Logistic Maps for Illogic Music

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Abstract: Logistic map is a simple non-linear polynomial equation with chaotic behavior. Here is presented a dynamic computer model programmed in Pure Data (Pd) that generates melodic sequences using two Logistic maps. Although this is a simple music model, there seems to be a myriad of possible sound generational structuring and parameterizations capable of delivering indeterministic (*illogic*) music that is unpredictable while retaining an auditory similarity. This paper describes this so-called illogical music model, presents its algorithm and discusses its sound and musical results.

Keywords: Logistic map, Chaotic music, Algorithmic composition, Computer music performance.

1. Introduction

Chaotic music is a term that refers to a branch of algorithmic composition models that use non-linear equations presenting chaotic behavior to generate music, being it symbolic (music notation or control) or acoustic (audio). There are many developments on this field, such as the melodies created by chaotic algorithms that map music into the colors of the rainbow (SOTIROPOULOS, 2011, p. 388); a Chua's oscillator (a nonlinear system whose dynamical behavior is chaotic) used to generate sound and music (BILOTTA, 2005, p. 253); or a thorough comparison between the tonal possibilities of discrete dynamic systems for symbolic music composition in contrast with compositions made by humans (KALIAKATSOS-PAPAKOSTAS, 2013, p. 135). Among them, Logistic map is probably the simplest one; which is comprised of a polynomial mapping given by a single non-linear equation that can present chaotic behavior. Initially created in the late XIX century by the Belgian mathematician Pierre François Verhulst, to predict populations' size variation, Logistic map gained momentum after mentioned in Robert May's seminal article on how simple deterministic models, such as this one, can present complex indeterministic behavior (MAY, 1976, p. 459).

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An enticing artistic feature of chaotic models, such as the Logistic map, is its ability of easily presenting a wide range of bounded variability for any minute change of its single parameter "R". Logistic map is defined by: "$X_{n+1} = RX_n(1-X_n)$". When X_n is normalized between 0 and 1 and R is ranging from 3.06 to 4, this equation will output values of X_{n+1} whose behavior varies from single oscillatory patterns (like harmonic sine-waves or repeating melodic intervals) to pseudo-random sequences (like complex tones or fairly complicated melodic patterns). As music is the art of organized sounds, both in terms of its microstructure (timbre) or macrostructure (melody, harmony and rhythm), chaotic behavior has the potentiality of becoming particularly interesting as it can deliver perceptually meaningful sound patterns to be explored in aesthetic fashion, able to create timbres, melodies, as well as polyphonic counterpoints and harmonies, forming structures that are original while retaining a degree of perceptual similarity with each other, perceptually bounded together in a set of possibilities that confers to the output sound (in terms of pulses, tones and timbres), stylistic identity that is cognitively meaningful and hopefully even affectively pleasing. Further understanding on deterministic and indeterminism music cooperation, similarities and applications for Logistic Maps peculiar features, such as its bifurcation diagrams, are in (PAREYON, 2011: 298-300 & 346-354).

2. Music perception and meaning

It is attributed to the avant-garde 20th century composer Edgard Varèse, a quote on music being made of organizing sounds. Once that our auditory perception is multidimensional, simultaneously sensing dynamic and minute variations of sound intensity (loudness), tone (pitch), spectrum (timbre) and time (rhythm), this sound organization happens in several distinct and parallel cognitive layers. Sensorially, music organization takes place in its temporal microstructure which is formed by psychoacoustic dynamic elements (loudness, pitch and timbre). Sound sensation happens in the brain with the participation of echoic memory that also registers auditory sensory information and doing so defines the boundary between tone (the sensation of dynamic spectral sound composition), and time (the sensation of delay patterns that are perceived as sound pulses and music rhythm) (JENKINS, 1961, p.1550). As I see it, timbre patterns are the building blocks of music meaning; similar to phonemes the compound any spoken language. Music microstructure occurs within a timeframe smaller than 50ms, which is the period of the first detectable (heard) sound tone, whose frequency, for our human auditory system, is around 20Hz.

Cognitively, music organization happens in timeframes that are larger than the sensorial one, so that the sound information can now convey context given by its contrast with the listener's short-term memory, thus allowing the mind to identify, expect and even anticipate identifiable sound events within musical structure (HURON, 2006, p. 41). This conveys what I am here referring as the macrostructure of music. Its timeframe is around a cognitive time window known as "specious present", a term originally conceived by the 19th century psychologist William James, referring to the timeframe that mind consciously perceives as being the current moment, the "now" (JAMES, 1893, p.609). In terms of music perception, I understand this timeframe as occurring between echoic and working memory, which is around 1 second of length, and even may have been the reason why this particular time length was chosen along human history to represent a minute duration or interval that is cognitively perceptible and meaningful, as representing a moment apart from the previous or the next, the "second" one. The perception of time in music is fundamental and has inspired many investigations in the field of systematic musicology, for instance on how music is modulated by memory (SNYDER, 2009, p.107) or the different brain areas shared in the processing of music making and listening (PERETZ, 2005, p.89). In this cognitive layer of sound macrostructure, music gains an equivalent of syntactic meaning, where melodies, harmonies and rhythmic patterns are identified, organized in a timeline of events and eventually understood in a larger context, as a musician style or a musical genre. Finally, in terms of affection, or the ability of music to evoke emotions in the listener's mind (probably the single most important reason why music is so ubiquitous and fundamental for all humankind), its timeframe of occurrence is widen to a length where the sound structure gains contextual narrative which is mediated by the intervention of long-term or auto-biographical memory, which not only allows the listener to appraise musical styles and genres but also to eventually being emotionally affected by them (SCHULKIND, 1999, p. 948). From my experience, it seems that music emotional context can be identified (appraised, not evoked) after about 3 to 5 seconds of continuous listening. However it usually takes longer (at least 30 seconds to one minute of music listening and favorable conditions) to have emotions properly evoked in the listener's mind. Further reference on memory time dynamics are found in (WICKELGREN, 1974, p. 775–780). Coincidentally, the average duration of a song is about 3 minutes and its musical form is usually made of 3 parts: A (introduction), AB (development) and A (conclusion). With that structure and timeframe, music has been able to quite effectively evoke emotions in the mind of listeners and therefore has been used as an efficient form of mood regulation (SAARIKALLIO, 2008, p.291).

3. An *Illogic* model of music

As said before, music making starts in a deterministic micro-structural sound sensorial level described by acoustic laws and empirical evidence of psychoacoustics that are well determined and understood. As longer timeframes of sound are listened, more perceptual strategies of its understanding are activated in the listener's mind, going into a manyfold of organizational perceptual layers, at each step getting less deterministic and logic while involving more participation of attention, memory, and context, till it reaches a sort of sound super-macrostructure of music where emotions can be evoked in/by the listener's mind, according to many *illogic* (in the sense of nondeterministic) aspects, such as: sociocultural background, music taste, mood, intention, and so forth. These are complex psychological fuzzy aspects still not fully mapped and understood (and probably never will be) but they are influenced by minute changes in the listener's body (hearing, vision, health, wellbeing, etc.), mind (conscious and unconscious thoughts and memories), experiences, situations, implications, attention, intention, consent, and current mood that impacts and is impacted by music listening. I believe to be possible that moods made by music listening are a type of emergent mind behavior generated by the complexity of a chaotic system output. If so, they may eventually be modeled by a simple nondeterministic equation.

Inspired by these thoughts, I created a dynamic computer music making model with two Logistic maps. One operates in the sound micro-structural level, creating new spectral distributions which are perceived as novel sound timbres. The other one operates in the musical macrostructure, creating regular sequences of context-bounded tones (musical notes) which are easily perceived as regular monophonic melodies. Together they generate the *illogic* music model that makes unpredictable melodies and timbres. This is a simple model that is here presented more as a process rather than an end, mostly aiming to demonstrate what further chaotic models with more sophisticated music compositional rules might be able to achieve.

The model here presented was programmed in Pure Data (www.puredata.info). Pd, for short, is an open-source computer platform for the visual programming of multimedia. Pd was specially designed taking into consideration real time (performance) in the parametrization (control), digital processing and generation (synthesis) of musical digital audio. Its creator, Miller Puckette, is the mathematician and musician who developed Pd during a project at IRCAM, back in the 1990s. Since then, Pd has been used and also developed by a community of volunteers that maintain this free digital environment of interactive computer music that aims to be the technological equivalent of a musical notation system (PUCKETTE, 2007, p.13).

Pd has a different programming logic style, specially designed to take performance into account. These algorithmic structures in Pd are called *patches*. I've observed that Pd's programming logic are sometimes, at first glance, seemd obscure, specially by the ones used to work with textual command-line programming languages, such as: C, Python or JavaScript. Pd patches can be created and edited in runtime which makes it possible for computer artists to explore "live coding", where the performance is the programming building process and its real time generated sound. The next figure shows an implementation of the Logistic map as a patch, and two different output behaviors, for small variations of R.

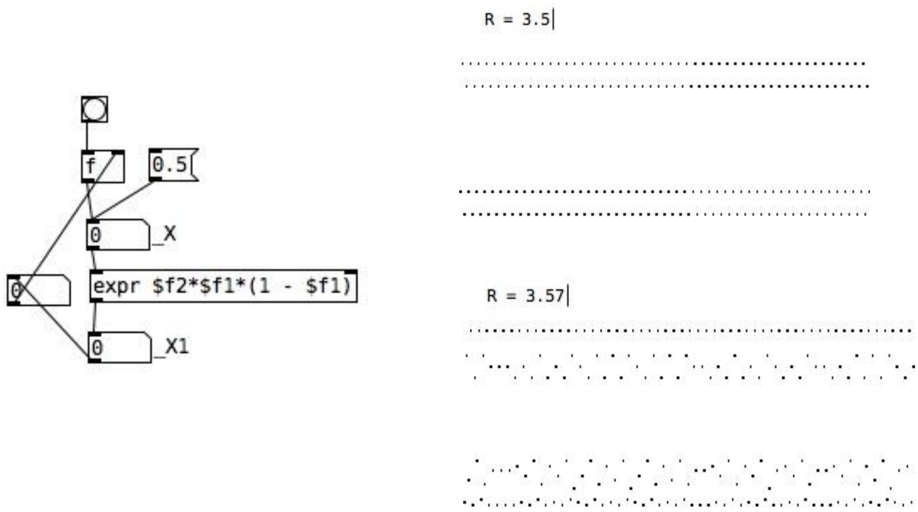

Figure 1: (left) Logistic map programmed in Pd. (right) Typical melodic distribution results, which is periodic for R=3.5 (top) and aperiodic for R=3.57 (bottom).

R is the value fed into the right inlet of the expression box (variable $f2 in the object "expr"). When R = 3.5 the output is periodic, oscillating between four different values, which are depicted by the 4 parallel horizontal dotted lines shown in the top right figure. When R = 3.57 (a minute difference from its previous value) the output pattern is quite different, now nearly aperiodic, depicted by the scrambled dots in a region between two horizontal lines, as shown in the above figure, at the bottom right. These two outputs are common examples of result possibilities that this simple deterministic equation is able to deliver. Periodicity is related to tonal sounds (presenting a clear pitch), like the ones normally generated by melodic monophonic instruments (ex: flute, trumpet, saxophone, etc.). Aperiodicity is related to non-tonal percussive instruments (without clear pitch, such as: shakers, drums, cymbals). As seen, these types of output organizations can be used to build sound microstructure (timbre), but also musical macrostructures (melodies). For that, the output can be mapped into a musical diatonic scale where each element represents a musical note fundamental frequency (pitch). For timbre, the output can be mapped to a frequency domain array and then converted back to time domain where each element will represent one partial in this sound timbre. The next figure shows a simple implementation of this patch, now adapted to the generation of timbres. Note that the partials amplitude linearly decreases proportionally to their frequencies, which is done to mimic the behavior of partials generated by natural resonating bodies, such as in tensioned strings or in the air columns inside pipes.

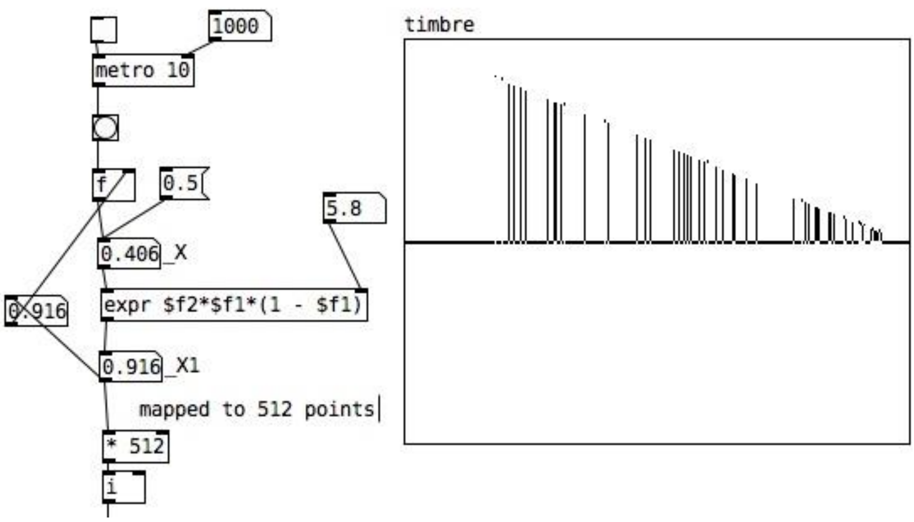

Figure 2: Logistic map model in Pd for timbre dynamic design.

4. Discussing results

As described, this simple musical experiment used two Logistic maps implemented in Pd; one to generate dynamic timbres (music microstructure) and another one to generate dynamic melodies (macrostructures with the dynamic timbres generated in the previous stage). Both timbre and melody can present periodic or chaotic behavior, according to the values of their respective variable R. When the models are within the chaotic range, their outputs are bounded within limits that make them original yet easily recognizable by our auditory perception, which is a welcome feature also found in natural soundscapes (DAVIES, 2003, p. 224).

To demonstrate it, a video was recorded and uploaded to the Youtube site, as an unlisted file (meaning that only someone with the following link can have access). The link for this video is: https://youtu.be/PPP9TYouJnE. As seen in this video, the computer model is played somehow like a musical hyperinstrument, as described in (MACHOVER, 1992) however still without the polishing of an instrument-like control interface. The main parameters that are used to control what is here called *illogic* music generation are the 2 growth rates R (for timbre and melody), the note generation rate (in the video, it starts at 500ms) and the melody's musical scale mapping (whose default is 3 octaves of equal-temperament chromatic scale). The video starts with a simple tone (waveform) and melody rate R=3. This generates a single note. As this R is slowly raised to 3.6, the melody becomes richer, going from a repeating musical interval to a complex bounded yet unpredictable path of melody, and even beyond, when R is a little above 3.65, where the model becomes chaotic, generating outputs similar to the one seen in the right bottom graph of figure 1. Around the third minute of this video, the second Logistic map is started, and new spectral distributions are created, thus generating new timbres. This is depicted in the top graph of the video that shows the sound spectral distribution being populated by new partials. The sound of the melody becomes increasingly “metallic”, resembling typical sounds generated by a tensioned metallic string struck by a hard baton (like in the Brazilian instrument “berimbau”) or pinched (like in a harpsichord). The melodic rate and mapping parameters are then raised, expressing new musical possibilities of this model where the melodic generation speeds up until reaching (at around the sixth minute) it's major complexity where the perception of melody is almost disrupted into what seems now approaching to white noise. The timbre is then reset where the sound looses its metallic-like texture (now resembling pink noise) but still with a background rhythmic structure that, although complex, is recognizable. At around six and a half minutes the parameters are slowly decreased where the music texture slowly is restituted and a notably repeating melody can once again be perceived aa slowing down until finally halting.

5. Conclusions

This paper presents a simple yet powerful computational model for dynamic music making based on the chaotic behavior of Logistic maps and its ability of generating ordered outputs whose sound organization falls into the frontier between linear and chaotic, deterministic and indeterministic, periodic and aperiodic, tonal and percussive, logic and *illogic*. This feature can make this type of model very enticing for artistic purposes, while easily implemented and controlled. For me, the musical output of this model holds an analogy with the beautiful patterns found in the boundary between laminar and turbulent flows. Laminar flows make deterministic, well behaved and predictable patterns, but are rare in nature, being a condition only achieved when specific aspects are met; like tonal regular melodic patterns. Turbulence, on the other hand, is unpredictable, complex and is the rule followed by most natural patterns, from micro to macrocosmos; which I compare to the sound organization found in natural soundscapes. Art, as I see it, is a human activity that occurs within this frontier. Arts and specially music (the most immaterial, informational and emotional art) is the result of a well balanced proportion that a skilled composer can achieve in pieces of music delivered by performers in a thread of organized sounds that have informational predictability while permeated by unexpected patterns which together engrave the musician personal style and a musical social genre, also conveying originality which entices the listener's mind to unveil and wonder with its hidden meanings, in a fashion somewhat similar to what the philosopher Dan Dennett's defines as humor (another human activity strictly related to time and emotion), like comedy, good music listening may also be understood as the "joy of debugging".

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