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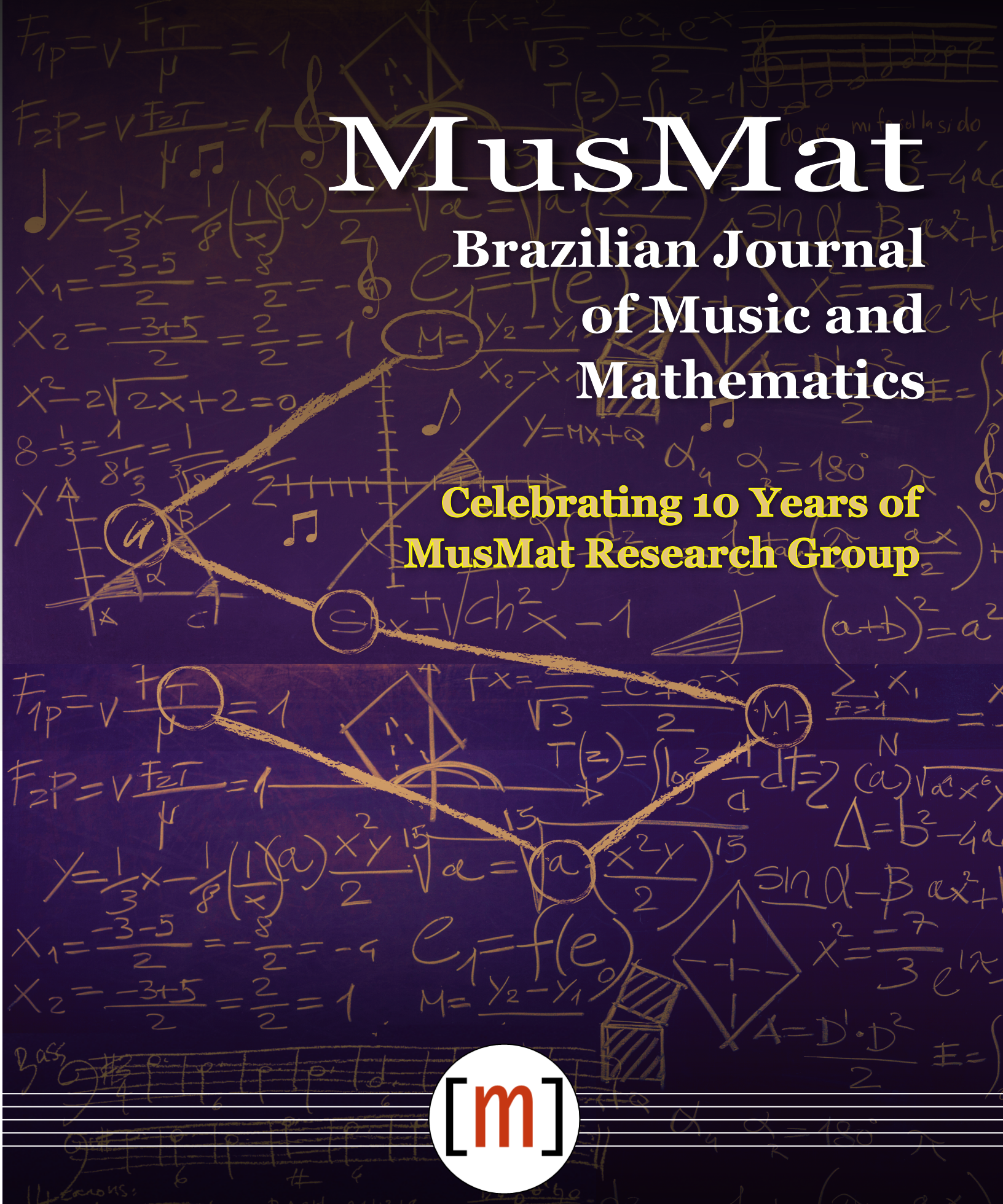

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Foreword

This year, the MusMat Research Group is completing 10 years of its existence, and we are very delighted to celebrate it with the release of this very special issue of the *MusMat • Brazilian Journal of Music and Mathematics*. Being aware of the greatness of the Latin America culture and its contrasting underrepresentation in the academic scenario, we devoted most of this issue to contribute and stimulate research on Music and Mathematics in our subcontinent. The first paper, by Gabriel Pareyon, is an exciting text on three questions about the anthropological and historical foundations for the indigenous relationships of Music and Mathematics in Latin America, where deep connections between both subjects are traced to the very historical roots of our culture. The second paper is an extensive (but not exhaustive) overview on the major developments of research on Music and Mathematics in Latin America in the last 25 years, made by a joint effort between the MusMat Group, Gabriel Pareyon and the fellow researchers that contributed with information regarding their respective countries, regions, and research groups. This is the first comprehensive overview on the research on Music and Mathematics in Latin America, and we hope that its publication stimulates similar initiatives. From now on, the MusMat Journal will dedicate a section to publish information with similar content: so, feel free to submit a document about your research group, region, or country, if you think this information is missing here. The third text on this issue is a compendium on the activities of the MusMat Group on the last 10 years, as well as current activities and future perspectives. Finally, we close this number presenting an interview with Richard Cohn, an outstanding researcher that published a paper in the first number of the MusMat Journal. We hope that the publication of this issue stimulates students and researchers to dive into this fascinating intersection of fields. Enjoy the reading!

MusMat Group

June 2022

Three Open Questions from the Indigenous Epistemology, over Music and Mathematics in the Latin American 21st Century

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Abstract: *It is a fact that the ancient peoples of the Americas had, for many centuries, robust mathematical and musical cultures. Although much of these cultures was not clearly recorded in writing by the first European settlers, this does not mean that their existence is insignificant, and much less, that they have been lost forever. Due to the importance of publishing studies on music and mathematics from a Latin American perspective, the following text, initiatory on the subject, proposes three key questions to promote a discussion from anthropological and historical foundations for the indigenous relationships of music and mathematics, and with the intention of motivating more questions, or formulating them in a better way, in further research.*

Keywords*: Mathematical Musicology. Latin America. Mesoamerican Mathematics and Music.*

I. Introduction

The following notes are motivated by an insight of music and mathematics from the viewpoint of regional history, in Latin American countries where colonization has not been fully achieved. In other words, from the living testimonial notions of an autochthonous knowledge within its own right to exist by itself, and in possible harmony with intellectual and artistic practices imported to build an enriched experience of the so-called Western civilization.

Many questions may arise from this discussion. However, here are only three of them as few among the capital ones, in order to develop further discussion on the history, education and transformation of mathematics related to music. These three questions are framed into the subjects of (firstly) the recognition of a cultural Mesoamerican and Andean heritage useful and refreshing for a wider comprehension of the bonds between music and mathematics; (secondly) music and mathematics by their relationship with ethics and a social frame for peace among the peoples, and the links of the latter, with life survival on Earth, and (thirdly) the search for a non-conflictive coexistence between musical practices and theories from different origin and contexts, without the imminent elimination of the non politically and economically predominant ones.

II. On the Heritage of Native Peoples

The history of mathematics for the ancient civilizations of Latin American countries such as Bolivia, Peru, Mexico and Central America is actually under intense scrutiny and resignification. We may recall that in 4th century BC the number zero was invented by the Olmec civilization[1], and later, along six or seven centuries from 2nd to 9th AD, the Maya developed a deep mathematical knowledge, on par with a refined literature and musical practices later absorbed by the cultures of Teotihuacan and Tula, which in turn are the basis of the Mexica civilization. Congruently, López-Austin [14] confirmed that Mesoamerican mathematics was elaborated aside with musical (*i.e.* sonic, aural, numerical, geometrical, rhythmic and algebraic) assumptions. This notion was previously suggested by Klein [10], Martínez del Sobral [16], Agustín-Aquino [1], Dehouve [8] and Romero-Murguía [23], within their appreciations of Mexican systematization of mathematics through geometrical design of musical instruments and architectural constructions for public performances during the Classic and Post-Classic Mesoamerican periods. In addition, López-Austin's article synthesizes how elementary conceptions of number and space in Mesoamerica are attached to deep axiomatics within a theoretical frame implicating musical and mathematical symbolization. Such symbolization also implies synesthetic codification and formal representation of time and space, with computation of long-term dates and astronomical investigation; a matter systematically investigated since C. Klein's publication from 1982 [10].

Klein [10, p. 25] proposed a model of the Mesoamerican universe "as a giant piece or pile of folded fabric", deducting its shape from a serpentine figure from the Dresden Codex (see Fig. 1). Thus, "the infinite strands of the universe must have been seen as integrated into a giant pile of *cloth*. The Maya manuscripts are full of references to the *folds of the k'atun*" (*loc. cit.*, my italics)[2]. López-Austin [14] concludes that these "folds of the k'atun" model was interpreted by Nahua cultures under the concept of the *cuencoliuhqui* (*op. cit.*: 139), "the cosmic flow that moves in ascents or descents [...] in a serpentine sliding of time and space", through different layers of synchronic complexity. This conceptualization would be enough popular in ancient times, to feed long-term transmission of cultural values. Then, for Klein [10, p. 22] (based on Boiles, [2]), the sunlight and cosmic rays are represented by "twisted cords" and "woven leaf rings", connected to the formalization of rhythms, pitches and melodies to be sung following to a ritual calendar of general use along a wide territory in North and Central America, during centuries in which these cosmic representations were displayed by cultural traits according to distinct epochs, and to levels of complexity more or less accessible in order to socially interpret this tradition, including music and dance as important symbolic conveyors.

As in the emergence of other cultures in the world, in Mesoamerica civilizations emerged from the cultivation of knowledge directly related to the measure of the heavens and meteorology, due to its great relevance to agricultural practices. However, in Mesoamerica the agricultural, the fungal and the phytomorphic were interpreted as a book, as an aesthetic, ethical, musical and mathematical lesson. Agricultural and astronomical practices were transformed, over the centuries, into systems of reading, interpretation and thought through numbers, geometry and the proportionality of phenomena.

As MacLaury states [15, p. 523], "The Aztec metaphor *people are plants* enables pervasive conception of human actions, processes, and creations [...this metaphor] equates the most beautiful achievement of people with that of plants". Thus, for these cultures, the nature of music and mathematics could not be of human exclusivity, but rather as a common trait to living beings and societies on Earth. The manifold organization of complexity would be composed by the same

[1] See: Meggers, 1975 [12, p. 2] & Dassonville, 2016 [7, pp. 107–108].

[2] A *katun* is a unit of time in the Maya calendar equal to 20 tuns or 7,200 days, equivalent to 19,713 tropical years.

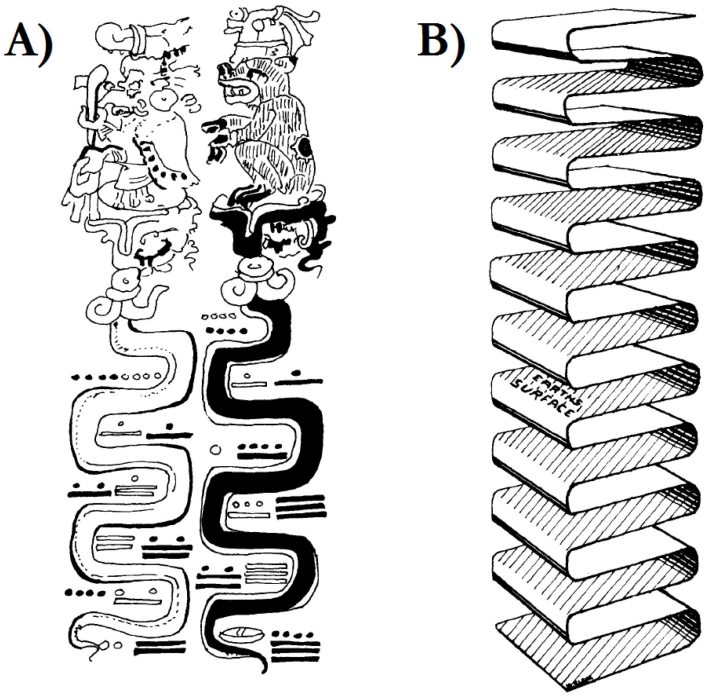

Figure 1: ***A)*** *Undulant serpent, Codex Dresden, plate 62. Maya. Post-Classic period (13th or 14th century). Drawing by Cecelia Klein [10, p. 62] based on J. E. S. Thompson [25]. Notice the writing of number zero in the black column-bottom, here represented by the abstract shape of an empty cocoa bean shell. After the Spanish invasion of Mexico, in the 16th century, nearly all the Maya texts were destroyed, in deference to writings that conform to Biblical doctrine; alongside, Western mathematics were introduced by scholars teaching in the universities following the European model, emerged from Christian and Aristotelian inspiration.*
B) *The same "Undulant serpent" chiral Maya model of the universe modeled as a "folded cloth" (*cuencoliuhqui *in Nahuatl language). Drawing by Henry F. Klein (from C. Klein, [10, p. 27]), with the text "Earth's Surface" at the central layer, referring to a present time-space.*

"elements" intertwined through a variety of combinations of "rhythms".

López-Austin [14] points out that the Mesoamerican folded fabric model of the universe does represent a "dynamics of crossed planes", in other words, a multidimensional time-space model intuitively perceivable for a local observer only in one of its layers (in Fig. 1-right, indicated as the "Earth's surface", perceivable as local time-space coordination). This interpretation arise a discussion on the history of modern cosmology and mathematics, since standard physical models in the so called Western academy are, if not "fully comparable", at least semiotically comparable. See for instance, in H. F. Klein (Fig. 2), the representation of "the universe's surface and the tangled underworld of the Mesoamerican cosmos", which strongly recalls the "standard model" of a flatten universe with gravitational loci of mass hyper-concentration ("black holes"); while the multidimensional model (Fig. 1) resembles conceptual structuration of M-theory, also a common place in modern Western cosmology.

Whether there is a mutual inspiration between anthropology, Mesoamerican archaeology and

standard cosmology, or rather we perceive mere superficial coincidences or serendipity, deep epistemics and cognitive anthropology should contribute to better understand the human structure of heuristics in physics and mathematics. Then, from this theoretical outlook, a comparativistic method for mathematical epistemics is urgent; since mathematical and semiotic ressemblances between the Mesoamerican *cuencoliuhqui* (i.e. the time-space *folded fabric* model) and M-theory reflects a deep analogy between mathematical abstraction and cognition of physics. This comparativism may depict a human ("universal") stem for the understanding of a "mathematical sense of the world", in parallel as music can be thought as the "mathematical sensation of the world". This approach allows us to study mathematics and music as cooperating systems for a deep cognition, or at least to investigate the nature of our cognition as/for/in the world.

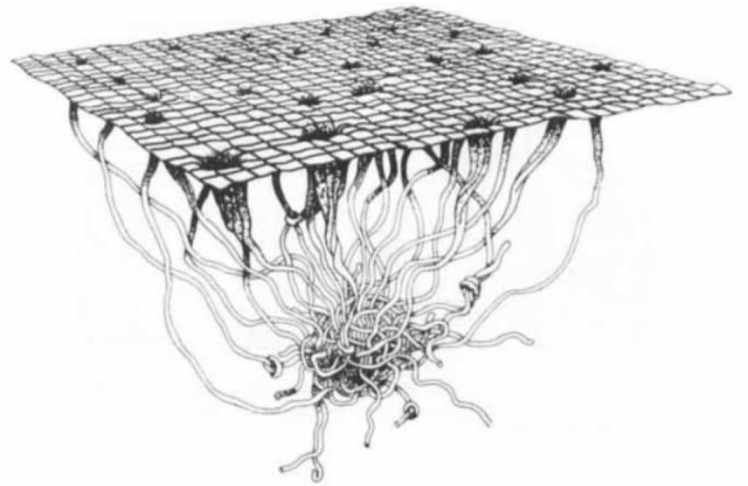

Figure 2: *The universe's surface and the tangled underworld of the Mesoamerican cosmos. Modelled by Cecelia Klein; drawing by Henry F. Klein (from [10, p. 12]).*

The charm of Maya numerical and geometric findings and codification, as we currently understand them, barely corresponds to the "visible top" of a complex culture with an original and peculiar interpretation of mathematics, formalized not only in numbers and geometric shapes, but also by color and sounding codes, as also happens in other Mesoaemerican and Andean cultures. In this sense, Liendo-Stuardo & Zalaquett [13], and Ramos-Amézquita [22] contribute to illuminate the fact that there where trigonometric and physical calculation in Maya, Nahua and Mixtec shrines and massive public spaces design, in order to produce resonance and amplification from voices and musical instruments performance during a rigorously organized calendar for cosmological rites and dance with dense symbolic contents, in a way opposite to Christian traditions where dance is forbidden as a religious practice, and seen as a non-scientific but "artistic" manifestation, splitting the world in a sensual and intellectual axiology; the classic Cartesian —if not Manichaean— axiology that tears our Modernity. Moreover, in Mesoamerican complexity we see strong coherence between mathematics and music through a wide gradient of social practices, in contrast with Western culture where the main focus is over strong individual practices, and its coherence as a weak device designed by a standardized agreement between politics and academy, or belief and institution.

Western modern culture also pays special attention to individuation and unification of symbols, methods and interpretations, to the extent that we may summarize this culture in one term: *unity*. Unity, unification and *unique standard* as the highest values in the context of an idiosyncratic

inertia from a religious biasing after Newtonian institutionalization. In contrast, in Nahuatl and other Uto-Mexican as well as Mayan languages, the concept of *music* cannot be captured in just one word, but in four conceptual fields decomposable in specific subsets (i.e. in "opposite categories" that are complementary among them, rather than absolute opposites; somehow in empathy with Greimasian tetrads, since we do not see an easier analogy proposed by a Western thinker). Even the word Nahuatl does present clear cognates with the concepts of *nahuati*, "to sound clearly and strongly", and *nahui*, the number four, implicating the other tree aural complementary "pluriverses" (instead of universes): *tzotzoniliztli* (rough musical, lower and dense sounds), *tzitzicaquiliztli* (acute and granular higher sounds), and *tlatzotzonaliztli* (melodic and harmonic elaborations on a rhythmic principle). This model —perceptual, aesthetic one— may be described in a non-Cartesian axiology and context, as Cartesianism conceives correct symmetry as the starting point for Euclidean mathematics, taking here the concept of *correctness* particularly by its etymological meaning ("to put straight, attempt to make (a crooked thing) straight, reduce to order, set right"; from *co-*, prefix for accompaniment or parallelism, and *regere*, "to lead straight, rule").

Pioneer European scholars in the Americas, specialized on music and rhetorics, or in musical theory and philosophy, arrived to Mexico in early 16th century; among them the friars Johan Dekkers (*ca*.1476–1525), during fourteen years professor of Theology at the Sorbonne; Peter van Gent (*ca*.1480–1572) singer from the Habsburg royal family, and Alonso de la Vera-Cruz (*ca*.1507–1584), author of *Physica speculatio* (1557), the first book of physics imprinted in the New World. As a part of a moral, spiritual and political "conquest", they established in Mexico, violently, the Cartesian axiology as a convenient abstraction of Christianity. Under this model, the first Mexican mestizo ordained as friar in the Americas was educated: Diego de Valadés (1533–1582), son of a Spaniard and a Tlaxcaltec, forced to deny and abandon the *nepoualtzitzin*, the native method for counting and associating numbers with colors, sounds and geometrical interpretations. Valadés edited *Rhetorica christiana* published in Perugia in 1579, a treatise where he summarized theological arguments about the nature of the indigenous ("Native Americans") and their "ability" to learn and practice Christianity. In *Rhetorica christiana* he abounded in the missionary methods of the mendicant orders and the means they used to evangelize, including the mandatory abandonment of the *nepoualtzitzin* and *Tonalpohualli* (Mesoamerican calendar) instead of the Christian calendar and Roman and Arabic numerals, and geometry and calculus—used, for instance, to construct the new temples and civil buildings of the New Spain. The final consequence of this lost, was the leave and condemnation of Teponazcuauhtla: a site simultaneously abstract and concrete, as the set of all musical instruments harmonized in a wide pluriverse of distinct and complementary harmonies and voices (see Fig. 3), which also represents an analogical and multiscalar-multidimensional harmonization that bonds music, mathematics and cosmology (a conceptualization closely related to the semiotics of previous Figures, 1 and 2).

Teponazcuauhtla (see Fig. 3, rightmost image) appears in mathematical thought as the fuzzy, self-(dis)similar and nearly-asymmetrical pluriverse where we find the Euclidean universe as a modest subset of perfect symmetries and linear well-ordered developments (a rather trivial exception). This notion is clearly exposed, in different words, by Prigogine's physical and cosmological theory, where "Euclidean geometry and linear order are just exceptional cases in a universe that we can only face through the study of probability, a rather humble discipline in the story of Western mathematics" (see Prigogine & Nicolis [21, pp. 13–14]). By resemblance and empathy with traditional mathematics and existential philosophy in ancient India and China, Teponazcuauhtla's theory may historically confirm the claim of Prigogine [21], in the sense that symmetrical and deterministic modeling in linear physics, are just a "small set" of special cases in physics and mathematics where we should pay special attention to probability if we want to

better understand what *our world* is. Actually this seems to explain why probability was the main concept behind the cyclical and periodic systems of prediction in ancient mathematical devices created in Mexico, including concepts such as *tonalpoa* (guessing by numbers or equivalent signs), *amana* (guessing by the behavior of water or any fluid) and *yolteouia* (guessing by what Charles S. Peirce named "abduction").

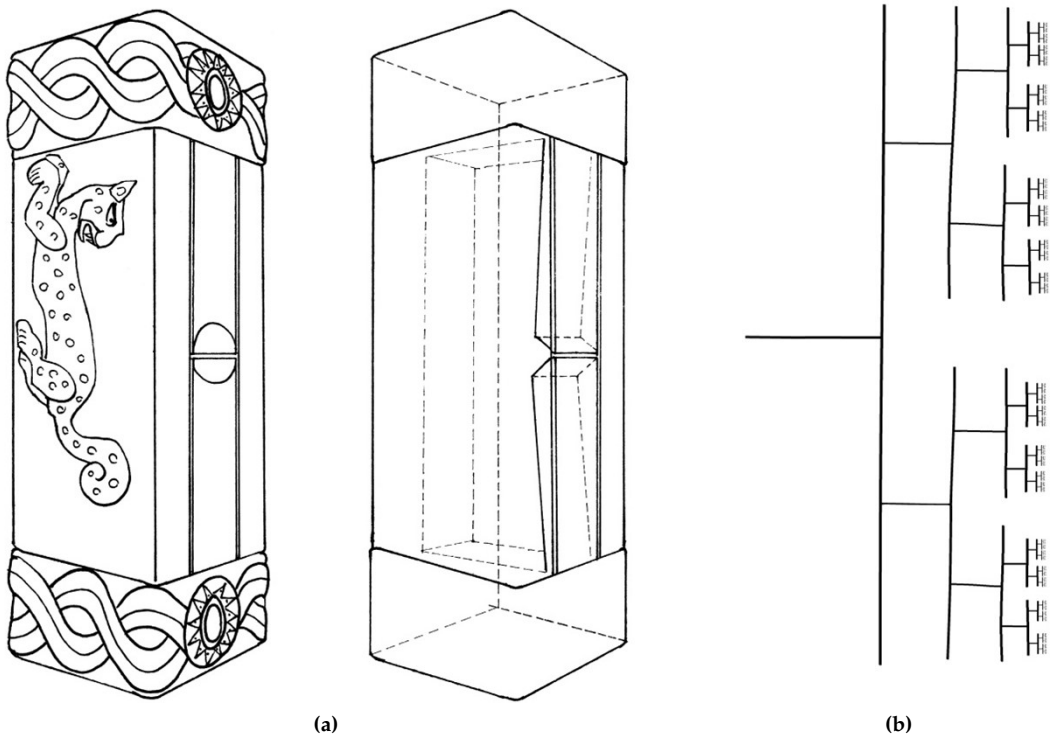

Figure 3: *A model of the sacred musical instrument called Teponaztli (vertical depiction for editorial purposes; the instrument is usually performed in horizontal position). This tuned percussive instrument has two characteristic resonance "tongues" physically coupled to the same wood block in a one-piece resonating system (i.e. the instrument's vibrating plates make part of the resonating cavity itself). Left: schematic surface, upper-frontal view of the instrument. Center: schematic structural view of the same object. Right: abstract self-similar structure of the universe of teponaztlis or Teponazcuauhtla, where the H or T shapes are coupling "all the teponaztlis" in a whole that represents the universe of quasi-symmetric geometrical relationships (slightly warped lines), including as a peculiar subset the set of radial or "perfect" symmetries, a particular case within this "forest" of symmetries (source: [19, p. 321]). For a physico-mathematical study of this instrument, see Pareyon & Pina-Romero [20]. Left and center drawings by Georgina Montes-Varela; the rightmost one by G. Pareyon.*

As a mathematical device, the Teponaztli (and its algebraic structuration in Teponazcuauhtla) has a correspondence with the Andean quipu (also *khipu*, i.e. recording devices fashioned from fiber strings used for storage and calculation data encoded as knots in positional, mathematical symbolization; see Fig. 4). Tun & Díaz-Sotelo [26] proposal on "recovering historic memory [through] Andean mathematics" is directly related to the recovering of an autochthonous, original music closely connected to Teponazcuauhtla. This is feasible because we may "take" the larger

line in Fig. 3 (rightmost image) and attach to it a virtually infinite set of lines of different lengths, using these lengths for "recording" harmonies as sets of proportions signed by knots in the quipu, and by hits in the teponaztli, a wooden percussion instrument. A quipu is a thread recorder with these computational features, whilst the *yupana* is its own interpretation although in the form of a "box of boxes" organized in positional stratification. Sandborn & Sandborn [24, pp. 38–41] study these devices for a comparative history of non-Western musical harmony, providing some clues for interpretation:

> Based on the observable structure of the yupana, that it was used to learn songs, and that it exists in circular form (a wheel), it must then be a calculating, learning, and compositional non-acoustic music technology device. [...] The geometric features of the yupana stones and beans are likely correlated to the quipu geometric color twining patterns of solid, spliced, spiral striped, mottled, or fade-in/fade-out, along with the multiple knot types and spin states.
>
> From this perspective, the yupana is then the calculating or theory composition instrument and the quipu is the recording medium of the composition. [...] It is strongly hypothesized that the quipu was a written language which was synonymous to music. [Its] interchangeability of numbers, colors, sounds and/or vowels, consonants, or morphemes is highly connective to the Hindu culture as well as the Aztecs and Mayas.

The dyadic relationship between understanding and teaching this knowledge —deep in both senses of anthropology and history, and ethics and aesthetics— should re-orient the skills and capacities of Latin American schools and universities, looking for a real independence and liberation of colonialism and mind subjection. The following section suggests that we need to start this task by revising our ethic-aesthetic grounds, before any attempt for complexity.

III. Peace as the starting point for a new understanding of music and mathematics

Not even in recent times, the musical-mathematical thought of the autochthonous peoples of the Latin American nations is considered to be included in the "states of the art" on music and mathematics. The reason for this neglect lies in a long tradition of discrimination and estrangement of native peoples seen as "useless" or "incomprehensible"; a kind of knowledge *incompatible* with the Eurocentric tradition of *true knowledge*, only associable with the so-called Western Civilization. It is from this perspective that Guido Adler (1855–1941) laid the conceptual foundations to split Musicology from Ethnomusicology, mirrored in the false symmetry of Mathematics and Ethnomathematics. Academic institutions, universities and official teaching and research institutes under the auspices of national governments inspired by the EU/US model, have aligned themselves to continue and strengthen the univocal nature of these perspectives. The result is a fragmented knowledge, where on the one hand the traditional knowledge related to the original peoples, simply do not have a place, and on the other hand the regional theories inspired by the western model *always* come behind and is not seen by the institutional academy but as marginal knowledge of "underdeveloped countries". This view is of a violent nature and does not contribute to any harmonization between ethics and aesthetics in mathematical research, nor in musicology. Then we should conceptualize the study of music and mathematics in a peaceful, constructive and collaborative model, as suggested by D'Ambrosio [6].

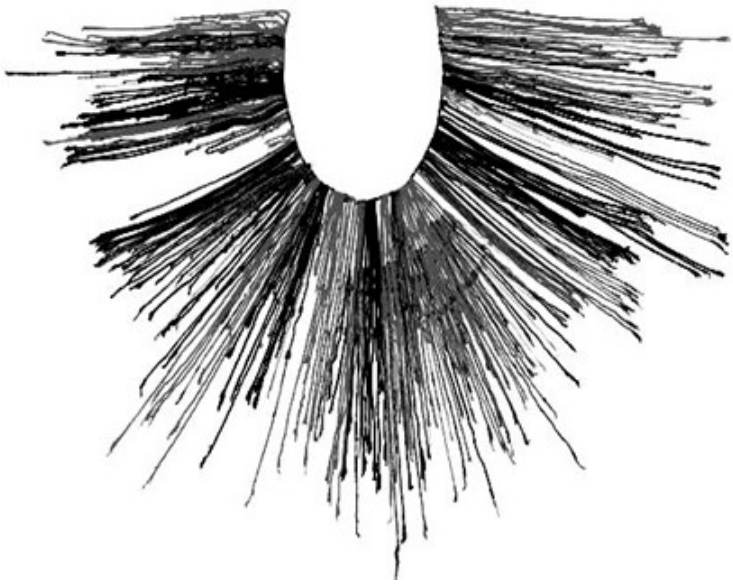

Figure 4: *Inca quipu, Larco Museum, Lima (Peru); dated probably from 16th century. According to Sandborn & Sandborn [24, pp. 38–41], this fabric string device was used to calculate and recording musical information for pitch scales, as well as for harmonic and rhythm combinations.*

Brazilian mathematics educator and historian of mathematics Ubiratan D'Ambrosio (São Paulo, 1932–2021) coined the concept of *Search for peace as a responsibility of mathematicians*, from the assumption that the relationship between Mathematics and Ethnomathematics must be conceptually empathetic and historically reversible in terms of mutual appraisal of epistemic categories, i.e. a non-ethnically centered Mathematics:

> Peace is understood in its four dimensions: Military Peace, Environmental Peace, Social Peace and Individual Peace. Reaching a state of Peace, in its four conceptual dimensions, is the most necessary and urgent task of humanity. We can affirm that this is the most universal problem that all of us face. It is recognized that Mathematics is the *most universal science*. The inquiry is how can these two universals be reconciled? It is naive to say that Peace is a concern of a different kind. The History of Mathematics shows us that the violations of Peace and the progress of Mathematics have mutually benefited from a strong association throughout the evolution of the human species. This [conceptualization] aims to discuss the responsibility of mathematicians in order to redirect their intellectual instruments, mainly Mathematics, for the search for Peace.

We may paraphrase this idea by its mirror in musicology, as follows: it is recognized that Music is the most universal social practice —especially if we interpret rhythm as a core component of Music. Indeed, the academy and institutionalization of (the study of) music documents how, respectively, Music and Mathematics were historically used to strengthen violence in its most varied forms (musical nationalism, musical fascism, political misuse of music). But, for the sake of a rational harmonization between ethics and aesthetics, Music and Mathematics must be redirected to a different kind of social construction, in order to grant a Peaceful understanding of different societies including their memories from their own histories and findings of numbers, geometry, algebra... as well as musical instruments, ways of tuning and singing, dancing... Then,

paraphrasing D'Ambrosio [6], we may say that *The search for peace is a responsibility of musicians (and musicologists)*. And as a consequence of this postulate, musical academics should accept no any fundamental distinction between "music" and "ethnomusic" or "mathematics" and "ethnomathematics", because, indeed, all music is in fact ethnomusic, and analogously all mathematics is ethnomathematics as a development and result of societies and individuals making part of and expressing through cultural groups of any ethnic composition.

IV. Teponazcuauhtla's musimatics in the 21st century

After the 16th century Conquest, the last splendors of Teponazcuauhtla were seen and heard by Sor Juana Ines de la Cruz (1648–1695), the Hieronymite nun who spoke Nahuatl and wrote a nevertheless incomplete axiomatics for music and mathematics. Thereafter, in spite of intense musical practice at Christian cathedrals and convents of the New Spain, a long silence prevailed on the subject, until violinist, composer and conductor Julián Carrillo (1875–1965) [3, 4, 5], of native origin, produced a notorious alternative over the European authorized systems of tuning and musical interpretation. Settled back in Teponazcuauhtla's blurry poetics and harmony, Carrillo invoked again "mathematical resonance" as a "forest of trees [that] resemble colored cataracts" [*sic*] [3, p. 101], specifically in the context of musical harmony. However, although Carrillo is respected as the reformer and refounder of modern music and mathematics in Mexico, his enormous and also unclear heritage demands today a huge effort, both in technical revision and in practice for rebuilding his musical instruments and performing his music using his musical scales based in 6th series of roots of 2 (i.e. $\sqrt[6]{2}$, $\sqrt[12]{2}$, $\sqrt[18]{2}$, and so on). An example of progress in this direction is García-Hurtado's PhD dissertation [9] about Carrillo's *Concertino* (1927) for microtonal ensemble nested within a symphonic (tonal) orchestra.

Carrillo's mathematical and musical challenge was also interestingly contrasted by another Mexican theorist and musical instruments constructor: Augusto Novaro (1893–1960), who based his own musical scales on the "natural" harmonic series [18], i.e. employing experimental proved behavior of vibrating strings, plates and tubes, in order to produce a set of useful harmonic intervals for music composition. This revolutionary conceptualization for a physically-based harmony did not dismiss a possible negotiation with psychoacoustic-based harmony, anticipating the use of Fourier analysis for studying both, the human ear's complexity and the material complexity in order to create and perform music from a perspective different to those employed along the Modern Era. Novaro's legacy is actually promoted by the Augusto Novaro Society founded by Californian composer/sound artist Kraig Grady (1952–), and also settled the foundations for the geometrical theory and compositional design developed by Mexican composer Ervin Wilson (1928–2016). In a certain manner, Wilson approaches also to Teponazcuauhtla's aesthetics, although not directly from archaeological or ethnological means, but after reviewing the symbolic trees of Charles S. Peirce (1839–1914), that historically produces, in the last third of the 20th century, two distinct lines of research: Wilson's diamond cross-sets, diamonds and combination-product sets (see Narushima [17]), and the well-known study on Generative Theory of Tonal Music (see Lerdahl & Jackendoff [11]). This culturally intersected music theorizing and knowledge opens a fruitful discussion on how Peirce united —consciously or unconsciously, by accident or not— a non-linear view on music (probably under Cantor's influence), with deep interests of Mesoamerican aesthetics where musical sound and mathematics are considered by necessary coexistence (i.e., again, Teponazcuauhtla's aesthetics).

Teponazcuauhtla: the forest of musical-mathematical signs and resonances is traced and plotted by the theories, musical instruments and compositions of Carrillo, Novaro and Wilson, and perhaps also perceivable in the most complex music of Conlon Nancarrow (1912–1997),

who adopted Mexican aesthetics as a meaningful component for his music; namely through non-linearity, hyper-plurality of harmonies, and the potential richness between symmetry and anti-symmetry. At this point, the books, musical scores and recordings, and software developed by Julio Estrada (1942–), in particular his *d1 theory* and harmonic permutahedron (described further in the next text of this same issue), seems to be a following chapter in the robust tradition of Carrillan trees and the (anti)symmetry and (dis)similarity relationships applied to a musical grammar.

The hardest current challenge for a Latin American reunion with a complex Native musical past, is actually to achieve a non-conflictive coexistence with Modern "Western" music theories and practice, that can be assimilated through the principle of symmetry and linear Cartesianism as a mathematical subset of a non-linear complexity (as Prigogine predicted for an extended yet "unified field" for human knowledge, including probability as a major topic). We may say that, in the meantime, music and mathematics research in Latin America, from California to Tierra del Fuego, is enriched through a hyper-diversity of which the following texts is rather a first and very wide view that necessarily excludes relevant topics for the sake of synthesis.

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Music and Mathematics in Latin America: Major Developments in the Last 25 Years

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Abstract

This text is an overview for Latin America across the field of music and musicology intersecting mathematics, including the advances from perspectives of experimentation, creation, analysis and pedagogy throughout interdisciplinary developments, particularly in the field of computational science and philosophy of science. Our main goal is to spread worldwide the richness and variety of research on music and mathematics in Latin America as well as stimulate further investigation in this fascinating intersection.

***Keywords**: Music and Mathematics. Latin-American Research. Survey.*

I. Introduction

The first text on this issue presents a compact and stimulating historical overview on the diversity of concepts and methods of the native people of Latin America on connections between music, mathematics, and the natural sciences. Despite the breakage of this evolutionary line due to the colonization and the consequent loss of information, the cultural richness of our subcontinent is element of study by scholars worldwide. This text is the result of a joint effort in collecting and summarizing some of the major developments made on research on music and mathematics in Latin America —at least on the last 25 years—, as means of providing an initial step on unifying and displaying our cultural and scientific richness on these subjects. By no means this text is exhaustive, nor it intend to be;[1] however its main importance lies on the fact that it is the very first document that broadly catalogues this kind of research in Latin America.

In order to facilitate the reading, the countries and/or regions are divided among the sections and subsections of this text, in alphabetical order. At the very end an extensive list of bibliographical references are provided, splitted in the same fashion as the rest of the text, but with continuous numeration. This work would not be possible without the collaboration of Rodolfo Coelho de Souza, Paulo de Tarso Salles, Marco da Silva Sampaio, Pedro Kroeger, Didier Guigue, Jônatas Manzolli and Luiz Biscainho, which provided us with information on their respective groups and research and to whom we thank a lot.

Finally, we think that the *MusMat • Brazilian Journal of Music and Mathematics* has a paramount responsibility on hosting not only this first overview but also on encouraging fellow researchers from Latin America on gathering and spreading similar information. In order to stimulate this, from now on the MusMat Journal will dedicate a section to publish information with similar content. Therefore, if your research group, region, or country is missing, please feel free to submit an analogous document to the Journal.

II. Argentina and Chile

Interest in the relations between mathematics and music has developed in the last fifteen years in Argentina. Musicological works have been published using statistical methods, complex networks, group theory, combinatorics, differential equations, signal analysis, deep neural networks and other computational methods. Here we present a brief overview of these investigations.

Physicist Damián Zanette works in statistical analysis of music and musical performance. In his paper "Zipf's law and the creation of musical context" [25] he considers the emergence of musical context from compositional decisions, privileging some elements over others. He shows that the assumption that the frequency of use of a musical "word" (a note with its associated duration in this case) increases proportionally to the number of its previous occurrences in the piece leads to a Zipf law for the vocabulary. A tonal context such as that of Bach's or Mozart's works is more consistent than Schoenberg's free atonality, and that is reflected in Zipf exponents: small exponents indicate a compact lexicon and, consequently, a robust, stable and well-defined context, while larger exponents reveal an abundant vocabulary.

In another article, Zanette and the composer and musicologist Fernando Benadon study microtiming in 48 performances of J.S. Bach's C Major Prelude from The Well Tempered Clavier, Book I, defining a measure of "rubato depth" and linking it with musical structure [2]. Moreover, applying the method of principal component analysis to individual performers, they find consistent tendencies of microtiming, groove-like, that differ from inconsistent timing inflections, rubato-like.

[1] In fact, this document is highly "biased" due to our own experiences, knowledge, areas of research, and geographical localization!

Between 2008 and 2011, composers Pablo Cetta and Oscar Pablo di Liscia worked in a research project on the application of Set Theory, Combinatorics and Group Theory to the organization of pitch class sets. Besides several publications [6, 9, 7, 8, 21, 5], the main production (still maintained and usable with free license and open code) is the *pcslib software* [4]. Psclib is a library of external objects for Pure Data, allowing for the application of pitch class sets and combinatorial matrices to analysis and composition [16]. It was initially developed as part of the project "Development of computational applications for the organization of tempered pitch in musical analysis and composition" (2007-2008), Universidad Nacional de las Artes, Buenos Aires; then it continued as part of the project "Musical applications of Sets and Combinatorial Matrices of Chromatic Pitches" (2009-2010), Universidad Nacional de Quilmes, Buenos Aires.

Pablo Amster y Bruno Mesz, mathematicians and musicians, studied the evolution of tango harmony in the period 1910–1960, using complex network methods [15]. Some meaningful tendencies of harmonic discourse that arise from this study are a progressive enrichment of harmonic transitions and a Zipf law for triadic chords, with exponents compatible with a very slow increase of vocabulary in time, according to the aforementioned Zanette interpretation of Zipf's law: the small world characteristics of the networks of harmonic transitions become more definite in the course of time. In a previous article [1] they consider a multi-agent model of emergence of musical styles, based on consensus models and an influence dynamics with delay between the agents (composers); that is, the agents influence each other based not just on their musical styles at a given moment, but also on the basis of their past styles. With this they seek to model delays in cultural propagation in, for instance, the emergence of jazz or tango, due to the slowness of information diffusion in the 19^{th} century. The effect of this delay is to produce more stylistic diversity at the end of the evolution process.

Pablo Riera, a physicist and musician, has studied timbre perception and sound synthesis using mathematical and computational tools. The work "Timbre spaces with sparse autoencoders" presented at the 16^{th} Brazilian Symposium on Computer Music [17] describes a non-linear dimensionality reduction technique to perform timbral analysis of musical pieces and sound synthesis using deep neural networks. Other methods explored for sound synthesis by Riera include the sonification of differential equations. The piece *Ode's ode* (ode to ordinary differential equations) was presented at the Live Coding Music event of the National Institute of Pure and Applied Mathematics (IMPA) [18]. There, he introduced a system for live coding with ordinary differential equations for the synthesis and control of sound, where you can write differential equations formulas and modify their parameters in real time. Riera also participated recently in a work carried out at IMPA, related to the analysis of musical networks. In this work scores of classical musical pieces were transformed into networks whose nodes are musical events. In several of the musical pieces, the networks presented properties of self-similarity or fractal structure [19].

In another line of work, Bruno Mesz, together with the physicists Marcos Trevisan and Mariano Sigman, investigated whether taste words elicited consistent musical representations by asking trained musicians to improvise on the basis of the four canonical taste words: sweet, sour, bitter, and salty. Their results showed that, even in free improvisation, taste words elicited very reliable and consistent musical patterns: "bitter" improvisations are low-pitched and legato, "salty" improvisations are staccato, "sour" improvisations are high-pitched and dissonant, and "sweet" improvisations are consonant, slow, and soft [13]. Projections of the improvisations of taste words to musical space (a vector space defined by relevant musical parameters) revealed that, in musical space, improvisations based on different taste words were nearly orthogonal or opposite. In other publications they design an algorithm to compose "flavorized music" [14], investigate taste-music correspondences of basic music structures (scales and chords) [22] and study the effect

of music in the perception of wine and other drinks [12, 23, 24, 10]. In a more artistic vein, they presented an installation based on [13] at the Cooper Hewitt Smithsonian Museum in New York.[2]

In his M.Sc. thesis, Nahuel Arca considers comparison methods of tuning systems using fuzzy logic.[3] He considers also the case of tuning systems that do not respect octave equivalence.

Jimena Royo Letelier, Chilean physicist and artist, has presented a sound installation at the Henri Poincaré Institute in Paris, associating sonorities to imaginary spaces by solving the wave equation in mathematical spaces such as $\mathbb{R}^3/\mathbb{Z}^3$.[4] This installation works by capturing the vibrations of the building that contains the exhibition and treating them digitally, to generate the sonorities that the building would have if it were embedded in the variety $\mathbb{R}^3/\mathbb{Z}^3$. The sound is reproduced through the vibration of some blackboards. Recently she presented *Muros Invisibles* (*Invisible Walls*) together with her collective artistic group IAKERI. This is an installation that proposes the perception of inequalities between women and men through immersion in a space where matter and sound are revealed and distorted by gender statistics.[5] As a scientist, Royo Letelier works on statistical models and signal processing, with applications to musical emotion detection and characterization of musical genres from audio [20, 3, 11].

In November 2016, the workshop "Recent mathematical and computational applications to music" was held at the University of Buenos Aires, bringing together researchers and students from all over Latin America and prestigious international guests. Organized by Bruno Mesz and Pablo Amster, with funding from the Latin American Center for Interdisciplinary Training (CELFI), it was the first experience of this magnitude at the regional level, which made possible the exchange of a large number of participants, including musicians, mathematicians, physicists and sound engineers, among other careers. The courses covered the following topics:

- *Tuning systems and fuzzy logic*, by Vicente Liern (University of Valencia)
- *Using mathematics and computers in music: a practical guide*, by Dmitri Tymoczko (Princeton University)
- *Introduction to discrete musical systems*, by Shlomo Dubnov (University Of California at San Diego)
- *Mathematical aspects in music*, by Pablo Amster (University Of Buenos Aires) and Bruno Mesz (University Of Tres de Febrero).

The following plenary conferences were also held:

- *Information-Theoretic Music Creativity*, by Shlomo Dubnov
- *Rock Logic*, by Dmitri Tymoczko.
- *Applications for the Theory of Sets of Classes of Heights and Combinatorial Matrices*, by Pablo Di Liscia
- *Analysis of Contemporary Music with Automatic Learning Techniques*, by Pablo Riera
- *Approximations to Language and Music Based on Information Theory*, by Damian Zanette
- *What Happened to the Music when the Spheres were Defaced?*, by Vicente Liern.

In addition, there were communications presented by researchers from various countries and a poster session in which doctoral students from across the region presented their progress. The closing ceremony consisted of a concert with a program linked to the subject of the workshop, including works by various participants and professors D. Tymoczko and S. Dubnov. The presentations and other contents can be found on the workshop page.[6]

[2] https://www.cooperhewitt.org/channel/senses/.
[3] http://cms.dm.uba.ar/academico/carreras/licenciatura/tesis/2015/.
[4] http://esthetopies.ihp.fr/works/text.html.
[5] http://iakeri.fr/en/project_en.html.
[6] http://mate.dm.uba.ar/~tallerdemusica/.

III. Brazil

Although the research on the intersection between music and mathematics in Brazil is relatively recent, it has been considerably developed with a number of ramifications in some research groups and universities of different points of the country, notably MusMat (UFRJ - Rio de Janeiro), NCIS (Unicamp - Campinas), M3 (UFPB - João Pessoa), UFBA (Salvador), and USP (São Paulo).

i. MusMat

The research group MusMat, associated with the Graduate Program of Music of Federal University of Rio de Janeiro (UFRJ), was founded in 2012 by Carlos Almada, Daniel Moreira, Liduino Pitombeira, and Pauxy Gentil-Nunes,[7] aiming mainly at applications of mathematical models to musical analysis and composition. After 2020, three new members – all mathematicians – have joined the group: Hugo Carvalho, Carlos Mathias, and Cecília Saraiva. The Group MusMat edits a biannual journal (*MusMat: Brazilian Journal of Music and Mathematics*[8]), organizes yearly a conference on these subjects, and hosts the first podcast in Brazil entirely dedicated to music and mathematics. In the paragraphs below the research done by current and former members of the MusMat is described, and for more information on other activities as well as historical information about the group, see the last text on this issue.

Carlos Almada pursues since 2011 a research on musical variation, based on Schoenbergian principles of *Grundgestalt* and *developing variation* through incorporation of original concepts, development of new methodological procedures and, specially, computational implementation and formalization, by considering two basic axes: analysis and composition. Concerning the former approach, a number of recent works contributed to the improvement of the analytical model: a study on developing variation applied to serial music [32], a proposal for formalization of metric organization of motivic unities [105], and a deep discussion and reformulation of the theoretical-methodological basis of the research, in a doctorate thesis of one of Almada's students, Desirée Mayr [106]. In 2019 Almada envisaged a new perspective for variation, by considering it under the lens of the Transformational Theory [38, 40], as well as by reinforcing the (not only metaphorical) links with biological variation [42]. These new approaches motivated the author to write a book entitled *Musical Variation: Toward a Transformational Perspective* (not yet published).

Concerning compositional issues, Almada elaborated a system formed by computational programs for algorithmic production of variations from a given basic source of material (*Grundgestalt*, in Schoenbergian terms). This system has been largely expanded in the last years, not only through formal connections with mathematical concepts [31, 35], but also by associations with evolutionary biology [30, 33, 34].

Since 2017, Almada and a team of undergraduate students have been also dedicated to investigate systematically the melodic and harmonic construction of Brazilian popular-music composer Antonio Carlos Jobim. A statistical analysis of a representative corpus of songs by Jobim by a robust computational system especially designed for the task provided a considerable amount of data that has been used for the elaboration of theoretical models, aiming to generalization and further application to other repertoires [36, 98, 37, 39, 41, 43, 107].

Liduino Pitombeira studies formal systems applied to musical composition, including both original and modeled ones. The latter were object of two four-year projects (from 2015 to 2018) in *Systemic Modeling* [142, 136]: in the first project, developed within UFRJ's graduate program in music (PPGM/UFRJ), several *Ponteios* by Camargo Guarnieri were modeled, producing, as

[7] Gentil-Nunes left the group in 2020.

[8] See https://musmat.org/en/musmat-journal/.

a result, compositional systems from which pieces for various instrumental formations were planned and composed [59, 60, 133, 137, 138]; in the second one, developed with advanced undergraduate students, eight short Brazilian pieces for piano or guitar were modeled, and also resulted in analytical papers and new original compositions [134, 135, 139]. The modeling methodology is accomplished through the application of several theoretical frameworks such as: *Developing Variation*, *Partitional Analysis*, *Contour Theory* [132], *Textural Contour*, *Motivic and Gestural Analysis* [140], *Markovian Chains*, [59], *Harmonic Endogeny*, [141], *Linear Partitioning*, *Pitch-Class Similarity*, *Voice-Leading Operations*, *Pitch Class Set Theory*, etc.

Starting in 2019, the present research project, titled *Development of Compositional Systems*, focus on original systems introduced as computational algorithms, graphical plannings, and formal definitions. The development of compositional systems encourages the production of diverse computational libraries and scripts to function as CAC (Computer-Assisted Composition), such as the *CompTools* package developed by Pedro Proença (doctoral student) in *Python*. An intense dialogue with other MusMat projects is also an essential part of this research. Recently, the Ph.D. student Roberto Macedo started compositional experimentation with morphogenetic (dynamic) systems, expanding the field and opening more creative possibilities. Students (graduate and undergraduate), post-doctoral fellows, alumni, and independent researchers connected to this project meet once a week to discuss and expand the theoretical framework, to read books and papers related to the subject, to develop programs, and especially to write papers on compositional systems

Pauxy Gentil-Nunes is responsible for one of the research axes developed by members of the MusMat group regarding the development and applications of *Partitional Analysis* or *Musical Partitional Theory* [81]. The research has produced concepts, methodological and computational tools, as the software applications *Parsemat* and *Partops* (*Partitional Operators*), and publications related to the mediation between musical composition and the *Theory of Integer Partitions*, which is itself a branch of *Number Theory*. Partitional Analysis (PA) has been developed since 2003 by Pauxy Gentil-Nunes and since 2011 by members of the MusMat Group.

The theory is based on the groupings of concurrent musical elements in a musical plot, using criteria arbitrated by the analyst [82]. The first analysis and applications [80] were inspired by Wallace Berry's *Textural Analysis*, but quickly move to applications on other domains, as melodic structure [81], orchestration [109, 90], form [78], analysis of musical gestures [61], and interaction with contour theory [118, 115]. One of the most striking features of the theory is the possibility of perform translations between distinct domains as it always convert the relations and elements to the same general framework.

At the moment, the research is moving to applications closer to musical practice. Moreira [112], for example, studies the stages of implementation of PA in the creative process itself, where the questions posed to the composer imply an amplification of the mappings and concepts. Ramos [143] on the other hand, propose the application of PA in the formalization of the instrumental technique and the partitional analysis of physical aspects of performance (finger, hands) and its relationship with texture.

In theoretical terms, all researchers are dealing, to a greater or lesser extent, with consideration of partitions as a surface element, which receives an organization at hierarchical levels of progressive depth, with roles similar to the Schenkerian levels to pitches. These efforts have led to the concept of *Partitional Complexes* [83, 84] — the coordinated and hierarchical use of a set of partitions, by an entire piece or some section, that ends up expressing at a broader level an implicit partition, called *referential*, that more legitimately reveals the nature of the textural discourse. The concept is being refined and expanded but has already proved useful in clarifying where PA seemed not to work fairly (in the sense of making the structure more visible).

Daniel Moreira is interested in the investigation of musical texture as both an analytical and a compositional tool. As an analytical tool, the first approach was presented in his Masters dissertation, with the formulation of a *textural contour* [118, 115, 113], a proposal that combines the *Theory of Musical Contours* with the *Partitional Analysis* [81, 80]. By using such methodology, Moreira analyzed the melodic contours of bassoon and flute in, respectively, Stravinsky's *Introduction* of *The Rite of Spring*, and Debussy's *Prélude a l'Après midi d'un faune*, to compare with their correspondent textural contours [118, 117]. Moreira also applied this methodology in Mussorgsky's *Promenade* from *Pictures at an Exhibition* and Schoenberg's *Three Pieces for Piano* Op.11 to investigate, respectively, the relation between texture and orchestration and the possible reflects of developing variation techniques in textural domain [116, 114]. Moreover, Moreira implemented two computational applications to facilitate the usage of his methodology (*Contour Analyzer* for general contours [119] and *Jacquard* for textural contour [122][9]), and introduced ornamental functions for texture according to its duration and position within the textural contour [121].

Concerning compositional approaches, Moreira has been formalized compositional strategies for using texture within the creative process [120, 111, 112]. To do so, Moreira has discussed the textural morphology, i.e., the internal components involved in the formation of any textural configuration. From this discussion, Moreira has proposed the formalization of three *textural spaces* to encode the morphology of a given textural configuration. In a general sense, a textural space is defined as the set of textural configurations connected (or related) by a given transformational process. Each space differs from one another in their level of details, using different codes to describe the textural morphology. The most superficial space, called *Textural-class space* (*tc-space*), consists of the simple division of textural components into two abstract structures (line and block), which is related to the general perception of what most people can aurally identify when listening to a piece of music. *Unordered partition space* (*up-space*) is the very proposal of Pauxy Gentil-Nunes' Partitional Analysis [81], in which the textural morphology is portrayed by integer partitions. Finally, *partition layout space* (*pl-space*), the last space, is the most refined description of a textural configuration since it includes the spatial (ordered) organization of its internal components. In his doctoral thesis [112], Moreira formulates a pre-compositional strategy, called *textural design*, in which codes from any of textural spaces are combined into a two-dimensional array ready to be implemented as music[10]. After creating a textural design, the composer may realize it as music, which may be influenced by the articulation of the other musical parameters, as well as the various aspects concerning the compositional practices. In order to discuss this relation between the codes within textural spaces and their realization as music in the score, Moreira defines five *Modes of Textural Realization* that map potential realizations of a given textural code according to different perspectives, such as, pitch content, rhythm, timbre, musical form, and the like. These five modes are not necessarily an exhaustive taxonomy of textural realization, but they cover the most striking features of compositional practices regarding texture.

Carlos Mathias is an associate professor at the Institute of Mathematics and Statistics of the Universidade Federal Fluminense, mathematician, and a professional drummer. His main areas of research are Philosophy of Mathematics Education, Assessment and Curriculum, the dissemination and popularization of Mathematics, and Music and Mathematics. His work in the latter area has two main branches. The first one relies on education, more specifically on the use of rhythm for developing metaphors and images around arithmetical and geometrical concepts for visually impaired students. Mathias has developed a pedagogy on mathematical education called Drummath [102, 103]. The second branch relies on the use of Number Theory and Finite Groups Theory on rhythmic mapping and generation. More specifically, his work

[9]Both are available in `www.musmat.org`

[10]This methodology is based on Morris' *Compositional Design* [124].

relies on building mathematical structures for rhythmic representation and the use of these very structures on computational generation of rhythms. Recent efforts on the latter approach have been recently made in partnership with Carlos Almada, with whom Mathias recently wrote an article proposing an encoding system for drum set's rhythms based on Gödel numbering [104].

The main work of Hugo Carvalho is more directed toward audio signal processing, being his main research area the application of statistical methods to digital audio restoration. In a nutshell, this problem consists on estimating the underlying original signal from a degraded version of it. The process is always performed in a digital computer, and usually there is only a single copy of the corrupted signal, that most commonly comes from the digitalization of recordings in old medias such as gramophone disks and wax cylinders. This lack of observed information makes the inverse problem quite hard to be solved, and therefore, statistical methods may be employed as means to ease this issue. Audio restoration *via* bayesian methods was the main topic on Carvalho's doctorate thesis [54]; some of his publications on the subject on journals and conferences are [52, 53, 48] and specially the more recent [56], where a solution to the problem of restoring audio signals degraded by severe pulses with significant low-frequency content[11] was proposed.

More recently, Carvalho became interested in music information retrieval (MIR), an area within audio signal processing concerned with obtaining high-level information (such as genre, mood, chords, and even the music sheet) from the raw audio signal. Clearly some pre-processing must be made in the time series containing this unstructured data in order to extract more meaningful information and potentially mimic our cognition when interpreting music.

Another of his more recent research interests are the application of probabilistic methods to music composition and analysis. More specifically, in the work [77], with Liduino Pitombeira, the authors employ Markov chains to model the evolution of rhythmic textural partitions and melodic contours of a piece (in this case, the second movement of the *Five Movements for String Quartet, Op. 5* by Webern), and as an example Pitombeira uses the extracted information on the planning and composition of an excerpt of the first movement of his composition *Plate Two: Érebo, Op. 255*. Carvalho also is also working with Carlos Almada in a markovian model applied to chord classes on Tom Jobim's music. Finally, he also wrote a musically-oriented introductory tutorial on Markov chains [55].

Cecília Saraiva is a mathematician and an amateur musician interested in several interactions between the formal language of mathematics and the musical world, specially in the power of algebraic tools to describe and explore musical possibilities, from analysis to composition. Her academic background in Algebra (noncommutative algebra and algebraic geometry) sets the direction of her interests toward the reach of languages such as group and set theory in music. Her main goal at the moment is to study ways in which algebraic tools can unveil musical aspects so that analysis and creation can be mingled. She joined MusMat by the end of 2021 MusMat Conference, after her talk in the round table about algebraic processes and music, in the context of the Twelve-Tone legacy.

In 2020, she advised an undergraduate thesis written by student João Lucas dos Santos Braga studying group theory applied to Neo-Riemannian basic language. João Lucas used the PLR group to analyse excerpts of music created by himself [50].

In 2021, Cecília presented a short talk in an event at UFRJ (Federal University of Rio de Janeiro) called *Celebrating Women in Mathematics*, designed to draw attention to female production and current activities or accomplishments in pure and applied mathematics. The talk *In search of Beauty: algebraic inquiries on the musical fabric* was inspired by questions set in Hermann Weyl's

[11] This problem occurs, for example, when the needle-arm mechanism of reproducing devices passes through very deep scratches or glued breakages.

classic "Symmetry". This talk gave birth to a four-day workshop in 2022 edition of the same event. The workshop *Introduction to Musmathics* primarily focused on presenting the area to mathematics undergraduate students, ended with a basic introduction to neo-Riemannian/transformational theory.

Cecília is currently working with undergraduate students in two projects: one is dedicated to explore some available tools from group and transformational theories in jazz and Brazilian popular music with former student João Lucas S. Braga. A second project, in a more advanced status, will soon be in its final form and uses the language of graph theory to study symmetries and interesting patterns in Brazilian popular music, and also proposes ways to use these tools to teach mathematics in elementary/high school. This work is a project carried out with student Paula Leal.

ii. GPA - UFRJ

The Audio Processing Group (GPA) is part of the Signals, Multimedia and Telecommunication Lab (SMT) at the Federal University of Rio de Janeiro (UFRJ). Officially created in 2000, it is a Brazilian Electrical Engineering research group exclusively dedicated to audio research. Over these years, GPA has produced 10 doctors and more than 30 masters in the field.

Covering as main topics audio restoration [56] and enhancement, spatial sound and acoustic localization [93], audio quality assessment [49], audio coding, and audio analysis and synthesis [58], since its origins the group is characterized by a strong link with Music. In addition to its ubiquity in applications from restoration to spatial sound and coding, Music has always motivated a considerable part of the Group's work, in the form of regular contributions in the field of Music Information Retrieval (MIR) involving alternative time-frequency representations for music [57], pitch tracking in polyphony [47], rhythm analysis [79], separation of musical sources [76], audio-to-score alignment and expressiveness in music, among others.

Current highlights in GPA's production are: the development of combined time-frequency representations for musical signals with improved resolution in both time and frequency aspects [74]; and a set of investigations in the rhythmic analysis of Afro-Latin American musical genres [144]. This last topic has permeated a lasting cooperation with the Uruguayan GPA (Grupo de Procesamiento de Audio), which included a two-year project with Télécom Paris and Centrale Supélec (France). In Brazil, GPA has collaborated with UFRJ, CEFET-RJ, IME, LNCC, UERJ, UFAM, UFSC and Unicamp among teaching and research institutions; and HP, Microsoft and Globo among the companies.

iii. USP

Since the beginning of the current decade, Paulo de Tarso Salles, a recognized expert in structural aspects of Villa-Lobos's music, has been especially interested in studies about symmetry and co-related subjects. Two of his recent papers are special noteworthy in the present review, due to their theoretical-methodological importance: the first one applies Salle's original formulation named *redes de transformações harmônicas* (*harmonic-transformation networks*) in the analysis of some of Villa-Lobos' pieces [146]. The second study addresses the concept of *regiões Euler* (*Euler regions*) which is connected to Richard Cohn's hexatonic and octatonic cicles [147].

Three of Salles's former students act as outstanding collaborators in the expansion of these researches, which mostly involve application of mathematical models for the understanding of Villa-Lobo's creative processes: Ciro Visconti investigates symmetric relations in Villa-Lobos' guitar *études* [153], and proposes an interesting spacial approach, named *axes of contextual inversion*, for

bringing up tonal relations in selected passages of the Post-tonal repertoire [154]. Walter Nery Filho works with intricate geometric representation of Neo-Riemannian operations [125]. Joel Albuquerque's recent PhD thesis [29], grounded in the Group and Transformational theories, presents an exhaustive typology of spatial schemes that intends to map symmetric connections between sets in Post-tonal music. Along the last years, Albuquerque has also been published some related papers in co-autorship with Salles and other colleagues [29, 28, 26].

Rodolfo Coelho de Souza, who hosts a degree in Engineering, coordinates two basic lines of research that dialogue with both music and mathematics. The first line addresses physical-mathematical modelings of sounding aspects of Brazilian native musical instruments, associated with algorithmic strategies, and involving methods of wave-terrain synthesis, additive synthesis with noise, among others, being also articulated with the fields of algorithmic composition and artificial intelligence [72, 73, 131, 66, 65, 62]. Simultaneously, Coelho de Souza has been developed several topics associated with pitch-class set theory, proposing original and particular views, aiming both to analytical and compositional applications. His main contribution for the expansion of this field is the formulation of a special theory addressing non-ordered-intervals classes, besides studies concerning intrinsic properties of some special pitch collections, as well as neo-Riemannian mappings [71, 69, 68, 70, 67, 64, 63, 152].

iv. NICS - Unicamp

The Interdisciplinary Nucleus of Sounding Communication (Núcleo Interdisciplinar de Comunicação Sonora – NICS[12]), created on April 5, 1983, focuses its research interests on the different manifestations of sound, under the perspectives of information, cognition and creativity. NICS brings together researchers from different fields, particularly arts and the sciences, and develops interdisciplinary projects that aim to broaden the relationship between musical conception and the proposals of new production, control and sonic analysis models. Since 1994, the Nucleus has focused on Mathematical Modeling, Computational Simulation, Musical Composition, and Computational Music [151, 108, 130, 145, 99, 100].

The NICS has established agreements with several national and international institutions. Among them, we highlight a partnership with the Institute of Neuroinformatics (INI) of the Swiss Federal Institute of Technology (ETHZ), in Zurich, in the production of the ROBOSER system, and VOX POPULI, an evolutionary system applied to algorithmic musical composition developed by researchers Jônatas Manzolli (NICS), Artemis Moroni (Division of Robotics and Computational Vision - DRVC of CTI Renato Archer, from Campinas), Fernando Von Zuben, and Ricardo Gudwin (FEEC); this system was awarded the Dream Centenary Computer Graphics Grand Prix 99 in Aizu, Japan, in the category of Best Interactive Installation. The current research lines are (1) Musical Analysis with Computational Support, focused on the computational support for musical analysis as a way for understanding the sound dimensions of music; (2) Multimodal Creation, as research on creative processes from the reflection on the interaction of the musical gesture, human-machine interaction, computational models applied to sound design and interaction with other languages in multimodal performances, and (3) Multimodal Interaction in Therapeutic Procedures, through applications of interactive musical and multimodal technology anchored in computational models and interdisciplinary studying, covering musical and neuroscience fields.

[12] https://www.nics.unicamp.br/

v. Genos Research Group

In Bahia, the relationship of music and mathematics are mainly associated to the Genos research group,[13] a multidisciplinary group of research of theory, composition, and musical computation linked to Federal University of Bahia (UFBA). Genos has developed research projects in four main areas: (i) codification, (ii) music algorithmics, (iii) Pitch-class set theory, and (iv) analysis supported by statistical data.

Pedro Kroeger, leader of Genos, has developed a research on codification of musical structures in a higher level than the traditional codification of pitches and rhythms through numbers and letters in such a way to express wider structures as all the fugue's subjects of Bach's well-tempered clavier or chords of a harmonic progression, considering the representation of chordal roots, harmonic rhythm, chordal qualitites, and their possible inversions, among others [97]. In his doctorate thesis, Kroeger developed a meta-language for sound synthesis [94], and he also explores automated harmonic analysis [95] and statistical analysis addressing occurrence of consecutive fifths and octaves, cadences, voices, augmented-sixth chords, voice leading and crossing, seventh resolution, and final cadences in Bach's 366 choirs [96].

Jamary Oliveira developed compositional applications such as *MUSICOMP* (1981), that prints a serial matrix, *SONG.DATA* (1985), which provides an analysis of melodies [127], and *Pitch-Class Processor* (PCN[14] – 1992), that consists of a pitch-class calculator for analysis based on pitch-class set theory. Oliveira has been also interested in pitch and rhythm codification [128] and twelve-tone composition systematization [126]. The MusMat Conference of 2020 was in homage to Jamary, who left us this same year.

Marcos Sampaio is, perhaps, the main researcher on the theory of *musical contour* in Brazil [101, 124, 123]. His doctoral thesis presents a complete review on contour theory, pointing out some problems and inconsistencies in some algorithms proposed by different authors [150]. Most of these concepts were implemented in the software called *MusiContour*.[15] Sampaio is also responsible to the refinement of the reduction [149] and similarity algorithm, proposing two new algorithms to enable the comparison of contours with different cardinalities [148].

Other former students related to Genos are Dennis Queiroz Carvalho, whose interests lies on axial symmetry of some pitch-class sets, like the set-class [8-17] [51], and Natanael de Souza Ourives, who is interested in reviewing combinatoriality of twelve-tone structures [129].

vi. Mus^3 Research Group

The Mus^3 group is linked to the Federal University of Paraíba (UFPB) and to the NICS - Unicamp (São Paulo) and develops researches in the areas of Musicology, Sonology and Music Computing. The lines developed are focused on the formalization of the timbre-related composing dimensions at their conceptual level, covering the research of Didier Guigue [85, 87, 88] and *Textural Planning*, referring to the work of José Orlando Alves [45, 46, 91, 92, 75, 110].

Estética da Sonoridade (*Esthétique de la Sonorité*) is the title of Didier Guigue's referential book [86, 87], which constitutes one of the essential publications in musical analysis in the country. The theory is constructed from the concept of *Composite Sound Unit* (*CSU*), defined as the combination and interaction of musical *primary* components (a collection of pitches) with *secondary* components – namely intensities, ranges, registering, densities, modalities of statistical distribution of pitches or other low-level elements, e.g. deviations, entropy and others. This theory led to the development of

[13] https://genosmus.com/

[14] See http://www.angelfire.com/music2/bahia/pcn/pcn2001ptb.htm

[15] See https://pypi.org/project/MusiContour/

a library of functions in the OpenMusic environment, the *SOAL* (*Sonic Object Analysis Library*) [89]. There is an ongoing collaboration between the groups Mus[3] and MusMat around the research on musical texture, generating sharing of concepts and tools in publications and software [90].

Textural Planning is developed by José Orlando Alves [44, 45, 46] and aimed at applications in the compositional process. Its starting point is the use of matrices to define modes of organization of durations and texture, from Wallace Berry's theory (1976, see [44]). The work has resulted in publications and papers, both by Alves and his advisers, as well as a software (*TexturalCalc*) developed by Felipe Grisi [92] in Java, which calculates the level of textural complexity of each instrumental configuration of the work, based on the number of real components and their relation to density-number.

IV. Colombia

In the context of Latin American systematic musicology, Colombian contributions are paramount for a clearer understanding of *subjective categorization, gesture theory, topos theory, homotopy theory* and *homology theory* applied to music, as well as computational developments for music composition and analysis. For a historical introduction on the latter, Cuéllar-Camargo's article [159] provides a first approach that obviously needs to be actualized. In respect to subjective categorization of music, with a great regional, potential interest, we find Pablo Mendoza-Halliday's PhD dissertation [162], *Musical genre as a dynamic process: a semio-cognitive theory of the categorization of music*, in short, an effort to explain to musicians how social use of music requires fuzziness criteria analogous to semantics and pragmatics in language, in order to produce "musical sense". This conceptualization involves complex dynamics for building and distinguishing musical genre and generic indications, being a novelty in Latin American literature:

> *Musical genre* is a concept widely used to name "kinds of music". However, in music theory this concept has provoked debates on how it is conceptualized, what is categorized, how is such categorization achieved, what is its function, and if it is necessary, or if it can be avoided or replaced. Regardless of the posture on these subjects, categorization of sensitive experiences such as music is cognitively inevitable. It is one of the ways to make knowledge apprehensible. For the same reason, the categorization of music is one more tool in the construction of the meaning that music acquires and that allows to make use of it.
>
> The aim of this thesis is to review the concept of musical genre through a theoretical proposal that reassesses the current paradigms of categorization and that, from a viewpoint of its socio-psychological dimension, explains how the category is comprised, how it can be analyzed and how it affects the processes of musical signification through generic indications. The cognitive (enactivist) and semiotic (Peircean) theories of categorization are integrated in this proposal in order to conceive genre as a dynamic process that includes phases of cognitive categorization, conventionalization, and taxonomization. This allows us to have a broader approach to this form of categorization of music, and it can contribute to rethink the role of genre in music theory. Furthermore, analyzing genre and generic indication as signs of music leads to an understanding of its function as hermeneutic guides to music. This can shed light on how to appropriate this semiotic strategy and optimize its use.

Whether Mendoza-Halliday does not develop mathematical formalization or computational direct applications, his logical schematization in fact paves the way for further research in

music and mathematics involving *category theory, algebraic topology* and *fuzzy set theory* with clear computational implications. Furthermore, another Colombian young scholar, Juan Sebastián Arias-Valero [155, 156, 157, 158] seems to fulfill the necessary complement for this task; so the harmonization of Mendoza-Halliday and Arias-Valero scientific goals would be a relevant next step within this domain. By now, Arias-Valero extended Mazzola's *topos theory* [158], both in philosophical and pragmatic perspectives for music theory, contributing to the Peircean and Grothendieckian expansions in musicology (exploring the conceptual fields of *hypergestures, gestures on locales, localic topoi, sites, morphisms of sites, presheaves* [i.e. sheaves on the trivial topology] and *sheaves*). The successful adaptation by Arias-Valero from Peirce, Grothendieck and Mazzola for a mathematical theory of music was encouraged by Valero's former supervisor, Colombian mathematician and philosopher Fernando Zalamea (Bogotá, 1959).

In recent years, Professor Zalamea published articles and books on the analogic deep relationship between *musical creativity* and *mathematical creativity*, with particular attention on concepts from Charles S. Peirce and Alexander Grothendieck, from a specific philosophical point of view:

> In fact, Peirce, Einstein and Grothendieck involve, with different techniques, the observer's framework and the partial cognitive dynamics of the agents. In particular, in the Grothendickean *making*, a network of incessant *transfers, translations, translations* between apparently distant regions of mathematics is introduced." (Zalamea, 2014 [165, p. 112]).
>
> In categories with some good properties of compositionality and coverage, an abstract topology (Grothendieck's topology) can be defined by (sub)collections of morphisms that "tie" well each other. The categories of pre-sheaves (categories of functors to values in the category of sets) verify these good properties of compositionality and coverage, and abstract topologies can be defined there. Grothendieck's topos come from categories of pre-sheaves that are "situated" around a given abstract topology (those categorical environments are also called then *sites*). *A geometry of the situation precedes logic*, against many postulates of analytic philosophy. (*loc. cit.*, footnote).

This theoretical stressing on how the "observer's framework" is related to "the partial cognitive dynamics of the agents" is an issue immediately associated to the self-referential complexity of music (as referred in Pareyon, 2011 [163], both in terms of musical *poietics* and *poiesis*). Furhermore, Zalamea's synthesis and clear explanation on Grothendickian categories, makes also clear how urgent is for regional musicology, building ties between Mendoza-Halliday and Arias-Valero's prospects. Moreover, the Peirce-Grothendiek adaptations to music theory and practice, unveils a shortcut for communication between Mesoamerican aesthetics (and more specifically from Tepon-azcuauhtla interpreted as a *sheaf*, in its Grothendiek-topologic sense) and modern mathematics in dialogue music. Finally, this topological conception of music can be seen as an analogy to the "mathematical *pointless* thinking" as explained by Đurđevich in terms of *quantum geometry* (see: Mexico and Central America, below; notice that the musical theory of Zalamea is also enriched by musicologist and composer J.S. Lach-Lau, mentioned in the following Section, too).

Recently, Colombian post-graduate researcher Nicolás Jaramillo-Ramírez (Julio Estrada's former student) successfully defended one of the best master's theses at the Faculty of Music - UNAM, in the area of composition [160]. This thesis, with the title "Infinite Points between Yes and No: Percepts in Musical Creation", explains Boolean logic with extensions to infinite-valent logic applied to musical composition, Fuzzy Set theory, and the Continuum-Discontinuum theory (based on Estrada's doctoral thesis). Jaramillo-Ramírez's dissertation paves the way to better understanding the link between Peirce's semiotic logic (*ponecipuum*, *percipuum* and *antecipuum*), towards category theory and its possibilities for a logic of intersemiosis and the intersemiotic

continuum (as postulated by Pareyon, 2011 [163, pp. 108–114]).

V. Mexico and Central America

Regarding the intersection of music and mathematics, the cultural dynamics of Mexico is one of the most complex in the region. This is due both to the cultural breadth of the so-called Greater Mexico (which includes the regional fringe that extends between California and Texas,[16] as well as Central America), as well as to the depth of its ethnic and cultural history. However, this section is limited to what Lluis-Puebla ([223, p. xxv]) defines as "modern mathematical theory of music from the 1980's". As a specialized academic issue, this activity is mainly centralized in six institutional clusters: (i) FaM/PMDM (UNAM), (ii) IIMAS/ICAT/FCiencias (also UNAM), (iii) Cenidim/CMult/ESM (CENART), (iv) FaM (UV-Xalapa), (v) CR/CMMAS/ENES (UNAM-Morelia), (vi) CUCEI/CuCosta/CUCBA (UDG). This section ending does explain how students and scholars from Guatemala and El Salvador participate on this network. Additionally, an appendix on Costa Rican specialists is provided.

i. FaM/PMDM

The Faculty of Music (FaM) and its Postgraduate Program in Music (PMDM) of UNAM (National Autonomous University of Mexico), historically leads the study and development of music and mathematics in Mexico. Within this framework, and for the last 25 years, as mentioned at the end of the first text on this issue, Julio Estrada (emeritus professor, Faculty of Music, UNAM, Mexico City, and IIE-UNAM researcher), adopted and adapted the heritage of Carrillo and Nancarrow as "cataracts and trees" of harmony in terms of space and time (see [191, p. 101]), through a variety of approaches between what Estrada himself defines as *discontinuum* and *continuum* [200]. However, a first attempt to reformulate music theory, particularly from its symmetries associated to a musical algebra and syntax, was introduced in the context of Boolean algebra and finite group theory; as Estrada explains [199, p. 159], in the sense of an analogic reasoning:

> Finite Group Theory operations can be described as logical transformations that arise when working with the elements of an abstract language which, when applied to other languages, prove to be of great interest and usefulness as they show some operative mechanisms that man practices throughout his physical and intellectual action.
>
> This theory studies the symmetries of forms. When applied to music, it shows the coincidence with those symmetries that have been traditionally used (retrograde, inversion, and retrograde inversion) and other symmetries that would pass unnoticed with regular systems of musical analysis.

Along most of his theoretical development —with very specific applications to musical analysis and composition in multilayered criteria for distance and scalar production— Estrada's starting point is a matrix design based on the symmetry of scales and their permutation/combinatoriality as pitch collection. As a matter of fact, one may trace this strategy through his first steps for modeling scales in terms of symmetric arrangements, partitions and *logical transformations* (see, e.g.: [199, pp. 167–168], [200, pp. 171–186]; with systematized employment in Estrada, 2001 [201],

[16] For an introduction to the problematic conceptualization of mathematical understanding in communities along this fringe, see Gutstein *et al.*, 1997 [210, p. 731], in parallel to the understanding of music theory, since mathematics and music theory "is usually seen as a neutral and objective subject, devoid of specific class, cultural, or political content; the connection to social activism may seem to be a stretch. Even when mathematics [and music theory] learning is seen as personally empowering, it is rarely thought of as having broader, social justice implications for those learning it".

2002 [202], 2006 [203]). When adapting these *symmetric principia* to independence of interval size/distance (a concept closely related, both to Carrillan aesthetics, and to the distinction between *discontinuum* and *continuum*), an infinite yet *well-formed* relationship is foreseen across pitch, scale, texture, timbre, rhythm and meter, with "great interest and usefulness" for creating, understanding and transforming music. Estrada also produced important research to connect the technical view of his abstract methods, with "cultural integration", avoiding the temptation of put rationalism forward social and historical values of regional culture (see Estrada, 2011 [204]).

Among Estrada's students and collaborators are Leticia Cuen [196, 197], mainly focused on the structural role of the Golden section (φ) in modern music; Mauricio Rodríguez [245, 246], occupied in the study of the *algorithmic expressiveness* in scores based on signal analysis and graphical transformations, and how such *expressiveness* is related to rhythmic and metrical complexity; and Víctor Adán [169, 170, 171, 172], post-graduated from The MIT, and specialized in the analysis, modeling and resynthesis of music from its hierarchical structures interpreted from a dynamical systems and signal processing approach, as well as interested in the concept of *control* in the context of musical symbolization and programming for scores production.

Adán collaboration with Estrada is particularly meaningful because of the implementation of Estrada's $d1$ *theory* and harmonic permutahedron, where $d1$ refers to any harmonic or interval distance (d) which minimal constructive and/or syntactic order equals 1. Given that, in practice, 1 may be defined as any "basic" interval, the $d1$ *theory* may be useful in any periodic lattice *kind of* structural harmony, and may also be implemented for any periodic structure of time. Its corresponding software allows the user to experiment, for example, the permutation of a series of time intervals in an oscillation with constant amplitude, or the representation (and sounding output) of different slopes of curves product of the modification of a *continuum/discontinuum* variable.

Another leading scholar historically active at FaM/UNAM is pianist, flutist, teacher, composer, mathematician and programmer Roberto Morales-Manzanares [236], later a full-time Professor at the Faculty of Music, University of Guanajuato, where he founded the LIM (Laboratorio de Informática Musical/Musical Computer Lab). During years 1980's and 90's, Morales-Manzanares settled the computational grounds for ESCAMOL, an algorithmic program for music composition and visual-spatial interaction, actually employed by the University of Berkeley, San Jose State University, Yale University and McGill University. Morales-Manzanares' post-graduate students include J.R. Cabezas-Hernández [190], Edmar Soria [248, 249], Mauro Herrera-Machuca [212], and J.E. Gómez-Elizondo [207], all of them occupied on audiovisual and "transmedia hybrid creation" employing mathematical models, mostly implemented in SuperCollider and ESCAMOL, and extensions to develop strategies of music and meta-music creation.

ii. IIMAS/ICAT/FCiencias

The UNAM Faculy of Sciences (FCiencias) located at Ciudad Universitaria (Mexico City's south) has a meaningful tradition crossing over music, mathematics, physics and other fields in natural sciences, and for academic and research in musical projects frequently collaborates with the Institute of Research in Applied Mathematics and Systems (IIMAS) and the Institute of Applied Sciences and Technology (ICAT).

An internationally recognized pianist, Emilio Lluis-Puebla is also —since 1980— Algebra Professor at the Department of Mathematics (FCiencias, UNAM), where he formed at least three generations of scholars within both domains of music and mathematics. His research and teaching expertise includes Theory of Homotopy, K-Algebraic Theory, Vectorial Sheaves, Cohomological Operations, Infinite Loop Spaces, Homological Algebra, and Groups Cohomology

(see [218, 219, 220]). His theoretical approach to music and musicology was firstly inspired by Birkhoff's (1933) *aesthetic measure* [188], Borel & Serre's (1958) view on the Riemann–Roch theorem [189], and Lendvai's (1979) geometric conceptualization of musical form analysis [217].

Since 2004 Lluis-Puebla closely collaborates with Guerino Mazzola and produced a wide number of articles, books and academic seminars and conferences on music and mathematics [221, 222, 194, 240]. Among Lluis-Puebla's most internationally renowned post-graduate students are Flor Aceff (see [167, 168]), Jesús David Gómez-Téllez (see [208, 209]), Mariana Montiel, and Octavio Agustín-Aquino. Montiel adds to her prolific research output, the merit of probably being the first female scholar in Latin America being completely devoted to the study of a number of facets linking music and mathematics with algebraic, geometric, combinatorial and topological methods, and pedagogic, psychological and other applied approaches "to understanding musical phenomena" (see [229, 230, 231, 232, 233, 234, 235]). Agustín-Aquino, in his turn, produced in the region the widest specialized literature on the mathematical theory of musical counterpoint (see [173, 174, 180]), including $2k$-tone equal temperament [176], microtonal counterpoint extensions [177], extended counterpoint symmetries and continuous counterpoint [179], as well as other computational counterpoint worlds [178] (see also Nieto-Ramos [237]).

Pablo Padilla-Longoria, Professor at UNAM's Department of Mathematics, and at IIMAS, granted his Ph.D. at CIMS, New York University; he also graduated in piano from Mannes College of Music, New York, and is an active performer of organ and harpsichord. He created the Seminar on Music, Computation and Mathematics (SEMIMUTICAS – IIMAS), and developed pioneering research, in collaboration with Prof. Alejandro Ramos-Amézquita, employing mathematical modeling for the study of Mesoamerican and New Spain buildings' archaeoacoustics. During 2016-2017, Padilla-Longoria was a visiting member of the Fitzwilliam College of the University of Cambridge, where a research project on mathematics and music was carried out in collaboration with F. Knights (Fitzwilliam College) and D. Tidhar (Center for Music and Science) (see [238]). His students include Chávez-Zamorate [195], J.R. Cabezas-Hernández [190], J.A. Lobato-Cardoso [224, 225], and Iván Paz [244], most of them programmers/analysts, familiar to cellular automata, neural networks and AI algorithms applied to musical performance and composition.

Felipe Orduña-Bustamante, is researcher of the Acoustics and Vibrations Group, ICAT-UNAM. He has a degree in Physics (UNAM, 1987), a Master's degree and PhD in Sound and Vibrations (University of Southampton, England, 1990 and 1995). He works on instrumentation and acoustic measurements, digital signal processing, musical acoustics and technology, and computational musical retrieval (Bañuelos & Orduña-Bustamante, [187]), and he also contributed to the acoustic investigation of musical instruments Native to Mexico, studied through physical, computational and mathematical methods (Herrera-Castro et al., 2018 [211]; for other technological and systematic applications on the same topic, see also García-Munoz [206], and Pareyon & Pina-Romero [241]).

Micho Đurđevich, researcher at the Institute of Mathematics (FCiencias), UNAM, is a specialist on C-Algebras, Galois theory, principal quantum sheaves, and quantum geometry, mainly focused on quantum circles and quantum spheres. This mathematical repertoire also includes Cohen forcing within the context of Grothendieck and Giraud theory of Topos (developed by Mazzola [161], and emphatically applied by Lach-Lau [214] to musical composition; an approach also used by Gómez-Téllez *et al.*, 2017 [209]). Đurđevich output [198] also facilitates an accurate mathematical context for the understanding of Teponazcuauhtla, the non-linear set of musical aesthetics described in the first text on this issue, especially with the use and conceptualization of the teponaztli conceived as an experimental (theoretical but mostly practical) device for quantum circles, where "the entire fabric of space is considered as the one and indivisible whole" (Đurđevich, *op. cit.: Introduction*; completely empathetic with the historical view in the first pages of the present study).

iii. Cenidim/CMult/ESM

The National Center for the Arts (CENART) in Mexico City, also constitutes an important cluster for musical research and interdisciplinary production. CENART is the home of the National Center for Music Research, Documentation and Information (Cenidim), the Multimedia Center (CMult) and the Escuela Superior de Música (ESM) that depends of the National Institute of Fine Arts (INBA). In fact, engineer Raúl Pavón developed at INBA, in the early 1960's, the first music synthesizer completely made in Mexico, and he also published in Cenidim the first Mexican book on music and computers (Pavón, [242]). The first years of the CENART building complex benefitted from the skills of José Antonio Amozurrutia (Mexico City, 1950), pianist, composer, mathematician and programmer who designed the first CENART intranet and computer clusters at CMult (see: Amozurrutia, 1997 [181]), before starting his pioneering project for applying music theory and computer science to the investigation of genetic epistemology. Later, the first decade of the 21st century was particularly fruitful at CMult, with mathematical/programming seminars and live coding concerts with a celebrated participation of computer scientists and music creators such as Ernesto Romero, Sergio Luque, Iván Paz [244] and Jaime Lobato-Cardoso [224, 225], implementing mechanic, algorithmic and multimedia devices with local developments.

In 2012 Gabriel Pareyon founded at Cenidim the first national Seminar of Science and Music Theories (originally SeCiTem), in cooperation with SEMIMUTICAS and the FaM-UNAM, with students of Pablo Padilla-Longoria and Emilio Lluis-Puebla. This organization was the ground for launching the International Congress on Music and Mathematics (Puerto Vallarta, 2014), with a homage to Julián Carrillo and Alexander Grothendieck synthesized in the book The Musical-Mathematical Mind (see: Pareyon *et al.*, 2017 [240]). Then in 2015 pianist Gabriela Pérez Acosta entered the Cenidim team after completing a neurological study on musical cognition (see: [243]), and she continued organizing SeCiTem, thereafter under the simplified name of Seminar of Science and Music, ever within the cooperation Cenidim-INBA/FaM-UNAM, and which is the most regular forum in Mexico for the discussion on topics bonding music and mathematics. Up to this point, the Seminar monthly coordinates lectures about pitch-class set theory, intervallic theory, microtonal tuning, harmony and counterpoint, music and computational experimentation, mathematical modeling of voice and instrumental spectra, musical syntax and semantics, musical form analysis, music perception and psychoacoustics, and other similar subjects.

iv. FaM/UV-Xalapa

Central-eastern Mexico's activity on music and mathematics is concentrated in Xalapa, the capital city of Veracruz state, mainly within the regular courses, seminars and concerts leaded by Professor Emil Awad [184, 185], graduated *magna cum laude* from Juilliard School and Manhattan School of Music, and PhD in Music from Harvard University. In words of Hebert Vázquez [250, p. 10, 465], Awad is a key scholar in modern history of regional music and mathematics from a North American perspective:

> Despite the fact that in the last 30 years [before 2006] the theory of atonal music has produced a huge stock of theoretical-analytical literature and has been firmly installed in the curricula of professional music teaching, especially in the Anglo-Saxon countries, in Mexico it is still practically unknown. [250, p. 10] In this regard, it is necessary to highlight the important academic work that composer Emil Awad has been doing since the 1990s; pioneer in Mexico, both in the field of teaching the theory of atonal music and in Schenkerian analysis. [250, p. 465]

Hebert Vázquez himself is author of the most thorough compendium on atonal music theory ever published in Mexico (see: [250]), both for analytic and compositional interest. Altogether with Vázquez, other Mexican composers of his generation that perceive their musical output rather as demonstrations of their mathematical postulates are Antonio Russek (1954–), Ana Lara (1959–), Víctor Rasgado (1959–), Juan Fernando Durán (1961–), Juan Trigos (1965–), Armando Luna Ponce (1964–2015) and Georgina Derbez (1968–), mostly active in Mexico City.

v. CR/CMMAS/ENES-UNAM-Morelia

Morelia city in Central-Western Mexico is the home of the oldest public music school in the country, the Conservatorio de las Rosas (founded 1743), which have had prestigious teachers in music composition and analysis, including Juan Sebastián Lach-Lau [213, 214, 215], a scholar that reunites to his musical abilities the domains of algebraic topology, sheaves and fibre bundles, connecting concepts and methods borrowed from Badiou [186] and Grothendieck (see: Zalamea, [251, 252]). As Lach-Lau explains [216]:

> The above investigation is inscribed in a larger project that seeks to develop a model based on ideas stemming from mathematical phenomenology (close to the idea of a *topos* by Grothendieck), from which the conditions of appearance of musical events and structures are stipulated and fixed in terms of perceptual and conceptual limits and gradations, derived from logical, geometrical and topological objects. What is sought is that the expressive range and the ways of setting these ideas in a musical context covers a wide aesthetic ground. The model invites the positing, at several time scales and in interaction with the *continuous* (morphological), as well as the *discrete* (structural) facets of musical form, their conditions of appearance, both real and ideal: on the one hand there is a conceptual aspect, extensible to all kinds of *forms* and abstract relations; on the other hand, there is the empirical feature, comprising selection and generation of *materials* that exert resistance: found objects, instrumental modes of playing, sonic experimentation, improvisation, etc. This conjunction lays the ground for a synthetic approach to composition, involving both intuition and rationality. It does not impose any kind of a-priori aesthetic and it is not reducible to a method or technique, but is a way of setting out abstract sonic spaces and their transformations in a framework of structural relations and operations that can guide and assist the compositional act.

In fact, Lach-Lau's research and output provides a clear and interesting network for a virtuous connection of theories formulated by Colombian scholar Fernando Zalamea (above mentioned), not only over Grothendieckian mathematics, but also importantly over Charles S. Peirce, as a key author for better understanding the link between non-linear algebras and regional historic interpretations of music (Teponazcuauhtla's extensions on Carrillo, Nancarrow, Estrada, to which we may add the quantum spaces and trajectories studied by Đurđevich, [198]). Lach-Lau's valuable work on music and mathematics pedagogy also impacts on CMMAS (Centro Mexicano para la Música y las Artes Sonoras) and the ENES-UNAM, where he also teaches.

vi. CUCEI/CuCosta/CUCBA (UDG)

Even when the University of Guadalajara (UDG) does not develop any musical research program in music and mathematics, in 2014 the CUCEI (University Center for Exact Sciences and Engineering) and the CuCosta (University Center for Jalisco State Coast) united forces to celebrate the International Congress on Music and Mathematics in Puerto Vallarta, already mentioned, with

Pareyon as a main organizer, and also with valuable support by E. Lluis-Puebla, S. Pina-Romero, O. Agustín-Aquino, J.S. Lach-Lau, R. Morales-Manzanares and P. Padilla-Longoria. Sharing this enthusiastical spirit, CUCEI's Department of Computation acquired an anechoic chamber and opened a series of seminars on acoustics, psychoacoustics, electronic music, music & sound analysis, and selected topics on related mathematics. Pareyon students at CUCEI included Emilio Ceja-Cárdenas and Rodrigo Castro López-Vaal (actually PhD student at the Postgraduate School of Music, Georgia State University, Atlanta, under de guidance of Mariana Montiel). A more recent student of Pareyon, both in UDG and FaM/PMDM, is Axel Avendaño (Guatemala City, 1980), devoted to construction and re-construction of a variety of aerophones and to explore their relationship with Mayan languages actually spoken in Central America, employing Fourier analysis and syntax modeling of speech in a tonal, Native language (K'iche') retrieving musical harmony, meter, rhythm and texture information (see: Avendaño, [183]). In recent years Pareyon permanently collaborates with Martha Georgina Orozco-Medina (from CUCBA, University Center for Biological and Farming Sciences), within her Seminar for Urban Noise Research. A fourth student of Pareyon, graduated as seismologist from CuCosta (Puerto Vallarta), is environmentalist and composer Yair López [226] who actually leads there a scientific program for sounding landscape register and ecological interpretation of music employing mathematical and computational tools.

vii. Costa Rica & El Salvador

Although Costa Rica has a long-range tradition of musical and mathematical communication with Mexico (e.g. through the education and career development of composer Rocío Sanz Quiroz (1933–1993), a former member of the Liga de Compositores de México who employed her own serial methods), the country has a noticeable independence for music research and production. For example with the output of composers Bernal Flores (1937–) whom combinatorial strategies for serial composition and mathematical modeling of popular rhythms continued to a following extent in the work of Mario Alfagüell (1948–), who composed a huge number of musical works and taught composition using his own system of numerical series (notice that a recent chapter on mathematical formalization of Latin American dance tradition, as already started by Miranda-Medina & Tro [228], and Rodríguez *et al.* [247], would need, however, an independent synthesis). Younger Costa Rican scholars include Mauricio Soto, graduated as Computer Scientist and Music Teacher at the University of Costa Rica, and M.Sc. in Software Engineering at Carnegie Mellon University, as well as Gabriela Chavarria-Soley [193], who works on mathematical modeling of absolute pitch hearing, in the Universidad de Costa Rica. A former student of Roberto Morales-Manzanares at the PMDM-UNAM, already mentioned J.R. Cabezas-Hernández [190], is also a distinguished Salvadorean scholar making part of a new generation of Central American specialists interested on topics connecting music, mathematics and computer science.

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Celebrating 10 Years of the MusMat Research Group: Survey of Activities and Future Perspectives

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Abstract

In the year of 2022 the MusMat Research Group completes 10 years of existence. During this time the group developed several paramount activities for stimulate, promote and propagate the teaching and investigation of music and mathematics, specially in Brazil. This document is a landmark in our history, presenting a comprehensive overview of our previous achievements and pointing out to future goals.

I. Introduction

The activities of the group MusMat (Fig. 1) were launched in 2012 by Carlos Almada, Daniel Moreira de Sousa, and Pauxy Gentil-Nunes, who realized that their individual research projects, developed in the Music Graduate Program at the Rio de Janeiro Federal University, although distinct from each other in essence, objectives, and methodology, orbited around a common denominator, namely, the intersection of music and mathematics.

Formally registered in 2013 as an academic research group, MusMat has, since then, enlarged its frontiers, by incorporating new ideas, lines of investigation, projects, as well as students, and, especially, members. In 2014 Liduino Pitombeira joined MusMat, which represented a

Figure 1: *Three logos of MusMat research group over the years.*

considerable expansion of the area of interests covered by the group. In 2020, due to personal reasons, Gentil-Nunes left MusMat. To replace him, the remaining members decided then to invite three mathematicians (who are also talented musicians), inaugurating a new phase for the group, a phase of extraordinary improvement and growth. They are Hugo Carvalho, Carlos Mathias, and Cecília Saraiva.

Besides acting at Undergraduate and Graduate levels (by giving disciplines, lectures, as well as supervising undergraduate, master, and doctorate students' projects and the research of post-doctoral fellows), the MusMat members have dedicated great part of their efforts to publish two annual editions of *MusMat • Brazilian Journal of Music and Mathematics* (since 2016), to organize an annual conference on Music and Mathematics (also since 2016), and, more recently, to produce a podcast intended to make specific researches and general information about the confluence of these two central subjects known to a wider public.

The next sections intend to concisely describe some of MusMat's main achievements along the last ten years.

II. MusMat Conferences

i. 2016

The first MusMat conference took place in November of 2016, at the School of Music of the Federal University of Rio de Janeiro (Figs. 2 and 3). In this first edition (as well as in the next two ones) the conference was still held nationwide. Noteworthy names from Brazilian academic scenery were invited to present their research projects.

In addition to sessions of oral communications, the 2016 MusMat conference included a concert with compositions for flute, bassoon and piano based on the researches of the three MusMat members Liduino Pitombeira, Daniel Moreira, and Carlos Almada.[1] The following lectures and round tables were also presented:

- First day (Nov/23/2016):
  - Keynote by Rodolfo Coelho de Souza (University of São Paulo), entitled *Conceitos Matemáticos Utilizados na Composição de* A Máquina de Pascal em Pernaguá (*Mathematical Concepts Used in the Composition of* A Máquina de Pascal em Pernaguá);

[1] Although the concerts included pieces by others composers, for the sake of conciseness here are included only pieces by members of the MusMat Group. For the same reasons, sections of oral communications are not listed.

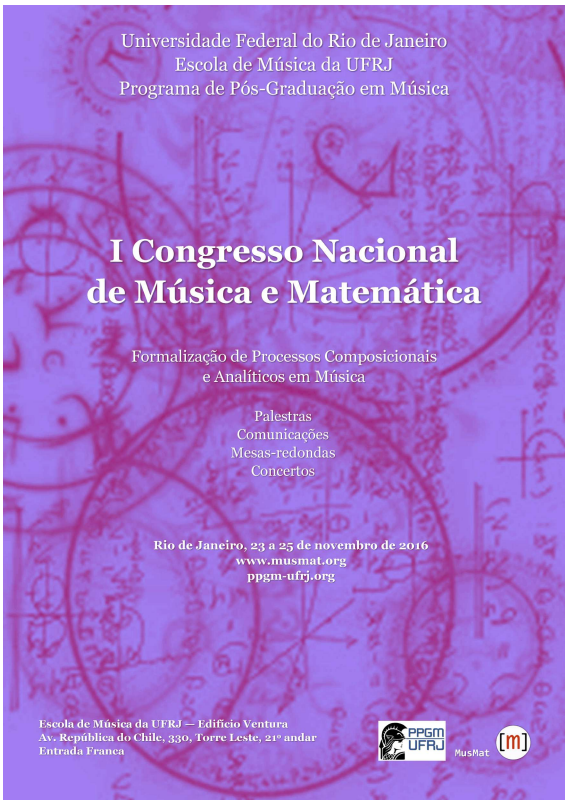

Figure 2: *Poster of the 2016 MusMat Conference.*

Figure 3: *Participants of the 2016 MusMat Conference.*

- Second day (Nov/24/2016):
  - Round table 1 – "Música e matemática: contextos" (Music and Mathematics: Contexts). Participants: Alexandre Reche (Federal University of Rio Grande do Norte) – *On the Relation of Quality and Quantity in the Context of Musical Composition*
    Carlos Mathias (Federal Fluminense University) – *Educação matemática de deficientes visuais: uma proposta por meio de sons, ritmos e atividades psicomotoras* (*Mathematics Education for the Visually Impaired: A Proposal through Sounds, Rhythms and Psychomotor Activities*)
- Third day (Nov/25/2016):
  - Round table 2 – "Música e matemática: conceitos" (Music and Mathematics: Concepts). Participants: Marcos Sampaio (Federal University of Bahia) – *Contour Algorithms Review*
    Carole Gubernikoff (Federal University of the State of Rio de Janeiro) – *Algumas considerações sobre as relações entre música e matemática* (*Some Considerations on the Relations between Music and Mathematics*)
  - Concert – Works for flute (Eduardo Monteiro), bassoon (Aloysio Fagerlande), and piano (Flavio Augusto)
    - *Patrono quieto*, Op. 207 (Liduino Pitombeira)
    - *Germinata III* (Carlos Almada)
    - *Sagração de um fauno na primavera* (Daniel Moreira)

ii. 2017

In the second MusMat conference, that took place in December of 2017 at the School of Music of the Federal University of Rio de Janeiro (Fig. 4), we invited outstanding researchers to cover different studies on the intersection between music and mathematics. Also, a concert performed by the Trio de Palhetas do Rio de Janeiro premiered pieces by Liduino Pitombeira and Carlos Almada that are deeply related to their respective research work.

- First day (Dec/5/2017):
  - Keynote by Gabriel Pareyon (Guadalajara University), entitled *Mathematical Analogy Versus Metaphor in Understanding of Music as a Biochemical and Biological Structure*;
- Second day (Dec/6/2016):
  - Lecture by Carlos Volotão (Military Engineering Institute of Rio de Janeiro) entitled *Composição musical inteligente* (*Intelligent Music Composition*).
  - Concert *Matemúsicas* (*Mathmusics*), performed by the Trio de Palhetas do Rio de Janeiro (Rodrigo Herculano, oboe; Igor Carvalho, clarinet; Carlos Bertão, bassoon)
    - *Vientos Tejanos*, Op. 203 (Liduino Pitombeira)
    - *Germinata I* (Carlos Almada)

iii. 2018

In 2018 (Fig. 5) the conference was considerably expanded, not only in the number of activities and guests – both from Brazil and overseas –, but also in the musical works especially composed for the event, which were distributed into two concerts.

- First day (Dez/3/2018):
  - Keynote by Robert Peck (Lousiana State University), entitled *The State of the Art: New Directions in Music and Mathematics*

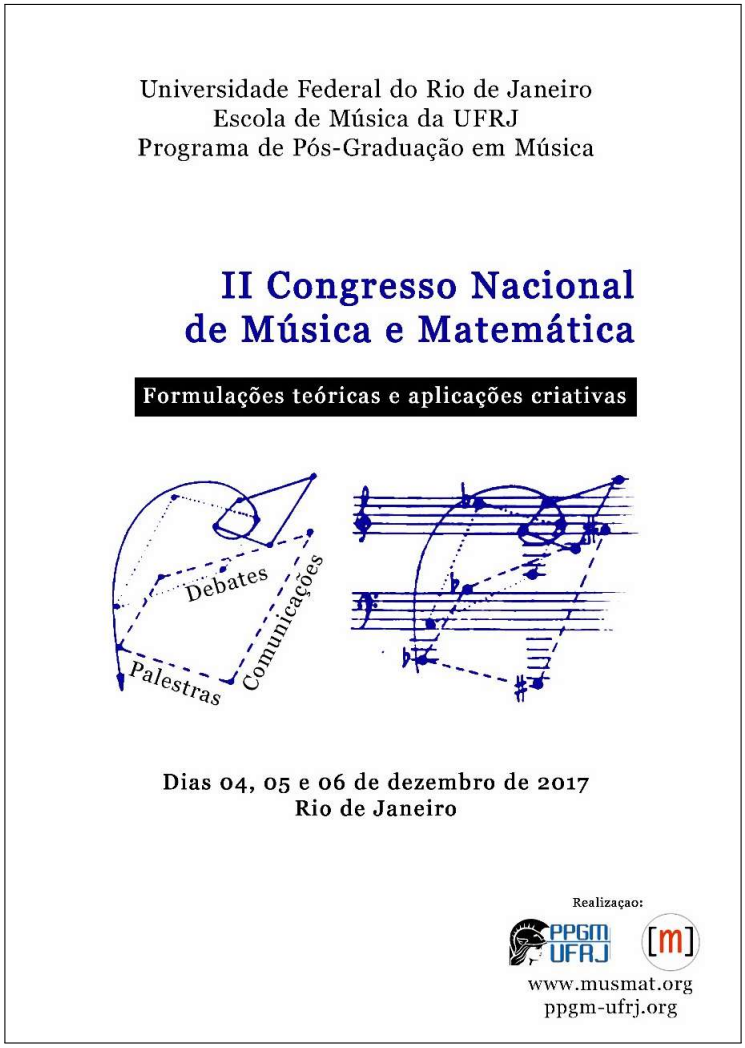

Figure 4: *Poster of the 2017 MusMat Conference.*

  - Round table 1 – "Inovações conceituais em análise musical" (Conceptual Innovations in Music Analysis). Participants:
    Didier Guigue (Federal University of Paraíba) – *Análise da Orquestração, uma proposta de formalização* (*Orchestration Analysis, a Proposal for Formalization*)
    José Augusto Mannis (University of Campinas) – *Análise como processo cognitivo musical* (*Analysis as a Musical Cognitive Process*)
    Athur Kampela (Federal University of the State of Rio de Janeiro) – *Modulação Micrométrica e Intuição Assimétrica: uma abordagem complementar* (*Micrometric Modulation and Asymmetric Intuition: A Complementary Approach*)

- Second day (Dez/4/2018):
  - Round table 2 – "Inteligência artificial em música" (Artificial Intelligence in Music). Participants:
    Carlos Volotão (Military Engineering Institute of Rio de Janeiro) – *O papel das redes*

Figure 5: *Poster of the 2018 MusMat Conference.*

neurais na composição musical (*The Role of Neural Networks in Music Composition*)
José Gabriel Rodrigues (Federal University of Rio de Janeiro) – Deep Learning *e redes convolucionais* (*Deep Learning and Convolutional Networks*)

- Concert 1 – *Homenagem a Debussy* (*Hommage to Debussy*), piano works performed by Tamara Ujakova, Thallyson Rodrigues, and Flavio Augusto.
  - *Impressões 1, 2 e 3* (Daniel Moreira)
  - *Liberjongo 2* (Pauxy Gentil-Nunes)
  - *Trois monuments*, Op. 234 (Liduino Pitombeira)
  - *Pop prelúdios* (Carlos Almada)

- Third day (Dez/5/2018):
  - Lecture, by Richard Cohn (Yale University), entitled *Glass Graphics*
  - Lecture, by Robert Morris (Eastman School of Music), entitled *4, 4, 4: A Talk on my String Quartet Quattro per Quattro*
  - Round table 3 – "Diálogos inovadores entre música e matemática" (Innovative Dialogues between Music and Mathematics). Participants:
    Cecília Saraiva (Federal University of the State of Rio de Janeiro) – *Musicalizando a matemática: um olhar interdisciplinar e humanista* (*Musicalizing Mathematics: An Interdisci-*

plinary and Humanistic View)
Carlos Mathias (Federal Fluminense University) – *Música e matemática, aritmética e ritmos* (*Music and Mathematics, Arithmetic and Rhythms*)
Ciro Visconti (University of São Paulo) – *Grafos neoriemannianos para além das tríades e tétrades* (*Neo-Riemannian Graphs beyond Triads and Tetrads*)

- Fourth day (Dez/6/2018):
  - Lecture, by Stephen Guerra (University of Miami), entitled *An Introduction to Afro-Diasporic Hemiolic Metric Space*
  - Concert 2 – Abstrai Ensemble – Pedro Bittencourt (saxes), Pauxy Gentil-Nunes (flute and electronics), Fábio Adour (guitar)
    * *Suarabácti* (Pauxy Gentil-Nunes)
    * *Mímesis* (Daniel Moreira)
    * *Artigo primeiro* (Carlos Almada)
    * *Eco morfológico* Op. 206 (Liduino Pitombeira)

iv. 2019

In that year the MusMat conference received international status, and English became the official language for the communications. Exceptionally, the event was also held in conjunction with the Third Conference of the Brazilian Association of Music Theory and Analysis (TeMa), at the Club of Engineering of Rio de Janeiro (Fig. 6). Distributed across the sessions, a record number of outstanding works concerning music and mathematics was presented along the whole week.

- First day (Oct/21/2019):
  - Round table 1 – "Tonal Theories". Participants:
    José Oliveira Martins (University of Coimbra) – *Non-octave Tonal Frameworks: Mistuning, Pitch Fields, and Transpositional Tonnetze*
    Patrick McCreless (Yale University) – *Theories of Chromaticism: Pedagogical Implications*
  - Round table 2 – "Theories of Textures". Participants:
    Didier Guigue (Federal University of Paraíba) – *A Proposal to Analyze Orchestrations from the Texture*
    Pauxy Gentil-Nunes (Federal University of Rio de Janeiro) – *Reading of Textural Functions, Instrumental Techniques and Space through Particionial Complexes*
  - Lecture, by Robert Morris (Eastman School of Music), entitled *Issues in Compositional Theory*
  - Concert 1 – works performed by Marina Spoladore (piano) and the Saxophone Quartet of UFRJ (Pedro Bittencourt, Vinicius Macedo, Fernando dos Santos, and Paulo Félix).
    * *Canções da velha era* (Pauxy Gentil-Nunes)
    * *Ponto, linha e plano* (Daniel Moreira)
    * *Berimbau*, Op. 216b (Liduino Pitombeira)
    * *Park Suite* (Carlos Almada)
- Second day (Oct/22/2019):
  - Round table 2 – "Post-Tonal Theories". Participants:
    Robert Morris (Eastman School of Music) – *Issues in Compositional Theory*
    Paulo de Tarso Salles (São Paulo University) – *Voice-leading among Pitch Class Sets: Revisiting Allen Forte's Genera*

Figure 6: *Poster of the 2019 MusMat Conference.*

  - Lecture, by José Oliveira Martins (University of Coimbra), entitled *Scalar Dissonance and the Polytonal/Modal Impulse in Twentieth-Century Music*
  - Concert 2 – works for flute (Eduardo Monteiro), bassoon (Aloysio Fagerlande), and piano (Flavio Augusto).
    * *Patrono quieto*, Op. 207 (Liduino Pitombeira)
    * *Quatro choro-fugatos para piano* (Carlos Almada)
    * *Sagração de um fauno na primavera* (Daniel Moreira)

- Third day (Oct/23/2019):
  - Round table 3 – "Theories of Popular Music". Participants:
    Gabriel Pareyon (University of Guadalajara) – *Non-Linear Approach to Popular Musicalities through a Non-Western Mathematical Understanding of Rhythm, Intonation and Locally Generated Harmony*
    Júlio Herrlein (Federal University of Rio Grande do Sul) – *Popular Music and the Rhythmic Set-Class Theory: A Survey*

- Fourth day (Oct/24/2019):
  - Round table 4 – "Current Theories and Tecnologies". Participants:
    Jean Pierre Briot (Centre National de la Recherche Scientifique – CNRS) – *Progress and Challenges for Music Generation by Deep Learning*
    Charles de Paiva (University of Campinas) – *Reflections on Algorithmic Models of Symbolic Representation for Computer-Assisted Musical Analysis*
    Rodrigo Schramm (Federal University of Rio Grande do Sul) – *Computational Information Retrieval Techniques applied to Musical Analysis*

- Fifth day (Oct/25/2019):
  - Lecture, by Gabriel Pareyon (University of Guadalajara), entitled *Peircean Mathematics for Musicology: Transcending Heraclitus Melancholy through Harmonic Synechism*

v. 2020

Due to COVID-19 pandemics, the 5th MusMat conference was held remotely (Fig. 7). For the first time, in addition to communications, lectures, and round tables, the conference included five concerts (with different instrumentations), whose video previously recorded were broadcasted through MusMat's channel in YouTube (as well as all the other live activities). In order to access them, please visit https://www.youtube.com/c/musmat.

- First day (Dec/8/2020):
  - Keynote, by Dmitri Tymoczko (Princeton University), entitled *Repeating Contrapuntal Patterns*
  - Round table 1 – "Algorithms and Music". Participants:
    Moreno Andreatta (University of Strasbourg) – *Processes and Techniques of 'Mathemusical' Learning: How to Teach Maths through Music via Computer Science*
    Marcos Sampaio (Federal University of Bahia) – *Computational Musicology, Algorithms, and Dataset*
    Jônatas Manzolli (University of Campinas) – *Dialogue between Composition and Analysis through Computer Models*
  - Concert 1 – works for solo clarinet (performed by José Batista Júnior)

* *Canopus*, Op. 253 (Liduino Pitombeira)
* *Uma bossa* (Carlos Almada)

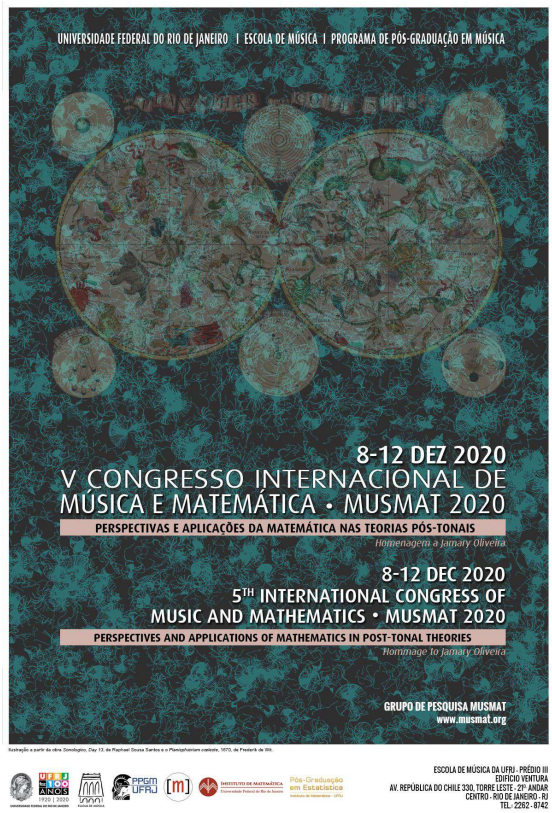

Figure 7: *Poster of the 2020 MusMat Conference.*

- Second day (Dec/9/2020):
  - Round table 2 – "Probability and Music". Participants:
    David Temperley (Eastman School of Music) – *Probabilistic Models of Musical Pleasure*
    Fabian Moss (École Polytechnique Fédérale de Lausanne) – *The Importance of Modeling in Computational Musicology*
    Hugo Carvalho (Federal University of Rio de Janeiro) – *Statistical Models for Music Emotion Recognition*
  - Concert 2 – works for voice (Andréia Adour) and guitar (Fábio Adour)
    * *Os 4 Elementos* (Liduino Pitombeira, text by Petrucio Viana)
    * *Fiboema 1* (Carlos Almada, text by Petrucio Viana)
    * *Epílogo* (Daniel Moreira, text by Stefan Zweig, translated by Manuel Bandeira)
- Third day (Dec/10/2020):
  - Round table 3 – "Logic and Music". Participants:
    Robert Peck (Louisiana State University) – *Beat-Class Set Classes and the Power Group Enumeration Theorem*
    Petrucio Viana (Federal Fluminense University) – *On the Logicity of Music*
    Francisco Aragão (Federal University of Ceará) – *Logic and the Logics of Music*

  - Lecture, by Scott Murphy (Kansas University), entitled *Common Musical Sets as Pareto-Optimal Peaks*
  - Concert 3 – works for solo bassoon (performed by Ariane Petri)
    * *Rocknária* (Carlos Almada)
    * *Alvorada* (Daniel Moreira)

- Fourth day (Dec/11/2020):
  - Round table 4 – "Temporal Organizations in Music". Participants:
    Carlos Mathias (Federal Fluminense University) – *Rhythmic Illusions*
    Arthur Kampela (Federal University of State of Rio de Janeiro) – *Time Suspended and Metric Grace-Notes: A Cognitive Short-Circuit between the Metronomic, the Metonymic and the Mnemonic*
    Marcelo Coelho (Souza Lima) – Suite I Juca Pirama: *Adapting José Eduardo Gramani's Rhythm to Ron Miller's Modal Jazz*
  - Lecture, by Rodolfo Coelho de Souza (São Paulo University), entitled *The Role of Mapping Function in the Algorithmic Musical Composition*
  - Concert 4 – *Enlarge Your Sax*: Works for saxes and electronics (performed by Pedro Bittencourt)

- Fifth day (Dec/12/2020):
  - Round table 5 – "Music Signal Processing". Participants:
    Rodrigo Schramm (Federal University of Rio do Grande do Sul) – *Improving the Classification of Rare Chords*
    Martín Rocamora (Universidad de la República del Uruguay) – *Signal Processing for Music Analysis from Audio Recordings*
    Bruno Masiero (University of Campinas) – *Spatial Audio and Object-Oriented Coding*
  - Lecture, by François Pachet (Spotify) and Jean-Pierre Briot (CNRS/ Lip6/ PUC - Rio de Janeiro), entitled *Some Reflexions about AI-Assisted Music Composition*
  - Concert 5 – *Electroacoustics and Alike*, organized by Marcelo Carneiro, Lilian Campesato, Paulo Dantas, and Mariana Carvalho

vi. 2021

The MusMat 2021 conference (also held remotely) celebrated the compositional production and theoretical thoughts of Arnold Schoenberg, particularly considering his contributions for the formalization of the use of mathematics for music composition (Fig. 8). An extraordinary number of experts ("Schoenbergians", in the strict sense), from several countries, presented works centered in analytical aspects of Schoenberg's production – both as a composer and theoretician –, as well as in their own studies, related in someway to Schoenberg's legacy.

- First day (Oct/18/2021):
  - Keynote, by Walter Frisch (Columbia University), entitled *Schoenberg's Creative Journey, 1897-1912*
  - Round table 1 – "Compositional Processes on Schoenberg's Music". Participants:
    Carlos Almada (Federal University of Rio de Janeiro) – *The Principles of Grundgestalt and Developing Variation in a Bio-Mathematical Model*
    Flo Menezes (State University of São Paulo) – *Schoenberg, the Harmony and its Colors*

Achille Picchi (State University of São Paulo) – Nacht, *Number 8 from* Pierrot Lunaire *by Arnold Schoenberg: A Perspective from the Artsong Theory on the Text-Music Relationship and its Unfoldings*

– Concert 1 – works for piano / "Hommage to Schoenberg (I)" (performed by Patrícia Mol, Ronal Silveira, Tamara Ujakova, Miriam Grosman, and Cristiano Vogas)
  * *Mente insana em Copacabana* (Carlos Almada)
  * *Elucubrações* (Daniel Moreira)

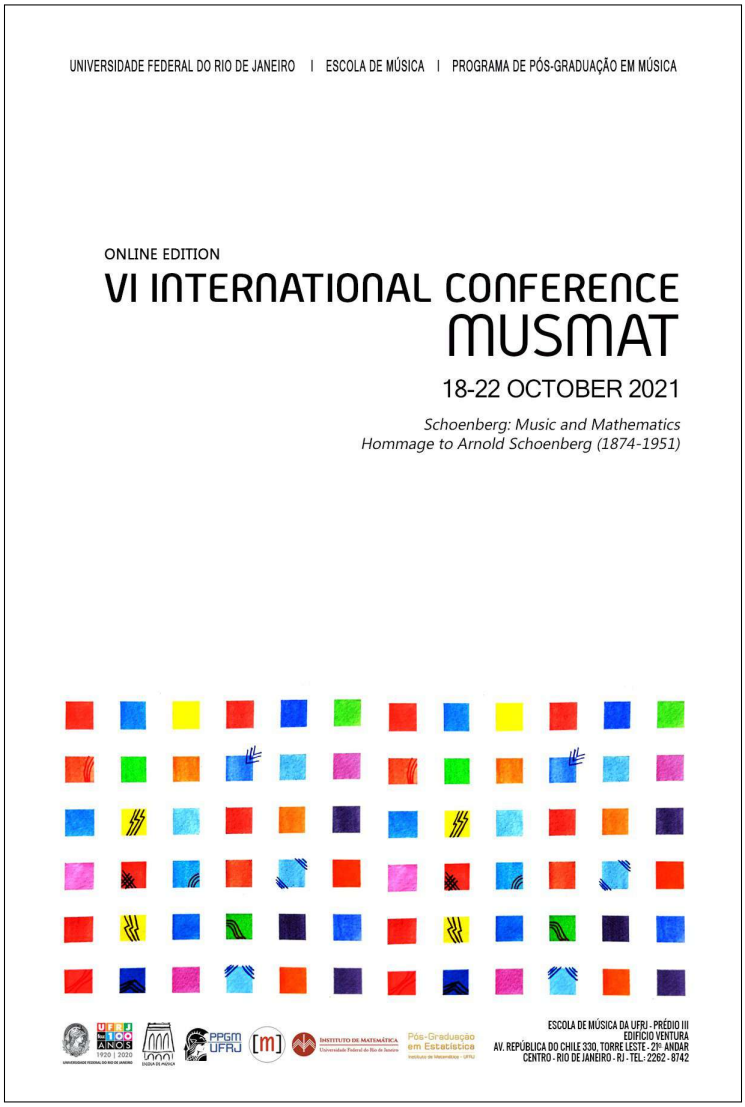

Figure 8: *Poster of the 2021 MusMat Conference*

- Second day (Oct/19/2021):
  – Round table 2 – "Dodecaphonism and Beyond". Participants:
  Robert Morris (Eastman School of Music) – *Form and Process in Schoenberg's Piano Piece, Opus 23, Number 3*

Sabine Feisst (Arizona State University) – *Tracing the Dissemination and Selected Manifestations of Dodecaphony, circa 1921–2021*
Gianluca Cascioli (Independent researcher) – *Alberto Colla's Concinnitas*
  - Lecture, by Guilherme Bueno (Federal University of Minas Gerais), entitled *The Painter Arnold Schönberg: Transfigured Artist*
  - Lecture, by Jack Boss (University of Oregon), entitled *Symmetry and the Musical Idea in Schoenberg's Op. 33a Piano Piece*
  - Concert 2 – works for string quartet / "Hommage to Schoenberg (I)". Performed by the Kalimera Quartet: Luísa de Castro (1st violin), Tomaz Soares (2nd violin), Daniel Albuquerque (viola), Daniel Silva (cello)
    * *Rondó alla Carioca* (Carlos Almada)
    * *Plate Three: Hemera*, Op. 263 (Liduino Pitombeira)

- Third day (Oct/20/2021):
  - Round table 3 – "Schoenberg and Analytical Processes". Participants:
    Ethan Haimo (Bar-Ilan University) – *Schoenberg, Mathematics, and the Fourth String Quartet*
    Severine Neff (University of North Carolina) – *Werker, Bach, Schoenberg, and Symmetry*
    Norton Dudeque (Federal University of Paraná) – *Three Motivic-Thematic Techniques by Schoenberg and their Analytical Application: Motivic unfolding, Developing Variation and Musical Prose*
  - Lecture, by Edgardo Rodriguez and Alejandro Martinez (Universidad Nacional de La Plata), entitled *Schoenberg Op. 11 No. 3: Working with Tones of the Motive*
  - Concert 3 – works for solo bassoon (performed by Ariane Petri)
    * *Vega*, Op. 295 (Liduino Pitombeira)
    * *Cinco variações sobre um tema (com variações) de Schoenberg* (Carlos Almada)
    * *Arboreal* (Daniel Moreira)

- Fourth day (Oct/21/2020):
  - Round table 4 – "Schoenberg and Group Theory". Participants:
    Robert Peck (Louisiana State University) – *Schoenberg's I-Combinatorial Space.*
    Cecília Saraiva (Federal University of the State of Rio de Janeiro) – *Some Dodecaphonic Musings with Homotopy and the Fundamental Group*
    Marco Feitosa (Independent researcher) – *Schoenberg and Group Theory: An Intervallic Approach to Tone Rows, Symmetry and Combinatoriality*
  - Lecture, by Jeffrey Perry (Louisiana State University), entitled *Grace and Clarity: Schoenberg's Other School*
  - Concert 4 – InterBrasilis Trio: Wladislaw Kreinski (flute), Jonatas Weima (saxophone), and Glenda Carvalho (cello)
    * *Arcturus*, Op. 262 (Liduino Pitombeira)
    * *Belo Monte* (Carlos Almada)

- Fifth day (Oct/22/2021):
  - Round table 5 – "Schoenberg and Set Theory". Participants:
    Marcelo Birck (Federal University of Santa Maria) – *Multimedia Collective Creation based on an Analysis of Arnold Schoenberg's Op. 6/19: A Pedagogical Experience*
    Ricardo Bordini (Federal University of Maranhão) – *Numbers in Schoenberg's Colors*
    Luigi Verdi (Conservatorio Santa Cecilia di Roma) – *Tiling Six-Part Double Canons on Trichords*

- Lecture, by Dominik Šedivý (Richard Strauss Institut), entitled *Harmonic Schemata in Contrapuntal Composition after Hauer and Steinbauer*
- Concert 1 – works for piano / "Hommage to Schoenberg (III)" (performed by Maria Di Cavalcanti, Tamara Ujakova, and Cristiano Vogas)
  * *Nocturne* Op. 266 (Liduino Pitombeira)

III. MusMat Journal

The *MusMat • Brazilian Journal of Music and Mathematics* (or MusMat Journal) was idealized by Liduino Pitombeira, Pauxy Gentil-Nunes, Daniel Moreira, and Carlos Almada in 2016. Its first number was issued at the end of that year, becoming a regular publication (two annual editions) in 2018. Since then, the journal has been counted with a considerable number of contributors, whose articles involved a wide spectrum of themes and approaches, especially connected with the use of mathematical concepts and models applied to musical analysis and composition, as well as to ground original theoretical formulations.

The next subsections present a basic overview of these articles, considering their authors and subjects.[2]

i. Volume I / number 1 (December, 2016)

Eight articles inaugurated this initial volume. The opening article of the journal was significantly written by Carlos Almada, the leader of the MusMat group. In this article, titled *Evolutionary Variation Applied to the Composition of CTG, for Woodwind Trio*, Almada integrates a broad research on musical variation introducing the original concept called "evolutionary variation", which is a convergence of Schoenbergian principles of Grundgestalt and developing variation and some ideas from Genetics and Evolutionary Biology. The concept is applied in a compositional system (Gr-S) resulting in an original piece for woodwind trio, which was premiered during the 2017 conference. The second article, *Teaching Atonal and Beat-Class Theory, Modulo Small*, was written by Richard Cohn. This paper advances a pedagogical program that models small cyclic systems before teaching the twelve-element chromatic system of atonal theory. In the third article, *Music as a Carbon Language: A Mathematical Analogy and its Interpretation in Biomusicology*, Gabriel Pareyon hypothesizes that music is more a feature and a consequence of chemical and biological constraints (not exclusive of humans), than a product "purely social" or "uniquely cultural". The fourth article, *All-(Generalized-) Interval(-System) Chords*, written Robert Peck, surveys the all-interval chords of small order and the interval systems in which they are situated, observing that these chords belong to three categories of difference sets from the field of combinatorics: $(v, k, 1)$ planar difference sets, $(v, k, 2)$ non-planar difference sets, and $(v, k, 1, t)$ almost difference sets. The fifth article, *On the Relation of Quality and Quantity in the Context of Musical Composition*, by Alexandre Reche e Silva, aims to highlight the connection between quality and quantity, from a musical point of view, by sketching a typology of musical qualities and presenting the J-Syncker, an assistant software for the generation of pre-compositional material. In the sixth article, *Contour Algorithms Review*, Marcos da Silva Sampaio and Pedro Kroeger present some problems of two Music Contour Relations Theory operations algorithms (one by Rob Schultz and the other by Elizabeth Marvin and Paul Laprade) and propose two alternative algorithms to solve these problems. The seventh paper, *Sound Shizuku Composition: a Computer-Aided Composition System for Extended Music Techniques*, by Ivan Eiji Simurra and Jônatas Manzolli, discusses a new environment

[2] Through a partial reproduction of the respective number's foreword.

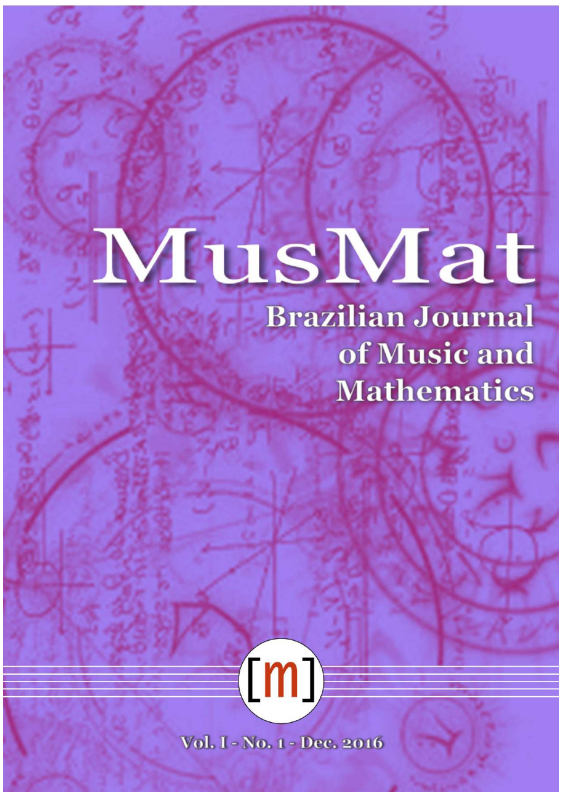

Figure 9: *Cover of the inaugural edition, v.1 n.1, published in 2016.*

for computer aid musical composition which is designed to create works centered on the creative use of instrumental extended techniques. The last article, *Discrete and Combinatorial Mathematics, Geometry and Mathematics of Continuous Functions Used in Some of my Compositional Projects*, by Rodolfo Coelho de Sousa, intends to demonstrate the different ways many of his compositional projects used mathematical tools, from the pre-compositional stage through a final product done with sound synthesis.

Table of contents:

Figure 10: *Cover of the second issue, v.2 n.1, published in 2018.*

ii. Volume II / number 1 (May, 2018)

After a gap in 2017, the second volume of MusMat Journal was released in 2018, presenting seven articles addressing different aspects of the various intersections between music and mathematics. In the first article, Guilherme Bertissolo discusses the idea of cycle and its various approaches within music theory to propose further compositional applications of it. Charles de Paiva presents a model for computational assisted-analysis that uses deterministic algorithms to reconstruct and simulate neighboring variants of existing musical scores. This methodology is demonstrated in the analysis of Steve Reich's *Clapping Music*. Luigi Irlandini discusses the context in which number and proportion relate to musical form in his own compositional practice considering concepts of mythic cosmologies. Luka Marohnić demonstrates how Hans van der Laan's plastic number can be used as a methodological approach to investigate the ratios between lengths of the vital parts of a sonata-form movement in a probabilistic perspective to determine the restrictions that are inherent to the temporal structure of the Eighteenth-Century Sonata form. This methodology is empirically applied to the set of sonata-form movements from the instrumental pieces of Mozart. Pauxy Gentil-Nunes discusses the way a given referential textural configuration, expressed in the form of an integer partition, can be presented by a composer in a piece in such a way that the various contiguous textures are understood as internal deviations of the referential partition, forming a hierarchical structure called Partitional Complex. Dmitri Tymoczko proposes an iterable voice-leading schema that enables the analysis of a wide range of repeating musical patterns in different repertoire. Finally, Didier Guigue presents the analysis of Webern's orchestration in his *Variationen* Op. 30 by mapping all sonic resources used in the piece and the way they are combined

in different setups in the orchestral design.

Table of contents:

- *Cycles in Music: Spaces, Experience and Applications in Music Theory and Composition*, by Guilherme Bertissolo
- *Rudiments of Simulation-Based Computer-Assisted Analysis Including a Demonstration with Steve Reich's* Clapping Music, by Charles de Paiva
- *Cosmicizing Sound Music — Cosmos — Number*, by Luigi Antonio Irlandini
- *A Study of Variation in Temporal Structure of Sonata Form*, by Luka Marohnić
- *Nestings and Intersections between Paritional Complexes*, by Pauxy Gentil-Nunes
- *Iterable Voice-Leading Schemas*, by Dmitri Tymoczko
- *The Function of Orchestration in Serial Modus: The Case of Webern's* Variations *Op. 30 and a Proposal of Theoretical Analysis*, by Didier Guigue

iii. Volume II / number 2 (December, 2018)

In this issue, Robert Peck presents his keynote lecture given at the 3rd National Conference of Music and Mathematics entitled *The State of the Art: New Directions in Music and Mathematics*, in which he covers the most recent trends in the convergent fields of music and mathematics, highlighting the formation of the Journal of Mathematics and Music, co-founded by the author in 2007. Ciro Visconti proposes a new approach for the representation of any contextual inversion operations between members of any set class. Liduino Pitombeira discusses the theoretical basis of the analytical-compositional methodology called Systemic Modeling, using Debussy's Prélude No.1 as a case study. Marco Sampaio proposes two new algorithms for melodic contour similarity that can be used with small and large contours. Robert Morris' article addresses two important issues in Pitch-Class Set Theory: *Z-Related Hexachords explained by Transpositional Combination and the Complement Union Property*.

Table of contents:

- *The State of the Art: New Directions in Music and Mathematics*, by Robert Peck
- *Axis of Contextual Inversion*, by Ciro Visconti
- *A Systemic Model for Debussy's Prélude No.1*, by Liduino Pitombeira
- *Contour Similarity Algorithms*, by Marcos da Silva Sampaio
- *Z-Related Hexachords explained by Transpositional Combination and the Complement Union Property*, by Robert Morris

iv. Volume III / number 1 (June, 2019)

This issue contains a paper by Marianthi Bozapalidou that introduces a machine model to describe fundamental music functions, such as transposition, inversion, retrograde, change of durations, pitch-class distribution, and move function. Guerino Mazzola proposes a mathematical construction of musical time, derived from mathematical gesture theory and examines its application to free jazz. Daniel Moreira de Sousa formalizes the concept of textural spaces, discussing aspects on their compositional implementation through the modes of textural realization. Rael Bertarelli Gimenes Toffolo presents a computational implementation of Pousseur's harmonic network in SuperCollider computer language. Michael Winter's paper shows how James Tenney's theory of harmonic distance in harmonic space were envisioned by Leibniz more than 300 years ago.

Table of contents:

- *Machine Representation of Fundamental Musical Functions*, by Marianthi Bozapalidou

Figure 11: *Cover of the third issue, v.2 n.2 published in 2018.*

- *Musical Time: A Gestural Construction*, by Guerino Mazzola
- *Composing with Textures: A Proposal for Formalization of Textural Spaces*, by Daniel Moreira de Sousa
- *Computational Implementation of Henry Pousseur's Harmonic Networks applied to Live-Electronic Music*, by Rael Bertarelli Gimenes Toffolo
- *A Few More Thoughts about Leibniz: The Prediction of Harmonic Distance in Harmonic Space*, by Michael Winter

v. Volume III / number 2 (December, 2019)

David Clampitt presents an overview of Scale Theory via combinatorics on words, particularly the interaction between notes and words. Hugo Tremonte de Carvalho presents a music-oriented introduction to Markov chains and their application to music composition and analysis. Stephen Guerra's paper examines the relationships between solo and timeline rhythmic structures of Afro/diasporic musics, presenting four techniques to understand some Baden Powell's solos as cycles of a samba timeline. Carlos Almada reformulates the basic structure of a recursive algorithm proposed by Douglas Hofstadter in order to introduce transformational-musical tools. Adolfo

Figure 12: *Cover of the fourth issue, v.3, n.1, published in 2019.*

Figure 13: *Cover of the fifth issue, v.3 n.2, published in 2019.*

Maia and Igor Maia present an analysis of Ligeti's *Musica Ricercata* I and II on a perspective of Information Theory and Complexity, including some of Ligeti's techniques of texture.

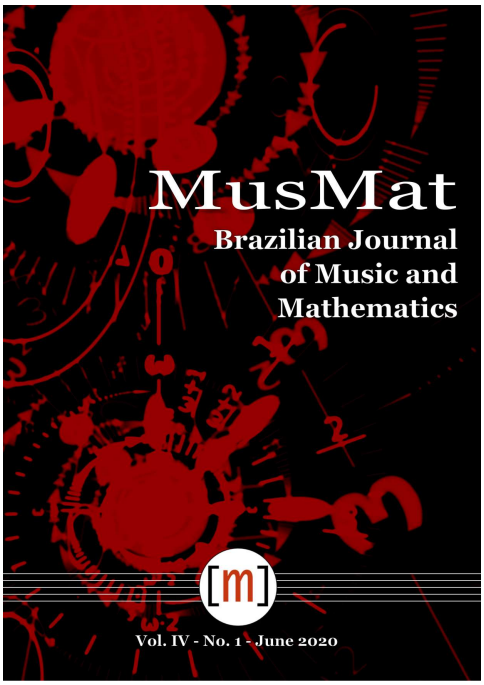

Figure 14: *Cover of the sixth issue, v.4 n.1, published in 2020.*

Table of contents:

- *An Overview of Scale Theory via Word Theory: Notes and Words, Commutativity and Non-Commutativity*, by David Clampitt
- *An Introduction to Markov Chains in Music Composition and Analysis*, by Hugo Tremonte de Carvalho
- *Toward a Theory of Structuring Rhythm in Improvisation in Timeline-Based Musics*, by Stephen Paul Guerra
- *PBach and Musical Transformations*, by Carlos Almada
- *An Information Theory Based Analysis of Ligeti's* Musica Ricercata: *Movements I and II*, by Adolfo Maia Jr. and Igor L. Maia

vi. Volume IV / number 1 (June, 2020)

Five articles integrate this number, covering diversified aspects from the rich confluence of musical and mathematical subjects. A study by Scott Murphy opens the issue, presenting an original approach of common-time meter, based on the properties of the correlate functions of metric weight and onset frequency. Jean-Pierre Briot examines deep-learning theory and techniques under the standpoint of autoencoder architectures, used for enhancing the compression of information for musical composition. Liduino Pitombeira presents a quite comprehensive survey concerning compositional systems, including the processes related to systemic modeling. Arthur Kampela discusses profoundly the processes associated with the Micro-Metric Modulation Theory. Marianthi Bozabalidou addresses the Theory of General Scale Systems through the prisms of algebraic groups, which involves the ideas of counterpoint groups and counterpoint spaces.

Table of contents:

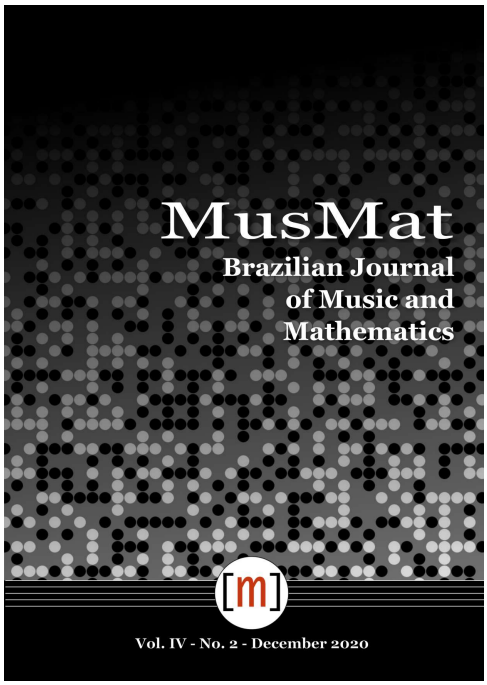

Figure 15: *Cover of the seventh issue, v.4, n.2, published in 2020.*

- *Common Rhythm as Discrete Derivative of Its Common-Time Meter*, by Scott Murphy
- *Compress to Create*, by Jean-Pierre Briot
- *Compositional Systems: Overview and Applications*, by Liduino Pitombeira
- *Rhythm and Entropy: The Exile of the Metric in the Deviation of the Pulse*, by Arthur Kampela
- *Scales, Counterpoint Triples and their Groups*, by Marianthi Bozapalidou

vii. Volume IV / number 2 (December, 2020)

Six original articles integrate this number. Marco Feitosa introduces the concept of *partitional harmony*, an original field of research that relates the Theory of Integer Partitions to several fields of Post-Tonal Theory. Robert Morris provides an in-depth analysis of Feldman's *Last Pieces* for piano solo, bringing to light information that can help the pianists perform the cancelling effect requested by the composer. Gabriel Pareyon combines Matthai philosophy with Category Theory, using Yoneda lemma, suggesting that the latter can support a robust philosophy of music within the scope of Category Theory. Robert Peck investigates the inversion operation, in terms of cycles, and its application to a dramaturgical context, through the examination of the Aristotelian concept of *peripeteia*, as observable in Birtwistle's opera *Punch and Judy*. Paulo de Tarso Salles explores Forte's Genera Theory (as well as other proposals that deal with similarities between pitch-class sets) and demonstrates the application of this theory in some works by Villa-Lobos. Pauxy Gentil-Nunes discusses the *partitioning complexes* and their application in musical practice by examining three situations: textural planning in the context of compositional processes, observation of the relationship between textural configurations and coupling of the body (performative partitioning), and spatial partitioning.

Table of contents:

- *Partitional Harmony: The Partitioning of Pitch Spaces*, by Marcos Feitosa
- *Aspects of Performance Practice in Morton Feldman's* Last Pieces, by Robert Morris
- *Philosophical Sketches on Category Theory Applied to Music-Mathematical Polar Semiotic*s, by Gabriel Pareyon
- *Time and Reversal in Birtwistle's* Punch and Judy, by Robert W. Peck
- *Voice Leading Among Pitch-Class Sets: Revisiting Allen Forte's Genera*, by Paulo de Tarso Salles
- *Reading Textural Functions, Instrumental Techniques, and Space Through Partition Complexes*, by Pauxy Gentil-Nunes

viii. Volume V / number 1 (June, 2021)

This issue comprises seven papers, that present the results of original and innovative research in the field. The number opens with a work by Robert Peck, where he applies the power group enumeration theorem to extend the Theory of Beat-Class Sets by also considering rhythms with more than one voice. Next, Gideon Effiong shows how quasigroups can be used as a unifying framework to describe musical objects and events, such as chord inversions, n-tone composition charts, and melodic motions. Ciro Visconti then discusses how Graph Theory can be used to describe all classes of trichords and tetrachords in Neo-Riemannian Theory, expanding beyond triads and seventh chords. Francisco Aragão presents an application of *Kripke semantics* to identify if a sequence of chords constitutes or not a tonal progression, which can be used to create a software to benefit students that do not have easy access to a harmony teacher. Subsequently, Juan Sebastián Arias-Valero and Emilio Lluis-Puebla develop deep relations and philosophical reflections between Gesture Theory and Category Theory. Next, Silvio Ferraz describes a series of patches in Max/MSP environment tailored to aid musical analysis and composition and exemplifies it with the composition of a piece based on Brahms' Op. 119. The issue closes with Daniel Moreira introducing the concept of *compositional entropy*, which deals with the amount of freedom of a composer when dealing with compositional choices, and such a concept is demonstrated in musical texture.

Table of contents:

- *Beat-Class Set Classes and the Power Group Enumeration Theorem*, by Robert W. Peck
- *Musical Quasigroups*, by Gideon Okon Effiong
- *Neo-Riemannian Graphs Beyond Triads and Seventh Chords*, by Ciro Visconti
- *Tonal Progressions Identification Through Kripke Semantics*, by Francisco Erivelton Fernandes de Aragão
- *A Conceptual Note on Gesture Theory*, by Juan Sebastián Arias-Valero and Emilio Lluis-Puebla
- *Modeling, Listening, Analysis, and Computer-Aided Composition*, by Silvio Ferraz Mello Filho
- *Measuring the Amount of Freedom for Compositional Choices in a Textural Perspectiv*e, by Daniel Moreira

ix. Volume V / number 2 (December, 2021)

This issue presents five articles discussing different aspects of the intersection between music and mathematics. Juan Sebastián Arias-Valero, Octavio Alberto Agustín-Aquino, and Emilio Lluis-Puebla present a theoretical description of a model based on the generalization of first-species counterpoint considering arbitrary rings, which results in a broader mathematical theory for contrapuntal intervals. Carlos Mathias and Carlos Almada introduce an original proposal to encode timelines as univocal integers by using arithmetic mapping so that drum-set timelines are encoded by using Gödel's Numbering algorithm. Paul Lombardi presents an interesting discussion

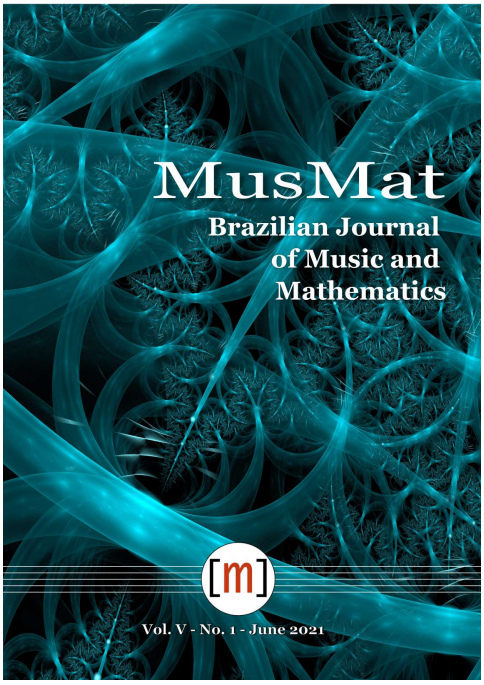

Figure 16: *Cover of the eighth issue, v.5 n.1, published in 2021.*

on feathered beams, examining their concept and notation to propose a graphing system to deconstruct them using examples from George Crumb's *Night Music* I. Hugo Carvalho proposes a new tool for performing time-frequency analysis on audio signals, the probabilistic spectrogram, that may allow for probabilistic interpretations related to the Discrete Fourier Transform and also the creation of new features for audio signal processing and music information retrieval. Finally, Juan Sebastián Arias-Valero and Emilio Lluis-Puebla present a specific and didactic application for gestural presheaves in the language of abstract gestures, dealing specifically with the relation thereof to the Yoneda embedding and Mazzola's idea and gestural sheaves, demonstrating the application in Mozart and Beethoven. In this issue, we are also glad to inaugurate a new section with an interview with Severine Neff, discussing her work on Schoenberg's music and theory.

Table of contents:

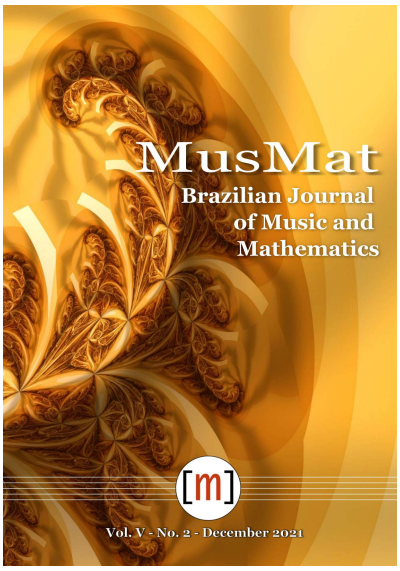

Figure 17: *Cover of the ninth issue, v.5 n.2, published in 2021.*

IV. MusMat Lectures

In 2019, the MusMat Group started a series of encounters, in which renowned researchers from different fields presented in-person lectures. Three lectures were held (Figs. 25, 26, and 27): the first one by Jean-Pierre Briot entitled *Revisitando a criação de conteúdo musical com técnicas recentes de aprendizagem profunda* (*Revisiting the creation of musical content with recent techniques of deep learning*); the second and third lectures were part of the preparation for a course given by MusMat Group, on the work Formalized Music, by Iannis Xenakis, in partnership with four professors in the mathematical field, within the scope of the Graduate Program in Music at UFRJ: Hugo Carvalho, Nei Rocha, Petrucio Viana, and Stafanella Boatto. Thus, Hugo Carvalho presented a lecture entitled *Cadeias de Markov aplicadas à composição musical* (*Markov Chains applied to musical composition*) and Petrucio Viana discussed the 6^{th} Chapter of Xenakis' book. In 2020, due to the COVID-19 pandemic, the lectures took a break, but it is a project that the group will regularly resume in 2023.

V. MusMat Podcast

To reach non-specialist teachers and younger students, the MusMat Group created the *MusMat Podcast*, the first podcast in Brazil that specifically offers content in Portuguese, about Music and Mathematics. The MusMat Podcast is hosted by Carlos Mathias and Hugo Carvalho and each episode consists of an interview with a guest, a specialist, teacher and researcher of the area. The episodes are available in audio format at `https://anchor.fm/musmat-podcast` and in video format at our YouTube channel `https://www.youtube.com/c/musmat`. Among the guests interviewed so far are Cecília Saraiva, Daniel Moreira, Carlos Almada, Hugo Carvalho, Liduino Pitombeira, Marco Feitosa, and Jack Boss (Figs. 18, 19, 20, 21, 22, 23, 24). The MusMat Podcast will start producing other kinds of content soon, such as videos of short duration, informal enough to be used on music/mathematics classes of Brazilian junior and high schools, hopefully.

VI. Future projects

Up to this point, the projects of the MusMat Group were directed mostly to research and teaching at Master and Doctorate level. One of our main goals from now on is to propagate the connections between music and mathematics to the undergraduate scenario, as well as to the community overall. A first step on this direction is the organization of a book with articles that contain or explain further details of each talk from the lectures and round tables that occurred during the Musmat Conference of 2021, which was a homage to Arnold Schoenberg and his legacy. We have asked the speakers to write or publish the content of their ideas in a volume that is being organized by the MusMat Group. Some texts are articles that were previously written but not published and some are fresh new, so that every text is original. We've had a lot of great talks and wanted to make them known to a wider range of people.

There are many books written in English and other languages about music and mathematics, such as David Wright's *Music and Mathematics* and Gareth Loy's *Musimathics*. Unfortunately, when the language in hand is Portuguese, finding books (or even introductory notes) about music and mathematics is a very hard task. This unavailability affects not only the brazilian students who lack proficiency in English, but also dampens the divulgation of more complex relations between music and mathematics to the common sense, that lay beyond acoustics/undulatory phenomena, for instance. One of our most cherished future projects also follows the trend of inclusion: a joint book written in Portuguese, that presents the basic elements of the area of music and mathematics and that explores its elementary and modeling properties and aspects.

We are also looking forward to having an expansion of the MusMat Group project that we call *Musmatinho* (little MusMat, translating from Portuguese), which is meant to be a gathering of MusMat, students and interested people who are not officially part of the research group, but may contribute with their ideas, creativity and enthusiasm. We've already had one meeting in which Doctorate, Post-Doctorate, Master, undergraduate and former students from both Music and Mathematics courses presented their interests, so we could get to know each other. Our aim is to grow our collaborative network and broaden students' perspectives toward the areas.

Figure 18: *Poster of the first MusMat Podcast, with Cecília Saraiva (available at* https://youtu.be/imszZsY0ezA*).*

Figure 19: *Poster of the second MusMat Podcast, with Daniel Moreira (available at* https://youtu.be/YuXhyFtPFCs*).*

Figure 20: *Poster of the third MusMat Podcast, with Hugo Carvalho (available at* https://youtu.be/nuXDXJO8Quc*).*

Figure 21: *Poster of the fourth MusMat Podcast, with Carlos Almada (available at* https://youtu.be/b02AbtxYcik*).*

Figure 22: *Poster of the fifth MusMat Podcast, with Liduino Pitombeira (available at* https://youtu.be/5VET-Eks_-E*).*

Figure 23: *Poster of the sixth MusMat Podcast, with Marco Feitosa and Cecília Saraiva (available at* https://youtu.be/fuxHT-Y8AO4*).*

Figure 24: *Poster of the sixth MusMat Podcast, with Jack Boss (available at* https://youtu.be/30BGDhKfqoY*).*

Figure 25: *Poster of the first MusMat Lecture, by Jean-Pierre Briot.*

Figure 26: *Poster of the second MusMat Lecture, by Hugo Carvalho.*

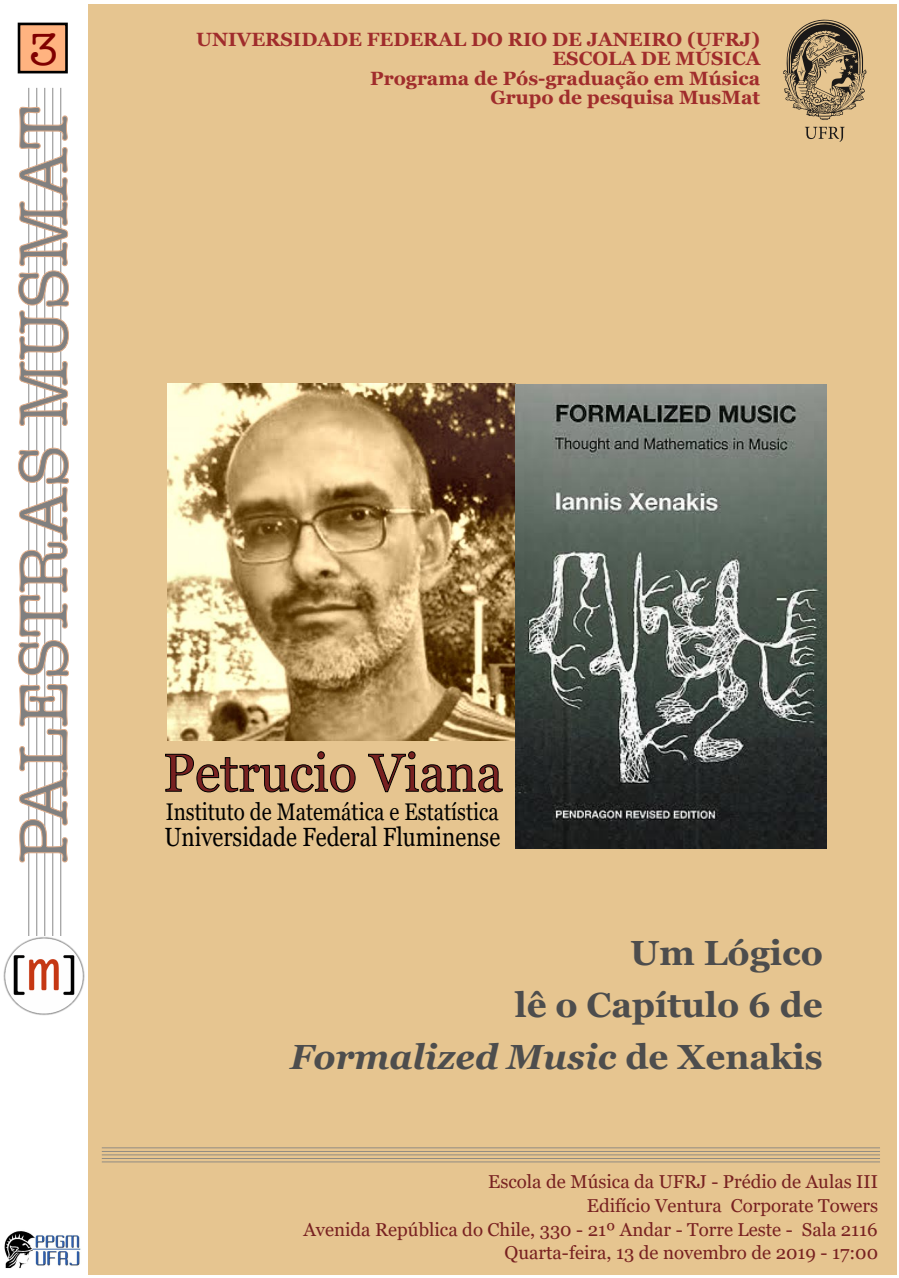

Figure 27: *Poster of the third MusMat Lecture, by Petrucio Viana.*

Interview with Richard Cohn

MusMat Research Group

29th June, 2022

I. How did you get interested in Mathematics in the beginning of your research?

As a child I always played with numbers in my head, and also enjoyed solving puzzles, but my formal math education ended in secondary school. My studies in atonal pitch-class theory with Robert Morris, as a PhD student, showed me how mathematical modes of exploration and expression could help me see more deeply into music that I cared about, and communicate my observations to others. My dissertation applied those modes to 20th-century music, with a focus on the music of Béla Bartók. My exposure to the writings of David Lewin and John Clough showed me how those modes of exploration and expression could be productively applied to historically earlier repertories, giving me the tools to understand some aspects of the music of Schubert, Brahms, and Wagner, which I loved, and for which my tonal and Schenkerian training were unsatisfactory. Lewin and Jeff Pressing, also suggested some ways to apply atonal pitch-class theory to rhythms and meters, and seeing the power of that generalization over time opened new doors for me.

Most of my mathematical work since then has emerged very clearly from specific music-analytic problems that I encountered. But the mathematics that I became comfortable with, in my analytic work, also merged with my pleasure in puzzle-solving, and sent me off in some more systematic directions, most specifically in my *Music Theory Online* article in 2004, in which I proposed a tetrahedral model for voice-leading among tetrachord classes. That really stretched my mathematical capacities to the limit.

All along the way I've had help from colleagues and students who got interested in musical problems that I identified, and taught me the mathematics to explore and express them. The late Jack Douthett, above all, was extraordinarily generous in teaching me how to think and write using math. I've also benefited, at different times, from correspondence or conversation with Julian Hook, Ian Quinn, and Dmitri Tymoczko, all of whom have done towering work. I've learned so much, both directly and indirectly, from their companionship and modelling, as well as from mathematically adept PhD students such as Adrian Childs and Clifton Callender, who studied with me at Chicago in the 1990's.

II. How do you see the collaboration between musicians and mathematicians, and how can it be more effective?

In my own experience, that collaboration just emerged naturally, as mathematicians became intrigued by the musical problems that I posed in a sort of quasi-mathematical language (which I'm sure seemed like a strange dialect to them). The resources now are so tremendous, largely thanks to the work of the Society for Mathematics and Computation in Music, since 2007, and

corresponding societies, such as yours. And they will be boosted by Julian Hook's forthcoming book from Oxford U.P., *Exploring Musical Spaces*, which is bound to be the starting point for both musicians and mathematicians concerning structuring of pitch and time. Jay's training is ideal, as he has doctorates in both fields, and he is also a skilled teacher. Of course it would be wonderful if everyone had that sort of cross-training, or if at least there were some post-graduate programs in mathematical music theory, where students were encouraged to pursue advanced work in both fields. The pioneering program at Georgia State is paving the way in the US. But the field needs to reach a certain level of critical mass for that possibility to become a reality more broadly.

III. In 1998, you wrote the introduction of the Journal of Music Theory volume dedicated to Neo-Riemaniann theory. How do you see the current status of Neo-Riemannian theory in the world today?

That's a complex question, because the boundaries of neo-Riemannian theory are difficult to determine. Within mathematical music theory, there is a steady stream of articles in the *Journal of Mathematics and Music*, and in the biennial proceedings of the Society for Mathematics and Computation in Music, that share neo-Riemannian concerns with voice-leading parsimony, with voice-leading properties of particular chords and scales, and with graphic and geometric models of musical systems. The most powerful recent work in this broad stream does not identify specifically with neo-Riemannian theory, but explicitly merges some of its questions, methods, and particular instruments with those of atonal pitch-class theory and diatonic scalar theory: the OPTIC model of Callender, Quinn, and Tymoczko; Tymoczko's multi-dimensional voice-leading orbifolds, as well as new approaches in his forthcoming book; the Discrete Fourier Transformation paradigm that that emerged from Ian Quinn's dissertation and is powerfully developed in a series of publications of Emmanuel Amiot and Jason Yust, among others; Leah Frederick's models of diatonic voice leading spaces, among many others that one might cite.

Within the field of music analysis proper, there are important recent applications to film music, by Frank Lehman and Scott Murphy. These writings are mostly rooted in a post-Lewinian transformational paradigm, though, that in the Buffalo sphere was already being absorbed into a more graphic and systematic orientation by the late 90's. As for analytical applications to 19th-century harmony, there has been a surge in the UK of work that traces roots in neo-Riemannian theory. There is little interest, though, within the American analytical community, in modelling music of the 19th century, or any historical repertories for that matter. To the extent that there is current interest in pre-1980 repertories, it is directed primarily to large questions of musical form rather than to small-scale questions of harmonic progression. But I have been heartened to learn that my *Audacious Euphony* is being translated into Czech and Chinese, and so by the time that American analysts find a new interest in 19th-century harmony there may well be some new international models from which they might take inspiration.

IV. Do you think that Music might someday provide problems to Mathematics the same way Physics did, historically speaking?

I think a mathematician is in a better position than I am to respond to this question. But my impression is that already in classical antiquity and the Middle Ages there was quite a bit of traffic in the pipeline from music to mathematics. In the early-modern era, one can point to Leonhard Euler, who already is beginning to work out graph-theoretic problems in his exploration of the properties of the *Tonnetz*, five years before his 1736 paper on graph theory initiated that branch

of mathematics. In our own time, the late Jack Douthett, in collaboration with Richard Krantz, productively applied their theory of maximally even sets to spin configurations of electrons. I understand that Jack's last paper, completed (with several collaborators) just days before he passed away in 2021, generalizes that work to solve a somewhat more complicated mathematical problem, the so-called "Three-color problem". We are really fortunate that music theory has, from time to time, drawn the interest of transcendentally brilliant people who are able to function across magisterially broad terrains!

V. We had the honor of publishing your article *Teaching Atonal and Beat-Class Theory, Modulo Small* in our first issue of **MusMat Journal**. In this number, we have been working in the state of the art of Music and Mathematics in Latin America. What is your opinion about the research in this realm and what kind of impact Latin American music may have on the research made in other countries?

There is so much tremendous music from so many different parts of South and Central America and the Caribbean islands; I regret that I have so far only become familiar with a small corner of this vast repertory. There is already a productive stream of research that applies beat-class set theory to the cyclic rhythms of many of these regional repertories. Since many of these repertories are improvised or orally disseminated rather than notated, and some of them are quite thickly layered, the biggest challenge is a musical one: to capture an initial representation through transcription. This requires the work of expert practitioners such as Stephen Guerra, who has transcribed a number of Baden Powell's solos, and is aided by the crafty pedagogical use of digital technology, as in the virtual *roda* of Jason Stanyek and Fábio Oliveria. Once the music is transcribed, there are significant research opportunities employing mathematical modelling, adapting techniques from pitch theory to achieve deeper understanding of multi-layered cyclic rhythms. Separately, there are also significant initiatives dealing with microtiming patterns as they unfold in continuous rather than digitally quantized time. Sophisticated technology broadly available to researchers has opened a number of doors, and there is an interest within the community of analysts, especially those associated with Analytical Approaches to World Music, and so this is a great time for learning more about these repertories.

VI. The entry "analysis" from New Grove presents a series of tools and analytical possibilities. Do you see an amplification of this fragmentation? Or maybe the opposite: is there a tendency toward synthesis or concurrent work of methodologies?

This is a really complicated question! I don't think I have a good answer for it, but maybe I can walk around it for a while. The intellectual paradigm under which music theorists operate has fundamentally changed since 1980, when Ian Bent's extraordinary entry on analysis was published in the New Grove. At that time the field was operating under an "unnatural confluence" (in William Benjamin's terms) of soft European connoisseurship and hard behaviorism. The cognitive revolution was just coming to the attention of the music-research community, and the fields of music perception and cognition were just beginning to operate on the remarkable international scale that they have since maintained, and intensified with the help of globalizing communication technology. Forty years later, the field of music theory and analysis lies cracked wide open in so

many different respects.

On the one hand, the cognitivist orientation led music theorists to see that even the simplest musical action, by the least trained of individuals, is unfathomably complex (I think here of the stunning 1991 book of Jeanne Bamberger, *The Mind Behind the Musical Ear*, which playfully explores how children mentally model *Twinkle Little Star*). Even after forty years of intensive work in behavioral sciences, and more recently in neuroscience, we still don't know how the human brain decides which pitch-class is the tonic, which beat-class is the downbeat, how the brain connects musical events into streams, how it processes multiple musical streams and multiple meters, how it groups events into motives, and how improvising musicians are able to keep it all going at such a fast pace in real time. And so there is no limit to the musical repertories that can be profitably studied, as the ethnomusicologists had been trying to persuade us throughout the 20th century.

On the other hand, the behavioral sciences have helped us to identify some aspects that are at the core of humanity's musicality. To some degree, those aspects seem to transcend cultural and other sorts of differences, and thus can stand as a starting place for the modelling of musical behavior and its artistic products. And those aspects, strangely, are the ones that benefit from mathematical modelling. They include categorical perception, which quantizes the continuum of pitch into discrete categories; bodily entrainment, which does the same for continuous time; pitch-class and beat-class equivalence, which convert linear phenomena into cyclic ones; and auditory streaming, which David Huron has brilliantly shown to underlie polyphonic practices. So whereas entropy has fractured the analytic enterprise into as many pieces as there are distinct musical cultures, the identification of quasi-universal attributes in the human body and mind has brought the enterprise back together again in a way that is well explored by mathematical concepts and modes of representation. Since those attributes exist at such an abstract level, they are ramified in so many varied ways in different micro-cultures that, as soon as one proposes a general mathematical model to unify the study of music, the whole enterprise then diffracts out again in an entropic explosion. I personally find the dialectic process exhilarating, and am happy to be a music theorist working in today's environment.

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FRONT COVER ART

Title: *Canvas' repository of images*
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