Contextual Dyadic Transformations, and the Opening of Brahms's Op. 5 as a Variation of the Opening of Schumann's Op. 14

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Abstract: *A contextual transformation that acts on the diatonic content of outer-voice dyadic harmony provides a way to demonstrate another correspondence between the openings of the F-minor piano sonatas by Robert Schumann and Johannes Brahms beyond those already recognized in current research. This contextual transformation also models some variation procedures within other music of Brahms and his eighteenth-century predecessors, suggesting an understanding of any influence of Schumann's opening on Brahms's compositional procedures within the framework of variation.*

Keywords*: Robert Schumann. Johannes Brahms. Sonata. Variation. Contextual transformation. Influence.*

I. Historical Context and Article Outline

30th September 1853 marks a significant date in the history of canonic western art music: on this Friday, the twenty-year-old Johannes Brahms first visited the home of Robert and Clara Schumann in Düsseldorf, Germany. With him, Brahms carried two completed slow movements—what would become the second and fourth movements—for his third piano sonata, but its fast movements were not yet composed, including the first movement ([3, p. 371]). During the four weeks of Brahms's stay in Düsseldorf, Clara played for him Robert's third piano sonata in F minor, op. 14, which begins as shown in Figure 1a. On 26 December 1853, Brahms sent off for publication as his fifth opus his completed third piano sonata in F minor in five movements, constituting the "most solid and impressive piece he had yet written" ([17, p. 126]). This sonata begins as shown in Figure 1b.

Some scholars recognize similarities between the openings of the two composers' sonatas (e.g. [7]). In arguably the most exhaustive study of Brahms's three piano sonatas, Gero Ehlert

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([6, p. 325]) proposes three likenesses, shown with colored shaded regions in Figure 1. The blue shading indicates an initiating three-note treble motive descending stepwise from A flat to F, the red shading indicates the bass's chromatic descent—-sometimes called a "lament bass"—-from F down to C, and the green shading indicates both an aggressive cadential arrival on a downbeat dominant triad with E in the treble and a trochaic ebb onto lower pitches on the second beat with C in the treble.[1] Nonetheless, Ehlert recommends against inferring too much from these similarities, concentrating instead on the considerable contrasts between their openings, such as the differences in their melodies.

In this article, I propose that these melodies, particularly the notes enclosed in Figure 1, closely correspond, lending additional support to the hypothesis that Brahms's opening is a variation of Schumann's. This proposed correspondence recruits a new type of contextual transformation **G** that expands an ordered pair of pitches while preserving its interval class. Section II of this article defines and demonstrates **G**, Section III shows how **G** serves as the basis for variation in three passages from Brahms's oeuvre composed before and after 1853, and Section IV returns to Schumann's and Brahms's sonatas, enlisting **G** to draw a connection between them.

II. The Contextual Dyadic Transformation G

A contextual transformation, relative to a usual (noncontextual) transformation, requires some additional knowledge—the "context"—about the input in order to determine the output. David Lewin ([14], [16]) cultivated this concept, which was further explored by Philip Lambert [13], Joseph Straus [20], and others. For an example, the transformation "invert around C" is a noncontextual transformation, because no knowledge is needed beyond the content of the pitches being transformed to calculate the result of the transformation. If the inputted pitch-class set contains a B, then the output of this "invert around C" transformation will contain a C sharp, regardless of the rest of the input. However, the transformation "invert around the major third," applied exclusively to a major or minor triad, is a contextual transformation, because it requires an extra assessment of the input to know the output.[2] If the input contains a B, then the output of this "invert around the major third" transformation will contain a C sharp if and only if the entire input is an E-major triad.[3]

To compare the openings of Schumann's and Brahms's F-minor piano sonatas, I present a contextual transformation that I call **G** (for Grow) that acts on ordered duples of pitches. For unequal pitches x and y, the transformation **G** changes <x, y> into <n, y> (and **G′** changes <y, x> into <y, n>) such that (1) the unordered sets {x, y} and {n, y} are pitch-class transpositions of one another, (2) x is between and unequal to both n and y, and (3) the difference between x and n is minimal. This transformation resembles Straus's generalized contextual inversions [20], in that they maintain both set class and one or more common tones (and, ordering aside, a transposition of a dyad could as easily be considered as an inversion). However, whereas Straus [20] concerns trichords, tetrachords, and pentachords, he does not address dyads. This transformation undergirds something resembling Tymoczko's dyadic form of circular voice-leading space [21], but Tymoczko privileges conjunct voice-leading distance—therefore, he favors the nearly even interval class 5 (ic5)—and he eschews transformational methodology.

[1] The first edition of Schumann's sonata has a D flat in the treble instead of the C at this point (m. 7).

[2] The "invert around the major third" contextual transformation, applied particularly to a major or minor triad, is called **REL** in Lewin [15], and abbreviated to **R** in neo-Riemannian theory ([12], [14]).

[3] To invert around a pitch or set of pitches is to invert such that the pitch or set inverts into itself, and the other pitches invert likewise. Inverting around a major third is to invert the two pitches of this interval into one another. The major third in an E-major triad is between E and G sharp. I_0 inverts these two pitches into each other; it also inverts B into C sharp.

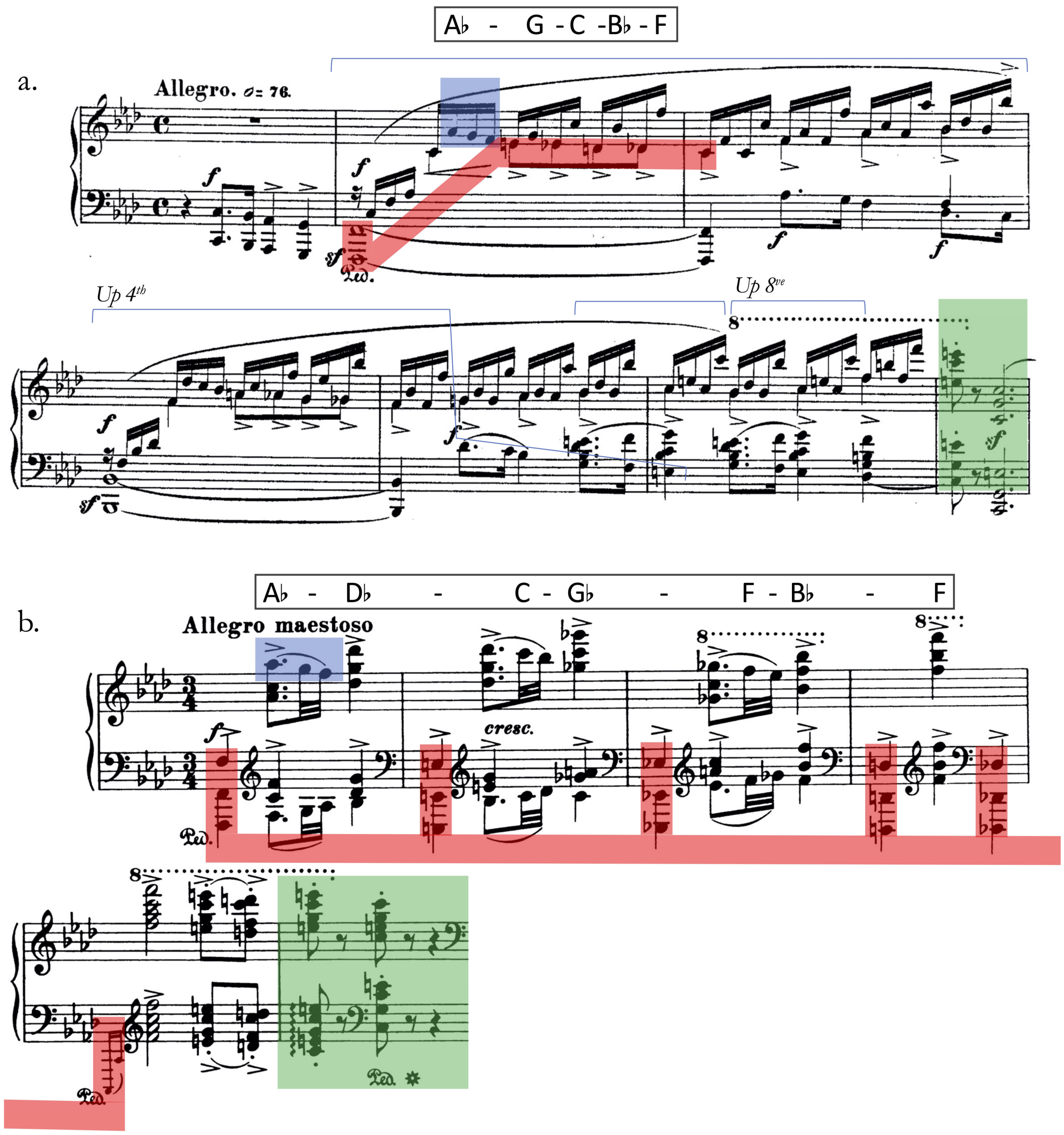

Figure 1: *(a) Robert Schumann, Piano Sonata No. 3 in F Minor, op. 14, mm. 1–7 (b) Johannes Brahms, Piano Sonata No. 3 in F Minor, op. 5, mm. 1–6 (coloration indicates correspondences put forward in [6]).*

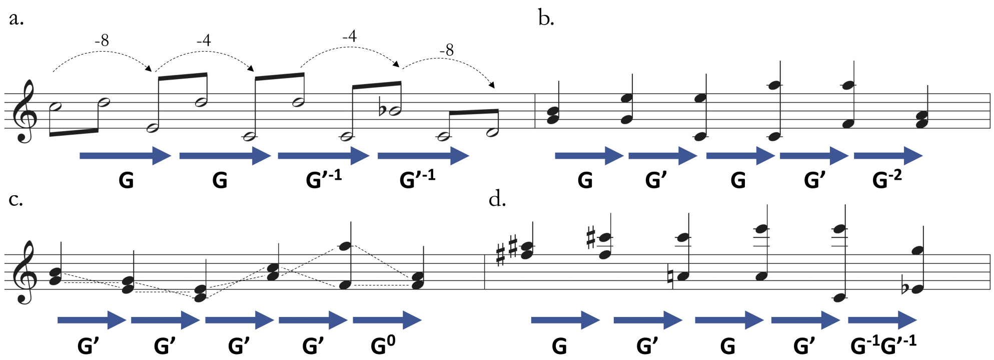

Figure 2: *Demonstrations of **G**: (a) temporal ordering <first, second>, chromatic or WT_0 scale, registral pitch (b) registral ordering <top, bottom>, white-note scale, registral pitch (c) prime-form ordering <root, not root> (à la Straus [20]), white-note scale, pitch class (d) registral ordering <top, bottom>, $OCT_{0,1}$ scale, registral pitch.*

G can be defined for any scale and any kind of ordering, act on any interval class, and involve pitch classes rather than registral pitches. If the inputs and outputs are pitch classes, the second and third parts of the definition provided above are immaterial. Following convention, compounds of **G** and **G′** are indicated with positive superscripts (e.g. $\mathbf{G}\bullet\mathbf{G} = \mathbf{G}^2$, $\mathbf{G'}\bullet\mathbf{G'}\bullet\mathbf{G'} = \mathbf{G'}^3$, the zero superscript ($\mathbf{G}^0$) indicates the identity transformation, and the inverse of **G** or **G′** is indicated with negative unit superscripts: $\mathbf{G}^{-1}$ and $\mathbf{G'}^{-1}$. When they act on registral pitches, $\mathbf{G}^{-1}$ or $\mathbf{G'}^{-1}$ shrink the interval between them; the inverse does not exist if and only if **G** acts on registral pitches and these registral pitches span the minimal distance permitted by the interval class they span within the governing scale.

Figure 2 provides some examples of **G** in action. The first dyad in Figure 2a is <C5, D5>, ordered in time. The **G** transform of this dyad changes only the first pitch (**G′** would change the second pitch), and lowers it in particular so that the distance between the two pitches will increase. This first pitch is lowered precisely to E4, because this is the one pitch both below C5 and closest to C5 that, when combined with D5, forms an unordered dyad that is a pitch-class scalar transposition of C5, D5 within, say, the chromatic scale or the whole-tone scale with C (WT_0). Therefore, **G**<C5,D5> = <E4,D5>. If **G** were applied to the <C5, D5> dyad within, say, the 2- or 3-flat diatonic scale, or the octatonic scale with C and D ($OCT_{2,3}$) instead, then G<C5,D5> = <Eb4,D5>. Although <C5,D5> and <Eb4,D5> as unordered sets are not chromatic pitch-class transpositions of one another, they are nonetheless pitch-class transpositions of one another within these other scalar contexts.

Figure 2a continues by using the output <E4,D5> as the input for another **G** transformation, which yields <C4, D5>, again within the chromatic scale or WT_0. Curiously, the moving pitch drops only four semitones (E4 down to C4) under the second **G**, whereas it dropped eight semitones (C5 down to E4) under the first **G**. This curiosity, highlighted with curved arrows and well manifested in both the two back-to-back **G** transformations that open Figure 2a as well as the two back-to-back $\mathbf{G'}^{-1}$ transformations that follow, illustrates a special feature of contextual transformations: homogeneity of contextual transformation has the potential to be realized as heterogeny of noncontextual transformation. In published transformational theory, this feature is most evident in neo-Riemannian theory's contextual transpositions. For example, **LP** is a contextual transposition that transposes a major triad by $\mathbf{T_4}$ or "up a major third" and a minor triad by $\mathbf{T_8}$ or "down a major third." Lewin [14] used **LP** to equate two different sounding harmonic progressions in two themes from Richard Wagner's *Das Rheingold*: "Tarnhelm" (**LP**{G♯,B,D♯} = {E,G,B}) and "Valhalla" (**LP**{G♭,B♭,D♭} = {B♭,D,F}). In these two progressions, the transpositional size (major third) remains the same, but the direction of transposition ("up" or "down") changes depending on the contextual structure of the input (major triad or minor triad). In the first two progressions of Figure 2a, the transpositional direction of the first note (down) remains the same, but the size (eight or four semitones) changes depending on the contextual structure of the input (major second or minor seventh). From this point of view that focuses on size and direction, neo-Riemannian contextual transformations and G contextual transformations resemble "duals" of one another, at least colloquially.

Figures 2b, 2c, and 2d furnish further demonstrations, varying type of ordering, scale, and attention or inattention to octave equivalence. The ends of Figures 2b and 2c show how a **G** transformation with an even superscript outputs a dyad with the same ordered pitch-class content as the input. Figure 2c's dyadic series has the same pitch-class content as that of Figure 2b, even though Figure 2c simply iterates **G′** while Figure 2b alternates between **G** and **G′**. Figure 2c's sleight-of-hand owes to the use of prime form (e.g. [03]) as the dyad's ordering (similar to [20]), whereby, in an ordered pair <x, y>, x corresponds to the first digit (the [0. . .]) of the dyad's prime form—-what might be called the "root" ([10])—-and y corresponds to the second digit (the [. . . n]) of the dyad's prime form. Some may see this as analytical overkill, saying instead that Figure 5 simply displays descending-third transpositions, beginning as parallel motion through register and continuing by departing from such. However, the use of **G′** suggests a certain type of voice leading, annotated with straight and mostly crossed lines in Figure 2c, that declines a parallel hearing in favor of another perhaps more covert, in the manner similar to how Lewin [14] approached some instances of parallel voice leading in Debussy's music.

III. G as Variation Mechanism in Some Music of Brahms

A configuration of **G** that is especially suitable for tonal music is a **G** that acts within some diatonic scale—-or more generally, on the seven diatonic letters irrespective of accidental—-on concurrent pitches or pitch classes in outer voices, with an arbitrary ordering of <soprano, bass>. This emphasis on outer voices is consistent with other prominent theories of tonal musical structure ([18], [8]). For tonal music more associated with "high" styles, a good choice for the one interval class spanned by these outer-voice pitches is the diatonic interval class of two steps (dic2), more commonly known as imperfect intervals: thirds, sixths, tenths, thirteenths, and so forth. This interval class tends to be used more often between simultaneous outer-voice pitches in these styles than each of the other three diatonic interval classes (unison/octave (dic0), second/seventh (dic1), fourth/fifth (dic3)) except at beginnings and endings of formal units like phrases and sections, where dic3 and dic0 are more common.

The transformation **G** works well as a variation mechanism in general because it preserves one pitch while changing the other, a balance of mutability and immutability that is at the heart of variation technique. It works well as a variation mechanism more specifically in two capacities: addition of pitches, and alteration of pitches. The first capacity supplies embellishments through applying **G** *syntagmatically* to one dyad to create the next dyad, as Figure 2 does. When **G** is configured as outlined above, this converts a one-to-one first-species texture into a many-to-one second-(or third-)species texture: a pitch of one of the outer voices remains fixed as a "cantus-firmus" note, and the paired pitch from the other voice shifts to another pitch idiomatically consonant with the fixed pitch. The second capacity makes changes through applying **G** *paradigmatically* to one dyad to create the corresponding dyad in the next variational rotation, as Figure 2 does not. For example, if the variation is over a ground bass, each application

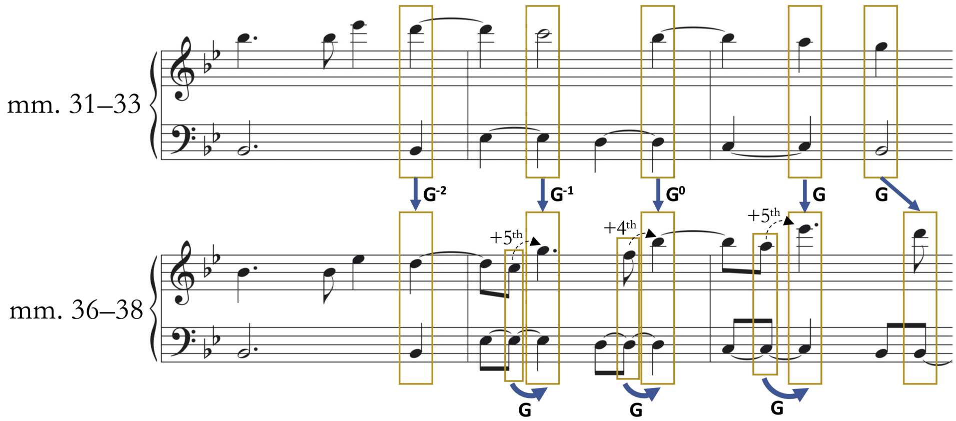

Figure 3: *Gradated application of* **G** *as variation technique: Brahms, Variations on a Theme of Haydn, op. 56, Finale, mm. 31—33 and 36-–38, soprano and bass lines only (registral ordering <top, bottom>, two-flat diatonic collection, registral pitch).*

of **G** would maintain a pitch in this fixed bass line while modifying the treble pitch above it (*mutatis mutandis*, consistent employment of **G'** works well for a reharmonizing variation of a fixed top line).

Figure 3 shown an instance of **G** operating in both paradigmatic and syntagmatic capacities. These excerpts come from the finale of Brahms's *Variations on a Theme of Haydn* of 1873, which is structured as a set of continuous variations on a repeating five-measure line in B-flat major (B♭-B♭-E♭-D-C-B♭-E♭-C-F-F) assembled from the outer voices of the theme's first phrase. This line first appears in the bass during the finale's first 65 measures, and moves to the treble for mm. 61-80 before returning to the bass in m. 81 soon before a coda in mm. 86–109. Measure 31 begins the seventh ground-bass variation, whose outer voices of its first three of five measures are shown in the top system of Figure 3. Here, a high treble line unhurriedly descends, mostly in parallel with the stepwise descending bass. Its syncopations create suspensions that decorate a series of imperfect harmonies above the bass line, enclosed in rectangles, that are displaced onto off beats in this cut-time music. By defining **G** as specified in this section, and additionally as acting on registral pitches, a succession of **G**-based transformations—shown in the middle of Figure 3—-paradigmatically converts the sixth variation's gently descending treble line of D-C-B♭-A. . . into the seventh variation's bouncily ascending treble line of D-G-B♭-E♭. . . , reduced in the lower grand staff of Figure 3. More specifically, this succession of transformations is gradated: the **G**-compound superscripts begin with their lowest value of -2, setting the treble line an octave lower, and incrementally increase through -1, 0, and 1, ultimately superseding the treble's pitch height in the earlier corresponding spot. Considering the seventh variation on its own, Brahms realizes this gradation syntagmatically with three distinct **G**s that fuel each embellishing leap in the melody, shown on the bottom of Figure 3. As another case of contextual-transformational heterogeny, these three leaps alternate between the size of a fifth and a fourth, indicated again with curved dashed arrows.

This same four-stage gradated succession of G-compounds undergirds another variation in a work Brahms completed three years after his *Haydn Variations*. The secondary theme in the finale of Brahms's first symphony begins as a set of continuous variations over a looped four-quarter-note bass line, first C-B-A-G in G major in the movement's exposition, and then F-E-D-C in C major in the movement's tonal resolution. The scale degrees ($\hat{4}$-$\hat{3}$-$\hat{2}$-$\hat{1}$) match those of the E♭-D-C-B♭

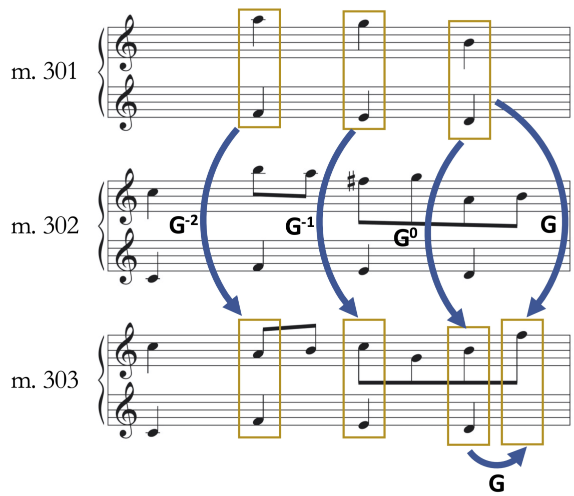

Figure 4: *Gradated application of **G** as variation technique: Brahms, Symphony No. 1, op. 68, iv, mm. 301—303, soprano and simplified bass lines only (registral ordering <top, bottom>, white-note scale, registral pitch).*

portion of the *Haydn Variations*'s ground bass upon which the **G** transformations were shown to act. Figure 4 transcribes and simplifies the outer voices for the beginning of the C-major version of this secondary theme. Measure 301 offers a first-species opening: two parallel tenths and a sixth that leads to a quasi-cadential arrival on an octave on the next downbeat, an interval that remains outside of **G**'s purview as defined here. The treble of m. 302 decorates the treble pitches of m. 301 with non-harmonic stepwise embellishments also outside of **G**'s purview. However, m. 303 relates to m. 301 with the same series of $\mathbf{G}^{-2}$, $\mathbf{G}^{-1}$, $\mathbf{G}^{0}$, and $\mathbf{G}^{(1)}$ transformations that was deployed for the analysis of the music in Figure 3: here, all four transformations are paradigmatic, and the last is also syntagmatic. As before, the result is a rising treble that starts an octave below the first pitch of its antecedent, and surpasses the antecedent by its end. While the line is significantly different, the preference for imperfect harmony before the cadence is preserved.

An outer-voice **G** that acts on pitch classes instead of registral pitches affords another perspective on more of this theme. In general, iterative applications of such a **G** on a dyad toggles its first (or, if **G'**, second) member between two states, which correspond to the two ordered-pitch-class-interval members of a non-zero interval class. For example, there are two diatonic pitch classes that form an imperfect interval above F: A and D. Therefore, **G**<A,F> = <D,F>, and **G**<D,F> = <A,F>. Such a **G** involutes, or, in other words, $\mathbf{G}^{-1} = \mathbf{G}$, and, more generally, $\mathbf{G}^{n} = \mathbf{G}^{n+2t}$, where $t, n \in \mathbb{Z}$. Figure 5 discloses this toggling in the theme's first nine measures, a span of time that comes full circle as m. 309 repeats m. 302, which ornaments the same consonant pitches from m. 301. The regions that are shaded yellow and purple keep track of the **G**-based toggling for each beat's dyad through these nine measures, where, above the fixed bass note, yellow signifies the starting imperfect harmony as the default and purple signifies the other (from a toggling perspective, antipodal) imperfect harmony. For example, the second beat's switch from the treble's initial A in mm. 301–3 (in yellow) to a contrasting D in mm. 304–6 (in purple) above the fixed bass note F can be indicated as **G**<A,F> = <D,F>. Applying **G** again restores the treble's F back to A for the next three measures: **G**<D,F> = <A,F>. (Figure 5 accommodates pitches outside of the C-major collection by defining **G** to act on the seven notated diatonic letters regardless of their inflections by accidentals.)

Not every outer-voice harmony exclusively expresses an imperfect quality during the second,

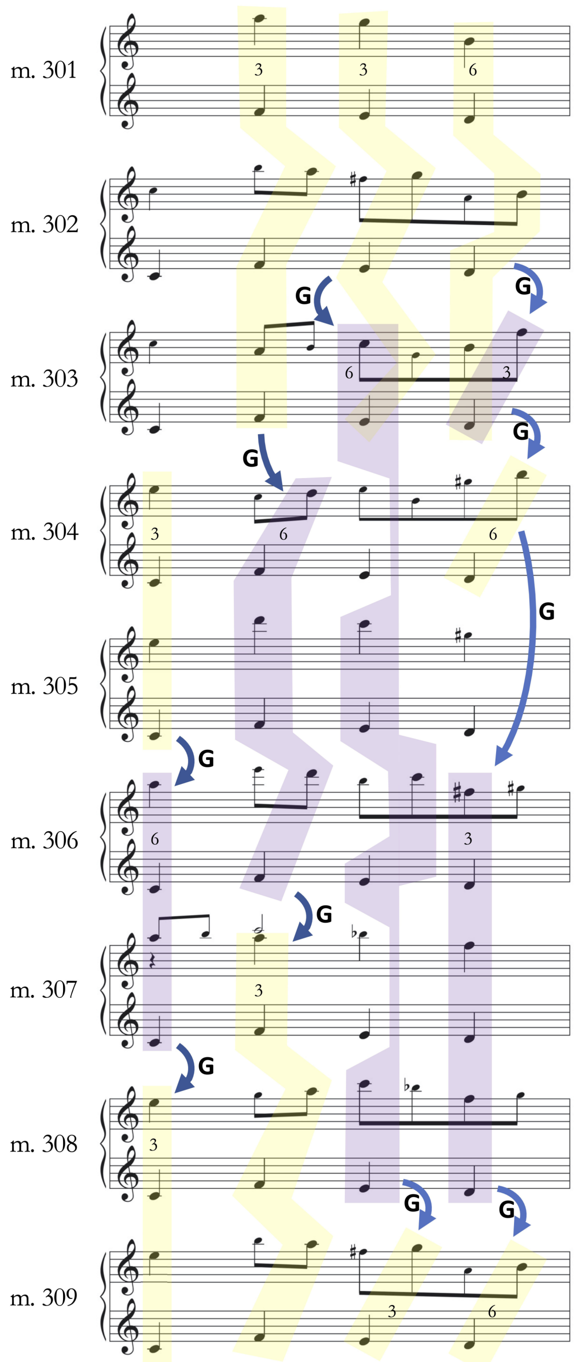

Figure 5: *Toggling application of **G** as variation technique: Brahms, Symphony No. 1, op. 68, iv, mm. 301—309, soprano and simplified bass lines only (registral ordering <top, bottom>, generalized diatonic scale, pitch class).*

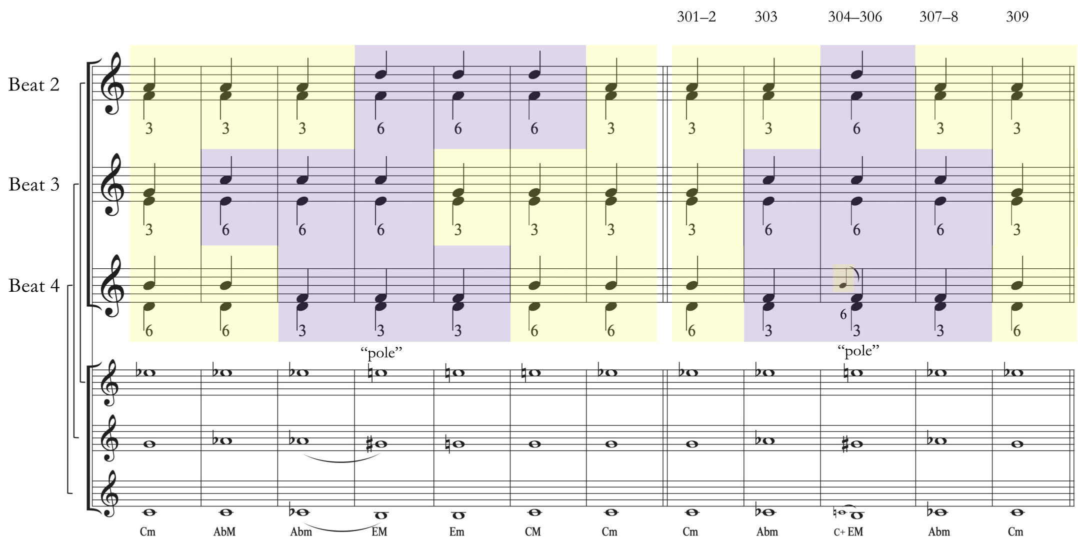

Figure 6: *(a) **G** toggling cycle as analogous to a hexatonic cycle (b) Portion of this cycle used by Brahms, Symphony No. 1, op. 68, iv, mm. 301—309 and its hexatonic analogue.*

third, and fourth beats of this theme; those treble pitches that express a different diatonic interval class with the ground bass receive a smaller font. However, there remains enough imperfect harmony to measure the advance through these nine measures using **G** as the unit of distance and energy. Each beat toggles between its two pitch classes at different times, not only within the measure, which is unavoidable, but also at different times between measures, which is not. The second and third beats toggle once, and the fourth beat toggles twice, before all three beats' pitches return to their default states. Even the treble pitches on the first beat, after they indulge octave quasi-cadences in the first two measures, join in the toggling, moving above the bass C from E to A in m. 306 and back to E in m. 308. This advance during the last three beats of each measure is analogous to motion along an instance of Richard Cohn's [4] hexatonic cycle, which Michael Siciliano [19] reinterpreted as staggered instances of toggling among three half-step dyads, as shown in the first part (a) of Figure 6, which reuses the default and contrasting coloration of Figure 5. Here, consonant triads are analogous to a mixture of harmonic thirds and sixths, whereas augmented triads are analogous to an exclusive use of one of these harmonic types or the other. However, Brahms's theme, summarized in Figure 6b, does not complete the cycle. Rather, it progresses halfway through the cycle to the completely purple "hexatonic pole" of the initial completely yellow default, which it embellishes with an "augmented triad"—-exclusively sixths during mm. 304–5, shown with grace notes in Figure 6b—-and then retraces its steps back to the default. This pole coincides with the transitory tonicization of A minor, the relative minor of the main key of C major.

In addition to serving as analyses in their own right, Figures 3–6 serve as documentation that **G** can model some of Brahms's variations, and therefore help to corroborate a claim that, if **G**, especially a gradated **G**, relates the opening of Schumann's op.14 to Brahms's op. 5, then one can think of the latter as a variation of the former. However, both the *Haydn Variations* and the First Symphony came after 1853: does **G** model variations before this date? One type of an eighteenth-century alteration modeled by **G′** (or occasionally $\mathbf{G'}^{-1}$), although typically governed by some other form besides variation, emerges when the composer presents two formally corresponding fixed stepwise rising treble motives, usually $\hat{3}$-$\hat{4}$-$\hat{5}$, twice: in one of its appearances (usually the

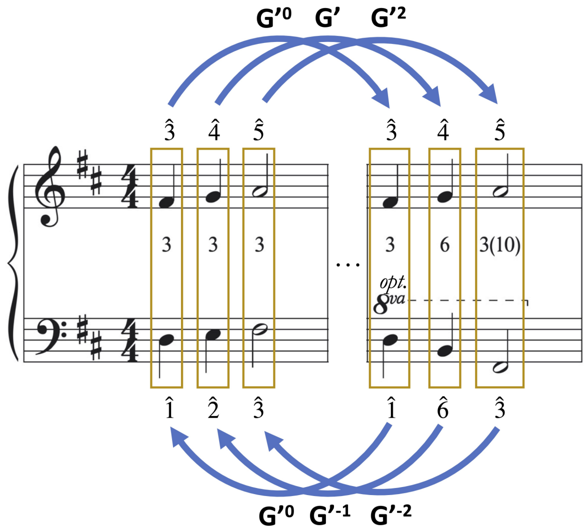

Figure 7: *An eighteenth-century motivic reharmonization using a three-stage gradated succession of* **G′**- *or* $\mathbf{G'}_{-1}$-*compounds.*

first), the bass harmonizes this motive with $\hat{1}$-$\hat{2}$-$\hat{3}$ in rising motion parallel to the treble; in the other; the bass harmonizes this motive with $\hat{1}$-$\hat{6}$-$\hat{3}$ in descending motion contrary to the treble. They may correspond as part of successive parallel basic ideas, phrases, or themes, or even as part of exposition and recapitulation.[4] As shown with an abstracted case of this in Figure 7, a three-stage gradated succession of **G′**–or $\mathbf{G'}^{-1}$–compounds relates these two moments, depending on which progression begets the other. While this transformation can serve practical benefits, such as connecting a part of this bass line to a higher or lower register, it also offers aesthetic diversity.

Another relevant precedent links this eighteenth-century practice to the next section's **G**-enabled analysis of the opening of Brahms's op. 5. Before visiting the Schumanns, Brahms had just completed his C-major piano sonata, which was published as his op. 1, although it was composed after his F-sharp-minor piano sonata, his second opus. The C-major sonata opens with a basic idea that rises from $\hat{3}$ (E) to $\hat{6}$ (A) in the treble, harmonized by parallel tenths with the bass. This basic idea is followed by its repetition that apexes one step higher on B flat, lightly tonicizing F Major, again with consistent parallel tenths below. The next two measures function as basic-idea repetition (the treble still rises by step), basic-idea variation (the bass departs from its strict parallel motion), and cadence on the dominant. As shown in Figure 8, **G′** well expresses the bass's transformation from conjunct and generally ascending motion in mm. 1-4 (Figure 8a) to disjunct and generally descending motion in mm. 5-6 and the two pickup notes (Figure 8c). As Figure 8b sets out, the outer voices in mm. 3-4 are transposed up four diatonic steps and the prevailing diatonic collection is shifted two accidentals sharpward from the one flat of F major to the one sharp of G major, indicated using Julian Hook's [11] nomenclature from his study of signature

[4] Examples of this altered repetition of the bass under the $\hat{3}$-$\hat{4}$-$\hat{5}$ portion of a formal unit include:

- J.S. Bach, Cantata, BWV 140, vi: **G′**(m. 1) = m. 3 (basic idea and its immediate repetition transposed to dominant)
- J. Haydn, Piano Sonata, XVI: 35, i: $\mathbf{G'}^{-1}$(mm. 40–41) = mm. 130–31 (exposition and recapitulation transposed from dominant)
- W.A. Mozart, Violin Concerto No. 2, K. 211, i: **G′**(m. 7) = m. 9 (basic idea and its immediate repetition)
- W.A. Mozart, Horn Concerto No. 3, K. 447, ii, **G′**(m. 3) = m. 11 (period and its immediate repetition)
- A. Salieri, Organ Concerto, IAS 27, i: **G′**(m. 116) = m. 117 (motive and its immediate repetition)

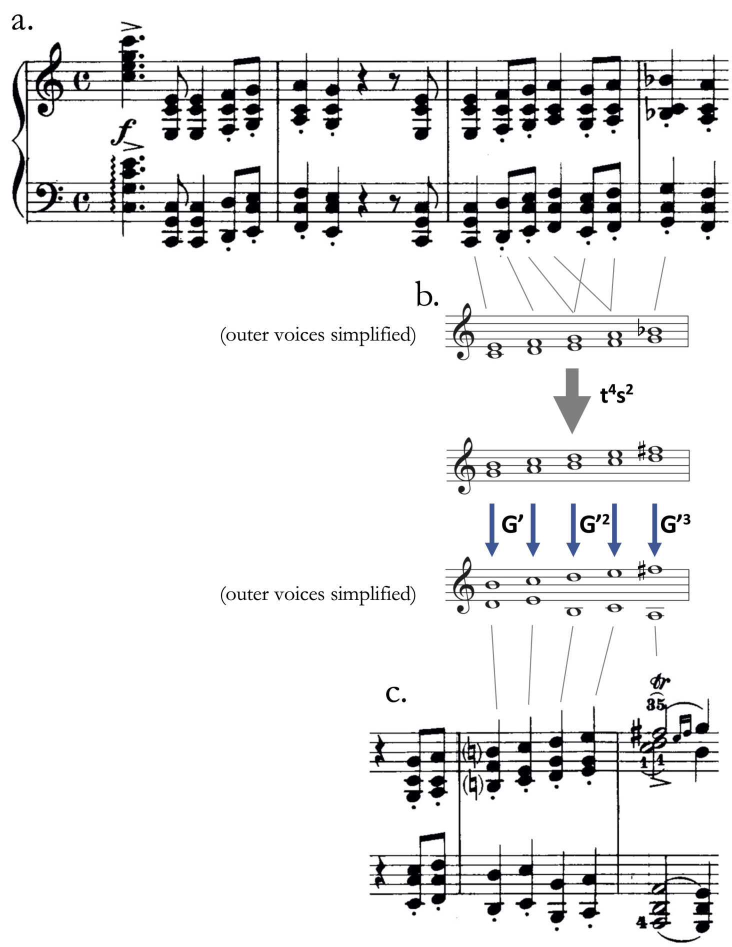

Figure 8: *(a) Brahms, Piano Sonata op. 1, i, mm. 1–4 (b) Figure 8a transformed by signature transformations and gradated* ***G′****(c) Brahms, op. 1, mm. 4–6.*

transformations. Then a three-stage gradated succession of **G′**-compounds, but with the first two stages operating on pairs of dyads, turns the outer voices from their incremental, lockstepped, and measured opening in mm. 1-4 to their vaulting, expanding, and dramatic conclusion in mm. 5-6.

IV. Schumann's Op. 14 and Brahms's Op. 5

In the second measure of the opening of Schumann's op. 14 piano sonata (Figure 1a), the local, mostly up-stemmed, maxima of the right-hand pitches—the jagged line A♭-G-C-B♭-F—bears the characteristic heterogenous stairstep design of a gradated **G**-series transform of parallel imperfect intervals (compare this treble line to that of mm. 36–38 of Figure 3). The left side of Figure 9 (a-c) shows the generation of this design from a succession of parallel thirds as a proposed precursory "deep structure." This is a mirrored form of the transformation from parallel thirds to an expanding wedge in the opening measures of Brahms's op. 1 in Figure 8, reprinted on the right side of Figure 9 (d, e) with corresponding parts connected with dashed lines for ease of comparison: Schumann's **G**-varied jagged line goes up in the treble, and Brahms's **G′**-varied jagged line goes down in the bass.[5] If the pattern in Schumann's second measure were to continue, a C in the bass and an E in the treble would appear on the downbeat of the third measure, an expectation that could be made more palpable if the performer would indulge even the slightest of ritardandos at the end of the second measure. As visualized with the left-pointing arrows and "no" symbol in Figure 9, this dyad does not materialize here despite this expected continuation. Yet it does appear on the downbeat of m. 6 as the culmination of the opening phrase albeit with the E two octaves higher. Schumann gets to this climactic cadence via transposed repetitions, shown in Figure 1 with brackets and italic labels: most of mm. 2–3 transpose up a fourth to make most of mm. 4–5, and two beats spanning the downbeat of m. 6 transpose up an octave to make the next two beats.

In comparing the first six measures of Brahms F-minor piano sonata to the first seven measures of Schumann's F-minor piano sonata, I hear a rerouting of the path from the starting dyad <A♭,F> to the finishing dyad <E,C>.[6] Whereas Schumann maneuvered this course with the aforementioned extensions, Brahms does so through expansions: specifically, an expansion of Schumann's second measure, aligning its <E,C> continuation, unrealized on the downbeat of m. 3, with its realization four measures later. This expansion is most clearly accomplished in the bass, as the chromatic descent F-E-E♭-D-D♭-C is stretched out from Schumann's single measure to Brahms's five measures. But a rerouted expansion needs to involve not only time but also register, especially in the treble, which in Schumann's second measure starts at A♭4 and rises around an octave to prepare for the unrealized E5, but then rises around two more octaves in the next four measures to peak at E7. How does Brahms's expansion of Schumann's treble make up for this two-octave difference?

One half of the solution is simply to move the melody's initial A♭4 up to A♭5, which is Brahms's first treble pitch. The other half of the solution is, using the apparatus developed in this study, to apply a three-stage series of **G**-compounds to Schumann's second measure. Figure 10 presents a detailed schematic of this application, reusing coloration from Figures 1, 5, and 6. The outer voices have been simplified, with the treble in particular down an octave in the Brahms portions of Figures 10c and 10d. Figures 10a and 10b inspects the left portion of Figure 9 from a slightly different angle. Figure 10a shows a "deep structure" of parallel thirds over a lament bass: as Figure 10's **G** operates on the seven diatonic pitch names without reference to a specific key, this "deep" bass line could be F-E-D-C, F-E♭-D♭-C, or F-E-E♭-D-D♭-C. Figure 10b performs a three-stage series of **G**-compounds to this "deep structure," and superimposes the two-voice design onto Schumann's first seven measures, highlighting the four-measure gap. The horizontal gray arrows marked **G** add a syntagmatic component to the interpretation, rather than the purely paradigmatic replacements of Figure 9, and the diagonal lines show the shift to the next stage, which always take place during a syntagmatic embellishment, at least of a diatonic framework.

The E at the end of Figure 10b is still one octave shy of the treble's cadential goal. To get there, one might imagine that Brahms applies a three-stage series of **G**-compounds ($\mathbf{G}^0$, $\mathbf{G}$, $\mathbf{G}^2$) to Schumann's second measure, varying Schumann's music in a manner that I have shown him to

[5] Here, "mirror" counterpoint refers to two lines that both inverted as individual lines, and swap positions in register, as if the notation of the two-voice construction has been reflected in a horizontal mirror.

[6] Instead of rerouting, one could apply the metaphor of repair, in that Brahms is "fixing" Schumann's detachment between the second measure's generation of an expectation of <E,C> to immediately follow the second measure, and <E,C>'s eventual appearance, displaced by four measures and two octaves. Following Harold Bloom's [2] theory of poetic influence, this metaphor would hypothesize that Brahms's response to Schumann's music is agonistic to some degree, that Brahms commits misprision and "misreads" Schumann's opening. Of Bloom's revisionary ratios, this misreading comes closest to what Bloom calls a *tessera* or fragment, such that Brahms's solution is "to retain its terms but to mean them in another sense, as though the precursor had failed to go far enough." [2, p. 14]. This approach is also consistent with Cohn's [5] interpretation of the opening of Brahms's op. 5 as itself involving "cracked and mended hemiolas," parenthetical insertions that replace one form of well-formedness with another.

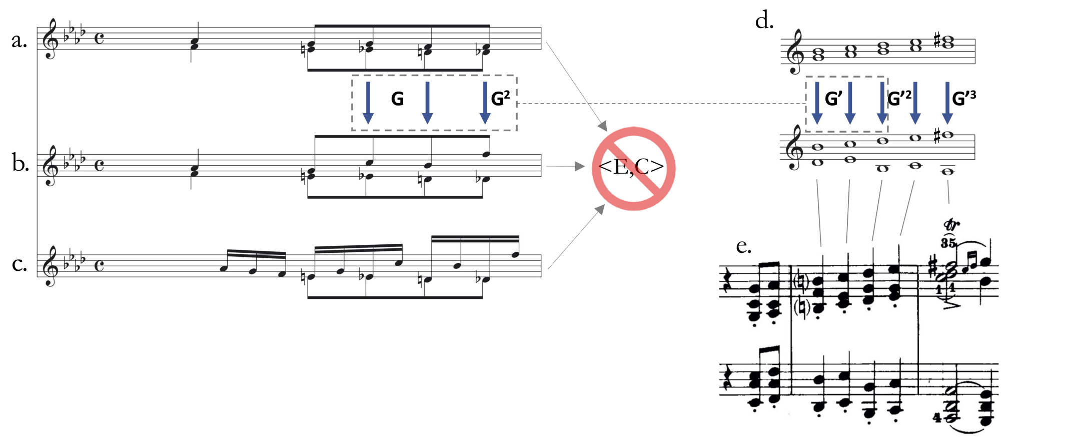

Figure 9: *(a) Chromatic lament bass harmonized in parallel thirds (b) Figure 9a transformed by gradated **G** (c) Schumann, op. 14, m. 2 (an unfolding of Figure 9b) (d) Figure 8a transformed by signature transformations and gradated **G** (e) Brahms, op. 1, mm. 4–6.*

use both before and after the fall of 1853, and producing the music of Figure 10c from the music of Figure 10b. I particularly value this way of relating these two passages, because it puts forward the notion that the hypothetical series of transformations that Schumann applied to a "deep" structure of purely stepwise parallel thirds to produce the contrary and stairstepping "surface" of the second measure of his F-minor sonata ($\mathbf{G}^0$, $\mathbf{G}$, $\mathbf{G}^2$) is the same series of transformations that Brahms applies to Schumann's "surface" to create the even steeper stairstepping "supersurface" of the first six measures of his F-minor sonata. Moreover, the link between "deep" and "surface" is more than hypothetical for Brahms: he compositionally realized this link in the first six measures of the piano sonata he had just completed months earlier, albeit with the stairsteps in the bass instead of the treble. The steeper ascent in the treble perfectly bridges the second octave, achieving the high E7 (again, shown down an octave at the end of Figure 10c and 10d for ease of reading).

One difference between the transformation from "deep" to "surface," and from "surface" to "supersurface," is the pacing by which the output moves through the three stages. The distribution of $\mathbf{G}^0$, $\mathbf{G}$, and $\mathbf{G}^2$ in the former is fairly regular with two dyads for each stage, as can be seen reading the transformations from left to right in between Figure 10a and 10b. However, for the latter—in between Figure 10b and 10c–the second stage of **G** predominates. Brahms's syntagmatic embellishment of <A♭,F> to <D♭,F> in the first measure passes the variation immediately into its second stage, in which it stays for five dyads, each a **G** transform of the rest of Schumann's second measure. Another syntagmatic embellishment of the final dyad—-<B♭,D♭> to <F,D♭>—-of these five commences the third and final stage right before the <E,C> conclusion. The result is a treble line whose pitch-class content only matches that of the very beginning and ending of Schumann's second measure and seventh downbeat, as shown with the yellow-as-default and purple-as-contrasting coloration employed earlier. All of the other treble pitches are different, which considerably obscures their **G**-enabled kinship.

These two syntagmatic additions to Schumann's treble content carry metric and rhythmic ramifications. Figure 10d summarizes how the pitches within the consonant framework of Figure 10c relocate into their final temporal and motivic positions; orange arrows indicate these displacements. Schumann's second measure expresses duple metrical relations on two levels: the even alternation of bass-then-treble within an isochronous compound melody puts a sixteenth pulse within an eighth pulse, and assigning two dyads per **G**-incremental stage puts this eighth

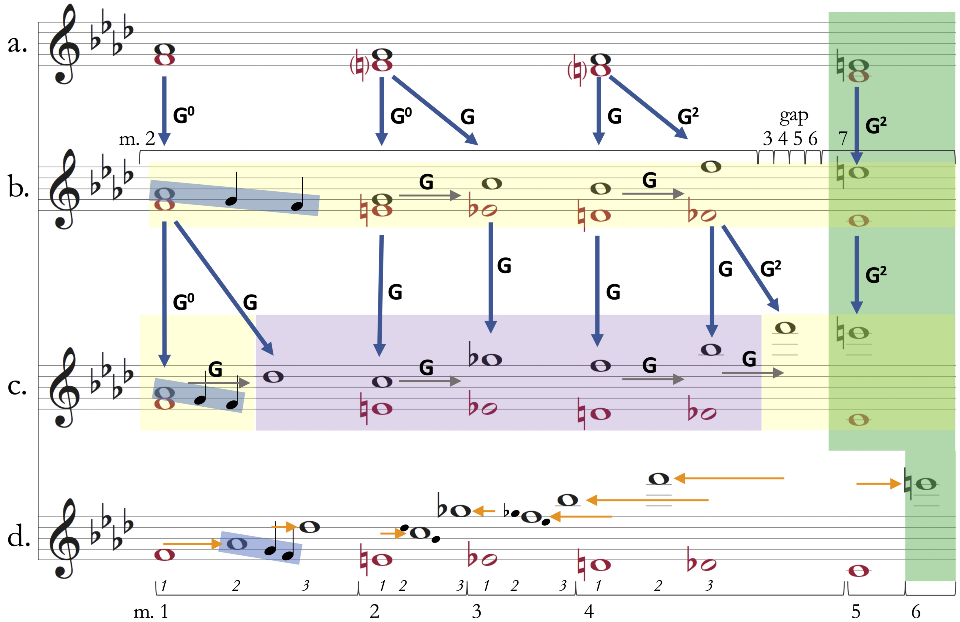

Figure 10: *(a) Lament bass harmonized in parallel thirds (b) Figure 5a transformed by gradated* ***G*** *to match Schumann, op. 14, i, m. 2 (compare to Figure 4b and Figure 4c) (c) Figure 5b transformed by gradated G (d) reduction of Brahms, Piano Sonata op. 5, i, mm. 1–6 (displacement of Figure 5c) (color scheme corresponds to that of Figure 1).*

pulse within a quarter pulse. However, the initial syntagmatic embellishment of <A♭,F> to <D♭,F> increments the number of pitch events during the first harmony from two to three—bass F, treble A♭, treble D♭-—which matches the younger composer's new meter of 3/4. Brahms maintains this same voice-beat assignment—bass-one, treble-two, treble-three—in the next two measures. Since this pair of measures remains in the second stage, where Schumann's pitches are only replaced rather than increased in number, Brahms must pull treble pitches earlier in time to meet each measure's two-treble-pitch quota, portrayed with left-pointing orange arrows. The resulting implicit bass suspensions create a powerful outer-voice torsion, especially in the context of the implicit treble D♭ suspension over the bass E on the downbeat of m. 2. The second and last-second syntagmatic embellishment of <B♭,D♭> to <F,D♭> furnishes an extra treble pitch, but three more would be needed to maintain the current voice-beat assignment: two treble pitches each for the bass pitches D and D♭. To square the uneven distribution, Brahms's fourth measure switches to a bass-one, treble-two, bass-three voice-beat assignment—bass D, treble F, bass D♭-—which also idiomatically accelerates the harmonic rhythm before the cadence.

V. Conclusions

This article presents one way in which mathematically-based music-theoretical tools may be mobilized to substantiate or qualify music-historical claims. Although this study involves a specific transformation as the tool and a specific intertextual influence in the claim, I suspect that other crossover opportunities await, not only through sophisticated statistical analyses of Big (Musical) Data (e.g. [9]), but also through close readings of individual works such as the one submitted herein. This article also endorses a transformational understanding of musical variation, a perspective especially explored by Carlos Almada (e.g. [1]) but also one that I suspect is far from exhausted in its scholarly and pedagogical utility.

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