Three Open Questions from the Indigenous Epistemology, over Music and Mathematics in the Latin American 21st Century

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DOI: 10.46926/musmat.2022v6n1.1-11

Abstract: *It is a fact that the ancient peoples of the Americas had, for many centuries, robust mathematical and musical cultures. Although much of these cultures was not clearly recorded in writing by the first European settlers, this does not mean that their existence is insignificant, and much less, that they have been lost forever. Due to the importance of publishing studies on music and mathematics from a Latin American perspective, the following text, initiatory on the subject, proposes three key questions to promote a discussion from anthropological and historical foundations for the indigenous relationships of music and mathematics, and with the intention of motivating more questions, or formulating them in a better way, in further research.*

Keywords*: Mathematical Musicology. Latin America. Mesoamerican Mathematics and Music.*

I. Introduction

The following notes are motivated by an insight of music and mathematics from the viewpoint of regional history, in Latin American countries where colonization has not been fully achieved. In other words, from the living testimonial notions of an autochthonous knowledge within its own right to exist by itself, and in possible harmony with intellectual and artistic practices imported to build an enriched experience of the so-called Western civilization.

Many questions may arise from this discussion. However, here are only three of them as few among the capital ones, in order to develop further discussion on the history, education and transformation of mathematics related to music. These three questions are framed into the subjects of (firstly) the recognition of a cultural Mesoamerican and Andean heritage useful and refreshing for a wider comprehension of the bonds between music and mathematics; (secondly) music and mathematics by their relationship with ethics and a social frame for peace among the peoples, and the links of the latter, with life survival on Earth, and (thirdly) the search for a non-conflictive coexistence between musical practices and theories from different origin and contexts, without the imminent elimination of the non politically and economically predominant ones.

II. On the Heritage of Native Peoples

The history of mathematics for the ancient civilizations of Latin American countries such as Bolivia, Peru, Mexico and Central America is actually under intense scrutiny and resignification. We may recall that in 4th century BC the number zero was invented by the Olmec civilization[1], and later, along six or seven centuries from 2nd to 9th AD, the Maya developed a deep mathematical knowledge, on par with a refined literature and musical practices later absorbed by the cultures of Teotihuacan and Tula, which in turn are the basis of the Mexica civilization. Congruently, López-Austin [14] confirmed that Mesoamerican mathematics was elaborated aside with musical (*i.e.* sonic, aural, numerical, geometrical, rhythmic and algebraic) assumptions. This notion was previously suggested by Klein [10], Martínez del Sobral [16], Agustín-Aquino [1], Dehouve [8] and Romero-Murguía [23], within their appreciations of Mexican systematization of mathematics through geometrical design of musical instruments and architectural constructions for public performances during the Classic and Post-Classic Mesoamerican periods. In addition, López-Austin's article synthesizes how elementary conceptions of number and space in Mesoamerica are attached to deep axiomatics within a theoretical frame implicating musical and mathematical symbolization. Such symbolization also implies synesthetic codification and formal representation of time and space, with computation of long-term dates and astronomical investigation; a matter systematically investigated since C. Klein's publication from 1982 [10].

Klein [10, p. 25] proposed a model of the Mesoamerican universe "as a giant piece or pile of folded fabric", deducting its shape from a serpentine figure from the Dresden Codex (see Fig. 1). Thus, "the infinite strands of the universe must have been seen as integrated into a giant pile of *cloth*. The Maya manuscripts are full of references to the *folds of the k'atun*" (*loc. cit.*, my italics)[2]. López-Austin [14] concludes that these "folds of the k'atun" model was interpreted by Nahua cultures under the concept of the *cuencoliuhqui* (*op. cit.*: 139), "the cosmic flow that moves in ascents or descents [...] in a serpentine sliding of time and space", through different layers of synchronic complexity. This conceptualization would be enough popular in ancient times, to feed long-term transmission of cultural values. Then, for Klein [10, p. 22] (based on Boiles, [2]), the sunlight and cosmic rays are represented by "twisted cords" and "woven leaf rings", connected to the formalization of rhythms, pitches and melodies to be sung following to a ritual calendar of general use along a wide territory in North and Central America, during centuries in which these cosmic representations were displayed by cultural traits according to distinct epochs, and to levels of complexity more or less accessible in order to socially interpret this tradition, including music and dance as important symbolic conveyors.

As in the emergence of other cultures in the world, in Mesoamerica civilizations emerged from the cultivation of knowledge directly related to the measure of the heavens and meteorology, due to its great relevance to agricultural practices. However, in Mesoamerica the agricultural, the fungal and the phytomorphic were interpreted as a book, as an aesthetic, ethical, musical and mathematical lesson. Agricultural and astronomical practices were transformed, over the centuries, into systems of reading, interpretation and thought through numbers, geometry and the proportionality of phenomena.

As MacLaury states [15, p. 523], "The Aztec metaphor *people are plants* enables pervasive conception of human actions, processes, and creations [...this metaphor] equates the most beautiful achievement of people with that of plants". Thus, for these cultures, the nature of music and mathematics could not be of human exclusivity, but rather as a common trait to living beings and societies on Earth. The manifold organization of complexity would be composed by the same

[1] See: Meggers, 1975 [12, p. 2] & Dassonville, 2016 [7, pp. 107–108].

[2] A *katun* is a unit of time in the Maya calendar equal to 20 tuns or 7,200 days, equivalent to 19,713 tropical years.

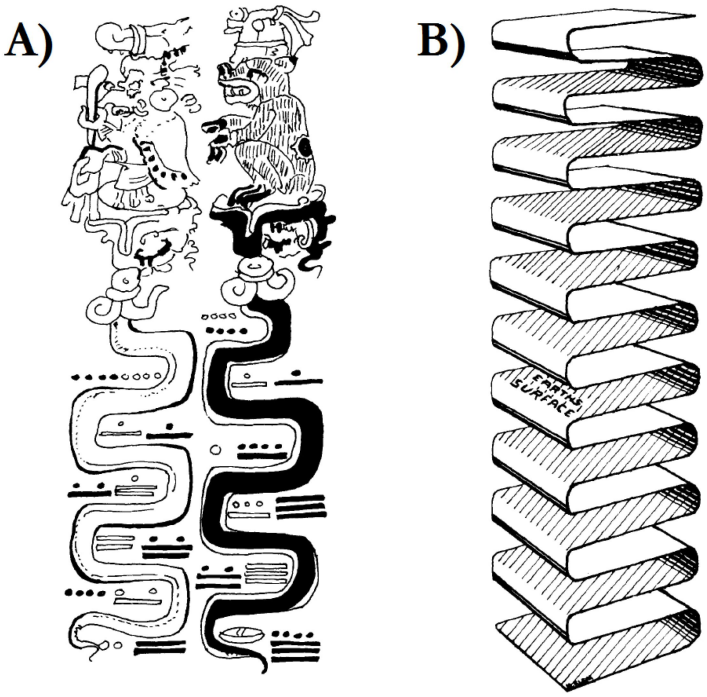

Figure 1: ***A)*** *Undulant serpent, Codex Dresden, plate 62. Maya. Post-Classic period (13th or 14th century). Drawing by Cecelia Klein [10, p. 62] based on J. E. S. Thompson [25]. Notice the writing of number zero in the black column-bottom, here represented by the abstract shape of an empty cocoa bean shell. After the Spanish invasion of Mexico, in the 16th century, nearly all the Maya texts were destroyed, in deference to writings that conform to Biblical doctrine; alongside, Western mathematics were introduced by scholars teaching in the universities following the European model, emerged from Christian and Aristotelian inspiration.*
B) *The same "Undulant serpent" chiral Maya model of the universe modeled as a "folded cloth" (*cuencoliuhqui *in Nahuatl language). Drawing by Henry F. Klein (from C. Klein, [10, p. 27]), with the text "Earth's Surface" at the central layer, referring to a present time-space.*

"elements" intertwined through a variety of combinations of "rhythms".

López-Austin [14] points out that the Mesoamerican folded fabric model of the universe does represent a "dynamics of crossed planes", in other words, a multidimensional time-space model intuitively perceivable for a local observer only in one of its layers (in Fig. 1-right, indicated as the "Earth's surface", perceivable as local time-space coordination). This interpretation arise a discussion on the history of modern cosmology and mathematics, since standard physical models in the so called Western academy are, if not "fully comparable", at least semiotically comparable. See for instance, in H. F. Klein (Fig. 2), the representation of "the universe's surface and the tangled underworld of the Mesoamerican cosmos", which strongly recalls the "standard model" of a flatten universe with gravitational loci of mass hyper-concentration ("black holes"); while the multidimensional model (Fig. 1) resembles conceptual structuration of M-theory, also a common place in modern Western cosmology.

Whether there is a mutual inspiration between anthropology, Mesoamerican archaeology and

standard cosmology, or rather we perceive mere superficial coincidences or serendipity, deep epistemics and cognitive anthropology should contribute to better understand the human structure of heuristics in physics and mathematics. Then, from this theoretical outlook, a comparativistic method for mathematical epistemics is urgent; since mathematical and semiotic ressemblances between the Mesoamerican *cuencoliuhqui* (i.e. the time-space *folded fabric* model) and M-theory reflects a deep analogy between mathematical abstraction and cognition of physics. This comparativism may depict a human ("universal") stem for the understanding of a "mathematical sense of the world", in parallel as music can be thought as the "mathematical sensation of the world". This approach allows us to study mathematics and music as cooperating systems for a deep cognition, or at least to investigate the nature of our cognition as/for/in the world.

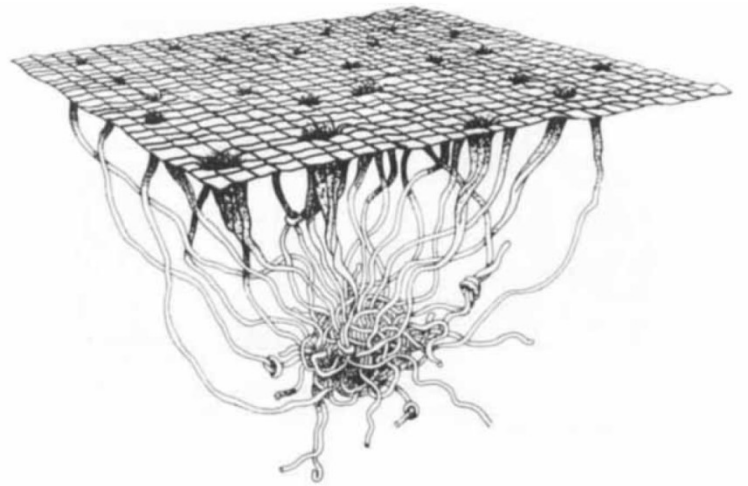

Figure 2: *The universe's surface and the tangled underworld of the Mesoamerican cosmos. Modelled by Cecelia Klein; drawing by Henry F. Klein (from [10, p. 12]).*

The charm of Maya numerical and geometric findings and codification, as we currently understand them, barely corresponds to the "visible top" of a complex culture with an original and peculiar interpretation of mathematics, formalized not only in numbers and geometric shapes, but also by color and sounding codes, as also happens in other Mesoaemerican and Andean cultures. In this sense, Liendo-Stuardo & Zalaquett [13], and Ramos-Amézquita [22] contribute to illuminate the fact that there where trigonometric and physical calculation in Maya, Nahua and Mixtec shrines and massive public spaces design, in order to produce resonance and amplification from voices and musical instruments performance during a rigorously organized calendar for cosmological rites and dance with dense symbolic contents, in a way opposite to Christian traditions where dance is forbidden as a religious practice, and seen as a non-scientific but "artistic" manifestation, splitting the world in a sensual and intellectual axiology; the classic Cartesian —if not Manichaean— axiology that tears our Modernity. Moreover, in Mesoamerican complexity we see strong coherence between mathematics and music through a wide gradient of social practices, in contrast with Western culture where the main focus is over strong individual practices, and its coherence as a weak device designed by a standardized agreement between politics and academy, or belief and institution.

Western modern culture also pays special attention to individuation and unification of symbols, methods and interpretations, to the extent that we may summarize this culture in one term: *unity*. Unity, unification and *unique standard* as the highest values in the context of an idiosyncratic

inertia from a religious biasing after Newtonian institutionalization. In contrast, in Nahuatl and other Uto-Mexican as well as Mayan languages, the concept of *music* cannot be captured in just one word, but in four conceptual fields decomposable in specific subsets (i.e. in "opposite categories" that are complementary among them, rather than absolute opposites; somehow in empathy with Greimasian tetrads, since we do not see an easier analogy proposed by a Western thinker). Even the word Nahuatl does present clear cognates with the concepts of *nahuati*, "to sound clearly and strongly", and *nahui*, the number four, implicating the other tree aural complementary "pluriverses" (instead of universes): *tzotzoniliztli* (rough musical, lower and dense sounds), *tzitzicaquiliztli* (acute and granular higher sounds), and *tlatzotzonaliztli* (melodic and harmonic elaborations on a rhythmic principle). This model —perceptual, aesthetic one— may be described in a non-Cartesian axiology and context, as Cartesianism conceives correct symmetry as the starting point for Euclidean mathematics, taking here the concept of *correctness* particularly by its etymological meaning ("to put straight, attempt to make (a crooked thing) straight, reduce to order, set right"; from *co-*, prefix for accompaniment or parallelism, and *regere*, "to lead straight, rule").

Pioneer European scholars in the Americas, specialized on music and rhetorics, or in musical theory and philosophy, arrived to Mexico in early 16th century; among them the friars Johan Dekkers (*ca*.1476–1525), during fourteen years professor of Theology at the Sorbonne; Peter van Gent (*ca*.1480–1572) singer from the Habsburg royal family, and Alonso de la Vera-Cruz (*ca*.1507–1584), author of *Physica speculatio* (1557), the first book of physics imprinted in the New World. As a part of a moral, spiritual and political "conquest", they established in Mexico, violently, the Cartesian axiology as a convenient abstraction of Christianity. Under this model, the first Mexican mestizo ordained as friar in the Americas was educated: Diego de Valadés (1533–1582), son of a Spaniard and a Tlaxcaltec, forced to deny and abandon the *nepoualtzitzin*, the native method for counting and associating numbers with colors, sounds and geometrical interpretations. Valadés edited *Rhetorica christiana* published in Perugia in 1579, a treatise where he summarized theological arguments about the nature of the indigenous ("Native Americans") and their "ability" to learn and practice Christianity. In *Rhetorica christiana* he abounded in the missionary methods of the mendicant orders and the means they used to evangelize, including the mandatory abandonment of the *nepoualtzitzin* and *Tonalpohualli* (Mesoamerican calendar) instead of the Christian calendar and Roman and Arabic numerals, and geometry and calculus—used, for instance, to construct the new temples and civil buildings of the New Spain. The final consequence of this lost, was the leave and condemnation of Teponazcuauhtla: a site simultaneously abstract and concrete, as the set of all musical instruments harmonized in a wide pluriverse of distinct and complementary harmonies and voices (see Fig. 3), which also represents an analogical and multiscalar-multidimensional harmonization that bonds music, mathematics and cosmology (a conceptualization closely related to the semiotics of previous Figures, 1 and 2).

Teponazcuauhtla (see Fig. 3, rightmost image) appears in mathematical thought as the fuzzy, self-(dis)similar and nearly-asymmetrical pluriverse where we find the Euclidean universe as a modest subset of perfect symmetries and linear well-ordered developments (a rather trivial exception). This notion is clearly exposed, in different words, by Prigogine's physical and cosmological theory, where "Euclidean geometry and linear order are just exceptional cases in a universe that we can only face through the study of probability, a rather humble discipline in the story of Western mathematics" (see Prigogine & Nicolis [21, pp. 13–14]). By resemblance and empathy with traditional mathematics and existential philosophy in ancient India and China, Teponazcuauhtla's theory may historically confirm the claim of Prigogine [21], in the sense that symmetrical and deterministic modeling in linear physics, are just a "small set" of special cases in physics and mathematics where we should pay special attention to probability if we want to

better understand what *our world* is. Actually this seems to explain why probability was the main concept behind the cyclical and periodic systems of prediction in ancient mathematical devices created in Mexico, including concepts such as *tonalpoa* (guessing by numbers or equivalent signs), *amana* (guessing by the behavior of water or any fluid) and *yolteouia* (guessing by what Charles S. Peirce named "abduction").

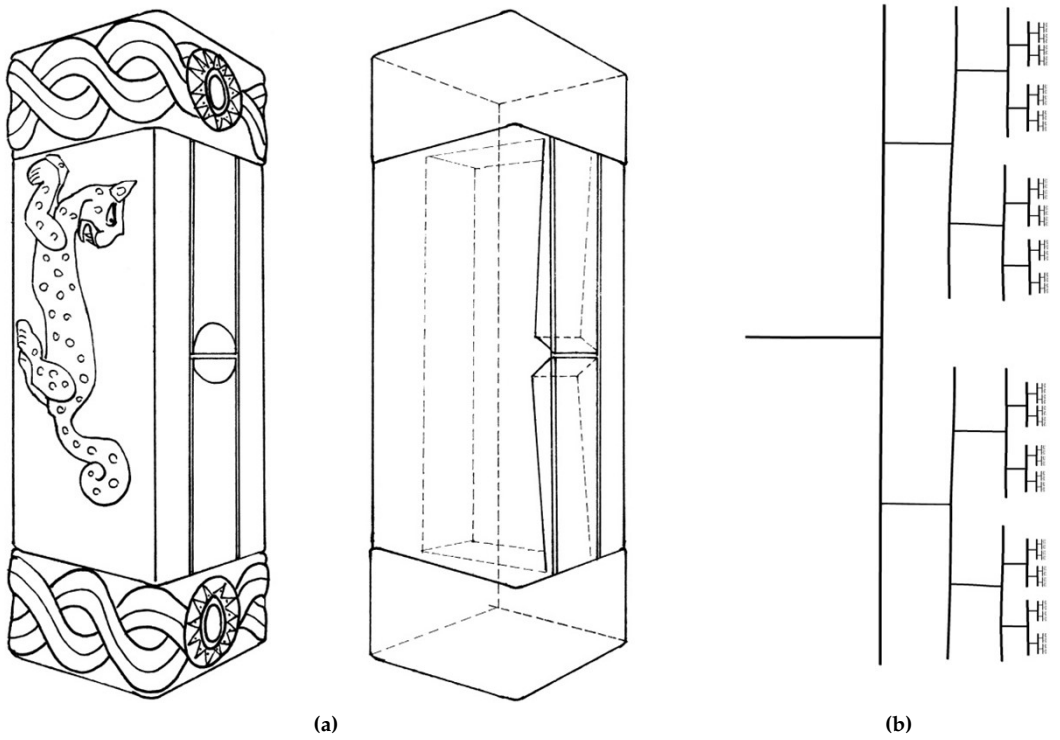

Figure 3: *A model of the sacred musical instrument called Teponaztli (vertical depiction for editorial purposes; the instrument is usually performed in horizontal position). This tuned percussive instrument has two characteristic resonance "tongues" physically coupled to the same wood block in a one-piece resonating system (i.e. the instrument's vibrating plates make part of the resonating cavity itself). Left: schematic surface, upper-frontal view of the instrument. Center: schematic structural view of the same object. Right: abstract self-similar structure of the universe of teponaztlis or Teponazcuauhtla, where the H or T shapes are coupling "all the teponaztlis" in a whole that represents the universe of quasi-symmetric geometrical relationships (slightly warped lines), including as a peculiar subset the set of radial or "perfect" symmetries, a particular case within this "forest" of symmetries (source: [19, p. 321]). For a physico-mathematical study of this instrument, see Pareyon & Pina-Romero [20]. Left and center drawings by Georgina Montes-Varela; the rightmost one by G. Pareyon.*

As a mathematical device, the Teponaztli (and its algebraic structuration in Teponazcuauhtla) has a correspondence with the Andean quipu (also *khipu*, i.e. recording devices fashioned from fiber strings used for storage and calculation data encoded as knots in positional, mathematical symbolization; see Fig. 4). Tun & Díaz-Sotelo [26] proposal on "recovering historic memory [through] Andean mathematics" is directly related to the recovering of an autochthonous, original music closely connected to Teponazcuauhtla. This is feasible because we may "take" the larger

line in Fig. 3 (rightmost image) and attach to it a virtually infinite set of lines of different lengths, using these lengths for "recording" harmonies as sets of proportions signed by knots in the quipu, and by hits in the teponaztli, a wooden percussion instrument. A quipu is a thread recorder with these computational features, whilst the *yupana* is its own interpretation although in the form of a "box of boxes" organized in positional stratification. Sandborn & Sandborn [24, pp. 38–41] study these devices for a comparative history of non-Western musical harmony, providing some clues for interpretation:

> Based on the observable structure of the yupana, that it was used to learn songs, and that it exists in circular form (a wheel), it must then be a calculating, learning, and compositional non-acoustic music technology device. [...] The geometric features of the yupana stones and beans are likely correlated to the quipu geometric color twining patterns of solid, spliced, spiral striped, mottled, or fade-in/fade-out, along with the multiple knot types and spin states.
>
> From this perspective, the yupana is then the calculating or theory composition instrument and the quipu is the recording medium of the composition. [...] It is strongly hypothesized that the quipu was a written language which was synonymous to music. [Its] interchangeability of numbers, colors, sounds and/or vowels, consonants, or morphemes is highly connective to the Hindu culture as well as the Aztecs and Mayas.

The dyadic relationship between understanding and teaching this knowledge —deep in both senses of anthropology and history, and ethics and aesthetics— should re-orient the skills and capacities of Latin American schools and universities, looking for a real independence and liberation of colonialism and mind subjection. The following section suggests that we need to start this task by revising our ethic-aesthetic grounds, before any attempt for complexity.

III. Peace as the starting point for a new understanding of music and mathematics

Not even in recent times, the musical-mathematical thought of the autochthonous peoples of the Latin American nations is considered to be included in the "states of the art" on music and mathematics. The reason for this neglect lies in a long tradition of discrimination and estrangement of native peoples seen as "useless" or "incomprehensible"; a kind of knowledge *incompatible* with the Eurocentric tradition of *true knowledge*, only associable with the so-called Western Civilization. It is from this perspective that Guido Adler (1855–1941) laid the conceptual foundations to split Musicology from Ethnomusicology, mirrored in the false symmetry of Mathematics and Ethnomathematics. Academic institutions, universities and official teaching and research institutes under the auspices of national governments inspired by the EU/US model, have aligned themselves to continue and strengthen the univocal nature of these perspectives. The result is a fragmented knowledge, where on the one hand the traditional knowledge related to the original peoples, simply do not have a place, and on the other hand the regional theories inspired by the western model *always* come behind and is not seen by the institutional academy but as marginal knowledge of "underdeveloped countries". This view is of a violent nature and does not contribute to any harmonization between ethics and aesthetics in mathematical research, nor in musicology. Then we should conceptualize the study of music and mathematics in a peaceful, constructive and collaborative model, as suggested by D'Ambrosio [6].

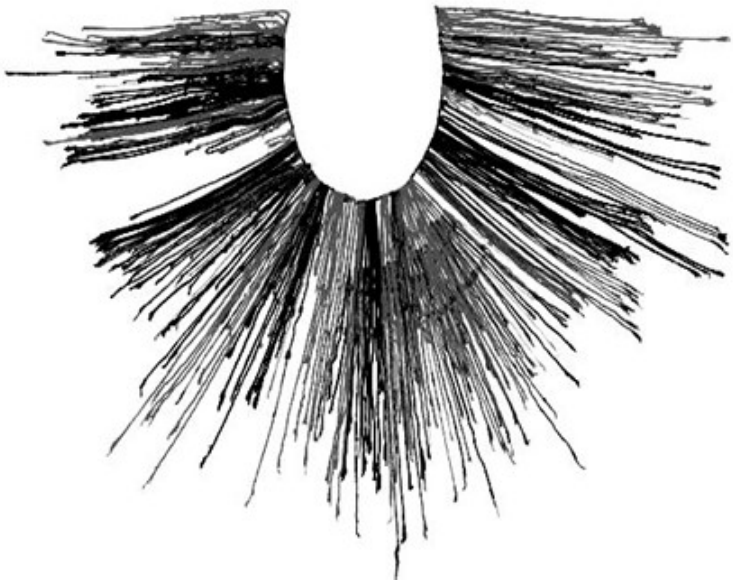

Figure 4: *Inca quipu, Larco Museum, Lima (Peru); dated probably from 16th century. According to Sandborn & Sandborn [24, pp. 38–41], this fabric string device was used to calculate and recording musical information for pitch scales, as well as for harmonic and rhythm combinations.*

Brazilian mathematics educator and historian of mathematics Ubiratan D'Ambrosio (São Paulo, 1932–2021) coined the concept of *Search for peace as a responsibility of mathematicians*, from the assumption that the relationship between Mathematics and Ethnomathematics must be conceptually empathetic and historically reversible in terms of mutual appraisal of epistemic categories, i.e. a non-ethnically centered Mathematics:

> Peace is understood in its four dimensions: Military Peace, Environmental Peace, Social Peace and Individual Peace. Reaching a state of Peace, in its four conceptual dimensions, is the most necessary and urgent task of humanity. We can affirm that this is the most universal problem that all of us face. It is recognized that Mathematics is the *most universal science*. The inquiry is how can these two universals be reconciled? It is naive to say that Peace is a concern of a different kind. The History of Mathematics shows us that the violations of Peace and the progress of Mathematics have mutually benefited from a strong association throughout the evolution of the human species. This [conceptualization] aims to discuss the responsibility of mathematicians in order to redirect their intellectual instruments, mainly Mathematics, for the search for Peace.

We may paraphrase this idea by its mirror in musicology, as follows: it is recognized that Music is the most universal social practice —especially if we interpret rhythm as a core component of Music. Indeed, the academy and institutionalization of (the study of) music documents how, respectively, Music and Mathematics were historically used to strengthen violence in its most varied forms (musical nationalism, musical fascism, political misuse of music). But, for the sake of a rational harmonization between ethics and aesthetics, Music and Mathematics must be redirected to a different kind of social construction, in order to grant a Peaceful understanding of different societies including their memories from their own histories and findings of numbers, geometry, algebra... as well as musical instruments, ways of tuning and singing, dancing... Then,

paraphrasing D'Ambrosio [6], we may say that *The search for peace is a responsibility of musicians (and musicologists)*. And as a consequence of this postulate, musical academics should accept no any fundamental distinction between "music" and "ethnomusic" or "mathematics" and "ethnomathematics", because, indeed, all music is in fact ethnomusic, and analogously all mathematics is ethnomathematics as a development and result of societies and individuals making part of and expressing through cultural groups of any ethnic composition.

IV. Teponazcuauhtla's musimatics in the 21st century

After the 16th century Conquest, the last splendors of Teponazcuauhtla were seen and heard by Sor Juana Ines de la Cruz (1648–1695), the Hieronymite nun who spoke Nahuatl and wrote a nevertheless incomplete axiomatics for music and mathematics. Thereafter, in spite of intense musical practice at Christian cathedrals and convents of the New Spain, a long silence prevailed on the subject, until violinist, composer and conductor Julián Carrillo (1875–1965) [3, 4, 5], of native origin, produced a notorious alternative over the European authorized systems of tuning and musical interpretation. Settled back in Teponazcuauhtla's blurry poetics and harmony, Carrillo invoked again "mathematical resonance" as a "forest of trees [that] resemble colored cataracts" [*sic*] [3, p. 101], specifically in the context of musical harmony. However, although Carrillo is respected as the reformer and refounder of modern music and mathematics in Mexico, his enormous and also unclear heritage demands today a huge effort, both in technical revision and in practice for rebuilding his musical instruments and performing his music using his musical scales based in 6th series of roots of 2 (i.e. $\sqrt[6]{2}$, $\sqrt[12]{2}$, $\sqrt[18]{2}$, and so on). An example of progress in this direction is García-Hurtado's PhD dissertation [9] about Carrillo's *Concertino* (1927) for microtonal ensemble nested within a symphonic (tonal) orchestra.

Carrillo's mathematical and musical challenge was also interestingly contrasted by another Mexican theorist and musical instruments constructor: Augusto Novaro (1893–1960), who based his own musical scales on the "natural" harmonic series [18], i.e. employing experimental proved behavior of vibrating strings, plates and tubes, in order to produce a set of useful harmonic intervals for music composition. This revolutionary conceptualization for a physically-based harmony did not dismiss a possible negotiation with psychoacoustic-based harmony, anticipating the use of Fourier analysis for studying both, the human ear's complexity and the material complexity in order to create and perform music from a perspective different to those employed along the Modern Era. Novaro's legacy is actually promoted by the Augusto Novaro Society founded by Californian composer/sound artist Kraig Grady (1952–), and also settled the foundations for the geometrical theory and compositional design developed by Mexican composer Ervin Wilson (1928–2016). In a certain manner, Wilson approaches also to Teponazcuauhtla's aesthetics, although not directly from archaeological or ethnological means, but after reviewing the symbolic trees of Charles S. Peirce (1839–1914), that historically produces, in the last third of the 20th century, two distinct lines of research: Wilson's diamond cross-sets, diamonds and combination-product sets (see Narushima [17]), and the well-known study on Generative Theory of Tonal Music (see Lerdahl & Jackendoff [11]). This culturally intersected music theorizing and knowledge opens a fruitful discussion on how Peirce united —consciously or unconsciously, by accident or not— a non-linear view on music (probably under Cantor's influence), with deep interests of Mesoamerican aesthetics where musical sound and mathematics are considered by necessary coexistence (i.e., again, Teponazcuauhtla's aesthetics).

Teponazcuauhtla: the forest of musical-mathematical signs and resonances is traced and plotted by the theories, musical instruments and compositions of Carrillo, Novaro and Wilson, and perhaps also perceivable in the most complex music of Conlon Nancarrow (1912–1997),

who adopted Mexican aesthetics as a meaningful component for his music; namely through non-linearity, hyper-plurality of harmonies, and the potential richness between symmetry and anti-symmetry. At this point, the books, musical scores and recordings, and software developed by Julio Estrada (1942–), in particular his *d1 theory* and harmonic permutahedron (described further in the next text of this same issue), seems to be a following chapter in the robust tradition of Carrillan trees and the (anti)symmetry and (dis)similarity relationships applied to a musical grammar.

The hardest current challenge for a Latin American reunion with a complex Native musical past, is actually to achieve a non-conflictive coexistence with Modern "Western" music theories and practice, that can be assimilated through the principle of symmetry and linear Cartesianism as a mathematical subset of a non-linear complexity (as Prigogine predicted for an extended yet "unified field" for human knowledge, including probability as a major topic). We may say that, in the meantime, music and mathematics research in Latin America, from California to Tierra del Fuego, is enriched through a hyper-diversity of which the following texts is rather a first and very wide view that necessarily excludes relevant topics for the sake of synthesis.

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