\frac{2}{7} \cdot \frac{2}{3} \cdot \frac{5}{1} \cdot \frac{2}{3}$. Obviously, that this is a mathematical redundancy. But in musical terms some aspects of this mathematical nonsense can be extremely fertile as it will be soon shown, and express a new layer of information buried within a new metric shift, or configuration.
b) The distributive factor: Taking these numbers into account means that every arrangement of them is a valid configuration of the total of the metronome's product. At the first numeric row at the Figure 11, we can clearly see that we had distributed the factors of both metronomes into two fractions, each being a part of the total product found within their respective numbers. Since the order of the factor does not alter the final product, we are able to exchange their positions at will. Again, what is a mathematical redundancy becomes a fertile strategy to deal with rhythmic outcomes. Figure 12 illustrates the 4 possibilities of distribution found within the arrangement of the fractions. Note that there are 4 rotations for the fraction:
a) $\frac{8}{9} \frac{5}{7}$ First fractional rotation

b) $\frac{5}{9} \frac{8}{7}$ Second Fractional rotation

c) $\frac{5}{7} \frac{8}{9}$ Third fractional rotation

d) $\frac{8}{7} \frac{5}{9}$ Fourth fractional rotation


Figure 12: Fractional Rotations and novel rhythmic outcomes.

At the examples of Figures 11 and 12, all the operations are done in order to alter the speed of $\mathrm{MM}=40$ such that a certain rhythmic output is generated that maps this metronome's speed onto the other. However, such mechanism can be easily distributed between both metronomic speeds such that operations are equally shared between both sides of the metronomic chain. To recap, from $M M=40$ to $M M=63$ we kept the numerator as the departure metronome and the
denominator's speed as the arrival point. The arrival metronome didn't go through any type of metric alteration to match the departure metronome. In order to distribute the rhythmic layers between both we need also to create the conditions to map the $M M=63$ speed onto the $M M=40$ metronomic speed. As we invert the metronomic positions (numerator goes to denominator and vice-versa) we are able to forge a rhythmical bridge towards the slower metronome. $\frac{63}{40}=\frac{337}{2225}$. It is important to see that mapping one number onto another implies cancelling each other's till no number is left. Figure 13 shows some examples where both metronomes are rhythmically twisted such that the resulting speeds of each separate rhythmical configuration maps onto one another. This is the principle of Micro-Metric Modulation as we have more than one layer to map at the sub-ratio level and we are not necessarily addressing just metronomic changes.


$$
\frac{40}{63}=\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{7} \cdot \frac{5}{1} \quad \frac{63}{40}=\frac{3}{2} \cdot \frac{3}{2} \cdot \frac{7}{2} \cdot \frac{1}{5}
$$



Figure 13: Mapping strategies to bridge speeds between both sides of the rhythmic chain. a) The two-level ratio at the $M M=40$ maps onto the one level ratio at $M M=63$. Note that instead of simply matching the Metronome's speed at the other side/bar, the two-level figure at he first bar addressed the one-level, irregular, rhythmic figure, at the other side under a distinct metronomic speed. b) The one-level ratio under MM=40 maps onto the two-level ratio at $M M=63$. Note that the rhtyhm is distributed between both metronomic speeds. Beyond that, the metric figure at the first bar does not match a similar rhtyhmic figure at the other. Its eighth-note configuration does not match the quarter-note figure on the other side, even if their speeds are the same.

Whichever fraction is used in one side it is immediately cancelled or eliminated on the other. Above, the first two fractions at the $\frac{40}{63}$ side are used to form both ratios (3 eighth-notes in the space of 2 eighth-notes); thus, they are scratched at the $\frac{63}{40}$ side. At the second bar we used the $\frac{7}{5}$ fraction cancelling out these numbers at the left side ( $\frac{40}{63}$ side). Proceeding that way, all the numbers that map one metronome onto the other were used in a distributive fashion. Such operation opens up huge metric possibilities using an elegant formula to bridge rhythms that present distinct metric configurations. And this led us to the Micro-Metric Modulation perspective/theory.

The method for finding similar speeds or a numeric equivalency between rhythms is clearly demonstrated by a simple mathematical device: the commutative and associative properties. Any number can be dismembered in smaller multiples. As seen in the previous examples, when fractional representation of larger sums were shown as the product of smaller quantities it became clear that in order to bridge and match rhythmic speeds, it was necessary to coordinate variables found within the metronomic pulse, the meter, and the metric figure context in order to bridge speeds and irrespective figural configurations. Mathematically, the commutative and associative property guarantees that the order of factors will not alter the final product. Therefore, [4 $\times 5 / 6$ ] $=[4 \times 6 / 5]=30$ for both. The final product of both equations can be easily seen represented in Figure 14.


Figure 14: Commutative and associative properties applied to rhythm: equivalency of speeds on both sides of the ratio chain.

At the first quintuplet figure the following result was produced: MM60 x 5 quintuplet-sixteenths $=\mathrm{MM}=300$ for each "leg" of the quintuplet sixteenth. Then, we need to acquire the span of 2 quintuplet-sixteenths to fit 3 sixteenth-note triplets within the same space. The span will be twice as slower $300 / 2=150$. After that, we multiply the span of $150 \times 3$ sixteenth-note triplets $=450$. That's the maximum speed reached by the first quintuplet figure. The same is done with the next 6:4 ratio. MM60 $\times 6=360 / 4$ (to calculate the span of 4 sextuplet-sixteenths $=90$. Within this span we need to fit 5 quintuplet-sixteenths. $90 \times 5=45$ which again confirms the matching of the sub-ratios' speed at both sides of the ratio chain. The operation illustrated above, a typical example of Micro-Metric Modulation, exemplifies some of the most important attributes of rhythmic/metric unfolding:
a) The common route or rhythmic bridge between apparently differing configurations is able to link rhythmically these configurations, offering enough ground for further subdivisions to occur since all the rhythms conjoined by the total span located between both rhythmic configurations can be subsumed to rhythmic deviation, or new metric subdivisions.
b) That in order to enter the next ratio (6:4) a new perspective is given to the performer. She/He has to be able to lift the overall metronomic pulse to blindly enter the next sub-ratio and consequently, the terrain of the next rhythm, attaching him/herself to the speed generated by the sixteenth-note triplets located under the first quintuplet figure. That way the performer can be sure that rhythmic precision linking both configurations is attained. As she/he enters the next rhythm through a sub-ratio the performer will be forced to reconfigure the span to be crossed. In the above case, when she/he enters the $6: 4$ ratio at the right through a sub-ratio of quintuplet sixteenths, it will be necessary to figure the "emergent/temporary metronome" of a quarter-note since she/he is crossing exactly 4 sixteenth-notes belonging to the 6:4 ratio above. At this point, there is a strategic counting inversion taking place. In order
to reach back to the rhythmic surface the performer has to understand that the emergent quarter-note just crossed indicates the rhythmic equivalent of 4 sixteenth-notes under a 6:4 ratio. But being currently subsumed to a subdivision that fits the surface's ratio rhythmic grid, it becomes easier to calculate the speed of the last two sixteenth-notes at the end of the 6:4 ratio - as they belong to the surface level of the very ratio (first level of subdivision).

The theory of Micro-Metric Modulation has a huge wealth of resources to bridge dicothomic figures of regular or irregular configurations as it calculates common rhythms with corresponding speeds. The most important feature opened up by such technique, one that brings immense possibilities to the development of Western Music's rhythmic canon, relates to the passage of sub-ratios between differing ratios' configurations. That is a first in Western Music showing the flexibility not yet thoroughly exploited within the musical metric system. I would like to end this essay with an example of this very last, unique rhythmic feature, I used in several of my pieces. ${ }^{2}$

In the piece "...B..." for 10 instrumental soloists, video and electronics premiered in 2012 in Darmstadt by the Linea Ensemble of Strasbourg, it is possible to notice the passing of sub-ratios within sub-ratios as the common rhythm between these configurations was known (Figure 15a and 15 b). First an isolated case to make the feature clear and subsequently the first metric page of the score where these novel rhythmic techniques are used simultaneously by the instrumental forces (Figure 16).


Figure 15: Fragment of "... B. . ." for 10 instrumental soloists, video and electronics (2012) from the horn and the trumpet parts. Premiered by Linea Ensemble of Strasbourg - Darmstadt (2012). Differently from the crossing of the sub-ratio at the top staff, the bottom staff places its sub-ratio of three sixteenth-notes triplet crossing two distinct metric figures: under the 7:6 ratio (where the triplet starts) we see a sixteenth-note rhythm (shown by the little pause above). On the other side at the top ratio configuration of 7:4, we see a dotted sixteenth-note (shown as well by a dotted sixteenth-note little pause). After factoring both top ratios, mapping one onto the other (as it was shown on previous examples), an important feature of MMM is foregrounded: one that clearly demonstrates the rhythmic flexibility of the figure, proving its rhythm is relative to context and not a cemented, given, rhythmic speed: both notes while exhibiting a diverse rhythmic configuration, have, nonetheless, the same speed.
${ }^{2}$ For detailed assessment of similar resources in other pieces, see [1], [2], [3], [4], [5], and [10],

Figure 16: First metric page of ". . B..." for 10 instrumental soloists, video and electronics (2012). Premiered by Linea Ensemble of Strasbourg - Darmstadt

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    ${ }^{1}$ The system and concepts presented in this paper are developed from the author's theoretical work on rhythm (i.e., Micro-Metric Modulation), and reflect some of his compositional and poetic practices (see [6] [7] [8] [9]).

