Tonal Progressions Identification Through Kripke Semantics

Francisco Erivelton Fernandes de aragão
Federal University of Ceará – *Campus* Quixadá
aragaoufc@gmail.com
Orcid: 0000-0002-2237-4877
DOI: 10.46926/musmat.2021v5n1.80-88

Abstract: *The automatic identification of tonal chord sequences has already been addressed through several formalisms. We return to this problem for didactic reasons, as we seek a formal solution that lends itself to automatic explanation of the way in which tonal sequences are identified. There is a search for a correspondence between the formal steps, which lead to the solution, and how a human agent does it to solve the problem him/herself. Given a sequence of chords, the task is to answer whether or not it constitutes a tonal progression, and how and why. It is an interesting problem because its formal solution, once easily automated, can give birth to educational software of real value in the case of young musicians whose access to harmony teachers is scarce or even null. This formalism applied to a large test body allows empirical proof of the fundamental idea that we can describe the tonal sequences by chaining together a minimal collection of basic tonal sequences. Students who do not have access to a harmony teacher will benefit from this harmonic analysis companion.*

Keywords: *Tonality identification. Modal semantics. Model checking. Automatic harmonic reasoning.*

I. Introduction

The classification of a chord sequence[1] as a tonal sequence is a fundamental task in the study of harmony and is one of the pillars of musical analysis. This is sufficient reason for it to be formalized. The difficulty of such a task can take on great proportions if long pieces are analyzed completely. This is a sufficient reason for it to be automated. In this study, sequences are defined as any concatenation of chords. Here are two situations of identification of a harmonic progression:

1) To identify that [Em - A^7 - Dm - G - C] is a tonal progression, in the key of C major, we must explain the role played by the non-diatonic chord, A^7.

Received: December 10th, 2020
Approved: June 2nd, 2021

[1]The term *sequence*, as defined in this work, is not related to the term *melodic sequence*, which consists of the repetition of motifs or phrases at different levels of pitch, maintaining the same interval pattern (modulatory sequence) or the same contour (diatonic sequence); neither is it related to the medieval sequence, which consists of "the most important type of addition to the official Catholic liturgical song" ([6, p. 739]). For a more in-depth examination of the term *sequence*, as it is traditionally used in the Western musical context, see [6, pp. 739–741].

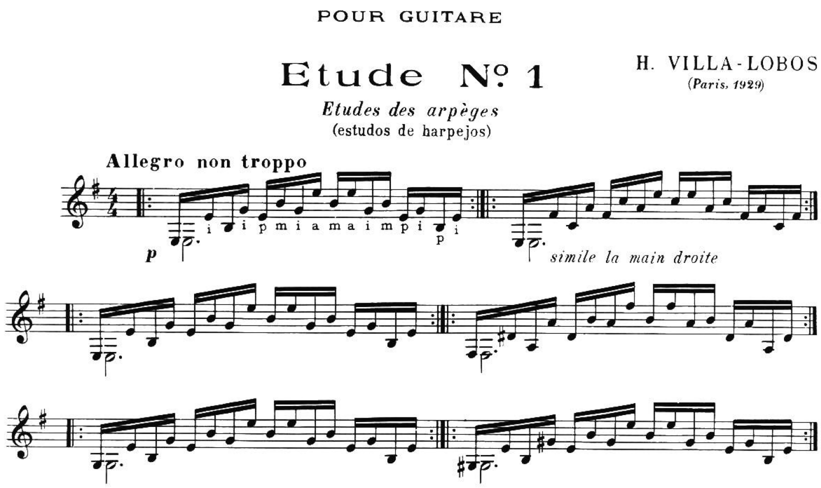

Figure 1: *The first six measures of Villa-Lobos' Étude No. 1, for guitar.*

2) Another type of difficulty is found in this excerpt from Étude No. 1, by Villa-Lobos (1953) ([5, p. 1]) shown in Figure 1, whose chord progression is: [Em - F#°/E - Em - B^7/F# - Em/G]. With the exception of the second chord, the others make up a traditional diatonic sequence in the key of E minor. The point here then becomes to explain the "tonal presence" of this chord.

II. The problem

It is worth remembering that the same sequence of chords can be constructed in more than one way, and therefore, it has more than one explanation. The problem faced here is not only if a sequence is tonal, but also why it is so. Moreover, the explanation must be formal: we are looking for a formal system to explain how the solution of a theoretical musical issue is resolved. In addition, as this question has several answers, the system must be able to find all of them.

i. Our contribution in a glance

We borrow the concept of *basic tonal sequence* from traditional harmony and, from there, we formalize the concept of *tonal sequence*. We achieve this through a recursive function inspired by Kripke Semantics ([8, pp. 83–94], [1, pp. viii–xi]) and Model Checking ([3, pp. 1–28]). Some consequences directly associated with these concepts are:

- We have developed an unprecedented formalism for identifying tonal progressions.
- Because it is a formalism similar to propositional logic in the syntactic part, and similar to Kripke's structure[2] in the semantic part, it is immediately convertible into an algorithm.
- We demonstrate that the proposed formalism is correct and complete in relation to the AHO formalism for detecting tonal harmonies based on context-free grammar ([7, p. 87–88]).

[2] A sophisticated example of applying Kripke structures in Game Theory can be found in [9, pp. 60–182].

III. Our proposal

To expose our idea, let us return to situation 1) in Section I (identification of a harmonic progression), and note that the chords in common with the two sub-sequences merge into one. To explain the role played by the non-diatonic chord A^7 in the sequence [Em - A^7 - Dm - G - C], we imagine that it is, indeed, two sequences [Em - A^7 - Dm] and [Dm - G - C]; and that both are chained together motivated by the presence of the common chord Dm. Note that the chords in common with the two subsequences merged together. This phenomenon occurs in natural language, in phonetics, and is called *crase*. In situation 2) in Section I, there is an ambiguity with respect to the second chord, which can be also interpreted as an Am6/E. These two types of solution are included in the formalism that we propose because what we are modelling are not sequences of chords but, rather, sequences of tonal functions.

IV. The proposed solution

In this work, we developed a formal system that performs the tonal harmonic analysis of musical pieces. The analysis is twofold: the conclusion about tonalness and the explanation how it was formed. This system produces a harmonic analysis from the chord sequence of a musical piece, identifying the key of the piece, its musical cadences, and the role of each chord in the cadences.

V. Formalization

i. Background

Model Check ([2, pp. 49—58]) is a procedure in which a semantic structure serves as a model representing a system. In addition, there is a specification that you would like the system to respect, any property that this system could have. Moreover, you have an automatic device called "Model Check Tool", an automatic tool that responds positively if that system has that property.

From a technical point of view, it answers whether that formal model respects the specification which is also written in a formal language. Model Check gives an answer "yes" if that model satisfies that specification.

Here we subvert this idea. The specifications, which are expressions of the formal language, for us, represent an object. That object will have a certain property. For us the model continues to represent the system, in the same way. However, instead of testing whether the system has a property, we are going to test whether that property is actually recognized by the system.

For us the model is the ruler and the specification is up to us. In this work here, in particular, the model represents the tonal system while the formal specification represents sequences of candidate chords for tonal harmonic progressions.

ii. Language

The syntax of our formal language consists of an alphabet formed by chord symbols considered as atomic constituents on which we abstract the details and consider them correctly written, and by a single rule of formation, the concatenation of these symbols.

iii. Semantics

We established three basic (primary) tonal sequences ([4, pp. 19–233]), formed as follows:

Definition 1 (Basic tonal sequences)**.** Basic tonal sequences show one of these formats:

- **Basic sequence 1**: *The dominant function, followed by the tonic function*;
- **Basic sequence 2**: *The subdominant function, followed by the tonic function*;
- **Basic sequence 3**: *The subdominant function, followed by the dominant function, followed by the tonic function*.

Example 1 (Basic tonal sequences)*.* For example, in the C major key, we have:

- **Basic sequence 1**: [G - C] or [B° - C].
- **Basic sequence 2**: [F - C] or [Dm - C].
- **Basic sequence 3**: [F - G - C] or [Dm - G^7 - C].

iv. Formation rules

Three different ways of grouping sequences are possible: by *juxtaposition*, by *elision*, and by *crase*. In the *juxtaposition*, two strings are joined only by concatenation; in the *elision* and in the *crase* there is a collapse between the sequences: one of the two sequences loses a component. More formally:

Definition 2 (Chaining rules)**.** Let $s = \langle s_1, s_2, ..., s_n \rangle$ and $r = \langle r_1, r_2, ..., r_m \rangle$ be sequences.

- The *juxtaposition* between them produces the sequence $\langle s_1, s_2, ..., s_n, r_1, r_2, ..., r_m \rangle$;
- The *crase* between them produces the sequence $\langle s_1, s_2, ..., s_n, r_2, ..., r_m \rangle$, if $s_n = r_1$;
- The *elision* between them produces the sequence $r = \langle s_1, s_2, ..., s_{(n-1)}, r_1, r_2, ..., r_m \rangle$ if $s_n \equiv r_1$.

Note that the crase requires equality between the extreme elements of the strings while the elision requires only equivalence. These two concepts are established by extension in formal language. For example, the equivalence between two points in the sequence can be defined as follows: $P \equiv Q$ when, for all label function $L \in \mathbb{L}$ and all paths π in a given structure K, there is

$$P \in L(\pi_i) \text{ if and only if } Q \in L(\pi_i). \tag{1}$$

In musical terms, if in the context of a key (L), both chords (points P and Q in a path) perform the same tonal function.

Example 2 (Chaining rules)*.* We have that:

- **Juxtaposition**: [G - C] and [B° - C] produces the sequence [G - C - B - B° - C].
- **Crase**: [Em - A^7 - Dm] and [Dm - G - C] produces the sequence [Em - A^7 - Dm - G - C].
- **Elision**: [Em - A^7 - F6] and [Dm - G - C] produces the sequence [Em - A^7 - Dm - G - C].

v. Structure and Representation

To model the idea of sequence, the states represent the elements with which the sequences are built. Relations between states represent the concatenation between the elements. The displacement between states obeying the dictates of accessibility relations, shapes the sequences.

To formally check out if they are indeed tonal sequences, one imagines each point of the sequence as a state[3]. Those states, which form sequences, are interconnected by accessibility relationships, and the *path*[4] between these states is used as a representation of linear sequences. The states are characterized by a *label function* that denotes their meaning. The following Kripke framework provides the necessary formalism.

[3]Each state is formally a node of a graph and represents a tonal function.
[4]The concept of *path* will be defined later in this work (Definition 4).

Definition 3 (Kripke Structure)**.** A *Kripke structure*, $K = \langle S, S_0, S_F, R, L \rangle$, is defined by:

- A non-empty set S of states;
- A set $S_0 \subseteq S$ of initial states;
- A set $S_F \subseteq S$ of final states;
- A total relation $R \subseteq S \times S$, i.e., for every state $s \in S$ there is a state $s' \in S$, such that $\langle s, s' \rangle \in R$.
- A collection $\mathbb{L}$ of label functions $L_i : S \to \Pi$, where Π is the set of all chords. The label function assigns to each state a chord symbol.

Example 3 shows an instance of Definition 3 with only 3 label functions:

Example 3. [Kripke Structure] Example of a Kripke structure $K = \langle S, S_0, S_F, R, L \rangle$, where

- $S = \{s_f, s_g, s_c\}$.
- $S_0 = \{s_f, s_g\}$.
- $S_F = \{s_c\}$.
- $\langle s_f, s_c \rangle \in R$, $\langle s_g, s_c \rangle \in R$, and $\langle s_f, s_g \rangle \in R$.
- $\mathbb{L} = \{L_1, L_2, L_3\}$ where

$$\begin{aligned} L_1(s_c) = \{\mathrm{C}\}, \quad & L_1(s_f) = \{\mathrm{F}\}, \quad & L_1(s_g) = \{\mathrm{G}\}, \\ L_2(s_c) = \{\mathrm{Am}\}, \quad & L_2(s_f) = \{\mathrm{Bm}\}, \quad & L_2(s_g) = \{\mathrm{E}^7\}, \\ L_3(s_c) = \{\mathrm{Cm}\}, \quad & L_3(s_f) = \{\mathrm{D}^7_{5\flat}\}, \quad & L_3(s_g) = \{\mathrm{G}^7, \mathrm{B}^\circ\}. \end{aligned} \tag{2}$$

In Example 3, $\Pi = \{\mathrm{C, F, G, Am, Bm, E^7, Cm, D^7_{5\flat}, G^7, B^\circ}\}$ is a set of chord symbols; the L_1 function associates the s_c state with a single chord symbol, the C symbol, and this means that in the key represented by the L_1 function, all other chord symbols are prohibited in this state, only L_1 is accepted. Note that the L_3 function associates with the state, s_g, both chord symbols, G^7 and B°.

Figure 2 shows the K^* structure with nodes labeled by the L_1 label function.

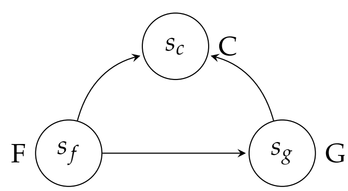

Figure 2: *Structure K^* labeled by L_1.*

The labeling of the K^* structure by the label functions L_2 and L_3 is illustrated in the Figures 3 and 4.

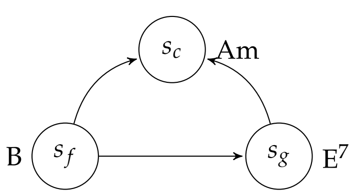

Figure 3: *K^* structure labeled by L_2.*

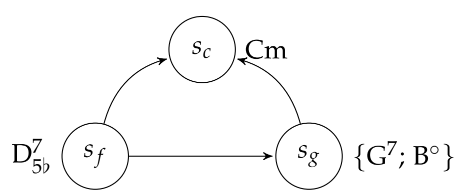

Figure 4: *K^* structure labeled by L_3.*

Note that the multiple labelling in K^* allows an interpretation of "worlds of structures" ,where each world represents a possible key. The accessibility relationship between states determine possible displacements over a Kripke structure. A series of two-by-two accessible states defines a *path* over the structure:

Definition 4 (Path)**.** A *path* in a Kripke structure from a state $s \in S$ is defined by:

- A sequence $\pi = s_0, s_1, ..., s_n$ such that $s = s_0$ and $\forall k \in \{0, 1, ..., n\}$, $\langle s_k, s_{k+1} \rangle \in R$;
- A path starting in the s state is said to be an s-path;
- The state s_k of path π is notated by π_k;
- The prefix $s_0, s_1, ..., s_k$ of π is notated $\pi_{0,k}$;
- The suffix $s_k, s_{k+1}, ..., s_n$ of π is notated $\pi_{k,n}$.

Example 4. [Paths in a Kripke Structure] In the structure of Example 3, from the state s_f we have the following paths: $\pi_A = [s_f, s_g, s_c]$ and $\pi_B = [s_f, s_c]$; from the s_c state we have the path: $\pi_C = [s_g, s_c]$. A succession of arrows represents a path in the graph.

Let Π be a set of chord symbols and P a chord symbol in Π. Let $K = \langle S, S_0, S_F, R, L \rangle$ be a Kripke structure and π a path of the K structure. Finally, let ϕ be a formal expression. We define now when K *satisfies* ϕ *on the path* π, denoted by $K \models_\pi \phi$:

Definition 5 (Satisfaction)**.** Let $K = \langle S, S_0, S_F, R, L \rangle$ be a Kripke structure, π and π' be paths in K, L and L' be label functions in K and ϕ a formal expression. We say that K satisfies ϕ when:

$$K, L \models_\pi P \iff P \in L(\pi_0) \tag{3}$$

$$K, L \models_\pi (P\phi) \iff K, L \models_\pi P \quad \text{and} \quad (K, L \models \pi_{1,n}\phi \quad \text{or} \quad K, L \models_{\pi'} \phi) \tag{4}$$

$$K, L \models_\pi (P\phi) \iff K, L \models_{\pi_0} P \quad \text{and} \quad K, L' \models_{\pi'_0} P \quad \text{and} \quad K, L' \models_{\pi'_{1,n}} \phi \tag{5}$$

An expression is said to be satisfied if it is read entirely, until the last of its chord symbols without resulting in error[5].

We assume a function of labels for each possible key, assigning the chords that perform the tonic function to the s_c state, those who fulfills the function of dominant to the state s_g, and those who fulfill the function of subdominant to the state s_f.

VI. Examples of identifying tonal progressions

The sequence of triads will be interpreted in reverse, s_0 being the last triad of harmony. The following are examples of satisfaction of sequences whose formation took place in different ways. In them, satisfaction occurs according to the definition 5 in view of the structure of example 3, labeled, each time, with the proper label function.

In the following examples, we will use the next label functions that will be numbered to facilitate exposure

$$L_1(s_c) = \{\mathrm{C}, \mathrm{Am}\}, L(s_f) = \{\mathrm{F}, \mathrm{Dm}\} \text{ and } L(s_g) = \{\mathrm{G}^7, \mathrm{B}^\circ\} \tag{6}$$

$$L_5(s_c) = \{\mathrm{F}, \mathrm{Dm}\}, L(s_f) = \{\mathrm{B}^\flat, \mathrm{Gm}\} \text{ and } L(s_g) = \{\mathrm{C}^7, \mathrm{E}^\circ\} \tag{7}$$

$$L_{10}(s_c) = \{\mathrm{A}, \mathrm{F}\sharp\mathrm{m}\}, L(s_f) = \{\mathrm{D}, \mathrm{Bm}\} \text{ and } L(s_g) = \{\mathrm{E}^7, \mathrm{G}\sharp^\circ\} \tag{8}$$

We are going to study three chord sequences: [B$^\flat$ - C - F], [G - C - F - G - C] and [G - C - B$^\flat$ - C - F - G - C]. The third example exposes the basic procedure for identifying modulations ([4, ppd. 169-280]).

[5] We say that K is a model for ϕ.

Let ϕ be a chord sequence, $K = \langle S, S_0, S_F, R, L\rangle$ be a Kripke structure and $\pi = [s_f, s_g, c_c]$ be a path in K.

Example 5 ([B$^\flat$ - C - F] – Perfect cadence in the key of F major). The sequence of triads will be read in reverse: [F - C - B$^\flat$]. We want to show that $K \models_\pi$ F(CB$^\flat$). To Show that $K \models_\pi$ FCB$^\flat$, we have, by Equation 4 in Definition 5:

$$K \models_\pi \mathrm{FCB}^\flat \iff \mathrm{F} \in L(\pi_0) \text{ and } K \models_{\pi_{1,2}} \mathrm{CB}^\flat. \tag{9}$$

Taking $L = L_5$, and $\pi = \pi_A$ we have F $\in L(\pi_{A_0})$. Let's show that $K \models_{\pi_{1,n}}$ CB$^\flat$, by Equation 4 in Definition 5:

$$K \models_{\pi_{1,n}} \mathrm{CB}^\flat \iff \mathrm{C} \in L_5(\pi_1) \text{ and } K \models_{\pi_{2,n}} \mathrm{B}^\flat. \tag{10}$$

Taking again $L = L_5$, we have C $\in L_5(\pi_{A_1})$. Let's show that $K \models_{\pi_{2,2}}$ B$^\flat$, by Equation 3 in Definition 5:

$$K \models_{\pi_{2,2}} \mathrm{B}^\flat \iff \mathrm{B}^\flat \in L(\pi_2). \tag{11}$$

Taking again $L = L_5$, we have B$^\flat \in L_5(\pi_2)$.

There is a "B flat major" interpretation for L, i.e., L_5 which, together with the "*perfect cadence*" path π_A, recognize the sentence FCB$^\flat$ as a tonal progression. The recursive dynamics of Definition 5 takes that interpretation and recognize the sequence FCB$^\flat$ as a tonal progression.

Example 6 ([(GC)F]GC)] – Dominant cadence followed by a perfect cadence in the key of F major). The sequence of triads will be read in reverse: CGFCG. We want to show that $K \models_\pi$ C (G [F (C G)]). To show that $K \models_\pi$ C (G [F (C G)]), we have, by Equation 4 in Definition 5:

$$K \models_\pi \text{C (G [F (C G)])} \iff C \in L(\pi_0) \text{ and } K \models_{\pi_{1,n}} \text{G[F(CG)]}. \tag{12}$$

Taking $L = L_1$ and $\pi = \pi_A$, we have C $\in L(\pi_{A_0})$. Let's show that $K \models_{\pi_{1,n}}$ G[F (C G)], by Equation 4 in Definition 5:

$$K \models_{\pi_{1,n}} \text{G[F(CG)]} \iff \mathrm{G} \in L_1(\pi_1) \text{ and } K \models_{\pi_{2,n}} \text{[F(CG)]}. \tag{13}$$

Taking again $L = L_5$, we have G $\in L_1(\pi_{A_1})$. Let's show that $K \models_{\pi_{2,n}}$ [F(CG)], by Equation 4 in Definition 5:

$$K \models_{\pi_{1,n}} \text{[F(CG)]} \iff \mathrm{F} \in L_1(\pi_2) \text{ and } K, L^* \models_{\overline{\pi}} \mathrm{CG}. \tag{14}$$

Taking again $L = L_1$, we have F $\in L_1(\pi_{A_2})$. Let's show that $K, L^* \models_{\overline{\pi}}$ CG, by Equation 4 in Definition 5:

$$K, L^* \models_{\overline{\pi}} \mathrm{CG} \iff \mathrm{C} \in L^*(\overline{\pi}_0) \text{ and } K, L^* \models_{\overline{\pi}_{1,n}} \mathrm{G}. \tag{15}$$

Taking again $L^* = L_1$, and $\overline{\pi} = \pi_C$, we have C $\in L_1(\pi_{C_0})$. Let's show that $K, L^* \models_{\overline{\pi}_{1,n}}$ G, by Equation 3 in Definition 5:

$$K, L^* \models_{\overline{\pi}_{1,n}} \mathrm{G} \iff \mathrm{G} \in L^*(\overline{\pi}_1). \tag{16}$$

Taking again $L^* = L_1$ and $\overline{\pi} = \pi_C$, we have $G \in L^*(\overline{\pi}_1)$.

There are a "B flat major" interpretation for L, i.e., L_5 which, together with the "*perfect cadence*" path π_A, recognize the sentence FGC as a tonal progression. There is also another "B flat major" interpretation for L, i.e., L_1 which, together with the *plagal cadence* path π_C, recognize the sentence GC as a tonal progression. The recursive dynamics of Definition 5 take those two interpretations and recognize the entire sequence CGFCG as a tonal progression.

Example 7 ([$\mathrm{CG^7DmA^7E^7Bm}$] – Progression with diatonic and non-diatonics chords in C major). The sequence of triads will be read in reverse: we want to show that $K, L \models_\pi \mathrm{C}, \mathrm{G}^7, \mathrm{Dm}, \mathrm{A}^7, \mathrm{E}^7, \mathrm{Bm}$. By Equation 4 in Definition 5 this happens if, and only if, we have:

$$K, L \models_\pi \mathrm{C}\ \mathrm{G^7Dm(A^7E^7Bm)} \iff \mathrm{C} \in L(\pi_0) \text{ and } K, L \models_{\pi_{1.n}} \mathrm{G^7Dm(A^7E^7Bm)}. \tag{17}$$

Taking $L = L_1$ and $\pi = \pi_A$, we have $C \in L_1(\pi_{A_0})$. Let's show that $K, L \models_{\pi_{1.n}} \mathrm{G^7Dm(A^7\{E^7Bm\})}$, by Equation 4 in Definition 5:

$$K, L \models_{\pi_{1.n}} \mathrm{G^7Dm(A^7E^7Bm)} \iff \mathrm{G}^7 \in L(\pi_{1,n}) \text{ and } K, L \models_{\pi_{2,n}} \mathrm{Dm(A^7E^7Bm)}. \tag{18}$$

Taking $L = L_1$ and $\pi = \pi_A$, we have $\mathrm{G} \in L_1(\pi_{A_1})$. Let's show that $K, L \models_{\pi_{2,n}} \mathrm{Dm(A^7\{E^7Bm\})}$, by Equation 5 if Definition 5:

$$K, L \models_{\pi_{1.n}} \mathrm{Dm(A^7E^7Bm)} \iff \mathrm{Dm} \in L(\pi_{2,n}) \text{ and } \mathrm{Dm} \in L'(\pi'_0) \text{ and } K, L' \models_{\pi_{1.n}} \mathrm{A^7E^7Bm}. \tag{19}$$

Taking again $L = L_1$, and $\pi = \pi_A$, we have $\mathrm{Dm} \in L_1(\pi_{A_2})$. Taking $L' = L_5$ and $\pi' = \pi_A$, we have $\mathrm{Dm} \in L_5(\pi'_{A_0})$. Let's show that $K, L' \models_{\pi'_{1.n}} \mathrm{A^7E^7Bm}$, by Equation 5 in 5:

$$K, L' \models_{\pi'_{1.n}} \mathrm{A^7E^7Bm} \text{ iff } \mathrm{A}^7 \in L'(\pi'_1) \text{ and } \mathrm{A}^7 \in L''(\pi^*_0) \text{ and } K, L'' \models_{\pi^*_{1,n}} \mathrm{E^7Bm}. \tag{20}$$

Taking again $L' = L_5$ and $\pi' = \pi_A$, we have $\mathrm{A}^7 \in L_5(\pi_{A_1})$. Taking again $L'' = L_{10}$, and $\pi* = \pi_A$, we have $\mathrm{A}^7 \in L_{10}(\pi^*_{A_0})$. Let's show that $K, L'' \models_{\pi^*_{1,n}} \mathrm{E^7Bm}$, by Equation 4 in Definition 5:

$$K, L'' \models_{\pi^*_{1,n}} \mathrm{E^7Bm} \iff \mathrm{E}^7 \in L''(\pi^*_{1,n}) \text{ and } K, L'' \models_{\pi^*_{2,n}} \mathrm{Bm}. \tag{21}$$

Taking again $L'' = L_{10}$, and $\pi^* = \pi_A$, we have $\mathrm{E}^7 \in L_{10}(\pi^*_{A_1})$. Let's show that $K, L'' \models_{\pi^*_{2,n}} \mathrm{Bm}$, by Equation 3 in Definition 5:

$$K, L'' \models_{\pi^*_{2,n}} \mathrm{Bm} \iff \mathrm{Bm} \in L''(\pi^*_2). \tag{22}$$

Taking $L'' = L_{10}$ and $\pi^* = \pi_A$, we have $\mathrm{Bm} \in L''(\pi^*_2)$.

In this Example we take the "C major" label function L_1, the "D minor" label function L_5 and the "A major" label function L_{10} (see equations 6, 7, and 8). We also take the paths π, π', and = π^* all equal to π_A ("*perfect cadences*").

The recursive dynamics of Definition 5 take those interpretations and recognize the entire sequence CGFCB♭CG as a tonal progression.

VII. Conclusions and further work

We prove the correctness and completeness between this system and AHO (context-free) grammar developed in [7, p. 32–77]. A body of 100 works between classical music and Brazilian popular music was used as a test. The formalization employed naturally provides the basis for immediate computational implementation. The methodology employed in this work proved satisfactory in identifying tonal harmonic sequences, showing that a minimal structure can synthesize the idea of tonal harmonic progression.

References

[1] Blackburn, P.; Rijk, M.; Venema, Y. 2001. *Modal Logic*. Cambridge University Press.

[2] Burck, J. R.; Clarke, E. M.; Long D. E. 1991. Modal Logic Symbolic Model Checking with Partitioned Transition Relations. (A. Halaas and P. B. Denyer, eds.). In: International Conference on VLSI, 1. *Proceedings*... , pp. 49–58.

[3] Clarke, E. M.; Grumberg, O. and Peled, A. 2000. *Model Checking*. Cambridge, MA : The MIT Press.

[4] Schoenberg, A. 1979. *Armonia*. Madrid: Real Musical Editores.

[5] Villa-Lobos, H. 1953. *Douze Études pour Guitare* (Score, Guitar). Paris: Max Eschig.

[6] Randell, H. 1986. *The New Harvard Dictionary of Music and Musicians*. London: The Belknap Press of Harvard University Press.

[7] Villar, P. 2002. *Um analisador harmônico otimista*. Dissertação (Mestrado em Ciências da Computação). Fortaleza: Universidade Federal do Ceará.

[8] Kripke, S. 1963. *Semantical Considerations on Modal Logic*. *Acta Philosophica Fennica*, 16, pp. 83–94.

[9] Vasconcelos, D. 2007. *Lógica modal de primeira-ordem para raciocinar sobre jogos*. Tese (Doutorado em Informática), Rio de Janeiro: PUC.