Time and Reversal in Birtwistle's
Punch and Judy

ROBERT W. PECK

Louisiana State University

rpeck@lsu.edu

Orcid: 0000-0002-6226-1631

Abstract: *We examine the occurrence of peripeteia in Harrison Birtwistle's 1967 opera* Punch and Judy, *as manifest in a reversal of cyclic time. Specifically, we extend a metaphorical association between the passage of cyclic time in the opera and discrete rotation in the complex plane generated by the imaginary unit* i. *Such a rotation moves alternately between the real and the imaginary axes, as scenes in the opera pass correspondingly through sacred and profane orientations. The instance of peripeteia results in a counter rotation, a dramaturgical inversion. To bring this reversal into the metaphor, we extend it from its situation in the complex plane to one in the space of Hamilton's quaternions, wherein such negation is obtained through the product of upper-level imaginary units. The scene that contains the reversal and that which consists of the opera's comic resolution epitomize the drama and occupy the highest level of dramatic structure.*

Keywords: *Birtwistle. Inversion. Peripeteia. Complex numbers. Quaternions.*

I. INTRODUCTION

The operation of inversion is of central importance in many music theories, dating at least to Guido d'Arezzo, who, in the eleventh century, likened melodic inversion to a reflection: "Note that when a neume traverses a certain range or contour by leaping down from high notes, another neume may respond similarly in an opposite direction from low notes, *as happens when we look for our likeness confronting us in a well*" ([9], emphasis added).[1] Like Guido, modern music theories have generally regarded inversion as a reflection (e.g., pitch-class inversion is often depicted as a reflection through an axis on a mod-12 clock face). More recently, Guerino Mazzola [16, p. 44] describes inversion not as a reflection, but in terms of a gesture that is a 180° rotation in the complex plane. Mazzola depicts this gesture as "leafing" (as in turning a page in a book). He writes, "Leafing turns the original figure to its mirrored version. The point is that instead of mirroring x to $-x$...lift it into a new dimension and rotate the point until it comes down to $-x$." In regard to the complex plane, such a gesture passes through an imaginary dimension.

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[1]In the original text: "*Uel lineam neuma una facit saliendo ab acutis. Talem altera inclinata eregione opponat. Respondendo a grauibus. sicut fit cum in puteo nos imaginem nostram contra spectamus.*"
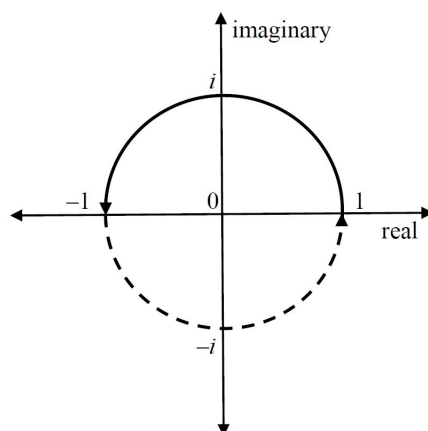
Figure 1: *Inversion as a half rotation in the complex plane.*

Mazzola's gestural interpretation situates inversion—or, more generally, negation—in the context of a cycle. Figure 1 illustrates this process: starting at the unit 1 on the real axis, the cycle passes through the imaginary unit i on its way to the negative unit -1, therewith obtaining inversion (negation). It continues through the negative imaginary unit $-i$, returning to 1 and completing the cycle (thus negating the negation). Hence, i is a square root of -1, which is in turn the square root of 1. Similarly, $-i$ is also a square root of -1. It generates the reverse [inverse] cycle of i. Such cycles may be continuous, as suggested by the circular path in Figure 1, or discrete, consisting of a finite number of points (e.g., 1, i, -1, and $-i$).

One may accordingly model musical inversions in terms of cycles, either continuous or discrete.[2] By not limiting ourselves to operations in pitch and pitch-class spaces, a more general interpretation of inversion as a process of negation yields a richer set of musical experiences for analysis. One such approach applies a cyclic interpretation of inversion to a dramaturgical context, specifically to that of Aristotle's concept of *peripeteia* (reversal of a situation). In *The Poetics*, Aristotle [2, p. 72] describes peripeteia as an integral part of tragic plot. "Peripeteia is a change by which the action veers round to its opposite, subject always to our rule of probability or necessity,"[3] the quintessential example's being Oedipus's reversal of fortunes in *Oedipus Rex*. Peripeteia, Aristotle notes, is attended by *anagnorisis* (or, recognition): "Recognition, as the name indicates, is a change from ignorance to knowledge, producing love or hate between the persons destined by the poet for good or bad fortune."[4] In Sophocles' play, it is Oedipus's recognition that his wife, Jocasta, is his own mother that precipitates the peripeteia.

In this study, we examine peripeteia as manifest in Harrison Birtwistle's 1967 opera *Punch and Judy*, with libretto by Stephen Pruslin,[5] in terms of a metaphorical association with the processes of negation described above. The opera—"a tragical comedy or a comical tragedy"[6]—is replete with cycles: cycles of seasons and zodiacal signs, times of day and hours on the clock,

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[2]For instance, [18] studies pitch-class inversion in terms of discrete cycles.

[3]"Ἔστι δὲ περιπέτεια μὲν ἡ εἰς τὸ ἐναντίον τῶν πραττομένων μεταβολὴ καθάπερ εἴρηται, καὶ τοῦτο δὲ ὥσπερ λέγομεν κατὰ τὸ εἰκὸς ἢ ἀναγκαῖον."

[4]"Ἀναγνώρισις δέ, ὥσπερ καὶ τοὔνομα σημαίνει, ἐξ ἀγνοίας εἰς γνῶσιν μεταβολή, ἢ εἰς φιλίαν ἢ εἰς ἔχθραν, τῶν πρὸς εὐτυχίαν ἢ δυστυχίαν ὡρισμένων· καλλίστη δὲ ἀναγνώρισις."

[5]At the time of *Punch and Judy*'s composition, Pruslin was pianist in Birtwistle's and Peter Maxwell Davies's new-music ensemble The Fires of London.

[6]The opera's subtitle, "a tragical comedy or a comical tragedy," is adapted from the title to an 1828 script by John Payne Collier: *The Tragical Comedy or Comical Tragedy of Punch and Judy*, illustrated by George Cruikshank [5].

compass directions, colors on a color wheel, and so on. It also features a large-scale instance of dramaturgical inversion. The character Punch undergoes an Ebenezer Scrooge-like transformation from villain to hero—or, perhaps Punch remains essentially the same, and it is the world he inhabits that becomes so-inverted.[7]

Birtwistle's music incorporates a number themes, many of which are in evidence in *Punch and Judy*. Several authors have discussed the roles of dramaturgy, symmetry, cyclic structures, spatial imagery (motion), geometrical interpretation, ritual and sacrifice, and order and randomness throughout his oeuvre [12], [7], [8], [1], [3], [4]. Among these topics, the treatment of time—especially regarding the recurrence of time and the reversal of time—is particularly salient. Among Birtwistle's works that engage significantly with aspects of time are *Refrains and Choruses* (1957), *Précis* (1960), *Tragoedia* (1965), *Chronometer* (1971–72), *The Triumph of Time* (1972), *The Mask of Orpheus* (1983), *Pulse Field* (1977), *Bach Measures* (1996), *Pulse Shadows* (1996), and *Exody '23:59:59'* (1997).

II.   *Punch and Judy*

Punch and Judy is a one-act opera of approximately 100 minutes. It is based loosely on the popular glove-puppet shows, which have remained a vital part of British seaside culture since at least the eighteenth century. These puppet dramas feature the antics of the principal character, Mr. Punch, in his (usually violent) interactions with various other puppets. In the opera, the principal characters are as follows:

- Punch, a puppet, and an infantile, sadistic mass murderer.
- His wife Judy, who is Punch's first murder victim. She appears later in the opera in the guise of a Fortune Teller.
- The pair of characters Doctor and Lawyer, who always appear together. The original Punch and Judy puppet shows occasionally included a lesser character named The Doctor; the Lawyer here seems to be Birtwistle and Pruslin's invention.
- Pretty Polly, a minor character and Punch's mistress in the classic Punch and Judy puppet dramas, and the object of Punch's infatuation in the opera. She also appears as a Witch in one scene.
- Perhaps most significantly, a character named Choregos, a Greek term that refers to a choral impresario in Athenian theatre. In *Punch and Judy*, Choregos fills the role of the Greek chorus; he is also the one character in the opera who is not a puppet. Choregos also appears later in the opera as Jack Ketch—a lesser character from the original Punch and Judy puppet dramas, patterned on a real-life, infamously brutal executioner in England in the seventeenth century by the same name. Jack Ketch's role in the opera is ambiguous: is he a villain, an instrument of the law, or both?

By including the character of Choregos, Birtwistle and Pruslin appear to be interpreting Aristotle's word in *The Poetics* literally: "The Chorus too should be regarded as one of the actors; it should be an integral part of the whole, and share in the action, in the manner not of Euripides but of Sophocles" [2, p. 92].[8]

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[7]Another interesting example of inversion in the context of puppet drama appears at the end of Heinrich Kleist's "On the Marionette Theatre" [14]. In an allegory for deification (transformation from puppet to god) at the end of this essay, Kleist describes the process of inversion that occurs as one approaches a concave mirror: the reflected image appears inverted at a distance, stretches into infinity at the focal point, and reappears upright up at a closer distance.

[8]"Καὶ τὸν χορὸν δὲ ἕνα δεῖ ὑπολαμβάνειν τῶν ὑποκριτῶν, καὶ μόριον εἶναι τοῦ ὅλου καὶ συναγωνίζεσθαι μὴ ὥσπερ Εὐριπίδῃ ἀλλ᾽ ὥσπερ Σοφοκλεῖ."

The plot of the opera derives only loosely from the original puppet dramas, which were not scripted, but often formulaic.[9] It might be summarized as follows:

- Punch throws his baby into fire and murders Judy.
- Punch begins stalking Pretty Polly.
- Punch murders Doctor and Lawyer.
- Punch continues stalking Pretty Polly.
- Punch murders Choregos.
- Punch has a nightmare in which all the characters he has murdered thus far intend to murder him.
- Punch awakens and resumes stalking Pretty Polly.
- Punch murders Jack Ketch, the evil alter ego of Choregos.
- Punch wins Pretty Polly, and they live happily ever after.

As we will see, the superficial aspects of the plot as described here are only part of the story that Birtwistle and Pruslin are telling. Friedrich Nietzsche writes in *The Birth of Tragedy*: "Everything which comes to the surface in the Apollonian part of Greek tragedy, in the dialogue, looks simple, translucent, beautiful"[10] [17, p. 67]. Another dimension to this opera exists—the Dionysian part, as typically characterized by the chorus in Greek drama—that is even more significant. As Nietzsche says: "We must understand Greek tragedy as the Dionysian chorus which over and over again discharges itself in an Apollonian world of images" [17, p. 58]. He continues, "Those choral passages interspersed through tragedy are thus, as it were, the maternal womb of the entire dialogue so-called, that is, of the totality of the stage world, the actual drama."[11] Pruslin writes: "[*Punch and Judy*] is an opera in quotation marks" [19, p. 7]. As such, the real drama of *Punch and Judy* takes place not on the stage, but rather in the opera's dramaturgical context.

Literary theorist Gabriel Josipovici describes *Punch and Judy* as "ancient Greek drama in the guise of popular puppetry" [13]. Jonathan Cross explores the ancient Greek basis for *Punch and Judy* in greater detail, including its incorporation of peripeteia as a structural device [7, 8]. Figure 2 is reproduced from [7].[12] Specifically, reading left-to-right and top-to-bottom, it demonstrates how the opera contains a number of passes through various recurring scenes: Melodrama, Passion Chorale, and Quest for Pretty Polly. The scenes appear in that order in the initial iterations, but their sequence inverts in the final iteration, in which Punch's fortunes also reverse. Cross gives the Nightmare scene as the work's instance of structural peripeteia. Cross goes further to suggest that the moment of anagnorisis that occasions this peripeteia is Punch's recognition in the nightmare of Judy—whom he has purportedly already murdered—disguised as a Fortune Teller. "At the height of the tarot game Punch literally recognizes Judy—she reveals herself; she is unmasked" [8, p. 76].

British conductor and music critic Michael Hall offers another account [12], which is somewhat more in keeping with the dramaturgical basis we are establishing here. Hall invokes Nietzsche's dualism between Apollonian and Dionysian forces in relation to the characters of Punch and Choregos. Nietzsche writes:

> Let us think about our own surprise at, and unease with, the chorus and the tragic hero of those tragedies, both of which we did not know how to reconcile with what we

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[9]Collier presents an account of a typical Punch and Judy show "[a]s told to John Payne Collier by Giovanni Piccini in 1827"[5]

[10]"*Alles, was im apollinischen Theile der griechischen Tragödie, im Dialoge, auf die Oberfläche kommt, sieht einfach, durchsichtig, schön aus.*"

[11]"*Nach dieser Erkenntniss haben wir die griechische Tragödie als den dionysischen Chor zu verstehen, der sich immer von neuem wieder in einer apollinischen Bilderwelt entladet. Jene Chorpartien, mit denen die Tragödie durchflochten ist, sind also gewissermaassen der Mutterschooss des ganzen sogenannten Dialogs d.h. der gesammten Bühnenwelt, des eigentlichen Dramas.*"

[12]Our Figure 2 appears as Figure 1 in [7].

Prologue

Melodrama I | Passion | Quest for Pretty
Chorale I | Polly I

Melodrama II | Passion | Quest for Pretty
Chorale II | Polly II

Melodrama III

Nightmare
Quest for Pretty
Polly III

Passion
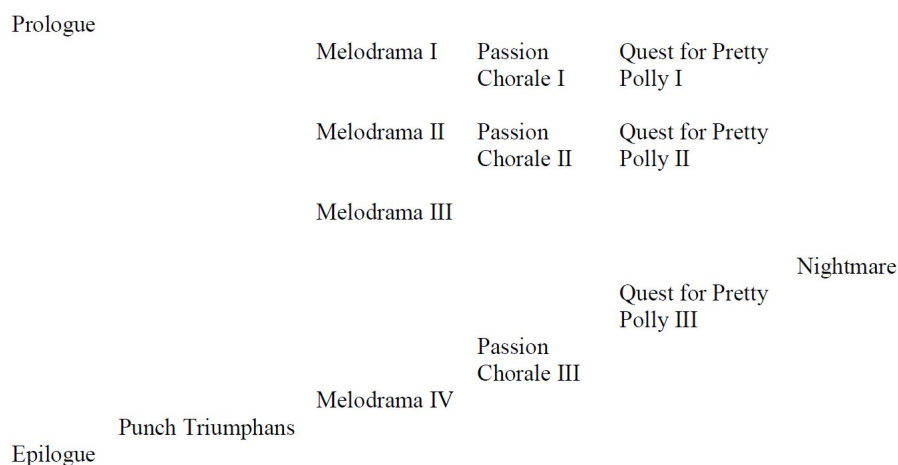Chorale III

Melodrama IV
Punch Triumphans
Epilogue

Figure 2: *Punch and Judy, Overall Design [7, p. 204].*

are used to, any more than with the tradition—until we again recognized that duality itself as the origin and essence of Greek tragedy, as the expression of two artistic drives woven together, the Apollonian and the Dionysian.[13] [17, p. 75]

According to Hall, Punch, the tragic hero, represents Apollonian individualism, whereas the chorus, embodied in the character of Choregos, represents Dionysian universalism. Just prior to his nightmare, Punch murders Choregos, which precipitates a nightmare; then, in the Nightmare scene, he envisions his own execution at the hands of all the characters he has murdered to that point. Hall suggests that Punch's killing of Choregos disrupts in a Jungian sense the balance of his own ego and the collective unconscious. To restore the balance, Punch *recognizes* unconsciously that he too must die, bringing on the substance of his nightmare. In Hall's interpretation, as in Cross's, the dream is the locus of the anagnorisis, the recognition that results in the peripeteia. For Hall, however, the peripeteia is manifest merely in terms of the plot—for instance, in the reversal of Punch's fortunes with Pretty Polly—not in the ordering of scenes.

Our reading here combines certain aspects of Cross's and Hall's analyses: in particular, Cross's idea that peripeteia has a structural manifestation with regard to the order of scenes, and Hall's Jungian notion that anagnorisis is associated with the murder of Choregos and Punch's resultant nightmare. However, we find a further, deep connection between the two aspects beyond those that these authors address.

III.   Time in *Punch and Judy*

The repetitive, cyclic nature of Birtwistle's score, and particularly of Pruslin's libretto, situates the drama in what we might call *time-outside-time*. The passage of time in the opera is cyclical rather than linear, following solar, lunar, and similar patterns; plot events and their musical settings recur correspondingly. The Romanian anthropologist and historian of world religions Mircea Eliade likens such natural cycles to repetitions of the cosmogonic act, the original act that gave birth

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[13]*"Denken wir an unsere eigene Befremdung dem Chore und dem tragischen Helden jener Tragödie gegenüber, die wir beide mit unseren Gewohnheiten ebensowenig wie mit der Ueberlieferung zu reimen wussten - bis wir jene Doppelheit selbst als Ursprung und Wesen der griechischen Tragödie wiederfanden, als den Ausdruck zweier in einander gewobenen Kunsttriebe, des Apollinischen und des Dionysischen."*

to the world, to the cosmos. "The creation of the world, then, is reproduced every year" [11, p. 62]. Eliade refers to this conception of time-outside-time as "*in illo tempore*" (a Biblical Latin term, meaning "in that time")—the continually renewable and renewing time of ritual, of sacred acts. He writes, "Any ritual...unfolds not only in a consecrated space...but in a 'sacred time,' 'once upon a time' (*in illo tempore*, *ab origine*)" [11, p. 21].

Eliade links the ritual of sacrifice with the ritual of creation; both are embedded in a culture's collective memory [11, pp. 74–75]. Among numerous examples in world religions, he notes that the Judeo-Christian site Golgotha (meaning "place of the skull," and also known as Mount Calvary) is simultaneously the place where Adam was created and buried (the skull in "place of the skull" is Adam's) and the location of Christ's crucifixion on the Cross. For Eliade, it is an example of "The Sacred Mountain—where heaven and earth meet...situated at the center of the world" [11, p. 12]. This sort of association does not appear to have escaped Birtwistle and Pruslin: at the end of the opera, Jack Ketch's hangman's gallows is transformed into a maypole. On the one hand, we have, in effect, a sacrificial altar—on the other, a symbol of vernal rebirth. These two poles, then, describe Eliade's *axis mundi*, the Center of the world.

The passage of time in the opera is discrete in terms of its global organization. The successions of sections are not segued dramatically; one does not proceed seamlessly into another. In certain sections, such as in the Passion Chorales, time stands still: a "continual present," which "completely ignores what is especially characteristic and decisive in a consciousness of time" [11, p. 86]. Within other sections, as in the Melodramas and Quests for Pretty Polly, is a greater sense of a temporal continuum—these sections are driven by their own internal dramas, but they are not linked to their surrounding material. The overall affect is similar to that of Karlheinz Stockhausen's concept of Moment Form [20], particularly as described by Jonathan Kramer [15], except that the non-continuous arrangement of sections in *Punch and Judy* does have a very specific function.

In his two published analyses of the opera, Cross notes the ordered repetition of the sections Melodrama, Passion Chorale, and Quest for Pretty Polly, and their ultimate reversal. However, his discussion omits certain sections that serve as integral parts of the picture: specifically, sections that Edward T. Cone might describe as elements of the frame [6]. Figure 2 above includes the framing sections Prologue and Epilogue—incidentally, both sung by Choregos—but it does not include the following instrumental interludes: the Sinfonia between Quest for Pretty Polly I and Melodrama II, and the interlude, "A Little Canonic Prelude to Disaster," between Quest for Pretty Polly II and Melodrama III. Adding these sections into the succession of scenes, we have the following sequence (Figure 3).

The sung and instrumental framing sections have a function similar to that of the Center in Eliade's world view. They are sacred places of origination—time-outside-time; they are the domain of Choregos, of universalism. Likewise, the Passion Chorales exist on the *axis mundi*. They are also the domain of Choregos: they are sung by him and by the rest of the cast, save for Punch and Pretty Polly—the "Chorus," as is indicated in the score. Further, from an aesthetic point of view, Pruslin writes that the presence of the Passion Chorales in the work is an indication of "an overt debt to Bach," and to the St. Matthew Passion in particular [19, p. 7]. One might argue that this allusion to Christ's sacrifice situates the Passion Chorales on the same axis as the sacred, framing places of origination—the Cross and Adam's skull, respectively: the hangman's gallows and the maypole. Moreover, the Passion Chorales are themselves framed by instrumental toccatas, linking them further with the framing material.

Prologue	Melodrama I (murder of Judy)	Passion Chorale I	Quest for Pretty Polly I	
Sinfonia	Melodrama II (murder of Doctor & Lawyer)	Passion Chorale II	Quest for Pretty Polly II	
Interlude	Melodrama III (murder of Choregos)			Nightmare
			Quest for Pretty Polly III	
		Passion Chorale III		
	Melodrama IV (murder of Jack Ketch)			
Punch Triumphans & Epilogue				

Figure 3: *Sequence of scenes (modified from [7]).*

Let us now examine the texts of the first two Passion Chorales:

PASSION CHORALE I (Choregos and Chorus):
Day murdered fame one game lost
Dreamer dread flaming lust
Deforming lameness
Deaf or nameless
Demon dared
Dam-ned
Dumb

PASSION CHORALE II (Choregos and Chorus):
Two times too lost four her sake
Totem stool for hearse ache
Tempest quicksilver
Tempts evil hearts
Tambour vile
Tumbril
Tomb

Both these chorales exhibit a subtractive syllabic process: they begin with lines of seven syllables, and decrease line-by-line to one syllable (one with a rather negative connotation). The third Passion Chorale, following the peripeteia, is different: it features an additive process, building from one syllable (one with a very positive intention) to seven. Structurally and aesthetically, it is the inverse of the previous two.

PASSION CHORALE III (Choregos and Chorus):
Love
Live on
Liven leaf

Life heaven feel
Lost haven unveil
Lust ever unavail
Last eve arun adumbrate

The additive process of the third Passion Chorale resembles remarkably the structure of the Adding-Song, which serves as the climax of the Nightmare scene. In the Adding Song, Choregos and the Chorus—in a call-and-response manner—enumerate various "tricks" that they intend to "treat" on Punch, as reparations for the treats he has tricked on them. Beginning with a single trick, they add tricks progressively until they reach seven, the same number as syllables involved in the additive and subtractive processes of the Passion Chorales.

ADDING-SONG (from NIGHTMARE SCENE) (Choregos and Chorus):
a fractured skull
a fractured skull, a bleeding face
a fractured skull, a bleeding face, a severed limb
a fractured skull, a bleeding face, a severed limb, an oozing eye
a fractured skull, a bleeding face, a severed limb, an oozing eye, a twisted neck
a fractured skull, a bleeding face, a severed limb, an oozing eye, a twisted neck, a gangrene foot
a fractured skull, a bleeding face, a severed limb, an oozing eye, a twisted neck, a gangrene foot, a burning sore

These treats with which you tricked us
We'll now treat as tricks on you!

Just as the Passion Chorales are associated with sacrificial death, the Adding-Song has a sacrificial aspect: Punch must die to restore balance, to realign Apollonian and Dionysian forces. If the Passion Chorales exist on the *axis mundi*, it would appear that so too does the Adding-Song, and, by extension, the entire Nightmare scene it epitomizes. Figure 4 incorporates these further modifications to Cross's original figure.

Rather than in tabular form, we can represent the cyclic passage of time suggested above in another way (Figure 5), using as a metaphor the illustration of negation as a half-rotational gesture in the complex plane from Figure 1. The discrete cycle of the imaginary unit i describes four points, passing alternately through the real and imaginary axes. Now, replace the real axis with the *axis mundi*. Map the unit 1 to the framing material—i.e., to the origin, to the bed of creation. Map the negative unit -1, also on that axis, to the Passion Chorales (including the Adding-Song)—to ritual sacrifice, to the paradox of life from death. These points lie on the sacred axis, the axis of true and enduring reality, of time-outside-time. It is the domain of Choregos, the one "real" character, who is not a puppet.

Likewise, replace the imaginary axis with the profane axis. This is the illusory axis of day-to-day existence; it is the puppet's axis. Map the imaginary unit i to the Melodramas, and map the negative imaginary unit $-i$ to the Quests for Pretty Polly. The surface, Apollonian elements of the plot describe paths that appear to direct the drama through the profane dimension. They are driven initially by Punch's obsessive-compulsive needs: the drive to murder (in direct contrast to regenerative sacrifice), alternating with his lustful ambition for Pretty Polly (a mockery of nuptial sacrament). Following the peripeteia, everything veers round to its opposite, including

Axis mundi	Profane axis	Axis mundi	Profane axis
Prologue	Melodrama I (murder of Judy)	Passion Chorale I	Quest for Pretty Polly I
Sinfonia	Melodrama II (murder of Doctor & Lawyer)	Passion Chorale II	Quest for Pretty Polly II
Interlude	Melodrama III (murder of Choregos)	Nightmare (Adding Song)	
			Quest for Pretty Polly III
		Passion Chorale III	
	Melodrama IV (murder of Jack Ketch)		
Punch Triumphans & Epilogue			

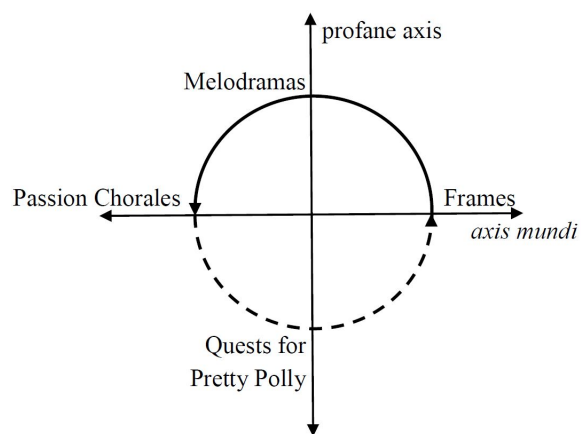**Figure 4:** *Sequence of scenes (modified from Figure 3).*

Figure 5: *Dramaturgical cycle.*

these seemingly ordered paths. The drive to murder becomes the need for justice, hence, Punch's execution of the villain Jack Ketch; and licentious pursuit becomes sincere courtship, with Punch's ultimately winning Pretty Polly.

IV. A Four-Dimensional Model

This metaphor, while assimilating the periodic passes through -1, does not yet address the hierarchical distinction between the Passion Chorales on the one hand, and the Adding-Song—which results in the peripeteia—on the other. Further, it does not account for the section Punch Triumphans, which appears in tandem with the final element of the frame, the Epilogue. Like the Adding-Song, Punch Triumphans seems to function on a higher architectonic layer. To bring these aspects into the metaphor, we add another level of structure that incorporates the space of the quaternions.

The quaternions, $\mathbb{H}$, discovered in 1843 by Irish mathematician William Rowan Hamilton, extend the idea of the two-dimensional complex numbers to a four-dimensional space: one real dimension and three imaginary dimensions. Their algebra is characterized by Hamilton's famous equation:

$$i^2 = j^2 = k^2 = ijk = -1 \tag{1}$$

in which i, j, and k are all imaginary units: they are all square roots of -1. (Further, each has its own unique negative, which is also a square root of -1.) As they lie in different dimensions from one another, the respective cycles of i, j, and k visit points on the circumferences of three mutually perpendicular circles that intersect in 1 and -1. Hence, the unit quaternions lie on the surface of a hypersphere. Further, the nodes of the discrete cycles generated by i, j, and k describe eight points on the hypersphere's surface: $\pm i$, $\pm j$, and $\pm k$ These eight points comprise a finite mathematical group, Q_8.

Each iteration of the dramaturgical cycle contains consecutive subsections titled "Travel Music," "Weather Report," and "Punch's Serenade." These subsections establish a temporal and spatial orientation for the iteration, as well as a section of a color wheel, such as appears in the following text of Travel Music I, Weather Report I, and Punch's Serenade I.

TRAVEL MUSIC I
Punch is seen traveling on Horsey in a picture frame on the Murder Altar.
Choregos (at his booth):
Suspended between Heaven and Earth.
Punch travels eastward to the land of eternal innocence.
Under the sign of the Crab.
Punch serenades his beloved on a shining summer afternoon.

WEATHER REPORT I
Punch descends and assumes a frozen serenade-pose at Pretty Polly's Pedestal.
Chorus:
3-o'clock east. 3-o'clock east.
A tempest swept by, then suddenly ceased.
3-o'clock east. 3-o'clock east.

PUNCH'S SERENADE I
A green spotlight reveals Pretty Polly dancing mechanically around her pedestal in disregard

of Punch. Toward the end of Punch's Serenade, Choregos approaches Pretty Polly as Punch's intermediary and offers her a huge sunflower.
Punch:
The world is blinded by lightening of green.
Silence and sounds and song of flaming green.
Greenness of sun and greenness of moon.
Green, how I long for you flaming in green.

Travel Music I, Weather Report I, and Punch's Serenade I situate the first iteration in the east (or in an eastward direction), under the astrological sign of the Crab (i.e., at 90° longitude), in the summer, at 3-o'clock, in the hue of green. The subsequent iterations of the dramatic cycle feature corresponding subsections with other coordinates, directions, and colors. It is as if the drama navigates a new, colored path in space/time in each of the iterations. We might say that these each of these paths visits a different imaginary (profane) dimension of our quaternion system: in dimension *i*, Punch murders Judy; in dimension *j*, he murders the Doctor and Lawyer pair; and in dimension *k*, he murders Choregos. In the reversed fourth iteration, he murders, or executes, Jack Ketch, Choregos's evil alter ego. This reversed iteration also moves through dimension *k*, but via the cycle of $-k$.

Let us call each of these moves into the respective imaginary dimensions a *hyper-operator* (as indicated with angle brackets, e.g., $\langle i \rangle$). Such hyper-operators are higher-level structures, middleground versions of the (imaginary) units that generate their constituent foreground cycles. They are not a feature of the algebra of the quaternion group; rather, they reproduce its structure on a higher architectonic layer. Essentially, a hyper-operator functions as a collective memory of the cycle's impetus, a myth of its purpose. The first few cycles are initiated by Punch's murders of various characters, by sinful motivations. Such collective memories become myths that ultimately require the expulsion of evil through sacrifice [11, pp. 74–75]. The final cycle is different: it leads to Punch's triumph over Jack Ketch, a heroic feat. These collective memories are linked with creation myths, where heroes (along with gods, etc.) are among the originators of celestial archetypes for profane activities [11, pp. 28–34].

In addition to the equivalence to -1 of the squares of the three imaginary units, Hamilton's equation provides an additional relation that characterizes the algebra of the quaternions: that the product of *i*, *j*, and *k*, in that order,[14] is equal to -1. The cycle of *k* in the third iteration is left incomplete; the Passion Chorale is not possible, because Choregos is now dead. Yet, we might say that the initiation of the cycle is sufficient to establish its trajectory, to imply its continuation. If so, we have an upper-level sequence of hyper-*i*, hyper-*j*, and hyper-$(-k)$, the quaternion product of which is hyper-(-1) (see Figure 6). It is this deep-middleground or background -1 that gives structural significance to the Nightmare scene, and in particular to the Adding-Song, which appears in the Passion Chorale's stead. However, this interpretation is predicated on the notion that the initiation of the cycle is sufficient to imply its completeness. To quote Ebenezer Scrooge [10, p. 70], "Are these the shadows of the things that Will be, or are they shadows of things that May be, only?" We next explore an alternate interpretation.

Consider the first two cycles in Figure 6, those that advance the drama into dimensions *i* and *j*, respectively. In quaternion algebra, the product of *i* and *j* (performed in that order) equals *k*. Accordingly, we may describe hyper-*k* as the product of hyper-*i* and hyper-*j* (Figure 7). Next, consider the final, reversed iteration, in which Punch executes Jack Ketch: this iteration

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[14]To equal -1, the units *i*, *j*, and *k* must multiply in that order or in one of its cyclic permutations, *jki* or *kij*. Any other ordering, such as *jik*, equals 1, not –1, as $ji = -k$.
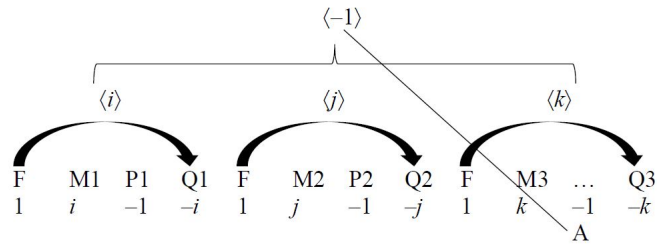
F = Frame; M = Melodrama; P = Passion Chorale; Q = Quest; A = Adding Song

Figure 6: $\langle i \rangle$, $\langle j \rangle$, and $\langle k \rangle$, and their product, $\langle -1 \rangle$.
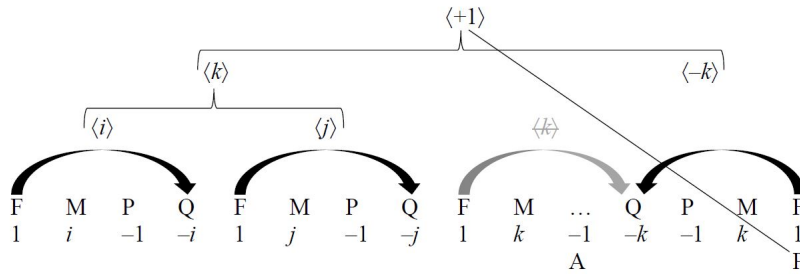
F = Frame; M = Melodrama; P = Passion Chorale; Q = Quest; A = Adding Song;
P = Punch Triumphans

Figure 7: *Generation of hyper-$\langle +1 \rangle$.*

is generated by $-k$, the inverse of k. Taken together, the quaternion product of k and $-k$ equals positive 1 (i.e., if $k^2 = -1$, then $k \cdot -k = +1$). Likewise, the product of their hyper-operators equals hyper-$(+1)$. This hyper-$(+1)$, then, yields the framing section Punch Triumphans, which elevates this instance of the frame to the same upper level of structure as the Adding Song in the previous interpretation.

V. Conclusions

These two sections, then—the Adding Song and Punch Triumphans—embody the true spectacle of *Punch and Judy*. They are Nietzsche's "maternal womb of the entire dialogue...the actual drama." They exist on the *axis mundi*, on the real axis, the realm of "the Dionysian chorus which over and over again discharges itself in an Apollonian world of images." Nevertheless, the role of the chorus is not limited merely to such utterances. It, too, as Aristotle asserts, is a character, and contributes to the drama. Yet, its participation exists on a level higher than the surface plot, a level unaffected by the plot's cyclic time. Eliade writes, "The life of archaic man (a life reduced to repetition of archetypal acts...), although it takes place in time, does not bear the burden of time, does not record time's irreversibility" [11, p. 86]. Hence, time can change course on the *axis mundi*, in the Center.

Taken as a mathematical group, $Q_8 = \{1, i, j, k, -1, -i, -j, -k\}$ is noncommutative, meaning that the specific ordering of its elements as factors in a product is essential; for example, $ij \neq ji$. In our model, these elements represent factors in the dramatic structure—their ordering denotes the sequence of temporal events. Hence, i's happening before j is not equal to j's happening before i,

suggesting that time in the metaphor is irreversible. In any mathematical group, however, there exists a subgroup that consists of the set of elements that commute with all members of the group. For the quaternion group Q_8, this subgroup consists of the two elements 1 and -1, the two points that lie on the real axis.

Above, we use these very points to represent the rituals of origin and sacrifice, as embodied in the framing material and the Passion Chorales—time-outside-time. "Through the paradox of rite, profane time and duration are suspended" [11, p. 53]. Further, each of these elements that is situated on the real axis commutes with those that appear in the various imaginary dimensions. For instance, $-1 \cdot -k = -k \cdot -1$, as in the elision of the end of the third iteration with the beginning of the fourth.

The consequences for this concept in our metaphor are significant: time, as an ordered sequence of temporal events, can "veer round to its opposite" in sacred time. Thus, it is not merely the presence of -1 in the Nightmare scene that engenders the peripeteia, but, more to the point, it is the situation of -1 in this particular subgroup. In mathematical group theory, such a subgroup is called the "center" of the group. The center of Q_8 is $\{+1, -1\}$; and, on that happy coincidence, "This comedy is at an end."

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